Capital requirements and portfolio optimization under solvency constraints: a dynamical approach

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Outline

1. Preliminaries and solvency constrained optimization
2. Modelling assets and liabilities
3. Empirical analysis
4. Conclusions
The setup

- A discrete-time framework with the set of trading dates $\mathcal{T} = \{t| t = 0, \ldots, T\}$.
- A portfolio of $n$ assets with the gross return process over the period $[t, t + 1]$ defined by $\mathbf{R}_{t+1} = (R_{1,t+1}, \ldots, R_{n,t+1})^T$.
- $\mathcal{F}_t = \sigma(\mathbf{R}_1, \ldots, \mathbf{R}_t)$ and we use the notation $E[\cdot|\mathcal{F}_t] = E_t[\cdot]$.
- Let $\mathbf{x}_t = (x_{1,t}, \ldots, x_{n,t})^T$ be the portfolio weights satisfying the budget constraint, $\sum_{i=1}^{n} x_{i,t} = 1$, and the no short sales constraint, $x_{i,t} \geq 0$, $i = 1, \ldots, n$.
- We introduce one-period optimization problems for a non-life insurance company over $[t, t + \tau]$, where $\tau$ is the solvency horizon.
- No rebalancing is allowed during the solvency period.
- $p_t$ - the aggregate premium available for investment at time $t$.
- $c_t$ - the regulatory initial capital provided by the shareholders.
- No other premiums are collected and no capital is issued or retired between $t$ and $t + \tau$.
- The insurer’s liability is modelled by a univariate random variable $Y_{t+\tau}$. It represents the aggregate claim amount over the solvency horizon which is assumed to be paid at time $t + \tau$.
- We define the insurer’s net loss:

$$L_{t,t+\tau} := L(c_t, x_t) = Y_{t+\tau} - (p_t + c_t)R^T_{t+\tau}x_t.$$ 

- No other sources of risk other than the ones modelled through $Y$ and $R$. 

Each optimization problem characterized by minimizing the capital requirement $c_t$, subject to two key constraints:

(i) **Solvency constraint**: capital requirement imposed by the insurer’s regulator and based on one of the following criteria: Ruin Probability ($RP$), Conditional Value-at-Risk ($CVaR$) and Expected Policyholder Deficit ($EPD$).

(ii) **Portfolio performance constraint**: target return on capital ($ROC$) provided:

$$ROC_{t,t+\tau} = -\frac{L_{t,t+\tau}}{c_t}.$$

- Santos, Nogales, Ruiz and Van Dijk (2012).
- Mankai and Bruneau (2012).
Optimization with \( RP \) constraint

- Motivated by Solvency II Regime which applies to EU based insurance companies.
- Identify the capital required to maintain a target level for the ruin probability over a specified period of time.

\[
\begin{align*}
\min_{c_t, x_t} & \quad c_t \\
\text{s.t.} & \quad E_t \left[ \mathbb{1}_{\{L_{t,t+\tau}>0\}} \right] \leq 1 - \alpha, \\
& \quad E_t \left[ ROC_{t,t+\tau} \right] \geq ROC^\alpha, \\
& \quad 1^T x_t = 1, \quad x_t \geq 0, \quad c_t \geq 0.
\end{align*}
\]

- \( \alpha \) represents the specified solvency level (e.g. \( \alpha = 99.5\% \)).
- \( ROC^\alpha \) is the lower bound for the expected return on capital.
The solvency chance constraint can be reformulated as a Value-at-Risk constraint, where the VaR of a loss r.v. \( Z \) at \( \alpha \) is defined:

\[
\text{VaR}_\alpha(Z) := \inf\{z \in \mathbb{R} : \Pr(Z \leq z) \geq \alpha\}.
\]

The constraint becomes:

\[
\text{VaR}_t^\alpha(L_{t,t+\tau}) \leq 0.
\]

There are two streams of literature dealing with solving chance constrained optimization.

1. Monte-Carlo estimators for the conditional expectation and perform further appropriate approximations.
2. Solve the chance constraint using the VaR representation.
   - Caliafore and Campi (2005), Nemirovski and Shapiro (2005)
   - scenario based approximations.

2. - Larsen, Mausser and Uryasev (2002) - algorithms based on iterative CVaR optimizations.
The Monte Carlo approximation of the solvency constraint:

\[
\frac{1}{m} \sum_{j=1}^{m} \mathbb{1}\left\{ Y_{t+\tau}(j) - (p_t + c_t)R_{t+\tau}^T(j)x_t > 0 \right\} \leq 1 - \alpha.
\]

This can be reformulated as a MIP problem. However, implementation becomes less efficient when \( m \) is large.

Our approach: *Semiparametric method*;

\[
\frac{1}{m} \sum_{j=1}^{m} E_t^{(j)} \left[ \mathbb{1}\left\{ Y_{t+\tau} - (p_t + c_t)R_{t+\tau}^T(j)x_t > 0 \right\} \right] \leq 1 - \alpha.
\]

We used: \( E \left[ \cdot | \mathcal{F}_t \cup \{ R_{t+\tau} = R_{t+\tau}(j) \} \right] = E_t^{(j)} \left[ \cdot \right] \).

A sufficient condition for convexity of our problem: \( Y_{t+\tau} \) has a conditional convex survival function.
Optimization with \textit{CVaR} constraint

- Rockafellar and Uryasev (2000) - alternative coherent risk measure to \textit{VaR}; quantifies the loss severity in case of default.
- Defined as a weighted average of the corresponding \textit{VaR} and conditional expected losses which strictly exceed \textit{VaR}.
- \textit{CVaR} coincides with the Expected Shortfall (\textit{ES}) for continuous distributions.
- \textit{ES} constitutes the basis for the target capital according to the Swiss Solvency Test (\textit{SST}) (EIOPA, 2011). We take $\beta = 99\%$.

\[
\begin{align*}
\min_{c_t, x_t} & \quad c_t \\
\text{s.t.} & \quad \text{CVaR}_t^\beta (L_{t,t+\tau}) \leq 0, \\
& \quad E_t [ROC_{t,t+\tau}] \geq ROC^\beta, \\
& \quad 1^T x_t = 1, \quad x_t \geq 0, \quad c_t \geq 0.
\end{align*}
\]
Rockafellar and Uryasev (2000) define CVaR:

$$\text{CVaR}^\beta(Z) = \inf_{s \in \mathbb{R}} \left\{ s + \frac{1}{1 - \beta} E[(Z - s)_+] \right\}.$$

The optimization problem becomes (only solvency constraint):

$$\min_{s, c_t, x_t} \quad c_t$$

s.t. 

$$s + \frac{1}{1 - \beta} E_t \left[ (L_{t,t+\tau} - s)_+ \right] \leq 0.$$

The traditional approach: use MC estimator and reformulate as a Linear Programming (LP) problem. Less efficient when $m$ is large.

Our approach: Semiparametric method:

$$s + \frac{1}{m(1 - \beta)} \sum_{j=1}^{m} E_t^{(j)} \left[ (Y_{t+\tau} - (p_t + c_t)R_{t+\tau}^{T}(j)x_t - s)_+ \right] \leq 0.$$
Optimization with \textit{EPD} constraint

- Introduced by Butsic (1994) as an alternative method to the ruin probability for measuring insolvency risk.
- Constitutes a useful tool in establishing the US Risk Based Capital (RBC) regulatory system (e.g. see NAIC, 2009).
- Defined as the expected loss in the event of insolvency:
  \[
  EPD(L_{t,t+\tau}) = E_t[(Y_{t+\tau} - (p_t + c_t)R_{t+\tau}^T x_t)_+] .
  \]
- Solvency criteria based on a target level of a deficit ratio:
  \[
  \frac{EPD(L_{t,t+\tau})}{E_t[Y_{t+\tau}]} \leq f .
  \]
- \( f \) is the maximum level for the \textit{EPD} ratio with \( 0 \leq f < 1 \). We arbitrarily take \( f = 0.25\% \).
A similar LP reformulations is available.

For consistency, we consider our semiparametric representation:

\[
\begin{align*}
\min_{c_t, x_t} & \quad c_t \\
\text{s.t.} & \quad \frac{1}{m} \sum_{j=1}^{m} E_t^{(j)} [(Y_{t+\tau} - (p_t + c_t)R_{t+\tau}(j)x_t)_+] \leq f E_t [Y_{t+\tau}], \\
& \quad E_t [ROC_{t,t+\tau}] \geq ROC^f, \\
& \quad 1^T x_t = 1, \quad x_t \geq 0, \quad c_t \geq 0.
\end{align*}
\]

A sufficient condition for convexity:

\[
E_t^{(j)} [(Y_{t+\tau} - (p_t + c_t)R_{t+\tau}(j)x_t)_+] \text{ is a convex function in } c_t \text{ and } x_t.
\]

This depends on the conditional distribution of \(Y_{t+\tau}\).
Modelling assets and liabilities

- Assets follow the DCC - GARCH model of Engle (2002)

\[
\begin{align*}
\log \mathbf{R}_{t+1} &= \mathbf{m}_{t+1} + \mathbf{\varepsilon}_{t+1}, \\
\mathbf{\varepsilon}_{t+1} | \mathcal{F}_t &\sim \text{MVN}(\mathbf{0}, \mathbf{H}_{t+1}), \\
\mathbf{H}_{t+1} &= D_{t+1}^{1/2} \Sigma_{t+1} D_{t+1}^{1/2}, \\
D_{t+1} &= \text{diag}(h_{1,t+1}, \ldots, h_{n,t+1}), \\
\Sigma_{t+1} &= \text{diag}(q_{11,t+1}^{-1/2}, \ldots, q_{nn,t+1}^{-1/2}) Q_{t+1} \text{diag}(q_{11,t+1}^{-1/2}, \ldots, q_{nn,t+1}^{-1/2}), \\
Q_{t+1} &= (1 - \theta_1 - \theta_2) \bar{Q} + \theta_1 \mathbf{u}_t \mathbf{u}_t^T + \theta_2 Q_t.
\end{align*}
\]

- \(D_{t+1}\) is the \(n \times n\) diagonal matrix formed with the univariate conditional variances GARCH(1,1).

- \(\Sigma_{t+1}\) is the time-varying conditional correlation matrix of \(\mathbf{R}_{t+1}\).

- Liabilities are LogNormal distributed:

\[
\mathbf{Y}_{t+\tau} \sim \text{LGN}({\mu}_{t+\tau}, {\sigma}_{t+\tau}).
\]

- \(\mathbf{Y}_{t+\tau}\) is independent of the enlarged filtration \(\mathcal{F}_t \cup \sigma(\mathbf{R}_{t+\tau})\).
Data description and estimation

- **Assets**: 3-asset portfolios formed with T-Bills, NASDAQ and NYSE.

  Table: Descriptive statistics for NASDAQ and NYSE log-returns from January 3, 2005 - July 29, 2011 for a total of 1,656 observations.

<table>
<thead>
<tr>
<th>Index</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Std</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASDAQ</td>
<td>-0.0959</td>
<td>0.1116</td>
<td>0.0001</td>
<td>0.0149</td>
<td>-0.1670</td>
<td>10.2725</td>
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<tr>
<td>NYSE</td>
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<td>0.1153</td>
<td>0.0001</td>
<td>0.0150</td>
<td>-0.3480</td>
<td>12.7329</td>
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</table>

- **Liabilities**: aggregate monthly claim amounts on property insurance for the same period used in the assets case.

  Table: Descriptive statistics for monthly claim amounts from January 3, 2005 - July 29, 2011 for a total of 79 observations (figures are in thousands Euros).

<table>
<thead>
<tr>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>StDev</th>
<th>Skewness</th>
<th>Kurtosis</th>
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<td>5.5068</td>
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</table>
(a) Conditional variance for NASDAQ

(b) Conditional variance for NYSE

(c) Conditional correlation

Figure: Conditional variances and correlations for the DCC-GARCH models based on the MLE estimates over the period January 3, 2005 - July 29, 2011 for a total of 1656 observations.
Figure: Efficient frontiers for DCC, CCC and UNI-GARCH models under the RP, CVaR and EPF-constrained problems.
Figure: Optimal allocation in Nasdaq.
Figure: Optimal allocation in NYSE.
Figure: Optimal allocation in T-Bills.
Rolling window implementation


- Given the estimates based on Samples $A$ and $A'$, find the optimal solution $(c_t^*, x_t^*)$.

- **Single rolling window**: drop the first $\tau$ obs. from Sample $A$ and replace them with the first $\tau$ observations from Sample $B$.

- **Double rolling window**: additionally, drop the first obs. from Sample $A'$ and replace it with the first obs. from $B'$.

- Repeat the estimation and optimization steps until the length of the out-of-sample data set is reached (i.e. $l_{B'} = 19$ times).
Preliminaries and solvency constrained optimization
Modelling assets and liabilities
Empirical Analysis
Conclusions

Data description and estimation
Efficient frontiers and capital allocation
Rolling window implementation
Out-of-Sample Performance

(a) $RP$ and single rolling window
(b) $RP$ and double rolling window
(c) $CVaR$ and single rolling window
(d) $CVaR$ and double rolling window
(e) $EPD$ and single rolling window
(f) $EPD$ and double rolling window

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Figure: Comparison of optimal portfolio allocation in NASDAQ, with single and double rolling window.
Using optimal solutions and time series of observed returns over the out-of-sample period, we test the performance of the DCC, CCC and UNI MGARCH models relative to two criteria:

1. **Solvency performance:**
   - Average total assets invested (premium + optimal capital).
   - Average solvency value.
   - Maximum solvency value.

2. **Portfolio performance:**
   - Average adjusted $ROC$.
   - Standard deviation adjusted $ROC$.
   - Sharpe Ratio.
   - Portfolio Turnover.
Average solvency values:

\[
\hat{RP} = \frac{1}{l_B} \sum_{k=1}^{l_B} \Phi \left( d_{t+k\tau} \right),
\]

\[
\hat{CVaR} = \frac{1}{l_B} \sum_{k=1}^{l_B} \left( \frac{E[Y_{t+k\tau}]}{1 - \beta} \phi \left( \sigma_{t+k\tau} - \Phi^{-1}(\beta) \right) - R^T_{t+k\tau} z^*_{t+(k-1)\tau} \right),
\]

\[
\hat{EPD} = \frac{1}{l_B} \sum_{k=1}^{l_B} \left[ E[Y_{t+k\tau}] \phi \left( d_{t+k\tau} + \sigma^2_{t+k\tau} \right) - R^T_{t+k\tau} z^*_{t+(k-1)\tau} \phi \left( d_{t+k\tau} \right) \right].
\]

\[
z^*_{t+(k-1)\tau} = (p_{t+(k-1)\tau} + c^*_{t+(k-1)\tau}) x^*_{t+(k-1)\tau},
\]

\[
d_{t+k\tau} = \frac{-\log R^T_{t+k\tau} z^*_{t+(k-1)\tau} + \mu_{t+k\tau}}{\sigma_{t+k\tau}}.
\]
### Table: Out-of-sample solvency performance under the single rolling window exercise.

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<tr>
<td></td>
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<td>DCC</td>
<td>2581.30</td>
<td>0.497</td>
<td>0.543</td>
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<tr>
<td></td>
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<td>CCC</td>
<td>2580.77</td>
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<td>2581.13</td>
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<td>0.585</td>
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</tbody>
</table>

| Problem 2. |                      | Covariance Model | Avg. CVaR | Max. CVaR |                     |                      |                   |
|------------|----------------------|------------------|-----------|-----------|---------------------|-------------------|                   |
| CVaR Constraint |                    |                  |           |           |                     |                   |                   |
|               |                      | DCC              | 2782.24   | -3.665    | 46.718             |                   |                   |
|               |                      | CCC              | 2782.06   | -3.236    | 53.702             |                   |                   |
|               |                      | UNI              | 2781.69   | -5.220    | 87.822             |                   |                   |

| Problem 3. |                      | Covariance Model | Avg. EPD Ratio (%) | Max. EPD Ratio (%) |                     |                      |                   |
|------------|----------------------|------------------|--------------------|--------------------|---------------------|-------------------|                   |
| EPD Constraint |                    |                  |                    |                    |                     |                   |                   |
|               |                      | DCC              | 2982.54            | 0.248              | 0.272               |                   |                   |
|               |                      | CCC              | 2982.33            | 0.249              | 0.275               |                   |                   |
|               |                      | UNI              | 2981.90            | 0.248              | 0.292               |                   |                   |
**Table:** Out-of-sample solvency performance under the double rolling window exercise.

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<th>Problem 2. CVaR Constraint</th>
<th>Avg. CVaR</th>
<th>Max. CVaR</th>
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<th>Problem 3. EPD Constraint</th>
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<td>UNI</td>
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Portfolio performance indicators:

\[ \hat{\mu}_{\text{AROC}} = \frac{1}{l_B l_B'} \sum_{k=1}^{l_B'} AROC_{t+(k-1)\tau},_{t+k\tau}, \]

\[ \hat{\sigma}_{\text{AROC}} = \sqrt{\frac{1}{l_B l_B'} \sum_{k=1}^{l_B'} (AROC_{t+(k-1)\tau},_{t+k\tau} - \hat{\mu}_{\text{AROC}})^2}, \]

\[ \hat{\text{SR}}_{\text{AROC}} = \frac{\hat{\mu}_{\text{AROC}}}{\hat{\sigma}_{\text{AROC}}}, \]

\[ \text{Turnover} = \frac{1}{l_B l_B' - 1} \sum_{k=1}^{l_B' - 1} \sum_{i=1}^{n} |X_{i,t+k\tau} - X_{i,t+(k-1)\tau}|. \]

These quantities are computed for the adjusted return on capital (AROC):

\[ AROC_{t,t+\tau} = \frac{(p_t + c_t^*) R_{t+\tau}^T x_t^* - E[Y_{t+\tau}]}{c_t} - 1. \]
Table: Out-of-sample portfolio performance under the single rolling window exercise.

<table>
<thead>
<tr>
<th>Portfolio Performance</th>
<th>Avg. AROC (%)</th>
<th>Std. AROC</th>
<th>Sharpe Ratio</th>
<th>Turnover</th>
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<tr>
<td>DCC</td>
<td>3.67</td>
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<td>DCC</td>
<td>3.32</td>
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<td>3.42</td>
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<td>3.05</td>
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<td>UNI</td>
<td>3.17</td>
<td>2.02</td>
<td>1.57</td>
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Table: Out-of-sample portfolio performance under the double rolling window exercise.

<table>
<thead>
<tr>
<th>Portfolio Performance</th>
<th>Avg. AROC (%)</th>
<th>Std. AROC</th>
<th>Sharpe Ratio</th>
<th>Turnover</th>
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Conclusions

- We propose three problems to jointly solve for the optimal capital requirement and its optimal portfolio allocation.
- We provide a novel semiparametric approach for solving these problems.
- Asset correlation plays an important role in the behaviour of the optimal capital required and the portfolio structure.
- Optimal required capital is very stable when the liability parameters remain constant over the rolling horizon; however, the variation is substantial when the liability is re-estimated at each step. The differences between the optimal portfolio weights are not as pronounced for the single versus double rolling exercises.
- The out-of-sample exercise indicates that the DCC model outperforms the CCC and No-Correlation model relative to both the solvency and portfolio performance criteria.