

**RISK-BASED CAPITAL REQUIREMENTS
FOR PROPERTY AND LIABILITY INSURERS ACCORDING TO DIFFERENT
REINSURANCE STRATEGIES AND THE EFFECT ON PROFITABILITY**

Rytgaard Mette

Danish Re
Kongevejen 100 B, DK-2840 Holte
Tel: +45 33 47 55 80
Fax: +45 70 27 55 10
E-mail: mette.rytgaard@privat.dk

Savelli Nino

Catholic University of Milan
Largo Gemelli 1, 20123 Milan
Tel: +39 02 7234 2943
Fax: +39 02 7234 2324
E-mail: nino.savelli@unicatt.it.

Abstract

A risk theoretical simulation model is here applied in order to assess the default risk for both Property and Liability multi-line insurers along a short-term time horizon. Further, some Risk-Based Capital requirements are analysed according to risk measures as VaR, TVaR, UES and Ruin probability, investigating the impact of either different horizon times and levels of confidence.

Moreover, the effect of different reinsurance strategies on the above mentioned RBC capital requirements is dealt with, also with reference to the profitability of the Insurer through a Risk vs Return trade-off analysis.

The results of the model show how new RBC requirements in non-life business should pay higher attention to the price of reinsurance covers, having not only a favourable strong effect on the risk but also on the expected value of the return, that might increase the total risk of insurer's insolvency even in a short time horizon. This may be useful in the present debate concerning the new EU capital requirements to be established in the Solvency II phase, and for internal risk modelling evaluating the financial strength of the company.

Keywords: Non-Life Insurance Solvency, Risk-Theory simulation models, Reinsurance strategies, Risk measures, Risk and Profitability trade-off.

1 INTRODUCTION AND FRAMEWORK OF THE MODEL.

The main target of the present paper is to analyse the risk profile of either a property and a liability multi-line insurer, on both solvency (or financial strength) and return benchmarks, and moreover to show also the effects of some traditional reinsurance treaties.

The framework of the model provides a risk theoretical approach where the underwriting risk is mainly dealt with, being the investment variables regarded as stochastic but with no credit risk and whereas the run-off risk arising from loss reserves is not considered as well as the correlation between different lines.

In classical Risk-Theory literature the stochastic Risk Reserve \tilde{U}_t at the end of the year t is given by:

$$(1) \quad \begin{aligned} \tilde{U}_t = & (1 + \tilde{j}_t) \cdot \tilde{U}_{t-1} + [(B_t - \tilde{X}_t - E_t) - (B_t^{RE} - \tilde{X}_t^{RE} - C_t^{RE})] \cdot (1 + \tilde{j}_t)^{1/2} + \\ & + \tilde{j}_t \cdot LR_{t-1} - TX_t - DV_t \end{aligned}$$

with gross premiums volume (B_t), stochastic aggregate claims amount (\tilde{X}_t) and general and acquisition expenses (E_t) realized in the middle of the year, whereas \tilde{j}_t is the stochastic annual rate of investment return. Furthermore, notwithstanding the claim reserving run-off is not considered here, the return coming from the investment of the initial Loss Reserve LR_{t-1} is taken into account, whereas the Loss Reserve at each time is assumed to be a constant coefficient of the gross premiums ($LR_t = l \cdot B_t$).

As to reinsurance, B_t^{RE} denotes the gross premiums volume ceded to reinsurer whereas \tilde{X}_t^{RE} and C_t^{RE} are the amounts of claims refunded by reinsurer and the reinsurance commissions, respectively.

Further, both taxation TX_t and dividends DV_t (the latter paid to stockholders at the end of the relevant year) are considered into the model.

It is worth to emphasize that for many general insurance lines (e.g. third-party liability) the run-off risk concerning the development of the initial estimate of claim reserve is not negligible at all and in practice it is an additive source of risk.

For each single line of business the gross premium amount is composed of risk premium $P_t = E(\tilde{X}_t)$, with safety loadings applied as a (constant) quota λ of the risk premium $\lambda \cdot P_t$ and expense loading as a (constant) coefficient c applied on the gross premiums:

$$B_t = P_t + \lambda \cdot P_t + c \cdot B_t$$

Notwithstanding the risk loading coefficient λ is kept constant over the whole time horizon; it is initially computed according to the standard deviation premium principle for the initial portfolio structure as follows, regarding both explicit and implicit risk loading:

$$(2) \quad (1 - tx) \cdot [\lambda \cdot E(\tilde{X}) + E(\tilde{j}) \cdot LR] = \beta \cdot \sigma(\tilde{X})$$

whereas tx denotes the (constant) rate of taxation. In practice, the insurer may ask for a total risk loading amount (net of taxation) equal to β for each unit of standard deviation of the stochastic total amount of claims of the line. Here the standard value $\beta = 35\%$ is considered for the computation of the total risk loading for each line of business.

For each line of business, the nominal gross premium volume increases yearly by the claim inflation rate (i) and the real growth rate (g):

$$B_t = (1 + i) \cdot (1 + g) \cdot B_{t-1}$$

assumed rates i and g to be constant in the considered time horizon but not necessarily the same for all lines of business.

Following the collective approach, the aggregate claims amount \tilde{X}_t (for each line of business) is given by a compound process:

$$(3) \quad \tilde{X}_t = \sum_{i=1}^{\tilde{k}_t} \tilde{Z}_{i,t}$$

where \tilde{k}_t is the random variable of the number of claims occurred in the year t for the relevant line and $\tilde{Z}_{i,t}$ the random claim size of the i-th claim occurred at year t for the relevant line.

Very often, the number of claims in general insurance is assumed to be Poisson distributed. Having a dynamic portfolio, the Poisson parameter for each line of business will be increasing (or decreasing) recursively year by year by the real growth rate g such that \tilde{k}_t is Poisson distributed with parameter $n_t = n_0 \cdot (1 + g)^t$ depending on the time.

On the other hand, the simple Poisson law frequently fails to provide a satisfactory representation of the actual claim number distribution. Usually the number of claims is affected by other types of fluctuations¹ than pure random fluctuations: trends, short-term fluctuations and long-term cycles. In the present paper trends as well as long-term cycles are disregarded and only short-term fluctuations are taken into account. For this purpose a structure variable will be introduced to represent short-term fluctuations in the number of claims. In practice the (deterministic) parameter of the simple Poisson distribution for the number of claims of year t will turn to be a stochastic parameter $n_t \cdot \tilde{q}$, where \tilde{q} is a random structure variable² having its own probability distribution depending on the short-term fluctuations it is going to represent. Since trends are disregarded, the only restriction for the probability distribution of \tilde{q} is that its expected value has to be equal to 1. Further, we will assume that \tilde{q} is Gamma distributed with parameters (h,h). Thus, the moments of the structure variable \tilde{q} are given by:

$$E(\tilde{q}) = 1 \quad \sigma(\tilde{q}) = 1/\sqrt{h} \quad \gamma(\tilde{q}) = 2/\sqrt{h} = 2 \cdot \sigma(\tilde{q})$$

Consequently, the number of claims \tilde{k}_t are Negative Binomial distributed, increasing the standard deviation of \tilde{k}_t as well as the skewness, thus increasing the risk of having excessive claim numbers.

For each line of business, the claim sizes, denoted by $\tilde{Z}_{i,t}$, are assumed to be i.i.d. random variables with a continuous distribution - having d.f. S(Z) - and to be scaled by only the inflation rate in each year. The moments about the origin are equal to:

$$E(\tilde{Z}_{i,t}^j) = (1+i)^{jt} \cdot E(\tilde{Z}_{i,0}^j) = (1+i)^{jt} \cdot a_{jz,0}$$

¹ See Beard, Pentikainen and Pesonen (1984).

² Here the variation of the Poisson parameter n from one time unit to the next is analysed. It is worth to recall that a "structure variable" have been also used when the variableness of the Poisson parameter from one risk unit to the next is to be investigated (see Bühlmann (1970)).

with \tilde{k}_t and $\tilde{Z}_{i,t}$ mutually independent for each year t . The expected claim size of a line has been simply denoted by m whereas r_{2Z} and r_{3Z} are the usual risk indices of the claim size distribution³.

For property, we distinguish between single-risk claims and catastrophe claims. Furthermore, we assume that there is a fixed sum insured for each property policy, and the single-risk claims can not exceed this amount. Catastrophe claims are assumed to affect a large number of policies representing all property lines of business.

Let L denote the number of lines of business. For each line of business, $l = 1, \dots, L$, let $K_l(t)$ denote the number of policies in year t and $SI_l(t)$ denote the sum insured of the each policy in year t . It is assumed that the portfolio is hit by two types of claims: Risk claims affecting only one policy and catastrophe claims affecting a large number of policies for all lines. The aggregate claims amount in year t , \tilde{X}_t is given by a compound process:

$$(4) \quad \tilde{X}_t = \sum_{l=1}^L \sum_{k=1}^{\tilde{k}_{l,t}} \tilde{Z}_{l,k,t} + \sum_{k=1}^{\tilde{k}_{cat,t}} \tilde{W}_{k,t}$$

where $\tilde{k}_{l,t}$ is the random variable of the number of claims for the l -th line of business occurred in the year t and $\tilde{Z}_{l,k,t}$ the random claim size of the k -th claim for the l -th line occurred at year t . Furthermore, $\tilde{k}_{cat,t}$ is the random variable of the number of catastrophe claims occurred in the year t and $\tilde{W}_{k,t}$ the random claim size of the k -th catastrophe claim occurred at year t . More details concerning the catastrophe losses simulation are reported in Appendix III.

2. A COMPARISON BETWEEN EXACT AND SIMULATION MOMENTS OF THE CAPITAL RATIO: AN EXAMPLE.

Usually, the capital ratio $\tilde{u}_t = \tilde{U}_t / B_t$ is preferred to be analysed instead of the total risk reserve amount.

Disregarding (only in this section) reinsurance, taxation, dividends, investment return from loss reserves and regarding the investment return rate j as deterministic and constant over the time and actual general expenses perfectly matched by the expenses loading ($E_t = c \cdot B_t$), for a single line of business the capital ratio in this particular theoretical framework is given by:

$$(5) \quad \tilde{u}_t = r \cdot \tilde{u}_{t-1} + p \cdot \left[(1 + \lambda) - \frac{\tilde{X}_t}{P_t} \right]$$

³ Risk indices of the claim size distribution are:

$$r_{2Z} = \frac{a_{2Z}}{(a_{1Z})^2} = \frac{a_{2Z}}{m^2} \quad \text{and} \quad r_{3Z} = \frac{a_{3Z}}{(a_{1Z})^3} = \frac{a_{3Z}}{m^3}.$$

where r and p denote the following two non-negative joint factors:

$$r = \frac{1+j}{(1+i) \cdot (1+g)} \qquad p = \frac{1-c}{1+\lambda} (1+j)^{1/2} = \frac{P}{B} \cdot (1+j)^{1/2}$$

The annual joint factor r is depending on the investment return rate j , the claim inflation i and the real growth rate g ; on the other hand factor p is depending on the incidence of the risk premium by gross premium (P/B), constant if expenses and safety loading coefficients (c and λ) are maintained constant along the time, increased of the return factor for half a year.

After some manipulations, the stochastic equation (5) of the ratio \tilde{u}_t turns to:

$$(6) \qquad \tilde{u}_t = r^t \cdot u_0 + p \cdot \left[(1+\lambda) \cdot \sum_{h=0}^{t-1} r^h - \sum_{h=1}^t \frac{\tilde{X}_h}{P_h} \cdot r^{t-h} \right]$$

The theoretical expected value, variance and skewness of the capital ratio U/B for a single line of business in the above mentioned scenario are reported in Appendix 1⁴.

Regarding a single line insurer (e.g. Motor Liability), having portfolio and general parameters as those reported in Table 1, the simulation model described in the previous section has been used and the results are figured out in Table 2, where they are also compared with the theoretical (exact) moments, derived in the formula reported in Appendix I, where also randomness of the structure variable \tilde{q} is included.

In that simple framework (as proved also in Appendix I), the initial capital ratio (u_0) is affecting only the expected values of the ratio U/B in the time horizon but clearly not standard deviation and skewness.

In addition to that, as to standard deviation of the capital ratio U/B at time $t=1$, as explained in the next formula:

$$\sigma(\tilde{u}_1) = (1+j)^{1/2} \cdot \sigma\left(\frac{\tilde{X}_1}{B_1}\right) = (1+j)^{1/2} \cdot \frac{1-c}{1+\lambda} \cdot \sigma\left(\frac{\tilde{X}_1}{P_1}\right) = (1+j)^{1/2} \cdot \frac{1-c}{1+\lambda} \cdot \sqrt{\frac{1+c_Z^2}{n_0 \cdot (1+g)} + \sigma_{\tilde{q}}^2}$$

we get it is given by the standard deviation of the loss ratio X/B at year 1 multiplied by the factor $1+j$ for half a year. Actually, for the Single-line liability Insurer described in Table 1, at time 1 the standard deviation of the loss ratio X/B is equal to 11.05% given by the standard deviation of the loss ratio X/B (15.05%) multiplied for the ratio (usually minor than 1) depending on expenses and safety loading $(1-c)/(1+\lambda)$ equal to 0.735. Finally we get the exact value of 11.27% contained in Table 2 for the standard deviation at time $t=1$ of the capital ratio U/B .

⁴ At this regard see also Pentikainen & Rantala (1982) and Savelli (2002)

TABLE 1: Parameters of the Single-line Insurer (Motor Liability)

Parameters :		STANDARD INSURER
Initial risk reserve ratio	u_0	0,25 (*)
Initial expected number of claims	n_0	18.000
Variance structure variable q	σ_q^2	0.02
Skewness structure variable q	γ_q	+ 0.28
Initial expected claim size (EUR)	m_0	6.000
Variability coeffic. of Z	c_Z	7
Safety loading coeffic.	λ	+ 2.10 %
Expense loadings coefficient	c	25.00 %
Real growth rate	g	5.00 %
Claim inflation rate	i	5.00 %
Investment return rate	j	4.00 %
Loss Reserve ratio	l	0 %
Taxation rate	tx	0 %
Dividends rate	dv	0 %

(*) This measure is equivalent to approximately 1.5 times the minimum EU solvency margin

TABLE 2: Single-Line Insurer (motor liability)– Results of 400.000 simulations EXACT AND SIMULATION MOMENTS OF THE CAPITAL RATIO U/B

TIME t	EXACT MOMENTS			SIMULATION MOMENTS (N=400.000)		
	MEAN	ST.DEV.	SKEW.	MEAN	ST.DEV.	SKEW.
0	25.00 %			25.00 %		
1	25.16 %	11.27 %	-0.384	25.12 %	11.26 %	-0.315
2	25.30 %	15.47 %	-0.268	25.26 %	15.46 %	-0.214
3	25.44 %	18.41 %	-0.217	25.41 %	18.35 %	-0.175

3. LIABILITY AND PROPERTY MULTI-LINE INSURERS: TWO CASE STUDIES IN THE GENERAL FRAMEWORK.

In the next sections the results of the simulation model illustrated in the previous sections are reported for two multi-line Insurers for a time horizon of 3 years (T=3).

The first Insurer has two different lines of liability insurances (motor and commercial liability) and the second one has three different lines of Property insurances (homeowners, agriculture

and commercial property). In the next these Insurers will be denoted by respectively Liability Multi-line Insurer (LMI) and Property Multi-line Insurer (PMI).

The features of these two multi-line Insurers (reported in Table n. 1) are not drawn by a specific real data set but are assumed on the base of practical market analyses and practical modelling made by working parties⁵.

As regards the main Insurer's parameters it is worth to emphasize that:

- claims number parameters: for each line according a Poisson distribution with a structure variable q distributed as a Gamma distribution with identical parameters (in practice the claim count has a Negative Binomial distribution). The distribution is different for each line, according different values of the expected number of claims and the variance of the structure variable (from 2% for motor liability to 4% for property lines);
- claim size parameters: for liability lines the claim size is distributed as a Lognormal variable, where the measure of variability coefficient c_z has a great relevance to explain the variability and skewness of the capital ratio distribution U/B. Regarding the property lines, the single risk claims are assumed to be Log-normal distributed – the only difference is that the claim amounts are truncated since they can not exceed the sum insured of the affected policy. The catastrophe claims are assumed to be Pareto distributed (one-parameter) and are affecting all lines of business in the same event. Since we will like to study proportional treaties with the ceded share of each policy to depend on the size of the policy (Surplus treaties), it is important that we are able to split each catastrophe claim on the class of business. Please refer to Appendix III to see how this problem is handled;
- investment return: the annual rate is regarded as a stochastic variable, by an autoregressive model (described in Appendix II), with an expected rate of 4%;
- real growth: is assumed constant and equal to +5% for all lines of business of both multi-line Insurers;
- claims inflation: the annual rate is assumed to be constant and equal to +5% for the liability insurances and smaller (+2%) for property insurances;
- loss reserves: are assumed to be a constant ratio of the gross premiums of the year, 120% for the liability Insurer and 30% for the Property Insurer;
- safety loading: having assumed $\beta=35\%$, the safety loading coefficient is computed according the formula (2) in section 1. The coefficients for the property lines (17.5%) take into account the reduced relevance of the loss reserve (30% of premiums, instead of 120% for the Liability insurer). The coefficient for the Commercial Liability (14.7%) is significantly larger than in case of Motor Liability (2.1%) because of the different c_z (16 instead of 7). It is worth pointing out that the coefficient λ are kept constant for the entire time horizon ($T=3$). Anyway, it is worth to point out that a periodical annual adjustment of the safety loading coefficient would have not affected significantly the results because of only a slight reduction of the ratio $\sigma(X)/E(X)$ as the expected number of claims are increasing year by year (as an effect of the positive real growth of the premium volume), reminding that the pooling effect is less relevant when a not negligible structure variable (no diversifiable risk) is present as in this case.
- expenses loading: 25% for both liability and property lines;
- taxation rate: flat rate of 35% is assumed

⁵ At this regard a special reference has been the model parameters used by the IAA Insurer Solvency Working Party in "A Global Framework for Insurer Solvency Assessment", draft May 2003

- dividends rate: at the end of each year is assumed to be paid to stockholders 20% of the (positive) annual result, net of taxation, Note that no reference to the capital ratio of the year is considered, because the measure of dividends is not affected by either a healthy or a dangerous state of the Insurer.

**TABLE 3:
PARAMETERS OF THE LIABILITY MULTI-LINE INSURER**

Parameters :		LOB 1 (MOTOR LIABILITY)	LOB 2 (COMMERCIAL LIABILITY)	TOTAL
Initial Capital ratio	u_0			0.0 %
Initial expected number of claims	n_0	18.000	2.000	20.000
Variance structure variable q	σ_q^2	0,02	0.03	
Skewness structure variable q	γ_q	+ 0.28	+ 0.35	
Initial expected claim size (EUR)	m_0	6.000	16.000	7.000
Variability coeffic. of claim size	c_z	7	16	
Loss Reserve Ratio	l	120 %	120.0 %	
Expense loadings coefficient	c	25 %	25.0 %	
Beta coefficient	β	35 %	35.0 %	
Safety loading coeffic.	λ	2.1 %	14.7 %	
Real growth rate	g	5.0 %	5.0 %	
Claim inflation rate	i	5.0 %	5.0 %	
Investment return rate (expect. value)	j	4,0 %	4,0 %	
Taxation rate	tx			35 %
Dividends rate	dv			20 %
Initial Risk Premium (mill Eur)	P	108,0	32,0	140,0
Initial Gross Premiums (mill Eur)	B	147,0	48,9	196,0

Regarding the reinsurance strategies, the Liability Multi-line Insurer will have the choice between Quota Share treaty with commission equal to the expense loading ($c_{RE}=c=25\%$) and an unlimited Excess of Loss cover with a premium rate equal the risk premium plus a factor (10%) of the standard deviation of the losses to the cover and including a further loading at 20% for profit, costs and brokerage .

TABLE 4:
PARAMETERS OF THE PROPERTY MULTI-LINE INSURER

Parameters :		LOB 1 (HOMEOWNERS PROPERTY)	LOB 2 (AGRICULTURE PROPERTY)	LOB 3 (COMMERCIAL PROPERTY)	TOTAL
Initial Capital ratio	u_0				0.0 %
Initial Number of policies		200.000	40.000	20.000	260.000
Sum Insured per policy (EUR)		200.000	400.000	1.200.000	
Expected loss frequency per risk losses		8.0 %	16.0 %	8.0 %	
Initial expected number of claims	n_0	6.000	2.000	2.000	10.000
Variance structure variable q	σ_q^2	0,04	0.04	0.04	
Skewness structure variable q	γ_q	+ 0.40	+ 0.40	+ 0.40	
Initial expected claim size (EUR) ⁶	m_0	8.000	12.000	24.000	12.000
Variability coeffic. of claim size	c_z	4	8	12	
Cataprophe PML (mill EUR)		80,0	128,0	48,0	256,0
Exp. n. of catastrophe claims					1
n_{CAT}					
Catastrophe claims – Pareto param	α				1.2
Cumulation factor					0.5
Loss Reserve Ratio	l	30.0 %	30.0 %	30.0 %	
Expense loadings coefficient	c	25.0 %	25.0 %	25.0 %	
Beta coefficient	β	35.0 %	35.0 %	35.0 %	
Safety loading coeffic.	λ	17.5 %	17.5 %	17.5 %	
Real growth rate	g	5.0 %	5.0 %	5.0 %	
Claim inflation rate	i	2.0 %	2.0 %	2.0 %	
Investment return rate (expect. value)	j	4,0 %	4,0 %	4,0 %	
Taxation rate	tx				35 %
Dividends rate	dv				20 %
<hr style="border-top: 1px dashed black;"/>					
Initial Risk Premium (mill EUR)	P	49,3	27,5	43,2	120,0
- hereof single risk losses		44,7	20,6	40,4	105,7
- hereof cat losses		4,6	7,0	2,7	14,3
Initial Gross Premiums (mill EUR)	B	77,2	43,2	67,6	188,0

Regarding the reinsurance strategies, it is assumed the Property Multi-line Insurer will have the choice on the market among Quota Share treaty with commission 27%, a Surplus treaty with commission 26% and an Excess of Loss per risk cover with a premium rate equal the risk premium plus a factor (10%) of the standard deviation of the losses to the cover and including a further loading at 20% for profit, costs and brokerage. In all cases the Property Multi-line

⁶ Before truncation in the sum insured.

Insurer's retention will be protected by a Catastrophe Excess of Cover with a rate on line set using a standard exposure rating method.

4. THE RESULTS OF THE SIMULATION MODEL REGARDING THE RBC REQUIRED ACCORDING SOME DIFFERENT RISK MEASURES.

A well known one-sided approach to risk evaluation is the Value-at-Risk (VaR) widely used when the risk relies on the occurrence of unfavourable events such as insolvency are to be estimated. That kind of approach has a sound background in actuarial literature, where Capital-at-Risk (CaR) and probability of ruin have usually been the main pillars in solvency analyses. The VaR measure is widely used in finance, but with a time span much more reduced (e.g. 10 days) compared with insurance risk analyses, in which a time span of either 1 or 2 years is usually adopted.

In insurance solvency, VaR can be summarized as the maximum loss for an insurer over a target horizon within a given confidence level (e.g. 99%); in other words it denotes a monetary amount for the risk of managing an insurance company.

Assuming to have no initial capital ($U_0=0$), the VaR for the horizon time (0,t) with a confidence level equal to $1-\varepsilon$ (e.g. 99.0%) is then given by:

$$VaR(0,t)_{1-\varepsilon} = -U_{\varepsilon}(t)$$

where $U_{\varepsilon}(t)$ is the ε -th percentile of the Risk Reserve amount at time t (with a confidence level required in the analysis usually rather large, at least 95%). In case this risk measure is used for RBC requirements at time 0, the investment return coming from the initial capital required should be also inserted into the formula. In this framework, where the investment return is regarded as stochastic, the expected value of the investment return has been adopted. Furthermore, RBC requirements should be compared with the initial gross premiums volume (B_0) of the Insurer, in order to be able to compare the output among different insurers and scenarios. Consequently, expressed as a ratio of the initial gross premiums, that risk measure can be written as:

$$(7) \quad rbc_{1-\varepsilon}^{VaR}(0,t) = \frac{RBC_{1-\varepsilon}^{VaR}(0,t)}{B_0} = \frac{-U_{\varepsilon}(t) \cdot [1 + E(\tilde{j})]^{-t}}{B_0} = -\frac{u_{\varepsilon}(t)}{\bar{r}^t}$$

where the percentile ratio $u_{\varepsilon}(t)$ and the joint factor \bar{r} are respectively:

$$u_{\varepsilon}(t) = \frac{U_{\varepsilon}(t)}{B_t} \quad \bar{r} = \frac{1 + E(\tilde{j})}{(1 + g) \cdot (1 + i)}$$

In case of a multi-line insurer with different growth rates g and claim inflation i , the factor \bar{r} will result as a weighted average according the relevance of each different line of business.

In case is preferred to use the TVaR as a risk measure in order to regard more properly the left tail of the risk reserve distribution (or equivalently the right tail of the loss distribution -U), the ratio of the RBC requirement can be expressed as follows:

$$(8) \quad rbc_{1-\varepsilon}^{TVaR}(0,t) = \frac{RBC_{1-\varepsilon}^{TVaR}(0,t)}{B_0} = \frac{-U_{\varepsilon}^{TVaR}(t) \cdot [1 + E(\tilde{j})]^{-t}}{B_0} = -\frac{u_{\varepsilon}^{TVaR}(t)}{\bar{r}^t}$$

where we have:

$$U_{\varepsilon}^{TVaR}(t) = E\left[\tilde{U}(t) \mid \tilde{U}(t) < U_{\varepsilon}(t)\right] = E\left[\tilde{u}(t) \mid \tilde{u}(t) < u_{\varepsilon}(t)\right] \cdot B_t = u_{\varepsilon}^{TVaR}(t) \cdot B_t$$

A third risk measure here regarded is the UES (Unconditional Expected Shortfall), taking into account either the probability of the event occurring and the magnitude of the resulting shortfall. The UES at year t is then given by:

$$\begin{aligned} UES(t) &= E[\max(0, -\tilde{U}_t)] = \Pr(\tilde{U}_t < 0) \cdot E\left[-\tilde{U}_t \mid \tilde{U}_t < 0\right] = \\ &= \Pr(\tilde{u}_t < 0) \cdot \left[E\left[-\tilde{u}_t \mid \tilde{u}_t < 0\right] \cdot B_t \right] \end{aligned}$$

As mentioned before, we can express also this risk measure in a relative way, compared to gross premiums of the year:

$$(9) \quad ues(t) = \frac{UES(t)}{B_t} = E[\max(0, -\tilde{u}_t)]$$

In the recent actuarial literature is emphasized how this risk measure can be regarded as the risk premium of an insurance contract which would cover the shortfall of the company in case it occurs, being the sum of the expected shortfalls (ruin deficits) weighted by their probabilities. In principle, this is the undiscounted premium to pay by the stockholders for a guarantee of solvency at time t without make available any initial risk capital.

Furthermore, the UES is a one-sided risk measure, like the semi-variance, in which deficits are included but surpluses are ignored.

As to strategy parameters (time horizon and level of confidence) to quantify the Risk-Based Capital by VaR or TVaR risk measures, the next simulation results have considered a time horizon of 1, 2 or 3 years, with a level of confidence of either 99.0%, 99.5% or 99.9%. The results are figured out also net of reinsurance according the formula (1). At this stage, only Quota Share arrangements are considered:

- for the Liability Multi-line Insurer a 10% Quota Share with reinsurance commissions equal to 25% (coincident with the direct expenses rate) of ceded premiums has been considered;
- for the Property Multi-line Insurer a 40% Quota Share cover with commission 27% has been considered. The retention is protected by a Catastrophe Excess of Loss cover in excess of the yearly expected amount of catastrophe losses up to 1.5 times the probable maximum loss (PML). However, it is assumed that it is possible to get losses up to 2 times the PML, such there is a (small) risk of having insufficient coverage.

TABLE 5. Liability and Property Multi-line Insurers – Results of 400.000.simulations
RBC AND UES RATIOS (%) ACCORDING DIFFERENT LEVELS OF CONFIDENCE AND TIME HORIZONS

NO REINSURANCE

RISK MEASURES	LIABILITY MULTI-LINE INSURER			PROPERTY MULTI-LINE INSURER		
	TIME HORIZON			TIME HORIZON		
	T=1	T=2	T=3	T=1	T=2	T=3
rbc(VaR) in %						
99.0 %	19.89	25.05	27.39	125.79	149.56	159.31
99.5 %	26.64	32.85	37.17	152.73	175.35	186.40
99.9 %	55.01	75.00	77.90	204.37	226.14	252.55
rbc(TVaR) in %						
99.0 %	33.35	41.32	46.01	161.35	184.71	200.31
99.5 %	44.01	54.25	60.38	184.22	209.02	229.99
99.9 %	85.70	94.68	96.59	235.71	263.16	313.63
ues in %	1.329	1.433	1.319	3.020	5.122	6.852

WITH REINSURANCE

Liability: 10% QS for each line and $c_{RE}=25\%$

Property: 40% QS for each line and $c_{RE}=27\%$. Retention protected by Cat XL EUR 220 mill xs EUR 8.4 mill at ROL 5.85%

RISK MEASURES	LIABILITY MULTI-LINE INSURER			PROPERTY MULTI-LINE INSURER		
	TIME HORIZON			TIME HORIZON		
	T=1	T=2	T=3	T=1	T=2	T=3
rbc(VaR) in %						
99.0 %	17.90	22.55	24.65	10.51	12.88	14.51
99.5 %	23.98	29.56	33.46	12.49	15.36	17.73
99.9 %	49.51	67.50	70.11	17.43	22.41	25.73
rbc(TVaR) in %						
99.0 %	30.01	37.19	41.41	13.64	16.95	19.25
99.5 %	39.61	48.83	54.34	15.89	19.90	22.51
99.9 %	77.13	85.21	86.93	22.69	27.40	30.82
ues in %	1.196	1.290	1.187	0.849	0.999	1.002

Note: also in case of Reinsurance the ratios are given as either RBC or UES amount divided by initial gross premiums.

5. EFFECTS OF DIFFERENT REINSURANCE STRATEGIES ON THE REQUIRED RBC.

For Liability lines different Quota Shares treaties and unlimited Excess of loss covers have been considered. The Quota Share Treaties are all having a reinsurance commission of 25% (equal to the expense ratio). The premium for the excess of loss covers are calculated as the risk premium for the layer plus 10% of the standard deviation, and finally loaded with 20% for expenses, brokerage and profit.

For Property lines different Quota Shares treaties, Surplus treaties and per risk Excess of loss covers covering up to the sum insured have been considered. The Quota Share Treaties are all having a reinsurance commission of 27%, and the Surplus treaties are all having a reinsurance commission at 26%. Given the retention R of the surplus treaty the retained share for each line of business is calculated as

$$\alpha_l = \min(1, R / SI_l) \text{ for } l = 1, \dots, L$$

by assuming for each line of business, all policies are having the same sum insured.

The premium for the per risk excess of loss covers are calculated as the risk premium (calculated exactly using the parameters in the d.f. $S(Z)$) for the layer plus 10% of the standard deviation, and finally loaded with 20% for expenses, brokerage and profit.

In addition to that, the retention is always protected by a catastrophe excess of loss cover in excess of the expected yearly expected cat losses **net** of proportional reinsurance (it is assumed that per risk excess of loss covers will have so large excess points that they will have no major impact on the catastrophe losses) and up to 1.5 times the total PML for all lines. Since we are assuming that it is possible to have losses up to 2 times the PML **net** of proportional reinsurance, there is a minor risk that the company in case of a very serious catastrophe will not have sufficient coverage in the top. In other words, the cat XL cover will change as the proportional programme changes.

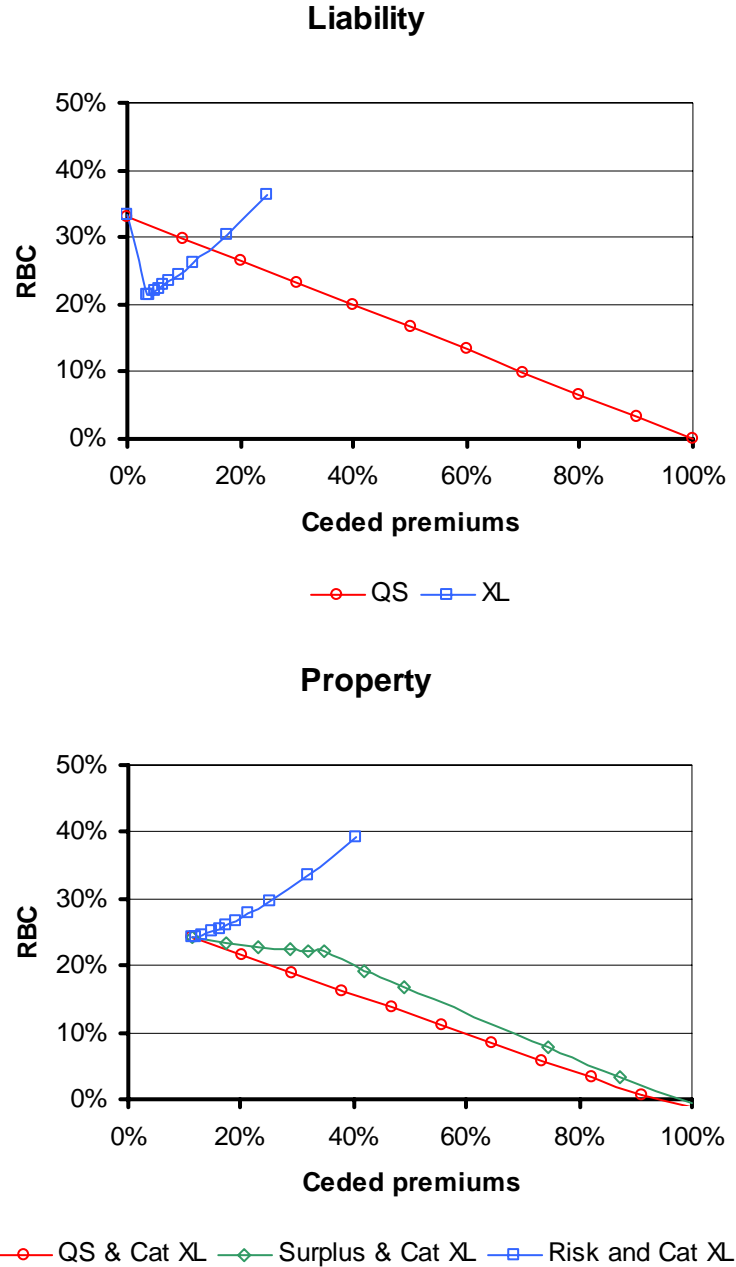
The premium for the catastrophe excess of loss covers are calculated as the risk premium (calculated using an exposure approach) for the layer plus 30% of the standard deviation, and finally loaded with 25% for expenses, brokerage and profit.

Figure 1 shows the correspondence between the RBC ratio (chosen through the TVaR 99% risk measure with $T=1$) and the ceded premium for different retentions on the reinsurance programmes. It is worth to mention that without any reinsurance cover the RBC ratio would be 33.25% and 161.35% of gross premiums for Liability and Property multi-line Insurer respectively. For the Property Insurer, net of only CaT XL cover the ratio is significantly reducing to 24.14%.

For the Liability multi-line Insurer, the graphs for the QS treaties are linear because of reinsurance commission (25%) identical to the expense ratio. Therefore the annual results net of the QS treaties are proportional to the annual results in case of no reinsurance. Clearly a linear behaviour would not be in force in case of either more favourable or more unfavourable reinsurance commissions,

In case of excess of loss, the RBC ratio increases for decreasing excess points (and thereby increasing ceded premiums). One reason is that the XL price is quite expensive and convenient (to reduce the risk) down only until a certain retention, and after that the cost is so high compared with the ceded risk that the expected result for the insurer is negative.

FIGURE 1: Correspondence between the RBC ratio (through TVaR 99% risk measure) and the ceded premium for different retentions on the reinsurance programmes for T=1 (results of 400.000 simulations).



For Property multi-line Insurer, all the graphs starts at ceded premiums equal to 11.5% which is the price for the catastrophe excess of loss in case of no proportional reinsurance. The graph for the QS treaties is no longer strictly linear: the reinsurance commission is no longer identical to the expense ratio, and the QS treaties are combined with a Cat XL covering the retention.

The graph showing the Surplus treaties breaks at 35% ceded premiums – this is because this point corresponds to a retention at EUR 400.000 which is the sum insured for the second line

of business (Agricultural). For retentions at EUR 400.000 or higher (i.e. ceded premiums less than 35%), homeowners as well as agricultural is fully retained by the insurer, and only the commercial line of business is affected by the surplus treaty – and there is no major increase in the RBC ratio by increasing the retentions to higher than EUR 400,000.

6. THE IMPACT OF THE RBC ON THE PROFITABILITY OF PROPERTY AND LIABILITY INSURERS.

Here the expected Return on Equity (RoE) will be used as a measure for the Insurer's performance and let $\bar{R}(0,T)$ denote the expected RoE all over the full time horizon (0,T). In the framework here assumed, where both dividends and taxations are present (as stochastic variables because of stochastic economic annual results), it is given by:

$$(10) \quad \bar{R}(0,T) = E \left[\frac{\left(\tilde{U}_T + \sum_{h=1}^T D\tilde{V}_h \right) - U_0}{U_0} \right] = E \left[\frac{\tilde{U}_T + \sum_{h=1}^T D\tilde{V}_h}{U_0} \right] - 1$$

It is worth to emphasize that this measure is not affected at all by the dividend policy adopted by the management in the time horizon, because if no dividends are distributed the risk reserve amount U is increasing of the same amount.

In this section we are considering the following reinsurance strategies:

Property Insurer:

- No reinsurance
- Quota share 40% with reinsurance commission 27% combined with catastrophe excess of loss EUR 222 mill xs EUR 8.4 mill at ROL 5.85%, 1 reinstatement at 100%
- Surplus treaty retention EUR 300.000 with reinsurance commission 26% (a surplus with this retention corresponds approximately to cede the same amount of premiums as the 40% QS above) combined with catastrophe excess of loss EUR 271.7 mill xs EUR 10.3 mill at ROL 6.27%, 1 reinstatement at 100% additional premium
- Per risk excess of loss EUR 840.000 xs EUR 360.000 at premium rate 7.51% combined with catastrophe excess of loss EUR 369.9 mill xs EUR 14.1 mill at ROL 5.85%, 1 reinstatement at 100% addition premium
- Catastrophe excess of loss EUR 369.9 mill xs EUR 14.1 mill at ROL 5.85%, 1 reinstatement at 100% addition premium

Liability Insurer:

- No reinsurance
- Quota share 10% with reinsurance commission 25%
- Quota share 20% with reinsurance commission 25%
- Unlimited excess of loss in excess of EUR 730.000 at premium rate 7.57%

For each reinsurance strategy applied, the minimum required rbc as described above is applied as initial capital ratio u_0 . The results of these reinsurance strategies applied on the 400.000 simulations are shown in Table 6 and 7.

TABLE 6. Liability Multi-line Insurer – Results of 400.000.simulations
RATE ON EQUITY (RoE) FOR DIFFERENT REINSURANCE STRATEGIES WITH INITIAL RISK RESERVE RATIO EQUAL TO THE REQUIRED RBC

REINSURANCE STRATEGY	INITIAL RISK RESERVE $U_0=RBC(T=1)$ VAR 99.5%			INITIAL RISK RESERVE $U_0=RBC(T=1)$ TVAR 99%		
	TIME HORIZON			TIME HORIZON		
	T=1	T=2	T=3	T=1	T=2	T=3
NO REINSURANCE						
rbc (%)	26.66	33.22	37.95	33.42	41.81	46.95
ues (%)	0.82	1.35	1.70	0.59	0.95	1.21
Finite ruin prob (%)	0.50	1.07	1.63	0.27	0.58	0.91
Expected finite-time RoE (%)	23.48	49.74	79.19	19.32	40.88	65.00
NET OF QS 10%						
rbc (%)	30.06	37.53	42.08	30.07	37.63	42.25
ues (%)	0.74	1.21	1.53	0.53	0.86	1.09
Finite ruin prob (%)	0.50	1.07	1.63	0.27	0.58	0.91
Expected finite-time RoE (%)	23.48	49.73	79.18	19.33	40.89	65.00
NET OF QS 20%						
rbc (%)	21.33	26.58	30.36	26.73	33.44	37.56
ues (%)	0.66	1.08	1.36	0.47	0.76	0.97
Finite ruin prob (%)	0.50	1.07	1.63	0.27	0.58	0.91
Expected finite-time RoE (%)	23.48	49.75	79.20	19.32	40.88	65.00
NET OF RISK XL						
rbc (%)	22.20	29.32	34.85	23.46	30.86	36.41
ues (%)	0.19	2.43	4.29	0.14	0.71	1.52
Finite ruin prob (%)	0.51	2.01	4.00	0.38	1.63	3.37
Expected finite-time RoE (%)	13.30	27.98	44.35	12.75	26.81	42.49

TABLE 7. Property Multi-line Insurer – Results of 400.000.simulations
 RATE ON EQUITY (RoE) FOR DIFFERENT REINSURANCE STRATEGIES WITH INITIAL RISK RESERVE
 RATIO EQUAL TO THE REQUIRED RBC

REINSURANCE STRATEGY	INITIAL RISK RESERVE $U_0=RBC$ VAR 99.5%			INITIAL RISK RESERVE $U_0=RBC$ TVAR 99%		
	TIME HORIZON			TIME HORIZON		
	T=1	T=2	T=3	T=1	T=2	T=3
NO REINSURANCE						
rbc (%)	152.68	177.80	190.65	161.41	187.33	205.30
ues (%)	1.53	3.25	4.96	1.17	2.55	4.04
Finite ruin prob (%)	0.50	1.10	1.65	0.38	0.86	1.32
Expected finite-time RoE (%)	7.72	15.84	24.64	7.45	15.28	23.76
NET OF QS 40%						
rbc (%)	12.50	15.49	17.98	13.67	17.07	19.50
ues (%)	0.17	0.43	0.70	0.12	0.33	0.55
Finite ruin prob (%)	0.50	1.38	2.35	0.33	0.99	1.74
Expected finite-time RoE (%)	19.48	40.64	63.76	18.08	37.71	59.13
NET OF SURPLUS						
rbc (%)	17.81	23.61	28.09	19.30	25.45	30.10
ues (%)	0.22	0.80	1.62	0.16	0.59	1.26
Finite ruin prob (%)	0.50	1.87	3.75	0.33	1.35	2.84
Expected finite-time RoE (%)	7.90	16.43	25.74	7.52	15.64	24.48
NET OF RISK XL						
rbc (%)	24.65	34.21	42.11	26.69	36.59	44.63
ues (%)	0.28	1.42	3.51	0.20	1.04	2.69
Finite ruin prob (%)	0.51	2.50	5.72	0.33	1.80	4.39
Expected finite-time RoE (%)	0.73	1.34	1.94	0.92	1.72	2.53
NET OF CAT XL						
rbc (%)	22.23	28.49	33.82	24.18	31.10	36.31
ues (%)	0.28	0.87	1.60	0.20	0.66	1.25
Finite ruin prob (%)	0.50	1.60	3.00	0.33	1.15	2.25
Expected finite-time RoE (%)	13.64	28.39	44.50	12.79	26.61	41.69

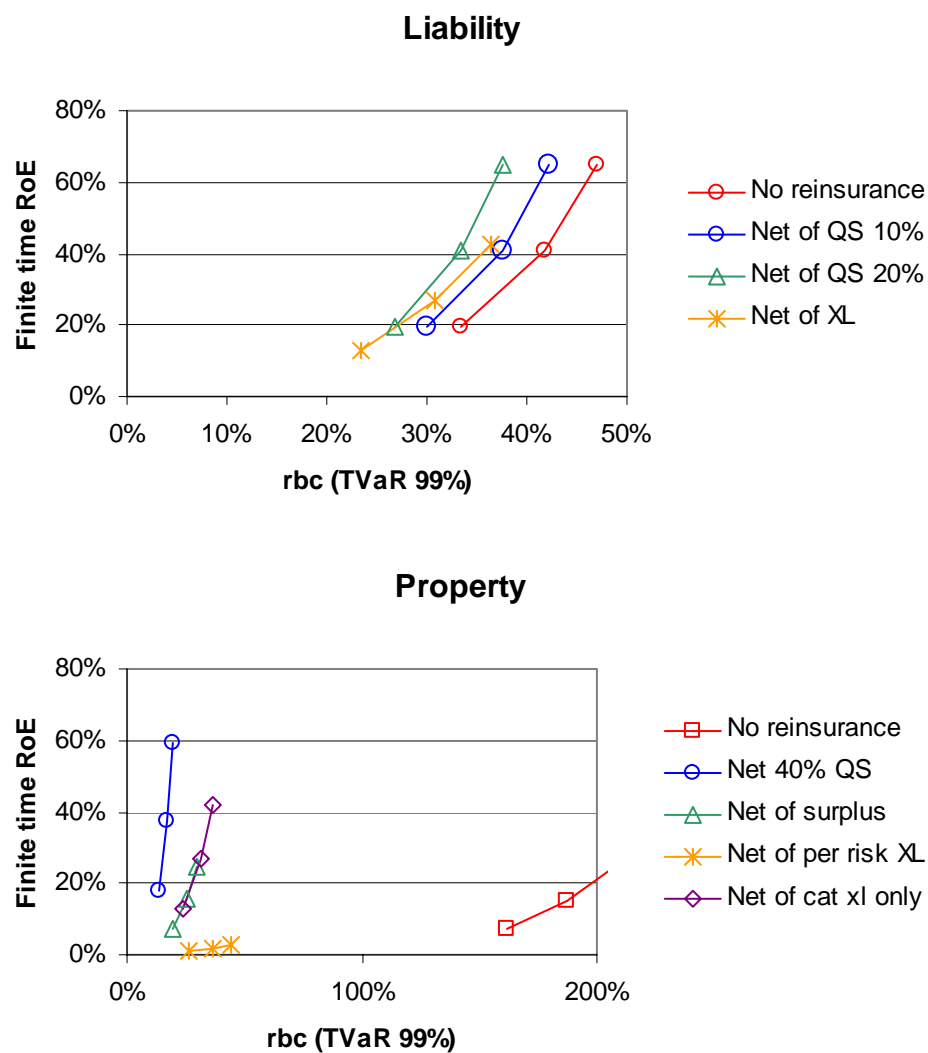
In **Figure 2**, the trade-off Risk vs Return of the two multi-line insurers is illustrated for all the 3 years of the time horizon, where the measure of risk has been denoted by RBC of the year with initial reserve ratio equal to the calculated TVaR 99% from Table 2.

For Liability Insurer the results show how the QS treaties have no impact on the finite time expected RoE $\bar{R}(0,T)$ because the commission rate is identical to the expense ratio. The only (natural) impact is that the RBC reduces according to lower retentions of the QS. The selected XL contract is slightly more effective in reducing the RBC, and the RoE is less because a loading is included in the reinsurance premium.

For Property it is seen that the 40% QS treaty is more effective in reducing the risk than the Surplus treaty with retention EUR 300.000 (the ceded premiums for the two treaties are

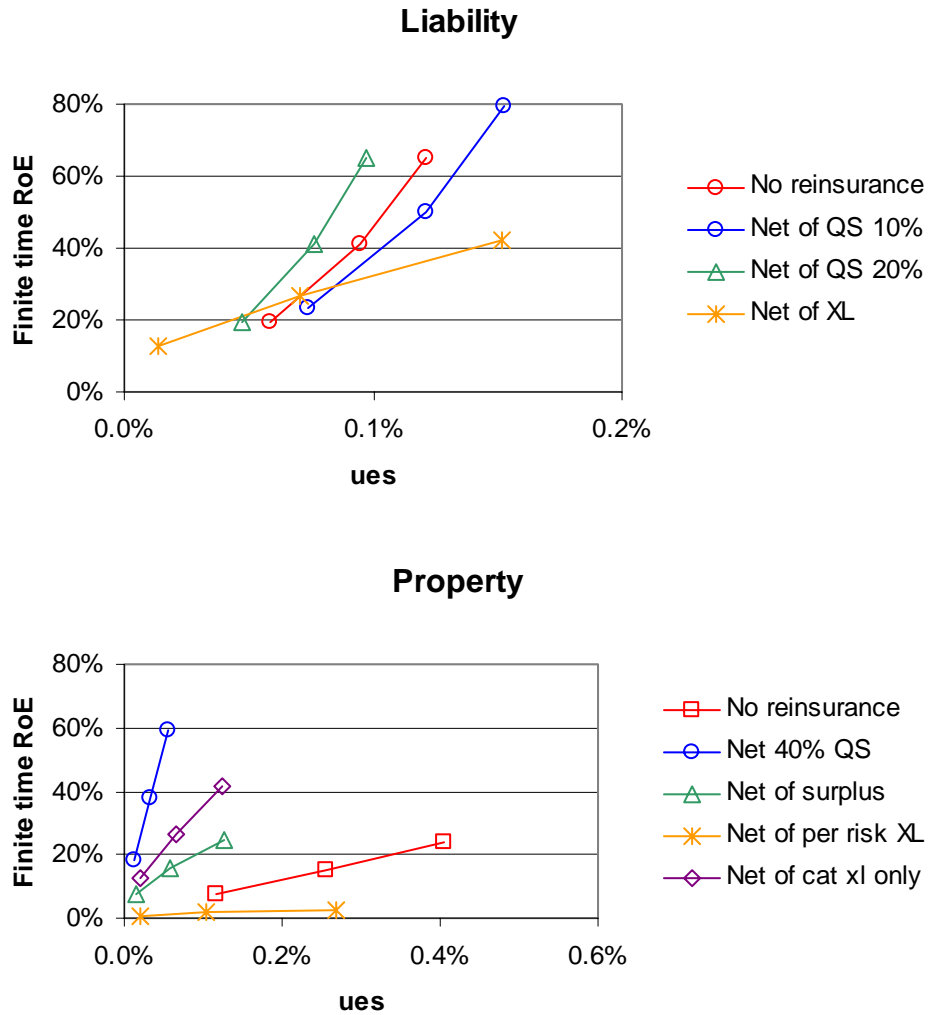
approximately the same). One of the reasons is that the surplus treaty does not protect against any variability in the yearly losses for homeowners line of business since this line is retained 100%. However, the expected RoE is better for the QS treaty – mainly because commission rate for the QS treaty is higher than for the Surplus treaty (the reinsurer will usually pay less commission for a more un-balanced treaty). The alternative replacing the proportional treaties by a per risk excess of loss protection is seen to be too expensive an alternative – the expected RoE is much less and the RBC is higher.

FIGURE 2 Risk vs Return Trade-off: finite-time Expected RoE vs RBC ratio (TVaR 99%), according to ceded premiums for different retentions on the reinsurance programmes for T=1 (results of 400.000 simulations with $u_0=rbc$ for each single reinsurance strategy) .



A similar trade-off is shown in Figure 3. Here the Unconditional Expected Shortfall (UES) of the year is used as risk measure and the profitability defined by the finite time expected RoE $\bar{R}(0,T)$. Similar conclusions can be made for this graph.

FIGURE 3. Risk vs Return Trade-off: finite-time Expected RoE vs UES ratio, according to ceded premiums for different retentions on the reinsurance programmes for T=1 (results of 400.000 simulations with $u_0=rbc$ for each single reinsurance strategy).



7. THE IMPACT OF THE RBC ON THE PROFITABILITY OF PROPERTY AND LIABILITY INSURERS.

Clearly, the Insurers here regarded are not representative as standard (liability and property) insurers in the market and furthermore the risk theoretical model here applied implies a certain degree of simplifications, concerning e.g. the claim reserving run-off risk, the dynamic premium rating, the correlation degree between the risks, the investment model, the asset allocation and the credit risk. However, the results of the model concerning RBC requirements and profitability in the previous sections show that the EU minimum solvency margin should be significantly increased by the EU legislation in the future, also paying major attention to pricing conditions included in the reinsurance treaties, affecting not only the variance of the process but the expected result of the year too, with unfavourable effects on insurer's solvency in some cases.

For instance, the results show that a Property Insurer should not underwrite any risk without a Catastrophe XL cover – otherwise an extremely high risk capital is required. The per risk XL cover seems only to be suitable for quite large retentions in order not to be too expensive, and it does not offer sufficient coverage of high variability in the number of claims. Furthermore, in case of a Quota Share treaty, the minimum capital requirement should not be always decreased accordingly the ceding quota to reinsurance; in case of unfavourable reinsurance commissions the insurer's risk is not decreasing in the same measure of the ceded quota and for larger time horizons (3 or 5 years) it might also be (not surprisingly) larger than the risk measure without that cover.

Finally, the new European RBC requirements should not reduce significantly the profitability of the company otherwise a minor interest on the insurance market would occur by financial investors.

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APPENDIX 1

Exact moments of the capital ratio U/B in a simplified framework

Expected value

$$\begin{aligned}
 & u_0 + \lambda p \cdot t && \text{if } r = 1 \\
 E(\tilde{u}_t) = & && \\
 & r^t \cdot u_0 + \lambda p \cdot \frac{1 - r^t}{1 - r} && \text{if } r \neq 1
 \end{aligned}$$

Variance

Reminding we have denoted by p the ratio $p = \frac{1-c}{1+\lambda} \cdot (1+j)^{1/2}$, the variance of the solvency ratio $\tilde{u}_t = \tilde{U}_t / B_t$ is given by:

$$\begin{aligned}
 \sigma^2(\tilde{u}_t) &= p^2 \cdot \sum_{k=1}^t \sigma^2\left(\frac{\tilde{X}_k}{P_k}\right) \cdot r^{2(t-k)} \\
 &= p^2 \cdot \sum_{k=1}^t \left(\frac{1+c_z^2}{n_0 \cdot (1+g)^k} + \sigma_q^2 \right) \cdot r^{2(t-k)}
 \end{aligned}$$

$$\text{with } s^2 = (1+g) \cdot r^2 = \frac{(1+j)^2}{(1+g)(1+i)^2}$$

In the usual case $r \neq 1$, $s \neq 1$ we have

$$\begin{aligned}
 \sigma^2(\tilde{u}_t) &= p^2 \cdot \left[\sum_{k=1}^t \frac{1+c_z^2}{n_0 \cdot (1+g)^k} \cdot r^{2(t-k)} + \sum_{k=1}^t \sigma_q^2 \cdot r^{2(t-k)} \right] \\
 &= p^2 \cdot \left[\frac{1+c_z^2}{n_0 \cdot (1+g)^t} \cdot \sum_{k=1}^t s^{2(t-k)} + \sigma_q^2 \cdot \sum_{k=1}^t r^{2(t-k)} \right] \\
 &= p^2 \cdot \left[\frac{1+c_z^2}{n_0 \cdot (1+g)^t} \cdot \frac{1-s^{2t}}{1-s^2} + \sigma_q^2 \cdot \frac{1-r^{2t}}{1-r^2} \right]
 \end{aligned}$$

Skewness

$$\begin{aligned} \gamma(\tilde{u}_t) &= -\gamma\left(\sum_{k=1}^t \frac{\tilde{X}_k}{P_k} \cdot r^{t-k}\right) = -\frac{\mu_3\left(\sum_{k=1}^t \frac{\tilde{X}_k}{P_k} \cdot r^{t-k}\right)}{\sigma^3\left(\sum_{k=1}^t \frac{\tilde{X}_k}{P_k} \cdot r^{t-k}\right)} = -\frac{\sum_{k=1}^t \sigma^3\left(\frac{\tilde{X}_k}{P_k} \cdot r^{t-k}\right) \cdot \gamma\left(\frac{\tilde{X}_k}{P_k} \cdot r^{t-k}\right)}{\left(\sigma^2\left(\sum_{k=1}^t \frac{\tilde{X}_k}{P_k} \cdot r^{t-k}\right)\right)^{3/2}} \\ &= -\frac{\frac{r_{3Z,0}}{(1+g)^{2t} n_0^2} \sum_{k=1}^t w^{3(t-k)} + 3\sigma_q^2 \frac{1+c_Z^2}{(1+g)^t n_0} \sum_{k=1}^t v^{3(t-k)} + \gamma_q \cdot \sigma_q^3 \sum_{k=1}^t r^{3(t-k)}}{\left(\frac{1+c_Z^2}{(1+g)^t n_0} \sum_{k=1}^t s^{2(t-k)} + \sigma_q^2 \sum_{k=1}^t r^{2(t-k)}\right)^{3/2}} \end{aligned}$$

$$\text{with } v^3 = (1+g) \cdot r^3 = \frac{(1+j)^3}{(1+g)^2 (1+i)^3}$$

We will always have a negative skewness $\gamma(\tilde{u}_t) \leq 0$ unless γ_q is negative, in that case the sign of $\gamma(\tilde{u}_t)$ is depending of the parameters. In general, we will have $\gamma_q \geq 0$.

In the usual case when $r \neq 1$, $s \neq 1$, $v \neq 1$, $w \neq 1$, the skewness of the solvency ratio can be written as

$$\gamma(\tilde{u}_t) = -\frac{\frac{r_{3Z,0}}{(1+g)^{2t} n_0^2} \cdot \frac{1-w^{3t}}{1-w^3} + 3\sigma_q^2 \frac{1+c_Z^2}{(1+g)^t n_0} \frac{1-v^{3t}}{1-v^3} + \gamma_q \cdot \sigma_q^3 \frac{1-r^{3t}}{1-r^3}}{\left(\frac{1+c_Z^2}{(1+g)^t n_0} \frac{1-s^{2t}}{1-s^2} + \sigma_q^2 \frac{1-r^{2t}}{1-r^2}\right)^{3/2}}$$

If \tilde{q} is Gamma(h, h) and \tilde{Z}_t is lognormal distributed (with two parameters), consequently \tilde{k} is Negative Binomial distributed and we have $\gamma_q = 2 \cdot \sigma_q$. Then

$$r_{3Z} = \frac{a_{3Z}}{m^3} = \frac{a_{3Z}}{a_{2Z}^{3/2}} \cdot \left(\frac{a_{2Z}}{m^2}\right)^{3/2} = \left(1+c_Z^2\right)^{3/2} \cdot \left(1+c_Z^2\right)^{3/2} = \left(1+c_Z^2\right)^3$$

and then the ratio:

$$\frac{r_{3Z}}{r_{2Z}^{3/2}} = \left(1+c_Z^2\right)^{3/2}$$

In this case we have (in the usual case with $r \neq 1$, $s \neq 1$, $v \neq 1$, $w \neq 1$):

$$\gamma(\tilde{u}_t) = - \frac{\frac{(1+c_Z^2)^3}{(1+g)^{2t} n_0^2} \cdot \frac{1-w^{3t}}{1-w^3} + 3\sigma_q^2 \frac{1+c_Z^2}{(1+g)^t n_0} \frac{1-v^{3t}}{1-v^3} + 2\sigma_q^4 \frac{1-r^{3t}}{1-r^3}}{\left(\frac{1+c_Z^2}{(1+g)^t n_0} \frac{1-s^{2t}}{1-s^2} + \sigma_q^2 \frac{1-r^{2t}}{1-r^2} \right)^{3/2}}$$

In the special case with no growth (i.e. $g = 0$), the formula becomes more simple (since $s = v = w = r$), and we are able to make further analyses.

$$\gamma(\tilde{u}_t) = - \frac{\left(\frac{(1+c_Z^2)^3}{n_0^2} + 3\sigma_q^2 \frac{1+c_Z^2}{n_0} + 2\sigma_q^4 \right) \cdot \frac{1-r^{3t}}{1-r^3}}{\left(\frac{1+c_Z^2}{n_0} + \sigma_q^2 \right)^{3/2} \cdot \left(\frac{1-r^{2t}}{1-r^2} \right)^{3/2}}$$

APPENDIX II

Simulating the stochastic return on investment

As regards the stochastic investment return, we assume the annual rate \tilde{j}_{t+1} of the relevant year ($t, t+1$) follows an AR process depending also on the inflation:

$$\tilde{j}_{t+1} = \bar{j} + b \cdot (\tilde{j}_t - \bar{j}) + (b_1 \cdot \tilde{i}_{t-1} + b_2 \cdot \tilde{i}_{t-2}) + c_j \cdot \tilde{\mathcal{E}}_{t+1}$$

where \tilde{i} denotes the inflation rate, having an AR(1) process as:

$$\tilde{i}_{t+1} = \bar{i} + a \cdot (\tilde{i}_t - \bar{i}) + c_i \cdot \tilde{\mathcal{E}}_{t+1}$$

and $\tilde{\mathcal{E}}_{t+1}$ is the noise term of the process for year $t+1$ (identical for both processes \tilde{i}_{t+1} and \tilde{j}_{t+1}), having assumed that all noise terms are i.i.d. and $\tilde{\mathcal{E}}_{t+1} + 2$ distributed as a Gamma(4;2). According the assumptions made, the noise term of each year has expected value 0, standard deviation 1 and skewness +1.

The values used in the simulations for the above mentioned parameters are as follows:

$$\bar{j} = 4\% \quad \bar{i} = 2\%$$

$b = 0.20$ = mean-reverting driving factor for return

$a = 0.65$ = mean-reverting driving factor for inflation

$b_1 = b_2 = 0.50$

$c_j = 0.15$ = noise term coefficient in the return process

$c_i = 0.80$ = noise term coefficient in the inflation process.

Furthermore, a minimum constraint has been fixed for either inflation (-1%) and return rate (+0.25%).

APPENDIX III Simulating the catastrophe losses

The number of catastrophe claims in year t $\tilde{k}_{cat,t}$ is assumed to be Poisson distributed with mean $n_{cat,t}$.

The catastrophe claim amounts, denoted by $\tilde{W}_{k,t}$, are assumed to be i.i.d. random variables with a continuous distribution function $F_{cat}(W)$ – and to be scaled by the claims inflation i as well as the growth rate g in each year. The s^{th} moments about the origin are equal to:

$$E(\tilde{W}_{k,t}^s) = (1+g)^{st} \cdot (1+i)^{st} \cdot E(\tilde{W}_{k,0}^s) = (1+g)^{st} \cdot (1+i)^{st} \cdot a_{cat,sZ,0}$$

with $\tilde{k}_{cat,t}$ and $\tilde{W}_{k,t}$ being mutually independent for each year t .

In order to be able to study the effect of various proportional contracts, hereunder surplus treaties, it is important to know the split of each catastrophe claim on each business line $(\tilde{W}_{1,k,t}, \dots, \tilde{W}_{L,k,t})$ with $\tilde{W}_{k,t} = \sum_{l=1}^L \tilde{W}_{l,k,t}$.

The marginal distribution of $\tilde{W}_{l,k,t}$ is assumed to be the Pareto distribution with cumulative distribution function $F_{W_{l,t}}$ or survivor function $S_{W_{l,t}}$ where

$$(AIII-1) \quad F_{W_{l,t}}(w_{l,t}) = 1 - \left(\frac{c_{l,t}}{w_{l,t}} \right)^\alpha, \text{ and } S_{W_{l,t}}(w_{l,t}) = 1 - F_{W_{l,t}}(w_{l,t}) = \left(\frac{c_{l,t}}{w_{l,t}} \right)^\alpha.$$

Furthermore, it is assumed that no catastrophe claim can exceed 2 times the 100 years PML (Probable Maximum Loss). Therefore, the Pareto distribution is truncated in 2 times PML for each line of business:

$$(AIII-2) \quad F_{W_{l,t}}(w_{l,t}) = \frac{1 - \left(\frac{c_{l,t}}{w_{l,t}} \right)^\alpha}{1 - \left(\frac{c_{l,t}}{2PML_{l,t}} \right)^\alpha}, \text{ and } S_{W_{l,t}}(w_{l,t}) = 1 - F_{W_{l,t}}(w_{l,t}).$$

For such catastrophe events, the L lines of business are not independent, and we will therefore use a method described in Wang (1998) to find the joint cumulative distribution function.

Let (U_1, \dots, U_L) be a L -dimensional uniform distribution with support on the hypercube $(0,1)^L$ and having the joint cumulative distribution function

$$(AIII-3) \quad F_{U_1, \dots, U_L}^{(\beta)}(u_1, \dots, u_L) = \left\{ \sum_{l=1}^L u_l^{-1/\beta} - L + 1 \right\}^{-\beta}$$

where $u_j \in (0,1)$, $l = 1, \dots, L$, and $\beta > 0$. As shown by Cook and Johnson (1981),

$$\lim_{\beta \rightarrow 0} F_{U_1, \dots, U_L}^{(\beta)}(u_1, \dots, u_L) = \min[u_1, \dots, u_L],$$

Thus, the correlation approaches to its maximum (i.e. co-monotonicity) when β decreases to zero; the correlation approaches to zero when β increases to infinity.

Cook and Johnson (1981) also gave the following simple simulation algorithm for the multivariate distribution given by (AIII-3):

Step 1. Let Y_1, \dots, Y_k be independent and each has an Exponential distribution with mean 1.

Step 2. Let Z have a Gamma $(\beta, 1)$ distribution.

Step 3. Then the variables

$$(AIII-4) \quad U_l = [1 + Y_l/Z]^{-\beta}, \quad l = 1, \dots, L,$$

have a joint cumulative distribution function.

For a set of arbitrary marginal distributions with survivor functions S_{W_1}, \dots, S_{W_L} , we can define a joint survivor function by

$$(AIII-5) \quad S_{W_1, \dots, W_L}(w_1, \dots, w_L) = \left\{ \sum_{l=1}^L S_{W_l}(w_l)^{-1/\beta} - L + 1 \right\}^{-\beta}$$

Consider the task of aggregating L lines of business $(\tilde{W}_{1,k,t}, \dots, \tilde{W}_{L,k,t})$. If we assume that $(\tilde{W}_{1,k,t}, \dots, \tilde{W}_{L,k,t})$ have a multivariate distribution given by (AIII-5), a simulation of $\tilde{W}_{1,k,t}, \dots, \tilde{W}_{L,k,t}$ can easily be implemented by

Step 4. Invert the (U_1, \dots, U_k) in (AIII-4) using $(S_{W_{1,t}}^{-1}, \dots, S_{W_{L,t}}^{-1})$ by calculating

$$\tilde{W}_l = \frac{c_l}{\left(1 - (1 - U_l) \cdot \left(1 - (c_l/2PML_l)^\alpha\right)\right)^{1/\alpha}}$$

or

$$\tilde{W}_l = \frac{c_l}{U_l^{1/\alpha}}$$

if the Pareto distribution is not truncated in 2 times the PML. In the multivariate uniform distribution given by (AIII-3), all correlations are positive.

European Windstorm Losses

The Geo Risks Research department in Munich Re has made some analysis of the winter storms in Europe in 1990 and 1999. The results are published in Munich Re (2002). For a number of countries in Europe – for house owner risks, for agricultural risks and for commercial risks, they have estimated the relation between loss ratio, the loss frequency and the average loss/affected policy, respectively, and the wind speed. They have also estimated maps showing the wind speed in various parts of each country for windstorm scenarios with return period of 100 years. When combining these numbers with the maps, the following table of “100 years windstorm PML’s” are found.

Based on these figures, the parameters for simulating the catastrophe claims are selected in order to obtain

- Catastrophe PML equal to EUR 80 mill, EUR 128 mill and EUR 48 mill for households, agricultural and commercial lines of business, respectively.
- A catastrophe factor (expected cat losses in pct of the gross premiums) at 6%, 16% and 4% for households, agricultural and commercial lines of business, respectively.