Insurance Contract Design and Endogenous Frailty

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Agenda

1. Motivation
2. Setup
3. Dynamic Adverse Selection
4. Pricing Implications
5. Simulation Study
6. Conclusion
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Costs for insurers due to policy termination (lapse/surrender): acquisition expenses, loss of premium income and associated investment/technical profits

Costs due to hidden information (adverse selection), e.g., ‘good’ risks more likely to lapse/surrender than the ‘bad’ risks

Our focus

- contract design and (endogenous) renewal/termination
- dynamics of renewal/termination
- representation in terms of ‘frailty process’ (immediate interpretation in case of Life, similar for P&C)
Literature

P&C
- Contract design with adverse selection: Rothschild/Stiglitz (1976), Janssen/Karamychev (2005)...

Life
- Deterministic setting, no contract design: Jones (1998), Valdez (2001)
- Adverse selection at inception: Finkelstein/Poterba (2004), Einav et al. (2010)...
- Dynamic policyholder behavior in VA: Milevsky/Salisbury (2005), Chen et al. (2008), Bacinello et al. (2010)...

Empirical evidence
- P&C: Chiappori/Salaniè (2000) Kim et al. (2009), Shi at al. (2011)...
Findings

- Framework for endogenous selection driven by the contract features
- Jointly allow for endogenous and exogenous termination
- Gauge the strength of dynamic adverse selection
- Implications for optimal contract design and fair pricing
  - Pricing measure as frailty-adjusted historical/baseline measure
  - Contract design shapes risk adjustment via endogenous adverse selection
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Filtered probability space \( (\Omega, \mathcal{F}, \mathcal{F} = (\mathcal{F}_t)_{t \in T}, \mathbb{P}) \)

- \( \mathcal{F} \supseteq \mathcal{G} \lor \mathcal{H} \)
- \( \mathcal{G} = (G_t)_{t \in T} \) provides information regarding all the relevant risk factors
- \( \mathcal{H} = (\bigvee_{i=1}^m H^i_t)_{t \in T} \) captures the actual occurrences of claims and terminations/surrenders

With \( T := [0, T] \)
Setup

Claims occurrence

$m$ contracts, single exposure insured (in case of life contracts think of individuals aged $x_0$ at inception)

1. $(\tau_i)_{i=1,\ldots,m}$ are the claims arrival times (i.e., individuals’ random times of deaths)
2. $(N^i_C)_{i=1,\ldots,m}$ with $N^i_C(t) = 1_{\tau_i \leq t}$ a $\mathbb{F}$-adapted process representing the claims occurrence indicator
3. $(\mu^i)_{i=1,\ldots,m}$ are $\mathbb{G}$-predictable claim intensities
4. $(\mu^i)_{i=1,\ldots,m}$ different processes, same law
5. policyholder $i$ can observe the realization of $\mu^i$, the insurer cannot (hidden information)
Termination/surrender times

\( \theta^i = \bar{\theta}^i \wedge \theta^i \) is the policyholder’s termination time

- \( \bar{\theta}^i \) is obtained endogenously (American option exercise)
- \( (\theta_i)_{i=1,...,m} \) are the individuals’ exogenous termination times
- \( (N_w^i)_{i=1,...,m} \) with \( N_w^i(t) = 1_{\theta^i \leq t} \) a \( \mathbb{F} \)-adapted process representing individual’s exogenous termination indicator
- \( (\lambda^i)_{i=1,...,m} \) are the \( \mathbb{G} \)-predictable termination intensities
Average claim intensity

\( \sigma^i = \tau^i \land \theta^i \land T \) is the individual’s exit time

- \((N_{\sigma}^i)_{i=1,\ldots,m}\) with \(N_{\sigma}^i(t) = 1_{\sigma^i \leq t}\) a \(\mathbb{F}\)-adapted process representing individual’s exit indicator
- \(\bar{\mu}(t) := \frac{\sum_{i=1}^{m} \mu^i(t)(1-N_c^i(t-))}{\sum_{i=1}^{m} (1-N_c^i(t-))}\) is the average claim intensity of the population
- \(\bar{\mu}_p(t) := \frac{\sum_{i=1}^{m} \mu^i(t)(1-N_{\sigma}^i(t-))}{\sum_{i=1}^{m} (1-N_{\sigma}^i(t-))}\) is the average claim intensity of the portfolio (not population)

We can think at \(\bar{\mu}\) as the intensity associated with a stopping time \(\bar{\tau}\)

Symmetrically \(\bar{\mu}_p\) is the intensity associated with a stopping time \(\bar{\tau}_p\)
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We want to quantify the divergence between the two probabilities of accident at any point in time

\[ P(\tau_p \leq T | G_t \lor H_t) \text{ and } P(\tau \leq T | G_t \lor H_t) \]

We then want to explain the divergence in terms of contractual features (i.e., endogenize the divergence as a result of contract design)

We will then introduce a frailty representation that is useful for valuation
Measures of divergence

An example: Csiszár’s family (Vonta and Karagrigoriou, 2010)

\[
D_{\overline{\tau}, \overline{\tau}_p}^\psi(t) = \int_t^\infty \frac{d\mathbb{P}(\overline{\tau} \leq s \mid G_t \lor H_t)}{ds} \psi \left( \frac{d\mathbb{P}(\overline{\tau}_p \leq s \mid G_t \lor H_t)}{d\mathbb{P}(\overline{\tau} \leq s \mid G_t \lor H_t)} \right) ds
\]

\[
= \int_t^\infty \psi \left( \frac{d\mathbb{P}(\overline{\tau}_p \leq s \mid G_t \lor H_t)}{d\mathbb{P}(\overline{\tau} \leq s \mid G_t \lor H_t)} \right) d\mathbb{P}(\overline{\tau} \leq s \mid G_t \lor H_t)
\]

where \( \psi(x) \) continuous, differentiable and convex for \( x \geq 0 \), and \( \psi(1) = 0 \), \( \psi'(1) = 0 \).
Endogenous frailty

- Contract configuration: element $u \in \mathcal{U}$
- We define
  \[ \bar{\mu}_p(t; u) = \bar{\mu}(t) \bar{\eta}(t; u) \]
- At the individual level we have, on $\{\sigma^i > t\}$,
  \[ \mu^i_p(t; u) = \mu(t) \eta^i(t; u) = \mu^i(t) \]
- $\eta^i$ unobservable (for the insurer) stochastic frailty process, arising endogenously from the contract
- $\eta^i$ shaped by the contract configuration $u$, which drives the wedge between the population and portfolio claims intensity
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Agents have access to a money market account $M$, yielding instantaneously the $\mathcal{G}$-predictable short rate $r$, so that

$$M(t) = e^{\int_0^t r(s)ds}$$

- Insurance contracts are characterized by a quadruple $(P, B^n, B^c, B^w)$.
- Parameterized by $u \in U$.
- $P$ denotes the single premium paid by the insured at inception.
- $B^x$ denotes $\mathcal{G}$-adapted no-claim/living $(x = n)$, claim/death $(x = c)$ and termination/surrender $(x = w)$ benefits.
The cumulative gains to the $i$-th insured are given by

$$G_i^i(t) = B^n(t^i -) 1_{\sigma^i \leq t} + B^n(t) 1_{\sigma^i > t}$$

$$+ B^c(t^i -) 1_{\tau^i \wedge T \leq t \theta^i > t} + B^w(\theta^i -) 1_{\theta^i \wedge T \leq t \tau^i > t}$$

- Under no arbitrage exists $Q^i \sim P$, under which $M^{-1}G^i$ is a (local) $Q^i$-martingale.
- As markets are incomplete, $Q^i$ is not unique.

The value of the contract to insured $i$ is

$$V^i(t; \theta^i) = M(t)E^{Q^i} \left[ \int_t^{\theta^i \wedge \tau^i} M^{-1}(s) dG^i(s) \bigg| \mathcal{F}_t \right]$$
Contract valuation II

By the conditional Poisson assumption, the pricing formula can be rewritten as

\[
V^i(t; \theta^i) = 1_{\tau^i > t} \tilde{M}(t) \mathbb{E}^{\mathbb{Q}^i} \left[ \int_t^{\theta^i} \tilde{M}^{-1}(s) d\tilde{G}^i(s) \mid \mathcal{G}_t \right] \\
= 1_{\tau^i > t} \hat{V}^i(t; \theta^i)
\]

- \( \hat{V}^i(t; \theta^i) \) is the pre-death price of the contract
- The fictitious money market account \( \tilde{M} \) and cumulative gains \( \tilde{G}^i \) are given by
  - \( \tilde{M}(t) = e^{\int_0^t (r(s) + \mu^i(s)) ds} \)
  - \( \tilde{G}^i(t) = B^n(t) + \int_0^t B^c(s) \mu^i(s) ds + \int_0^t B^w(s-) dN^i_w(s) \)
However

- $\bar{\mu}_d$ will not reflect the mortality experience of the underlying pool of policyholders.
- It will in general be incorrect to assume that the previous pricing formula applies.

The relevant price would instead be delivered by

$$V^i(t; \theta^i(u), u) = 1_{\tau^i > t} \hat{M}(t; u) \mathbb{E}^{Q^i(u)} \left[ \int_t^{\theta^i(u)} \hat{M}^{-1}(s; u) d\hat{G}^i(s; u) \bigg| G_t \right]$$

$$= 1_{\tau^i > t} \hat{V}^i(t; \theta^i(u), u)$$

with

- $\hat{M}(t; u) = e^{\int_0^t (r(s) + \mu^i_p(s; u)) ds}$
- $\hat{G}^i(t; u) = B^n(t) + \int_0^t B^c(s) \mu^i_p(s; u) ds + \int_0^t B^w(s- \) d$Nw^i(s)$
Endogenous frailty and change of measure

Define the likelihood process

\[
\frac{d\mathbb{Q}(u)}{d\mathbb{P}} \bigg|_{\mathcal{F}_t} = (1_{\tau > t} + \bar{\eta}(\tau; u)1_{\tau \leq t})e^{\int_0^{\tau \wedge t} \mu(s)(\bar{\eta}(s;u) - 1)ds}
\]

The average portfolio survival probability becomes

\[
\mathbb{P} \left( \bar{\tau}_p > T \left| \mathcal{G}_t \lor \mathcal{H}_t \right. \right) = \mathbb{E} \left[ e^{-\int_t^T \mu^{d|p}(s;u)ds} \left| \mathcal{G}_t \lor \mathcal{H}_t \right. \right] = \mathbb{E} \left[ e^{-\int_t^T \mu_d(s)\bar{\eta}(s;u)ds} \left| \mathcal{G}_t \lor \mathcal{H}_t \right. \right] = \mathbb{E}^\mathbb{Q}(u) \left[ e^{-\int_t^T \mu_d(s)ds} \left| \mathcal{G}_t \lor \mathcal{H}_t \right. \right] = \mathbb{Q} \left( \bar{\tau} > T \left| \mathcal{G}_t \lor \mathcal{H}_t; u \right. \right)
\]

\(\eta^i\) can be seen as originating from an equivalent change of measure from \(\mathbb{P}\) to \(\mathbb{Q}(u)\), and its dynamics is shaped by the contract design \(u \in \mathcal{U}\)
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Assumptions

- The short rate is constant, $r = 3\%$
- There is a penalty $\gamma$ on surrender, decreasing in time
- The individual force of surrender is constant and equal for all policyholders, $\lambda^i = \lambda = 4\%$
- The individual force of mortality is a non-mean reverting affine process
  \[ \mu^i = \beta \mu^i(t) dt + \kappa dW^i_d(t) \]
  with $\beta = 0.106$ and $\kappa = 3 \cdot 10^{-4}$
Traditional contracts

- \( u = (P, L, D, W) \in U \)
- The premium rate is given by
  \[
P = \frac{D \mathbb{E} \left[ \int_0^T e^{-rt} dN_c(t) \right] + L e^{-rT} \mathbb{P}(\tau > T)}{\int_0^T e^{-rt} \mathbb{P}(\tau > t) dt}
  \]
- Policyholder’s viewpoint is based on her own mortality experience
- The insured decides to surrender at time \( t \) if
  \[
  W(t) \geq D \mathbb{E} \left[ \int_t^T e^{-r(s-t)} dN^i_c(s) \mid \mathcal{F}_t \right]
  + L e^{-rT} \mathbb{P}(\tau^i > T \mid \mathcal{F}_t) - P \int_t^T e^{-r(s-t)} \mathbb{P}(\tau^i > s \mid \mathcal{F}_t) ds
  \]
- With, for example, \( W(t) = (1 - \gamma)tP \)
Average Survival Curves for Traditional Contracts, as a function of time and death benefit

Figure: Average Survival Curves, with $L = 1$
Average Survival Curves for Traditional Contracts, as a function of time and death benefit

Figure: Average Survival Curves, with $L = 1$
Average Frailty Processes for Traditional Contracts, as a function of time and death benefit

Figure: Average Frailty Processes, with $L = 1$
Average Frailty Processes for Traditional Contracts, with exogenous surrender

Figure: Average Frailty Processes, with \( L = 1 \)
Variable annuity

- $m$ policyholders purchasing the same index-linked contract with 
  $c = (\Pi, B^n, B^c, B^w) \in \mathcal{C}$

- Stock price process
  \[
  \frac{dS(t)}{S(t)} = \alpha \, dt + \sigma dW(t)
  \]
  with $\alpha = 5\%$ and $\sigma = 5\%$

- The account value of each policyholder is ($\phi$ is AMC)
  \[
  \frac{dF(t)}{F(t)} = (\alpha - \phi) \, dt + \sigma dW(t)
  \]
  with initial condition $F(0) = \Pi$

- The generic benefit is given by
  \[
  B^x(t) = \max\{F(t), \Pi e^{g_x t}\}, \quad x \in \{n, c, w\}
  \]
Variable annuity

The value of the contract at time $t$ for each policyholder is given by

$$V^i(t, \theta^i) = \mathbb{E}^Q \left[ \int_t^{\theta^i \land \tau^i} e^{-r(s-t)} dG^i(s) \mid \mathcal{F}_t \right]$$

The fair AMC $\phi$ is set by the insurer by solving

$$\Pi = V^i(0; \theta^i) = V(0)$$

In a dynamic programming framework, the insured decides to surrender at time $t$ if

$$(1 - \gamma) B^w(t) \geq V^i(t, \theta^i)$$
Average Survival Curves for Variable Annuities, as a function of time and guaranteed rate upon death

Figure: Average Survival Curves, $g_w = 0\%$ and $g_l = 2.5\%$ (baseline 1)
Average Frailty Processes for Variable Annuities, as a function of time and guaranteed rate upon death

Figure: Average Frailty Processes, $g_w = 0\%$ and $g_l = 2.5\%$ (baseline 1)
Simulation Study

Average Frailty Processes for Variable Annuities, with exogenous surrender

Figure: Average Frailty Processes, $g_w = 0\%$ and $g_l = 2.5\%$ (baseline 1)
Average Survival Curves for Variable Annuities, as a function of time and guaranteed rate upon death

Figure: Average Survival Curves, $g_t = g_w = 2.5\%$ (baseline 1)
Average Frailty Processes for Variable Annuities, as a function of time and guaranteed rate upon death

Figure: Average Frailty Processes, $g_l = g_w = 2.5\%$ (baseline 1)
Average Frailty Processes for Variable Annuities, with exogenous surrender

Figure: Average Frailty Processes, $g_l = g_w = 2.5\%$ (baseline 1)
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The surrender decisions shape the aggregate risk profile of the residual policyholders’ portfolio.

High sensitivity of endogenous exposure to contract design.

Framework to properly analyze trade-off between endogenous and exogenous drivers of surrender and adverse selection.

Endogenous adverse selection can be captured by change of measure and internalized by pricing functional.
Extensions and work-in-progress

- Testing for dynamic adverse selection
- Optimal contract design in P&C and health insurance
- Empirical applications
Adverse selection and moral hazard in insurance: Can dynamic data help to distinguish?

Mortality rates as a function of lapse rates.
Mortality, vol. 1.

Regression-based algorithms for life insurance contracts with surrender guarantees.

Stochastic mortality under measure changes.

The effect of modelling parameters on the value of gmwb guarantees.

Econometric models of insurance under asymmetric information.
In Handbook of insurance, pp. 365–393. Springer.

Testing for asymmetric information in insurance markets.
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Pricing a select and ultimate annual renewable term product.

Selection in insurance markets: Theory and empirics in pictures.

Optimal mandates and the welfare cost of asymmetric information: Evidence from the uk annuity market.

Adverse selection in insurance markets: Policyholder evidence from the uk annuity market.

Stochastic Solvency Testing in Life Insurance.

Is there dynamic adverse selection in the life insurance market?

Dynamic insurance contracts and adverse selection.
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Evidence of asymmetric information in the automobile insurance market: dichotomous versus multinomial measurement of insurance coverage.

Financial valuation of guaranteed minimum withdrawal benefits.

Equilibrium in competitive insurance markets: An essay on the economics of imperfect information.
The quarterly journal of economics, pp. 629–649.

Testing adverse selection with two-dimensional information: Evidence from the singapore auto insurance market.

Bivariate analysis of survivorship and persistency.

Generalized measures of divergence in survival analysis and reliability.
Thank you for your attention