

PRICING OF CATASTROPHE LINKED SECURITIES

BY

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ABSTRACT

A formula for the spread of Catastrophe Bonds is derived within a risk-pricing framework that deals with both systematic and non-systematic risk. The formula is as follows:

$$\text{Spread} = (\text{EL})^{\wedge(1/\rho)}$$

Here, EL is the Expected Loss as a percentage and $\rho \geq 1$, is a Risk Aversion Level (RAL).

In the absence of systematic risk, as is the case with these particular Insurance Linked Securities (ILS), the risk costs under the comprehensive framework correspond to the values calculated by the Wang PH Transform. This simplifies the analysis and facilitates both the derivation of the formula as well as the subsequent analysis.

The ρ 's implicit in over 70 of these ILS are then identified and the majority found to be in the range 1.65 ± 0.15 . A rationale for identifying appropriate values for this ρ is then developed by considering the role of the related frictional cost index in the comprehensive risk framework. This approach is demonstrated using data on US Hurricane losses.

The analysis indicates that current ILS spreads appear consistent with the assumption that the catastrophe model loss estimates are around the 1st or 2nd percentile of the 'true' loss distribution. This is both an extremely cautious approach for pricing these securities as well as a damning indictment on the work of the specialist catastrophe modelling companies.

A more realistic choice of the implicit pricing percentile, say to around the 10th percentile level (ρ around 1.3), would result in reductions of over 50% in current spreads. These new spreads should still provide fair returns to investors, whilst also becoming very attractive to issuers. A market that makes sense to buyers and sellers will prosper rather than decline.

KEYWORDS

Insurance Linked Securities, Catastrophe Bonds, Securitization, Reinsurance, Risk Premium, Systematic Risk, Non-Systematic Risk, Dynamic Financial Analysis, Frictional Costs, Proportional Hazards (PH) Transform, Unbundled PH Transform, Risk Aversion Index, Cobb-Douglas Production function, Generalised Pareto Distribution.

1 INTRODUCTION

The emergence of insurance linked securities (ILS), issued almost exclusively as catastrophe bonds, was seen as heralding the convergence of the insurance and capital markets. Some seven years later, this market is stagnating and appears to have gone into decline. The main problem seems to be the high spreads that these bonds attract, which appear to be based on catastrophe reinsurance Rates On Line (ROL) rather than spreads of high yield corporate bonds. These high spreads make ILS very attractive to investors but significantly unattractive to issuers, such as property insurers. Potential investors are also concerned with lack of liquidity and the costs involved in understanding the loss characteristics of these bonds.

ILS 'default' when a pre-specified event, or events, exceeding a pre-specified severity occurs. The amount by which the bond defaults then depends on either the actual monetary losses suffered or calculated by a pre-specified formula or model based on the parametric size of the event and its location. These loss profiles are estimated by one of the main catastrophe modelling companies using sophisticated models and detailed policy level exposure data. The majority of ILS issued to date have estimated annual default probabilities of around 1% (1 in 100 years) and exhaustion (total loss) probabilities of around 0.4% (1 in 250 years). Where rated, these characteristics place them at the Medium to High-Yield Grades of Corporate Bonds.

The pricing of these securities is generally made up of two components, a spread or margin over the risk free rate to cover the insurance risk and an underlying risk free rate, usually taken as LIBOR, to compensate investors for loss of interest. The spread component is equivalent, and directly comparable, to the traditional catastrophe reinsurance layer cost expressed as a Rate On Line (ROL). These bonds are held in a Special Purpose Vehicle to ensure there is no credit risk in addition to the insurance risk.

The spreads on ILS issued to date have been significantly higher than similarly rated corporate bonds. Over the last twenty years or so, the spreads on high yield US corporate bonds have been enough to, just about, make up for the incurred capital losses, spreading what are volatile results on an annual basis over the economic cycles. In comparison, the ILS, issued mainly over the last five years, have been priced at multiples of over 6 times their expected annual losses (see Tables 1 to 4 in Appendix 1).

The rationale behind the pricing of these securities is not given and one can only assume it is what the market is prepared to accept. The spreads for the ILS issued to date are almost indistinguishable from the Rates On Line for the equivalent catastrophe reinsurance layers. In essence, what is being transferred to the capital markets is the inefficient pricing requirements of the traditional aggregators of these risks, the catastrophe reinsurance companies, rather than transferring these risks to the market that can handle them most efficiently and cheaply.

The existence of such a pricing anomaly warrants closer investigation. The argument that this is still a new and untested market may be fair but dismisses the huge efforts and developments in catastrophe modelling achieved over this period. The lack of liquidity is likely to be less of an issue once investors understand the relatively short-term nature of these securities and realise that maturing ILS will be replaced by new securities for the same exposures.

Investors may also be reluctant to invest the time and effort needed to understand the risks associated with these new securities particularly when they are constantly reminded of the material and human cost of these horrendous events by the media. With such a background, how can the estimated return periods of these ILS, at say one in a hundred year levels, be trusted to be reliable? With such a small, illiquid market, such effort is currently confined to the few specialist investors who understand the opportunities afforded by the current high pricing levels.

What is needed to rekindle this market is a comprehensive and sound, technical, basis for determining appropriate spreads for these securities, based on the detailed catastrophe model loss estimates. This process should also deal with the confusing messages about these events and their insured costs that may be partly responsible for investor reluctance to accept the results of the catastrophe models as reasonable estimates of exposure. The rest of this paper is an attempt to provide this.

The next Section discusses the characteristics of ILS and Corporate Bonds, which appear similar in terms of loss experience but have significantly different sets of loss drivers.

Section 3 then discusses systematic and non-systematic risk and outlines a comprehensive methodology for risk pricing that deals with both types of risk. Variability from both market and company sources generate frictional costs described by a family of frictional cost curves. In the absence of market risk the average frictional costs are equivalent to the prices calculated by the well-known Wang PH transform. This observation simplifies subsequent calculations significantly.

Section 4 then demonstrates how this approach enables us to both calculate spreads as well as identify the risk index or parameter implicit in their spreads. This is then used to compare results on cheapness and dearness of the early ILS to those obtained by Lane (2000) and Wang (2002). Tables of results and charts on over 70 known ILS are given in Appendix 1.

Section 5 develops a simple, powerful, formula for the spreads of Catastrophe Bonds and demonstrates how this is a special, single parameter form, of the Cobb-Douglas Function.

Finally, Section 6 considers this pricing level parameter and uses an example to identify what it tells us about the level of conservatism that is implicit in current spreads. A more reasonable range is then suggested and the implications to spreads discussed.

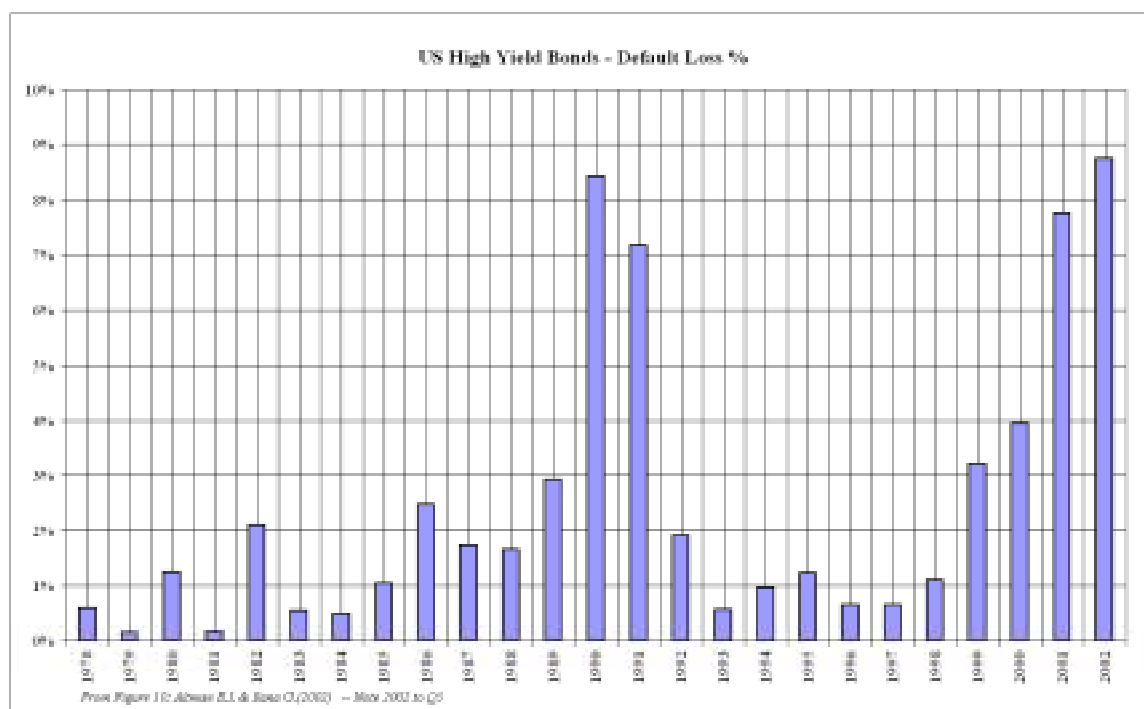
2 CHARACTERISTICS OF ILS AND CORPORATE BONDS

Taken at face value the loss characteristics of ILS are very similar to those of corporate bonds. An event triggers a loss that in turn results in a total or partial loss of capital. The only differences are then in what causes these events and then what determines how much of the capital is actually lost.

The issued ILS are subject to detailed loss analysis, usually carried out by one of the main specialist modelling companies. As can be seen from the Tables in Appendix 1, these ILS tend to have estimated default probabilities (or Probabilities of First Loss) of around 0.5% to 2.0% per annum. Recovery rates are estimated at around 30% and expected losses (to capital at risk) are just below 1%, overall. These characteristics are similar to those of high non-investment bonds (BB/Ba range) and the ILS that are rated are given ratings that correspond closely with the corporate bonds with similar default probabilities and loss characteristics.

The spreads for ILS have averaged just over 5% (or 500 basis points) making them look very attractive to investors given that the underlying expected losses are less than 1%. Over the last twenty or thirty years US high yield corporate bonds have attracted spreads of 3% to 4% (300 to 400 basis points) and have generated losses, as a percentage of capital, of around 3%. In other words, these corporate bond investors have just about got their money back over this long period but have enjoyed some very good years with some bad periods in between. They have most definitely not been receiving spreads of anything like 5 times the expected annual losses that appear the norm for the ILS issued this far.

The following chart shows the loss experience of the US high yield bonds over the last 25 years.



The average (\$ weighted) loss over this period was in-fact 3.16% - (Altman & Bana -2002)

The chart shows two high loss periods, when annual loss rates moved significantly, first around 1990 and then around 2000. These periods correspond very closely with the two recession periods, from 9/1990 to 3/1991 and then from 4/2001 to 12/2001. There were also two other recession periods during this time, from 1/1980 to 7/80 and then from 7/1981 to 11/82 and the chart also shows some increases in loss experience during these earlier periods, albeit not of the severity of the more recent ones.

History and economic arguments support the link between economic conditions and default rates of corporate bonds. These defaults are known to be industry dependent as different industries suffer to varying degrees through these periods. Predicting likely future defaults of these high yield bonds is a very complex process.

The chart also shows that there is no totally loss free year for high yield bonds. This is partly to do with the large number of bonds in existence at any one time and partly due to the potential for any company, that fails to obtain a top investment grade rating, to face financial difficulty over the short to medium term. There is then an attritional level of bond defaults that may, or may not, be directly related to economic conditions that arise from companies that find themselves in difficulty because of specific, company related, problems generally grouped under the operational or enterprise risk umbrella.

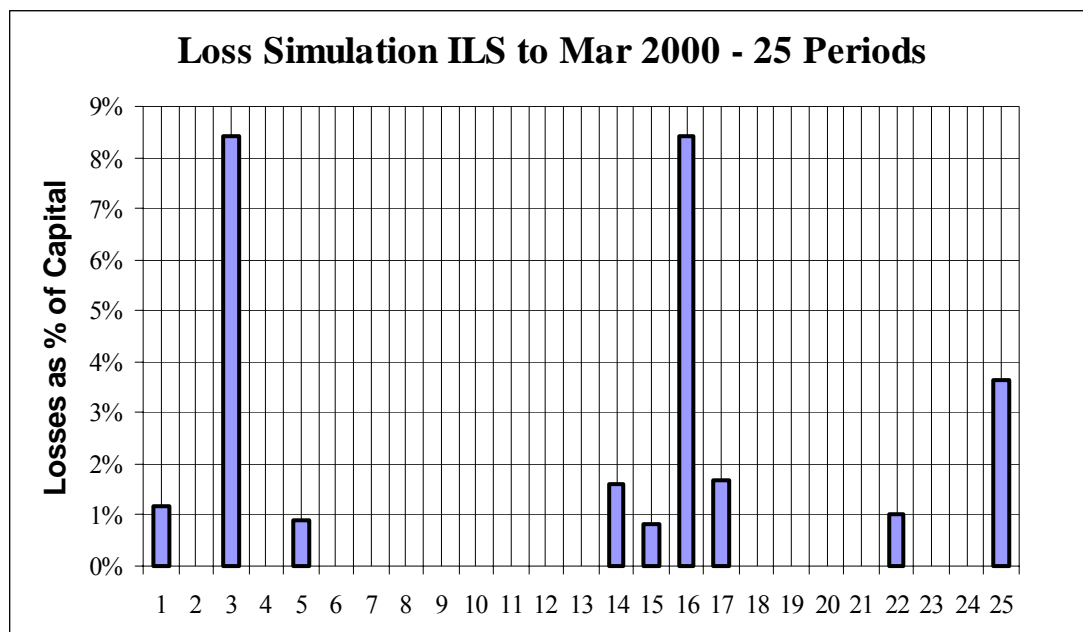
Corporate bonds are then subject to both market or systematic as well as company specific or non-systematic risk and a pricing methodology for these bonds has to deal with both elements in a unified, sound basis. A cash flow model, underpinned by an economic scenario generator with sufficient power to carry out any such analysis, will make the best of the current commercially available catastrophe models look like kindergarten toys and is beyond the scope of this paper. An approach along these lines will, however, be outlined in the next section.

Turning now to ILS, we find that there is very little actual ILS loss experience to date. This lack of actual, reliable data can be overcome to an extent by historic and scientific study of events stretching back hundreds of years as reflected in the continuing development and enhancement of the commercial catastrophe modelling tools. These models have powerful statistical simulation engines for generating events supplemented by sophisticated, engineering based, damage models that turn event severity characteristics at geographic locations into expected losses on a detailed property by property basis.

We can attempt to visualise how the experience of the first ILS issued up to March 2000 might have turned out if these bonds were issued for many years by running a simple simulation model based on the loss distributions implied by the known loss parameters.

The average losses over this period are close to the expected mean of 0.84% (see Table 1 in Appendix 1) but we would expect to see significant year-on-year variation and a high proportion of claim free years. Although this is partly due to the small number of ILS actually included in this sample, the reality is that the number of independent catastrophe exposure that are likely to be covered by ILS is no more than 10 to perhaps 15 on a world wide basis. Claim free years are then likely to be the norm, rather than the exception, at the current ILS default levels.

The following chart shows the results just one realisation (simulation) of 25 such periods.



The limited number of exposed peril/areas available to investors of ILS reduces the scope for risk diversification and even with spreads at 5% for an underlying loss rate of 1%, there is scope for losses to exceed the sum of spreads from an otherwise balanced portfolio of these securities. This situation may also exist with investors of high yield bonds who have the option to diversify their portfolios across perhaps 10 to 15 industry sectors.

So, the lack of diversification argument does not fully explain why ILS are being priced at over 5 times their expected loss rates when the capital markets appear happy to accept much lower spreads on the equivalent high yield corporate bonds.

This pricing anomaly and the resulting high spreads of ILS offers the most likely explanation for the relatively low level of activity in this new area. At these levels there is no financial incentive for issuers, property insurers, to consider such bonds as a part of their catastrophe protections. There is however, plenty of scope for significant reductions in the spreads of ILS that could still leave them attractive to investors and at the same time make them a real cost-effective alternative for issuers.

The key question is then to find a sound, rational basis for calculating spreads for ILS to either justify current levels or help determine new ones.

3 SYSTEMATIC AND NON-SYSTEMATIC RISK

In order to consider the pricing of any cash flows we would normally need to choose between the insurance and the capital market models for risk pricing. These two, apparently completely different approaches to pricing cash flows prompted the Casualty Actuarial Society (CAS) to initiate the Risk Premium Project (RPP). The RPP team were asked to review all the relevant research, across both disciplines, and report findings and conclusions - see Butsic et al (2000).

The first RPP Report represents a significant milestone in our understanding of risk. The report's key insight is the recognition that there are only two types of risk, systematic and non-systematic and that both generate risk costs. Systematic risk is risk correlated with capital markets and non-systematic risk is company specific risk, uncorrelated with financial markets. As both types of risk generate risk costs both need to be allowed in pricing. In the case of systematic risk an efficient market adjusts prices to compensate investors for the systematic component. The RPP also observed that risk or variability, from **both types of risk** generate unbudgeted internal costs or frictional costs but made no suggestions as to how these frictional costs are to be calculated. In the absence of systematic costs, non-systematic costs would then equate to frictional costs.

A comprehensive approach to risk pricing and valuation, consistent with the findings of the RPP, was developed by Christofides S. & Smith A.D. (2001) in their paper 'DFA - The Value of Risk'. For systematic risk they used a multiple factor arbitrage-free pricing approach, using explicit state deflator processes. Deflators provide competitive capital market pricing for any cash flow, including insurance cash flows. Consistent with capital market theory, this methodology implies a reward for investors who are exposed to systematic risk, that is, market risk that remains even in a diversified portfolio. For an introduction to this approach see Jarvis et al (2001).

For the frictional costs Christofides & Smith proposed a generic family of curves, the so-called 'Unbundled P-H Transform' function. This family of functions has the following form:

$$\theta(x) = \lambda \int_{-\infty}^x G(y)^{1-\lambda} dy + \int_x^{\infty} [(1-\lambda)G(y)^{-\lambda} + \lambda G(y)^{1-\lambda} - 1] dy$$

where:

- x is the ideal profit (defined as projected profit before frictional costs)
- θ is the frictional cost
- λ is a risk loading parameter between 0 and 1, and is determined by the overall level of costs in the market. $\lambda = 0$ corresponds to zero risk loading; $\lambda = 1$ implies that all risks are priced at their maximum value.
- y is a dummy integration variable
- $G(y)$ is a function, which increases from 0 to 1 as y moves from $-\infty$ to ∞ . The increase is not necessarily strict, nor continuous.

The function $G(y)$ is chosen so as to minimise the expectation of these frictional costs and this is shown to be achieved when:

$$G(x) = \frac{\int_{-\infty}^x D(y) dF(y)}{\int_{-\infty}^{\infty} D(y) dF(y)}$$

Here, $D(y)$ is the state price deflator and $F(x)$ is the cumulative probability function of the ideal profit X . This family of functions has the necessary characteristics, that is it is a convex function of ideal profit, so that businesses whose profits are more risky also attract higher frictional costs. It has a minimum value for some finite value of (ideal) profit and increases more steeply on the left than on the right. This is to ensure that unexpectedly low ideal profits generate more frictional costs than unexpectedly high ideal profits. For a good introduction to frictional costs see Varnell & Ng (2003).

Christofides & Smith show that the mean value of the realistic profit (ideal profit less frictional costs) is equal to the mean idealistic profit but under an adjusted risk neutral probability law. They then note that, with some sign changes, this is equivalent to the Wang proportional hazards transform (Wang 1995) with the risk loading parameter λ related to the risk aversion parameter ρ by $\lambda = 1 - 1/\rho$. This observation also confirms that the Wang PH transform deals with the non-systematic component of risk in the absence of systematic risk. The 'Unbundled PH Transform' for frictional costs is then a generalisation of the Wang PH Transform that applies in the more usual case where there are both systematic and non-systematic risk elements present. The frictional cost formulation also identifies the complete transformed distribution, rather than just calculate only the mean of the transformed distribution, so has application in risk capital evaluations as well as pricing. Note also that the systematic risk costs are in addition to these frictional costs as systematic risk creates the first and also contributes to the second.

The uniqueness of the Wang P-H Transform as an 'actuarial' premium principle was already known from Wang S., Young V. & Panjer (1997). They show how a distorted probability power function is the only form that can satisfy the five axioms set for 'actuarial' insurance pricing including that of additivity for comonotonic risks required to ensure consistency in pricing excess of loss reinsurance layers.

Most types of insurance business are prone to both systematic and non-systematic risk and their pricing needs to take account of both types of risk. For example insurance involving liability to third party injury that may take many years to settle will be open to market risk on the technical provisions invested over the period also be exposed to potential claims escalation from changes in the economic, fiscal and legal environments. Even short tail property insurance can carry a significant amount of systematic risk as, for example, theft claims tend to increase in times of economic downturns. Insurance related to natural catastrophe exposure is perhaps the only type of insurance coverage that has little if any exposure to systematic risk.

Given the above, we need look no further than the Wang PH Transform in order to price ILS whose underlying exposures are natural catastrophe ones. This is outlined in the next section.

4 PRICING CATASTROPHE BONDS

Catastrophe bonds pay risk free rate, such as LIBOR, to compensate investors for holding the capital for the period and in addition pay a spread to compensate them for the insurance risk. We argued that the Wang PH Transform is the appropriate model for the spread calculations.

The Wang P-H transform for a loss variable X is given as follows:

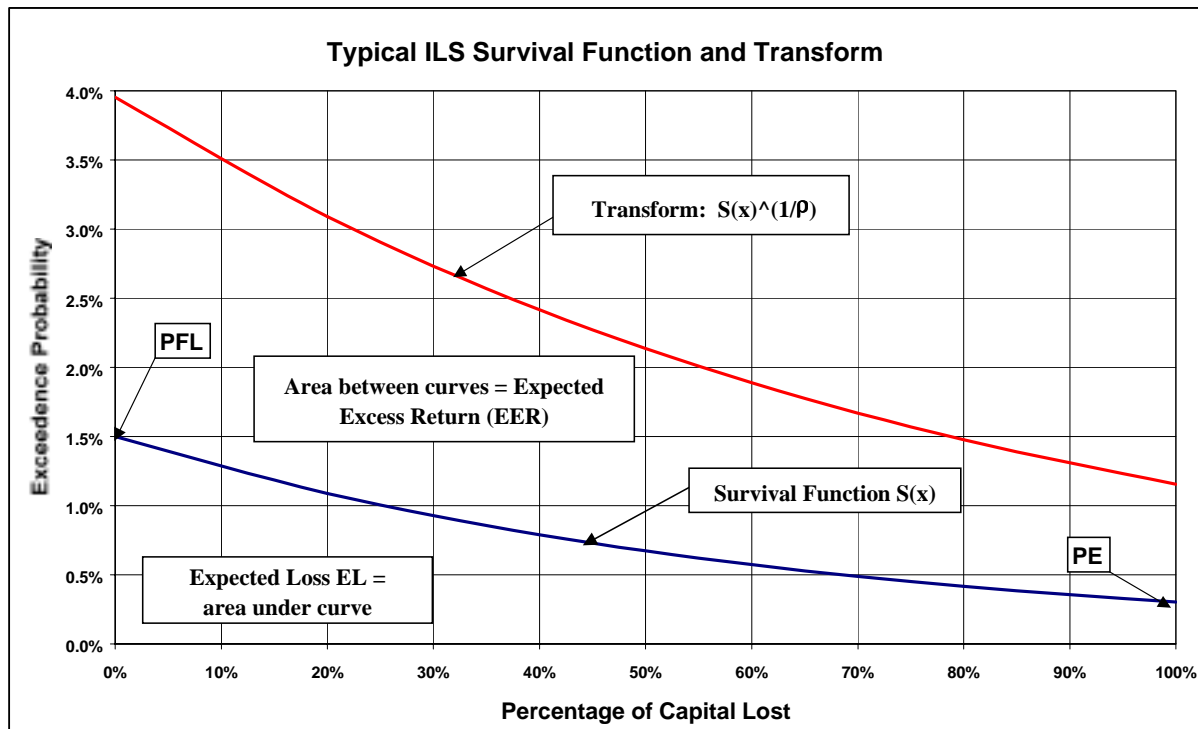
$$\Pi_{\rho}(X) = E[\Pi_{\rho}(X)] = \int_0^{\infty} (S(x))^{1/\rho} dx$$

Here, S(x) is the Survival Function of X, given by $S(x) = \text{Probability}(X > x)$ and $\Pi_{\rho}(X)$ is the risk adjusted premium at a Risk Aversion Level (RAL) $\rho > 1$. Note that the price increases as we increase the risk aversion level, ρ .

The survival function describes the complete distribution of the variable and carries as much information as the underlying probability distribution. We note that, for non-negative random variables X, the expected or mean value of X, E(X), is given by the integral of the survival function over the range of possible values that X can take.

$$E(X) = \int_0^{\infty} S(x) dx$$

The following chart shows a typical ILS Survival Function and its P-H transform. The areas under the two lines give the mean (risk premium) and the risk-adjusted premium respectively. The area between the two lines can be interpreted as the risk loading or Expected Excess Return (EER).



Examples of the use of this approach, including an analysis of the first USSA Residential Re securitisation can be found in Christofides, S. (1998).

In the case of each bond, we generally have estimates for the probability of first loss (PFL), the probability of cover exhaustion (PE) and the expected loss (EL). These values are estimated by detailed modelling of the underlying exposures, usually carried out by one of the main specialist catastrophe modelling firms, such as RMS, EQECAT and AIR.

These three estimates are all that is needed to identify a likely form of the Survival Function $S(x)$ of each ILS.

There are a number of possible choices for the actual form of this function, which has to be a non-negative decreasing one over the complete range of potential losses. Rebasing the values of X to the Unit Interval (0,1) enables us to represent the percentile losses and facilitates the analysis.

The simplest and most obvious form of the Survival Function is then given by a simple exponential decay of the form:

$$S(x) = \alpha * \exp(-\beta * x), \text{ where } \alpha \text{ and } \beta \text{ are parameters to be determined}$$

Under our formulation, we now have:

$$S(0) = \alpha = \text{Probability of First Loss (PFL)} \dots (1)$$

We also know that

$$S(1) = \alpha * \exp(-\beta) = \text{Probability of Exhaustion (PE)} \dots (2)$$

and that the Expected Loss (EL) as a percentage of the capital at risk, is given by:

$$EL = \int_0^1 S(x) dx = \int_0^1 \alpha * \exp(-\beta * x) dx = \alpha / \beta * (1 - \exp(-\beta)) \dots (3)$$

As this simple formulation only involves two parameters we will not necessarily find a unique solution unless we introduce a third parameter, say, a power parameter γ and redefine the Survival Function by:

$$S(x) = \alpha * \exp(-\beta * x^\gamma)$$

This can now be solved uniquely for α , β and γ . This is relatively easy. Equation (1) identifies α as the PFL, Equation (2) gives $\beta = \ln(\text{PFL}/\text{PE}) = \ln(\text{CEL})$ and Equation (3) can now be solved for the exponent γ using a numerical technique, such as the Goal Seek or Solver facilities available in spreadsheets. Appendix 3 contains an example of such a fit to an actual recent case where we have a dozen data points of the distribution.

The final stage is to identify the PH RAL ρ at implicit, according to the pricing basis, in the spread of the ILS. This is also easily done in a spreadsheet by numerical integration and Solver.

This approach then associates an index, the RAL ρ , to the spread of each ILS. This index can now be used in a relative sense to compare the cheapness or dearness of each of these securitisations and also enables us to look at changes in the implicit average of these indices over time to see if prices are getting harder or softer. A weighted average of this index values using, say, the Capital at risk as weight, estimates the overall pricing level. This average index can then be used to estimate spreads for each securitisation in the same way that Lane (2000, 2001, 2002) and Wang (2002) use for comparison purposes. Normally, we would stop the analysis with the derivation of each ρ as this recovers all the spreads exactly.

The following Table shows the results of this method for the early securitisations and compares these to those in Table 1 of Wang (2002). Shaded boxes indicate the ILS deemed 'expensive' according to each method.

SPV	Amount (US \$m)	Empirical Spread	Wang 2-F model	Lane Model	SC RAL Values	SC 'Av ρ ' Spread
1	2	3	4	5	6	7
Mosaic 2A	24.3	4.08%	3.88%	3.80%	1.760	3.85%
Mosaic 2B	20.0	8.36%	10.15%	11.83%	1.447	11.93%
Halyard Re	17.0	4.56%	4.82%	5.01%	1.643	4.89%
Domestic Re	80.0	3.74%	4.36%	4.45%	1.646	4.26%
Concentric Re	100.0	3.14%	4.01%	3.97%	1.616	3.85%
Juno Re	80.0	4.26%	4.15%	4.16%	1.714	4.01%
Residential Re	200.0	3.71%	4.08%	4.03%	1.653	3.95%
Kelvin 1stE	21.6	10.97%	12.80%	15.34%	1.456	15.56%
Kelvin 2ndE	23.0	4.82%	3.25%	3.02%	1.969	3.15%
Golden Eagle B	50.0	2.99%	2.81%	2.51%	1.817	2.25%
Golden Eagle A	126.8	5.48%	4.82%	5.03%	1.746	4.89%
Namazu Re	100.0	4.56%	5.20%	5.52%	1.590	5.42%
Atlas Re A	70.0	2.74%	2.35%	1.92%	1.903	1.74%
Atlas Re B	30.0	3.75%	3.15%	2.90%	1.851	2.69%
Atlas Re C	100.0	14.19%	11.01%	12.90%	1.766	12.90%
Seismic Ltd	145.5	4.56%	5.13%	5.38%	1.597	5.34%
Total or WgtAv	1188.2	5.11%	5.05%	5.32%	1.689	5.22%

Both Lane (2000) and Wang (2002) adopt a model to be applied to all securitisations under investigation and then fit the parameters to this model by minimising or optimising an overall statistic. In the case of Lane, the model is based on a version of the Cobb-Douglas function and we will return to discuss this in the next section. Wang uses a two-factor model that applies the t-distribution to offset inverse normal variates to also allow for parameter error. For more details on this see Wang (2002).

These results show consistency for this sample set. The ρ index basis is, however, more revealing as it also provides a sound basis for monitoring pricing changes over time. In Section 6 we will also outline how this parameter may be made specific to each issue.

Tables of results and charts for the ILS of the last four years are given in Appendix 1.

5 CATASTROPHE BOND SPREAD FORMULA

The Cobb-Douglas Function is the simplest production formula of classical economics and was introduced as far back as 1928 -see Bronfenbrenner et al (1932). It is usually expressed by the following formula:

$$Q=L^{\alpha}*K^{\beta}$$

Here, Q stands for output, L for labour, and K for capital and α and β are parameters to be estimated from the data.

Lane (2000) proposed a version of the Cobb-Douglas Function as an appropriate empirical basis for the Expected Excess Returns (EER) of ILS. In linking this function to insurance pricing Lane argues that the price, expressed as the Expected Excess Return (EER) (Spread - Expected Loss), has to be related to the frequency, as defined by the Probability of First Loss (PFL) and the and severity as defined by the Conditional Expected Loss (CEL, where $CEL=PFL/EL$). The empirical relationship proposed by Lane was then as follows:

$$EER=\gamma*(PFL)^{\alpha}*(CEL)^{\beta}$$

The parameters estimated for the 1999 transactions, were $\gamma=55\%$, $\alpha=49.5\%$, and $\beta=57.4\%$. and the fitted model was then:

$$EER=0.55*(PFL)^{0.495}*(CEL)^{0.574}$$

These parameters were somewhat different from those fitted to the 1998 data, particularly the beta, which had been around 2.2. This was interpreted as indicating a greater emphasis being placed on severity for the earlier transactions, albeit on a smaller data set with a relatively poor fit. Irrespective of the quality of the fit, this is an empirical formula and as such much care is needed in its use for setting future prices or comparing prices over time. A theoretical derivation could overcome these shortcomings.

The earlier discussion identified the Wang PH Transform as the appropriate pricing model for pricing these risks. We will now use this to derive a simpler, powerful formula for the spreads of catastrophe bonds. The derivation is based on the theory outlined in Section 3 and Section 4, and involves two minor approximations with insignificant impact on the results.

Firstly, we assume that the Survival function can be approximated by a simple exponential decay function. We define this function by setting the scale parameter α as the PFL and identifying the β implicit in the Expected Loss (EL) value. The resulting Survival function then starts at the correct place, α and produces the correct expected loss and so also the correct Conditional Expected Loss (CEL).

The fitted exponential survival function may or may not exactly match the known probability of exhaustion (PE). The values of p estimated using this simpler model for $S(x)$ are tabulated and compared with the values derived by the three-parameter exponential power version in the Tables in Appendix 1 for all the known ILS. The results show very clearly that this approximation has little if any impact on any of our results.

The simpler exponential decay version enables us to progress to an analytic pricing formula for these ILS as shown below.

$$\text{Spread} = \int_0^1 S(x)^{(1/\rho)} dx = \int_0^1 (\alpha * \exp(-\beta * x))^{(1/\rho)} dx = \alpha^{(1/\rho)} \rho / \beta * (1 - \exp(-\beta/\rho)) \dots (4)$$

Recalling Equation (3) we see:

$$EL = \int_0^1 S(x) dx = \int_0^1 \alpha * \exp(-\beta * x) dx = \alpha / \beta * (1 - \exp(-\beta)) \dots (3)$$

We now make the second approximation by linking Equations (3) and (4) as follows:

$$\text{Spread} = \alpha^{(1/\rho)} \rho / \beta * (1 - \exp(-\beta/\rho)) \approx \alpha^{(1/\rho)} * ((1 - \exp(-\beta)) / \beta)^{(1/\rho)} = EL^{(1/\rho)} \dots (5)$$

$$\text{Spread} = EL^{(1/\rho)}$$

We can verify the second approximation by Taylor expansion but it is easier to simply tabulate the percentage differences involved for the ranges of values of the underlying parameters we are likely to see in fitting ILS prices. The table below shows such a comparison.

% Diffs	Exponential Decay rate β						
	RAL ρ	0.00	0.50	0.75	1.00	1.50	2.00
1.4	0.00%	0.21%	0.47%	0.84%	1.85%	3.22%	4.88%
1.5	0.00%	0.23%	0.52%	0.91%	2.02%	3.52%	5.36%
1.6	0.00%	0.24%	0.55%	0.97%	2.14%	3.73%	5.69%
1.7	0.00%	0.25%	0.56%	1.00%	2.22%	3.87%	5.91%
1.8	0.00%	0.26%	0.58%	1.02%	2.26%	3.95%	6.04%
1.9	0.00%	0.26%	0.58%	1.03%	2.29%	4.00%	6.12%
2	0.00%	0.26%	0.58%	1.03%	2.30%	4.02%	6.16%

The table shows the percentage differences between the exact spread values calculated using Equation (4) and the approximate values according to Equation (5) for a range of values for both β and ρ . Note that excess layers typically have decay rates β of around 1 to 1.5, so the spread formula given by Equation (5) is accurate to around 1% to 2%.

This simple expression linking Spreads to Expected Losses enables us to estimate both the implicit ρ of issued ILS as well as provide an almost trivially simple but powerful formula for pricing future ILS. Simply take the Expected Loss and raise this to a power of 0.6 (1/1.65). For softer prices try 0.65 (1/1.55) and for harder prices try 0.57 (1/1.75).

We can turn this formula into its Cobb-Douglas form by observing that Expected Loss can be expressed as the product of PFL and CEL:

$$EL = PFL * CEL$$

So we now have,

$$\mathbf{Spread = EL^{(1/\rho)} = PFL^{(1/\rho)} * CEL^{(1/\rho)}}$$

The ILS pricing formula can now be seen as a special, single parameter, form of the Cobb-Douglas function. It was, however, derived from theory rather than empirical observation. Moreover, the formula identifies that the single exponent is the inverse of the Wang PH Transform RAL ρ and also demonstrates that the spread formula can be expressed as a function of frequency (PFL) and severity (CEL).

The implicit RAL ρ for all the known ILS's calculated using the three parameter exponential power curve, the two parameter exponential decay curve and the spread formula applied to just the Expected Losses for these bonds are also shown in Tables 1 to 4 in Appendix 1. These comparisons show that ρ 's estimated by the simple formula are slightly lower than the estimates obtained by using closer fitting shapes to the survival functions but these differences are generally less than 0.5%, quite insignificant in the context of the differences between expected losses and spreads.

There is, of course nothing to stop us using the actual derived survival function and numerical or curve fitting techniques to obtain more accurate estimates of these spreads. In practice this is unlikely to produce answers that differ from the ones estimated by the simple formula by more than a one or two percent. These differences will be dwarfed by the impact of the margins inherent in the choice of the appropriate index value ρ to be used in the final calculation. The example in Appendix 3 tabulates such differences for a recent ILS issue.

There is just one final hurdle to overcome, however, and that is to identify the appropriate pricing level, ρ , to apply. The whole question of what ρ represents and how appropriate values for it should be chosen is discussed in the next section.

6 DETERMINING ρ FOR CATASTROPHE BONDS

The final market price for any financial transaction is ultimately determined by market forces and only rarely by a standard pricing formula. The existence of a soundly based, easy to explain and understand, formula may however help to ensure that prices are set rationally so that the market has both willing buyers and sellers. This has not been the case with ILS, primarily because of lack of issuers who are put off by the high spreads being demanded. Investors argue that this is a new class that is illiquid and carries significant uncertainties.

As these bonds have a relatively short duration and are dependent on the occurrence of natural catastrophe events rather than market movements, it is unlikely that much activity will happen post issue although some trading may well occur immediately after an event has occurred. This should not prove such a big hurdle if there was a lively and expanding market of new or repeat issues. The high spreads are the biggest hurdle and the main reason discouraging companies from issuing these bonds as part of their catastrophe management programmes. We need to understand why these spreads are so high and the derivation of the pricing basis provides the framework for this investigation.

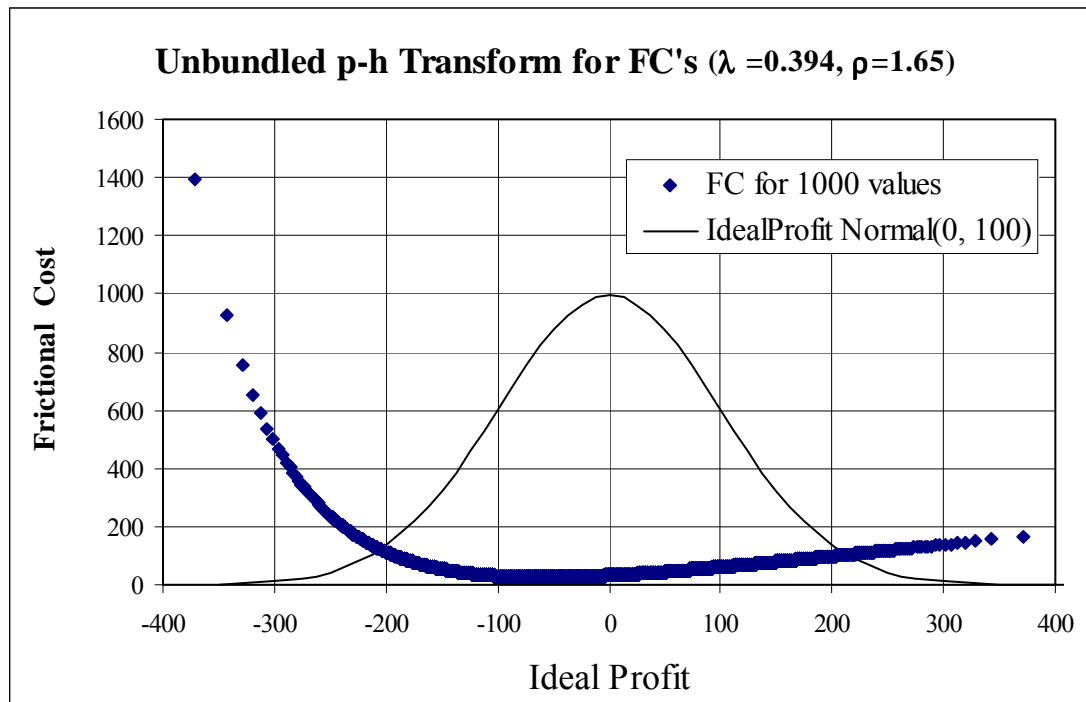
The simple formula for pricing ILS, $\text{Spread} = \text{EL}^{(1/\rho)}$ has just the single parameter to identify, so the search can be focussed on this parameter.

We have seen from the analysis and results shown in Appendix 1 that the ILS's issued to date have values of ρ centred on $\rho=1.65$. The ρ values implicit in a number of different types of simple financial transactions were discussed in Christofides S. (1998). In the case of a game of roulette (European Rules) he identified an implicit ρ of 1.00765. This compared to the range of ρ from 1.071 to 1.077, implicit in the UK Lottery prize allocations at the beginning of 1998. He also estimated a value of $\rho=1.77$, for the element with capital exposed, of the first issue of the Residential Re 1997 ILS. With hindsight, these transactions can be seen to be free of systematic risk so the use of the Wang PH transform is still valid. These results also demonstrate that some investors (casino owners) can work with prices set using values of ρ that are only marginally above its minimum value ($\rho=1$) and still make very good returns.

In order to progress this discussion we need to return to the complete risk evaluation and pricing framework outlined in Section 3 which covers both systematic and non-systematic risk, rather than attempt this in the narrow and rare instances where there is no systematic risk. In the complete framework we are looking at the Frictional Costs index λ , rather than the RAL ρ , but we saw that these indices are related by $\lambda = 1 - 1/\rho$.

In a DFA (Dynamic Financial Analysis) context, the FC index λ determines the level and shape of the frictional (or operational risk) cost distribution. These costs are an estimate of the 'unplanned or unforeseen costs' or costs that are left out of the normal plan projections. These FC distributions cover both the normal or attritional costs that risk management attempts to mitigate, but can never totally eliminate, the rare but extreme insolvency costs that a company may face from time to time and which may be big enough to lead to serious financial distress and ruin, as well as the upside costs that may be incurred following unexpectedly good results.

Although this is beyond the scope of this paper it may be helpful to illustrate the underlying characteristics of the 'unbundled p-h transform' form for the Frictional Costs distribution. The following chart shows the frictional costs associated with an underlying Normal distribution at the average index level of ILS, that is $\lambda=0.394$ or $\rho=1.65$. The underlying 'ideal profit' probability distribution function is also shown (on a different scale) to help visualise the relationship between this ideal profit and the contingent, associated, frictional costs.



The left side of the above chart shows the impact of the rare but extreme costs that would play a significant role in any risk based capital evaluation. The mean value of these frictional costs impacts value (rather than risk capital) and this average will be the same in the absence of systematic risk as the value calculated by the Wang PH transform. Some sign changes are needed to deal with losses rather than profits and we also need to move the index basis from λ to ρ but the values are then identical.

In a full DFA study we could, in theory at least, identify the λ embedded in the company's market value by comparing the future profit adjustments implicit in the market valuation with the company's own (ideal) profit expectations. This process turns profits projected in the internal plans, to a realistic or market basis.

In this context it is then helpful to look on the λ as a 'plan or management quality' index. When $\lambda=0$, the market believes that projected profits will be achieved and has no need to allow for frictional costs. This is often the view of management. Even where there is acceptance that 'operational risk costs' will invariably occur and past history is used to estimate and allow for the likely costs, such allowances will generally excluded the unseen extreme, insolvency, costs that only happen to others. These appear on the left-hand side of the chart. Interestingly, these rare but costly events account for approximately 50% of the overall frictional costs as defined by this family of functions.

Turning all of this to consideration of each of the Special Purpose Vehicles (SPV) set up for each of these securities, we see a company with a very simple structure and a very simple plan, so that specifying a DFA type model is almost trivial. All that is required is included in the issue prospectus and the only critical or variable elements are the loss parameters. Short of outright fraud, there is little operational management involvement that will influence any future outcomes. The only real uncertainty is with regard to the occurrence (or not) of a covered catastrophic event, its cost and possibly the timing of such an event.

This leads to the conclusion that in the case of an SPV, set-up especially for the issue of a catastrophe bond, the frictional costs are almost totally accounted for by the uncertainties in the catastrophe loss modelling. It is well known that different catastrophe models produce estimates of exposure, at any given probability or return period, that can vary by large percentages, particularly at the extreme return periods where securitisations are likely to be involved. This association of ILS risk pricing to model and parameter uncertainties is not new and is also discussed by Wang (2002) in justifying the two-factor transform. Uncertainty in the quality of these estimates is also one of the main reasons given for investors demanding such high spreads.

These uncertainties are statistical in nature and they arise, primarily, from the relatively small number of actual (extreme) events where good loss data is available to test and to calibrate the underlying catastrophe models. The accuracy of these models will improve over time but such improvements are almost certainly going to be slow and limited. In order to make any progress with the pricing of these securities, we need to accept the uncertain nature of these estimates and concentrate efforts in identifying the levels of this uncertainty so that investors and issuers can feel comfortable that what is being transacted is being priced 'fairly'.

So, in order to identify the corresponding risk index, λ or ρ , for ILS pricing purposes we need to estimate the error distributions of the catastrophe models loss estimates. Such a process has to take into account both the quality and amount of actual historic data on which the underlying catastrophe model is based and parameterised and will thus also depend on the peril covered, including its location. This sort of information should become an integral part of the catastrophe modelling and be made available to investors.

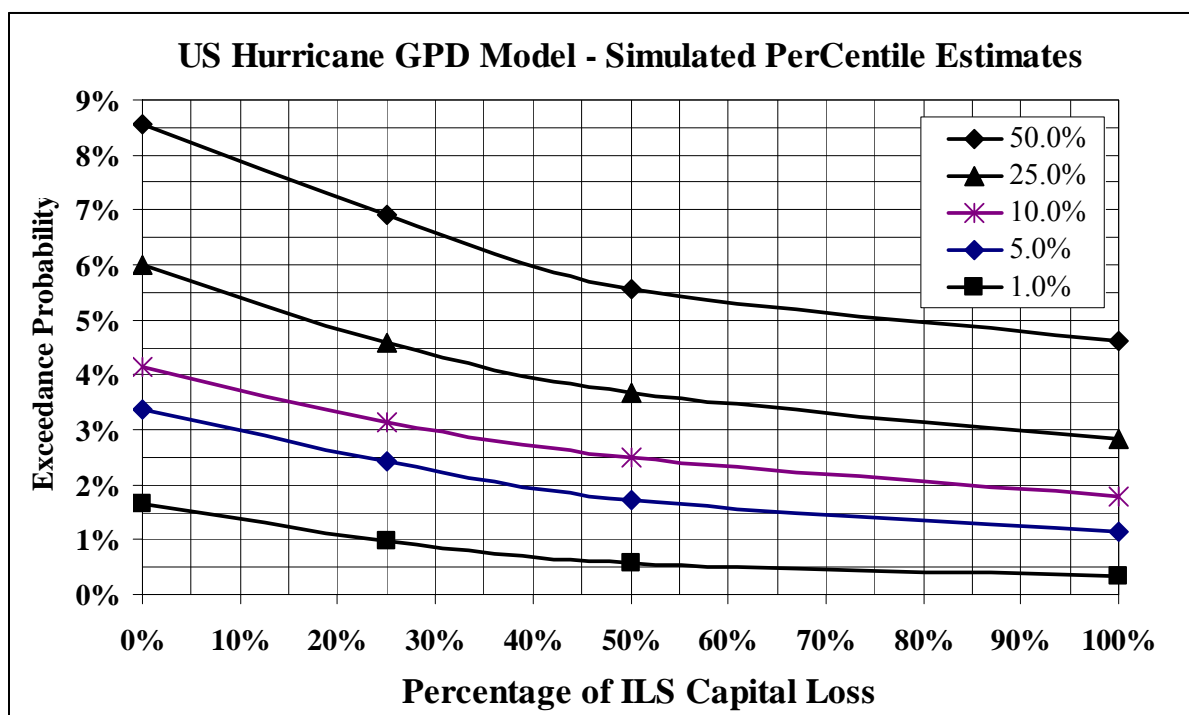
In order to demonstrate what such a process may involve and what new information it can provide to investors and issuers, we will outline an example using data on US Hurricane losses. We take the base data from Table A3.2 in Sanders D.A. et al (2002) which tabulates the 30 Costliest US Hurricanes of the last century, all of which had estimated costs of over \$1 billion (2000 US\$). These events include both Hurricane Hugo at \$10.9 billion and Hurricane Andrew at \$34.3 billion.

Looking at annual frequency first, the data shows a possible upward shift over the last forty years compared to the earlier periods, with 22 events out of 30 during this period. Over the later period, both the Poisson and Negative Binomial distributions provide a reasonable fit to the data. The Poisson is easier to deal with and we estimate the annual frequency as a random variable using Markov Chain Monte Carlo (MCMC) simulation. A reasonable fit is obtained with a the Poisson parameter given as Normal variable with mean of 57% and standard deviation of 12%, or $N(0.57, 0.12)$. Note this mean is slightly higher than the sample data mean of 55%.

For the severity distribution we have information only on a small number of extreme events, in excess of a very high threshold (\$1 billion). It is reasonable then to assume that the Picklands, Balkema-De Haan theorem applies and the distribution of the cost of these events can be described reasonably well by the Generalised Pareto Distribution (GPD). The usual Mean Excess plot, as shown in Appendix 4, supports this.

The parameters of the underlying GPD can also only be estimated as distributions rather than fixed and we need to do this before we estimate the likely error distributions around the key ILS trigger amounts. We do this by starting off with an assumed 'true' distribution and then draw many random samples from this 'true' distribution and estimate the parameters for each of these samples by Maximum Likelihood (ML). Each of these samples can then be combined with a frequency drawn randomly from the variable parameter Poisson described earlier and the probabilities associated with each trigger point of interest calculated for each such pairing. The assumed 'true' GPD distribution parameters are obtained from the actual data using ML. This process then estimates the frequency distribution at each of the trigger amounts. This process could be refined and improved by implementing it fully as a Bayesian MCMC simulation model of a compound Poisson-GPD process with all parameters as distributions derived from the data. The simpler approach is considered sufficient for now.

Typical results for a catastrophe layer are shown in the following chart. The chart shows, for example, that the median result (50th percentile) has a PFL of 8.6% and a PE of 4.6%, as shown by the top line. This represents the assumed 'true' expectations. The bottom line shows the estimates at the 1st percentile of these results and is at the near extreme of the possible range of results, given the assumed generating processes. If this is what is actually observed, or estimated by the catastrophe model as the likely (future) experience, then the calibration period has been an extremely light loss period, for hurricanes.



The first line (1st percentile) represents the investors' almost worst nightmare. The modelling results are extremely optimistic and the likely outcomes are almost certainly going to be much worse than these predictions. The process used to develop these estimates now facilitates further evaluation of what such a scenario warrants, in terms of risk costs.

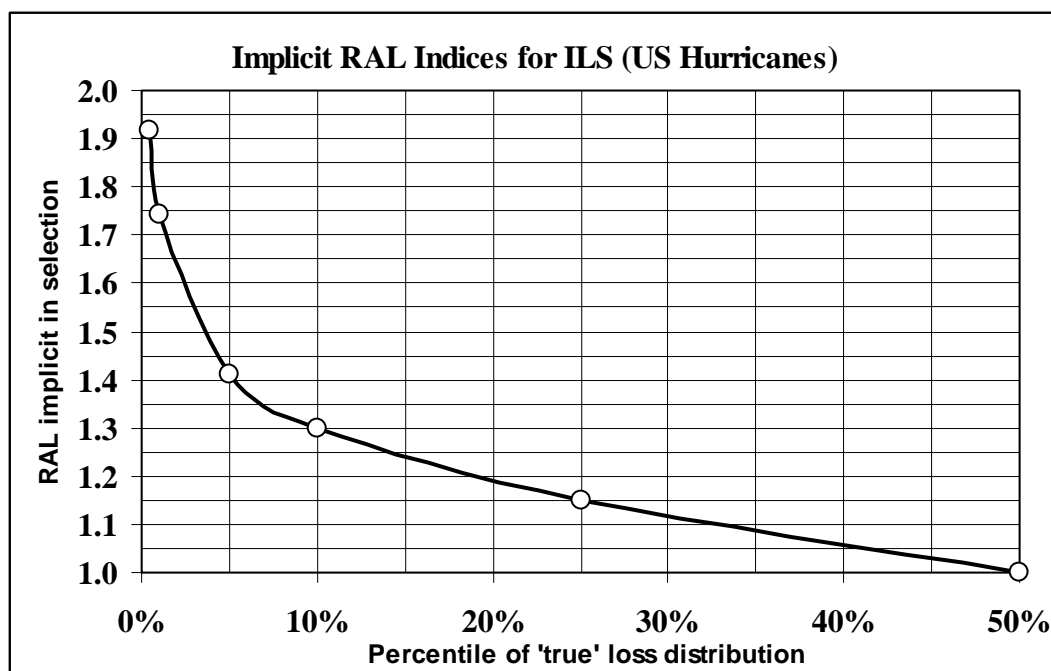
The next stage is to estimate the implied losses for each of these scenarios. This is done using the exponential form for these curves and Equation (3). The mean loss at the 50th percentile is 6.37% and at the 1st percentile it is 0.82%. The example securitisation limits were actually chosen to ensure that the expected loss at the 50th percentile was close to the typical spread of the issued ILS.

Taking the 50th percentile loss value as representing the 'fair' price, the RAL ρ implicit by each percentile distribution can now be estimated by the spread formula derived in Section 5. For example, the implicit ρ at the 1st percentile of the expected loss distribution is defined by the equation: $6.37\% = 0.82\%^{(1/\rho)}$. This produces a ρ value of 1.743, which is close to the current levels shown in Appendix 1.

The results for this and other percentile assumptions are given in the table below.

Item	Percentile of the re-sampled distributions					
	0.5%	1.0%	5.0%	10.0%	25.0%	50.0%
PFL	1.10%	1.66%	3.37%	4.13%	6.00%	8.57%
PE	0.18%	0.33%	1.15%	1.80%	2.84%	4.59%
Expected Loss	0.51%	0.82%	2.06%	2.81%	4.22%	6.37%
Multiple	12.5	7.7	3.1	2.3	1.5	1.0
Implicit value ρ	1.917	1.743	1.410	1.298	1.149	1.000

There is a shape to these results as can be seen from the following chart:



The above chart clearly illustrates the extremely cautious approach that is implicit in current ILS pricing. These results indicate that current spreads are centred on an assumption that catastrophe model estimates are around the 1st percentile level of their potential 'true' loss distributions. In other words, the current ILS pricing levels appear to almost totally disregard the results of the catastrophe modelling.

This observation still leaves the question of an appropriate choice for the pricing parameter, or the equivalent percentile of the underlying loss distribution, an open one. Moving from the 1st percentile to the 5th percentile is still a very cautious level for pricing and the results above indicate that this reduces ρ to around 1.41 from the current average of around 1.65. In pricing terms this will equate to a 38% decrease in spreads (based on EL= 0.9%). Moving to the lower quartile or 25th percentile level may be a more realistic choice and the ρ for such a choice is now down to 1.15, which would reduce spreads by as much as 70%, whilst still leave them some 50% above expected losses.

Securitisations which are triggered by event parameters, should have lower associated levels of uncertainty as the damage amounts will then be determined by a formula rather than actual damage costs. Such issues will then attract lower ρ 's and spreads compared to non-parametric securitisations with the same expected loss estimates. These lower spreads would then represent a discount to the issuer for retaining the basis risk associated with parametric triggers.

7 SUMMARY AND CONCLUSION

The financial markets have an almost infinite capacity to absorb and manage risk including catastrophe risk. Even the costliest catastrophe losses can look almost insignificant in relation to the daily movements in the global financial markets. The catastrophe reinsurance market on the other hand has to deal with both the aggregations of these risks as well as the maintenance of sufficient risk capital to enable it to trade in this business. This is clearly not an efficient business model and the natural place for catastrophe risk management has to lie with the capital markets in the medium to longer term. Securitisation, in the form of catastrophe bonds is the most obvious, viable means to achieving this goal.

The current spreads on these securities are very high in relation to what the investment markets normally enjoy for securities of similar loss characteristics. Insurers, as issuers have been very reluctant to enter this market as the required spreads and associated issue expenses are prohibitively high.

Our results confirm that current catastrophe bond spreads are grossly conservative. A reduction in ρ from 1.65 to 1.33 or a move from pricing based on 1st or 2nd percentiles to pricing using something nearer the 7th or 8th percentile levels of the underlying loss distribution would reduce spreads by 50%. These spreads will still be over 3 times expected losses and should still be very attractive to investors. Moving to more realistic assumptions about the choice of this implicit pricing percentile to, say, the lower quartile level or 25th percentile, reduces ρ to 1.15 and would result in a 70% reduction in current spreads. Even after such reductions these spreads will still be some 85% above expected losses.

In practice, it is unlikely that such a dramatic change to pricing levels will happen overnight. It is reasonable to expect, however, that a better understanding of this class and its associated risks can help to reduce these spreads to a more rational level, perhaps down to around 50% of current values.

Reductions in the current spreads of such proportions will provide property insurers with a significant incentive to issue such bonds as a very cost effective alternative to traditional catastrophe protections, at least for the top layers of their reinsurance programs. This may prove to be a significant threat to the specialist catastrophe reinsurers who will be faced with seeing their business transfer to the capital markets rather than see the two markets converge.

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APPENDIX 1

Summary Tables of Results by Issue Period *

Table 1: - to March 2000								
SPV	Amount (US \$m)	Annual Spread	Annual EL	PFL	PE	RAL Power	RAL Exp	RAL Simple
1	2	3	4	5	6	7	8	9
Mosaic 2A	24.3	4.08%	0.42%	1.15%	0.04%	1.760	1.743	1.711
Mosaic 2B	20.0	8.36%	2.84%	5.25%	1.15%	1.447	1.445	1.435
Halyard Re	17.0	4.56%	0.63%	0.84%	0.45%	1.643	1.643	1.641
Domestic Re	80.0	3.74%	0.50%	0.58%	0.44%	1.646	1.613	1.612
Concentric Re	100.0	3.14%	0.42%	0.64%	0.00%	1.616	1.585	1.581
Juno Re	80.0	4.26%	0.45%	0.60%	0.33%	1.714	1.714	1.712
Residential Re	200.0	3.71%	0.44%	0.78%	0.26%	1.653	1.655	1.647
Kelvin 1 st E	21.6	10.97%	4.45%	12.10%	0.50%	1.456	1.441	1.408
Kelvin 2 nd E	23.0	4.82%	0.30%	1.58%	0.07%	1.969	2.057	1.916
Golden Eagle B	50.0	2.99%	0.17%	0.17%	0.17%	1.817	1.817	1.817
Golden Eagle A	126.8	5.48%	0.63%	0.78%	0.49%	1.746	1.746	1.745
Namazu Re	100.0	4.56%	0.75%	1.00%	0.32%	1.590	1.586	1.585
Atlas Re A	70.0	2.74%	0.11%	0.19%	0.05%	1.903	1.902	1.894
Atlas Re B	30.0	3.75%	0.23%	0.29%	0.19%	1.851	1.851	1.850
Atlas Re C	100.0	14.19%	3.24%	5.47%	1.90%	1.766	1.769	1.756
Seismic Ltd	145.5	4.56%	0.73%	1.13%	0.47%	1.597	1.598	1.593
Total or WgtAv	1188.2	5.11%	0.84%	1.43%	0.44%	1.689	1.685	1.677

Table 2: Apr2000-Mar2001								
SPV	Amount (US \$m)	Annual Spread	Annual EL	PFL	PE	RAL Power	RAL Exp	RAL Simple
1	2	3	4	5	6	7	8	9
Alpha Wind FRN	52.2	4.62%	0.63%	0.99%	0.38%	1.653	1.653	1.648
Alpha Wind Prefs	37.5	7.10%	1.46%	2.08%	0.99%	1.601	1.601	1.598
Residential Re	200.0	4.16%	0.54%	0.95%	0.31%	1.648	1.650	1.642
NeHi	41.5	4.16%	0.70%	0.87%	0.56%	1.561	1.561	1.560
MedRe Class A	41.0	2.64%	0.22%	0.28%	0.17%	1.685	1.685	1.684
MedRe Class B	88.0	5.93%	1.16%	1.47%	0.93%	1.579	1.579	1.578
PRIME Hurricane	159.0	6.59%	1.27%	1.46%	1.08%	1.606	1.606	1.605
PRIME EQEW	129.0	7.60%	1.33%	1.69%	1.07%	1.678	1.678	1.676
Western Capital	97.0	5.17%	0.55%	0.82%	0.34%	1.761	1.761	1.756
Halyard Re	17.0	5.58%	0.22%	0.84%	0.04%	2.184	2.213	2.120
Gold Eagle 2001	116.4	5.58%	0.75%	1.18%	n/a	1.701	1.701	1.695
SR Wind CIA-1	58.2	5.83%	0.68%	1.07%	0.44%	1.761	1.762	1.756
SR Wind CI A-2	58.2	5.32%	0.76%	1.13%	0.53%	1.666	1.667	1.663
NeHi	8.5	4.56%	0.93%	1.00%	0.87%	1.515	1.515	1.515
Gold Eagle 2001	3.6	7.10%	1.18%	1.18%	1.18%	1.678	1.678	1.678
SR Wind CI B-1	1.8	7.10%	1.07%	1.07%	1.07%	1.715	1.715	1.715
SR Wind CI B-2	1.8	6.59%	1.13%	1.13%	1.13%	1.648	1.648	1.648
Total or WgtAv	1110.7	5.54%	0.86%	1.19%	0.58%	1.666	1.667	1.662

*All ILS Data from Lane (2000, 2001, 2002) - <http://www.lanefinancialllc.com>

SPV	Amount (US \$m)	Annual Spread	Annual EL	PFL	PE	RAL Power	RAL Exp	RAL Simple
1	2	3	4	5	6	7	8	9
Atlas Re II Class A	50.0	2.41%	0.050%	0.07%	0.03%	2.044	2.043	2.040
Atlas Re II Class B	100.0	6.84%	0.900%	1.33%	0.53%	1.762	1.761	1.756
Redwood Capital I	160.0	5.58%	0.530%	0.72%	0.34%	1.819	1.818	1.816
Redwood Capital II	194.0	3.04%	0.220%	0.31%	0.14%	1.755	1.754	1.752
Residential Re 2001	150.0	5.06%	0.680%	1.12%	0.41%	1.678	1.679	1.673
St. Agatha Re	33.0	6.84%	1.140%	1.55%	0.87%	1.670	1.671	1.668
Trinom Class A-1	60.0	8.11%	1.110%	2.42%	0.39%	1.815	1.815	1.792
Trinom Class A-2 (Pre)	97.0	4.06%	0.670%	1.01%	0.43%	1.566	1.566	1.562
Trinom Class A-2 (Post)	97.0	16.22%	0.670%	1.01%	0.43%	2.762	2.763	2.752
Redwood Capital I	5.0	7.10%	0.720%	0.72%	0.72%	1.865	1.865	1.865
Trinom (Pre)	4.9	10.14%	3.110%	3.11%	3.11%	1.516	1.516	1.516
Trinom (Post)	4.9	18.25%	3.110%	3.11%	3.11%	2.040	2.040	2.040
Total or WgtAv	955.8	6.17%	0.62%	0.95%	0.38%	1.854	1.854	1.848
Total Excl Trinom (Post)	858.8	5.04%	0.61%	0.94%	0.38%	1.752	1.751	1.746

SPV	Amount (US \$m)	Annual Spread	Annual EL	PFL	PE	RAL Power	RAL Exp	RAL Simple
1	2	3	4	5	6	7	8	9
Fujiyama	67.9	4.06%	0.670%	0.88%	0.42%	1.565	1.564	1.562
Pioneer A Jun-02	85.0	6.08%	1.280%	1.59%	0.97%	1.558	1.558	1.556
Pioneer A Dec-02	8.5	5.32%	1.280%	1.59%	0.97%	1.487	1.487	1.486
Pioneer A Mar-03	6.5	5.58%	1.280%	1.59%	0.97%	1.511	1.511	1.510
Pioneer B Jun-02	50.0	5.07%	1.270%	1.59%	1.05%	1.465	1.465	1.464
Pioneer B Sep-02	5.0	5.32%	1.270%	1.59%	1.05%	1.489	1.489	1.488
Pioneer B Dec-02	21.0	5.32%	1.270%	1.59%	1.05%	1.489	1.489	1.488
Pioneer B Mar-03	8.0	4.82%	1.270%	1.59%	1.05%	1.440	1.441	1.440
Pioneer C Jun-02	30.0	6.08%	1.280%	1.59%	0.98%	1.558	1.558	1.556
Pioneer C Sep-02	20.5	6.08%	1.280%	1.59%	0.98%	1.558	1.558	1.556
Pioneer C Dec-02	15.7	6.08%	1.280%	1.59%	0.98%	1.558	1.558	1.556
Pioneer C Mar-03	6.5	6.08%	1.280%	1.59%	0.98%	1.558	1.558	1.556
Pioneer D Jun-02	40.0	1.77%	0.220%	0.27%	0.186%	1.517	1.517	1.517
Pioneer D Sep-02	1.8	1.77%	0.220%	0.27%	0.186%	1.517	1.517	1.517
Pioneer D Dec-02	25.5	1.77%	0.220%	0.27%	0.186%	1.517	1.517	1.517
Pioneer D Mar-03	5.5	1.77%	0.220%	0.27%	0.186%	1.517	1.517	1.517
Pioneer E Jun-02	25.0	4.31%	1.290%	1.59%	1.01%	1.384	1.384	1.384
Pioneer E Dec-02	30.6	4.82%	1.290%	1.59%	1.01%	1.436	1.435	1.435
Pioneer E Mar-03	8.0	4.82%	1.290%	1.59%	1.01%	1.436	1.435	1.435
Pioneer F Jun-02	25.0	7.60%	1.310%	1.60%	1.02%	1.684	1.683	1.682
Pioneer F Dec-02	3.0	7.60%	1.310%	1.60%	1.02%	1.684	1.683	1.682
Pioneer F Mar-03	8.1	7.60%	1.310%	1.60%	1.02%	1.684	1.683	1.682
Residential Re 2002	125.0	4.97%	0.670%	1.12%	0.40%	1.673	1.675	1.668
St. Agatha Re	33.0	6.84%	1.140%	1.55%	0.87%	1.670	1.671	1.668
Studio Re Ltd.	150.0	5.17%	0.650%	1.38%	0.22%	1.719	1.717	1.700
Studio Re Ltd. Shares	25.0	8.11%	1.710%	2.13%	1.38%	1.621	1.621	1.620
Fujiyama (Equity)	2.1	7.10%	2.060%	2.06%	2.06%	1.468	1.468	1.468
Total or WgtAv	832.2	5.15%	0.94%	1.32%	0.66%	1.592	1.592	1.587

*All ILS Data from Lane (2000, 2001, 2002) - <http://www.lanefinancialllc.com>

APPENDIX 2

Relative Pricing Level Charts by Issue Period

Chart 1: ILS to March 2000

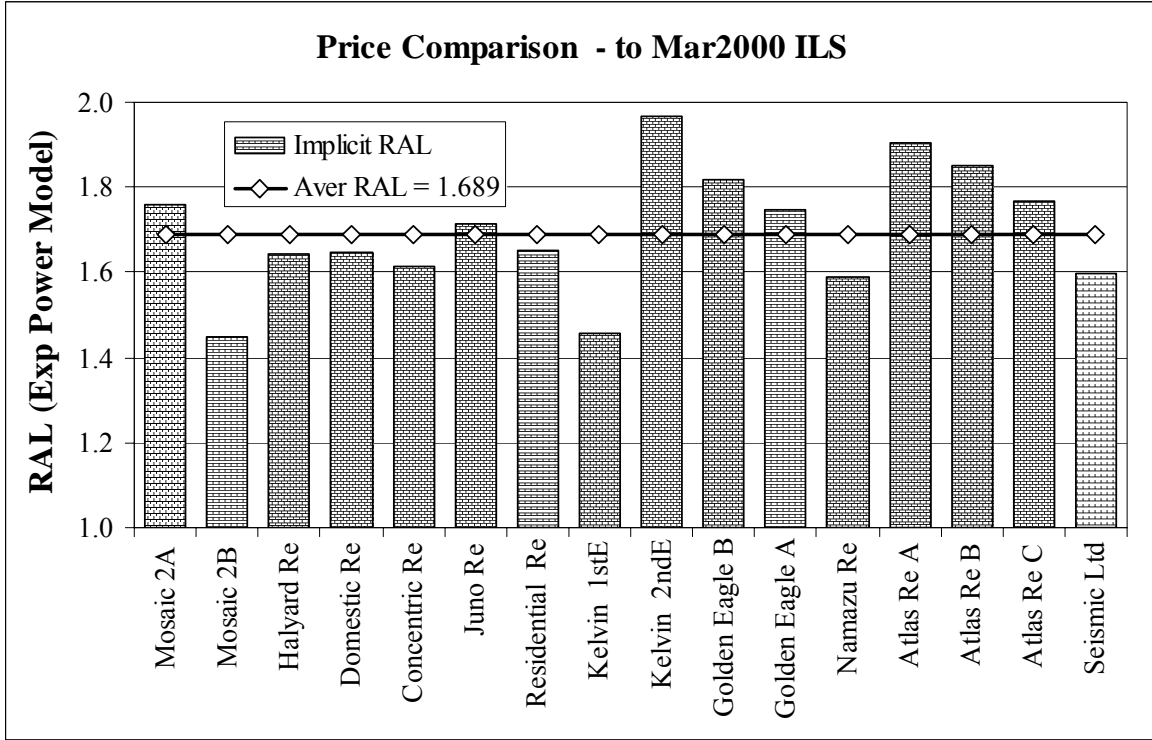


Chart 2: ILS Apr 2000 to Mar 2001

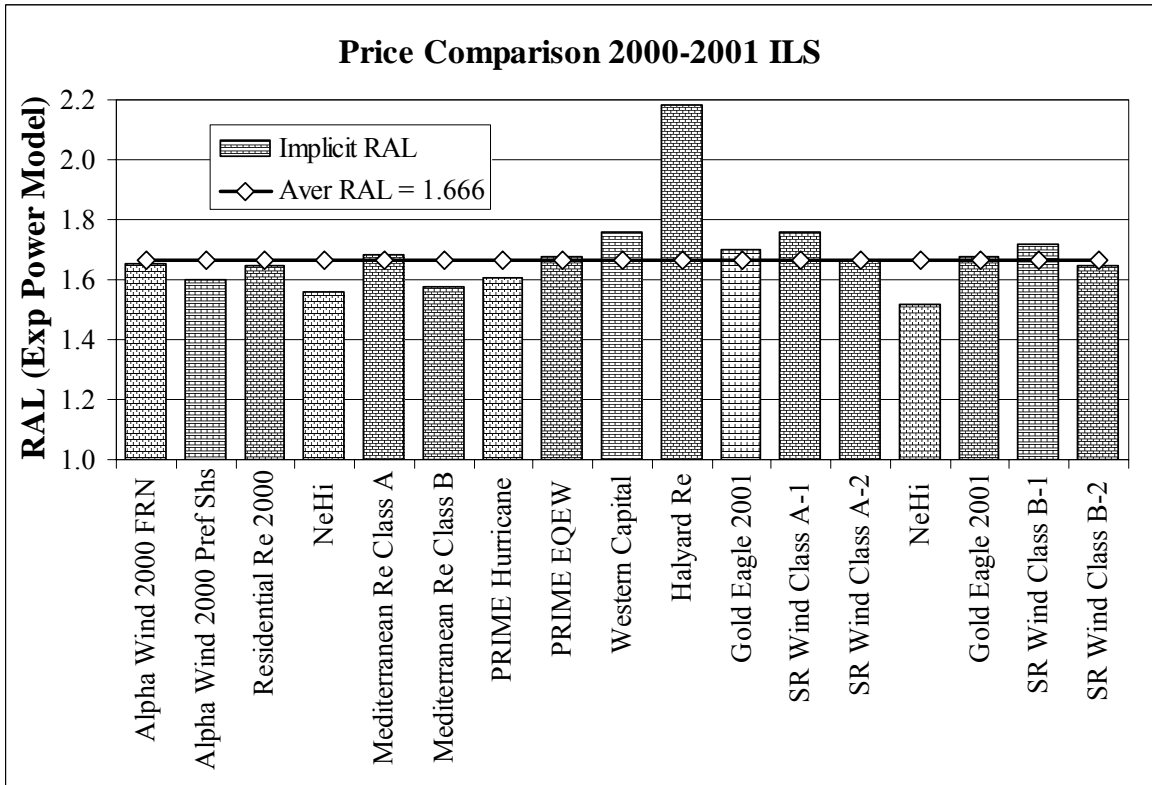


Chart 3: ILS Apr 2001 to Mar 2002

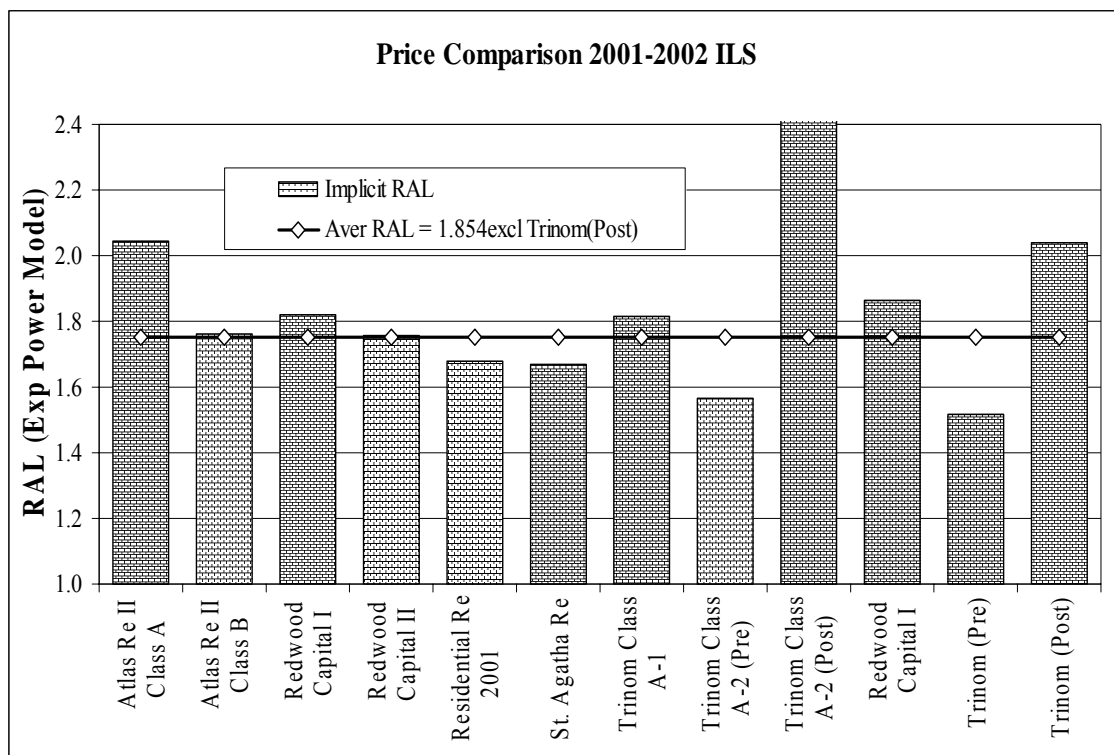
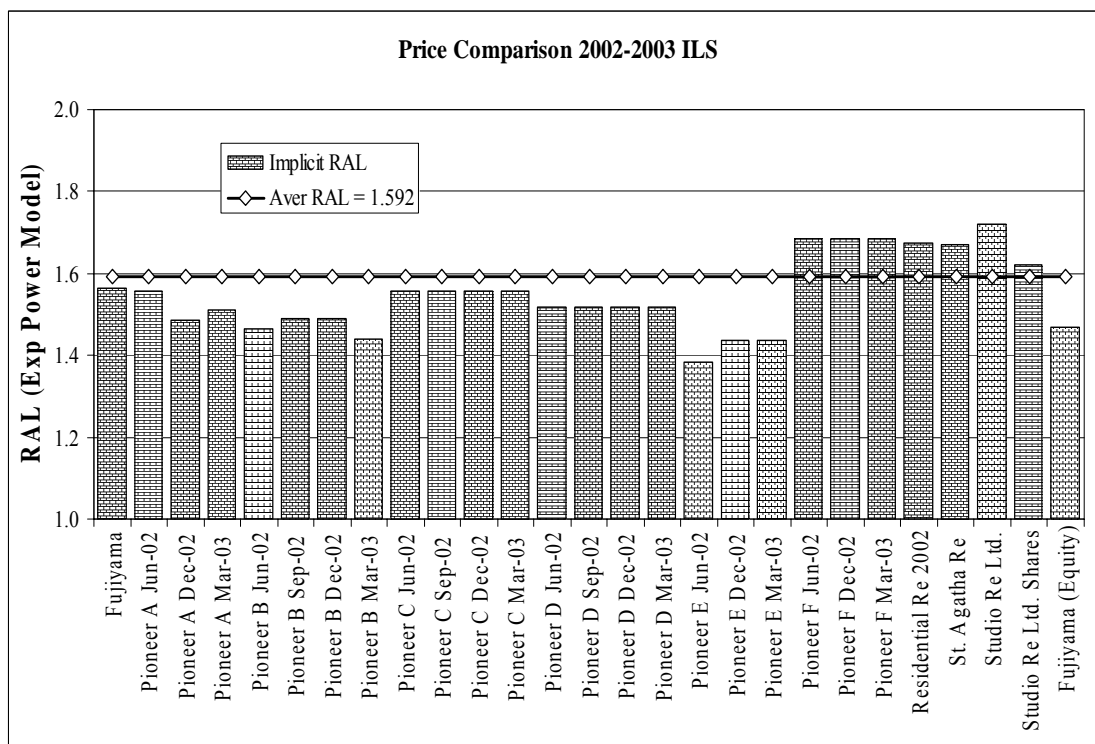


Chart 4: ILS Apr 2002 to Mar 2003



APPENDIX 3

Testing the accuracy of the Simple ILS Pricing Formula

The following table shows the Survival Function of a recent actual ILS derived from data published in the prospectus. The table also shows two fitted curves to this data, the first an exponential power curve and the second a quartic curve. These curves facilitate the numerical calculations of the spreads. Note both fit well, particularly the quartic polynomial.

% Lost x	Survival Function	Exp PowerFit	Quartic Fit
0.00%	1.10%	1.10%	1.10%
1.16%	1.05%	0.99%	1.06%
2.16%	1.00%	0.94%	1.03%
7.46%	0.90%	0.81%	0.89%
10.61%	0.80%	0.75%	0.82%
18.24%	0.70%	0.65%	0.68%
24.88%	0.60%	0.59%	0.59%
33.50%	0.50%	0.53%	0.50%
38.97%	0.45%	0.49%	0.46%
49.59%	0.40%	0.44%	0.41%
71.97%	0.35%	0.35%	0.34%
89.05%	0.30%	0.31%	0.30%
100.00%	0.28%	0.28%	0.28%

The spreads for this issue can now be estimated at various values of the Risk Aversion Level index ρ , by numerical integration and compared to those that are estimated by our relatively simple formula ILS pricing formula:

$$\text{Spread} = \text{EL}^{1/\rho} - \text{EL} \quad \text{some } \rho \geq 1 \quad (\text{currently around } 1.65 \pm 0.15)$$

The results, which are shown in the following table, indicate that the error in our simple formula, around the likely RAL met in practice, is no more than 2%. Note that the expected annual loss (EL) implicit in this survival function is 0.489% (Spread($\rho = 1$) = EL = 0.489%).

RAL ρ	Quartic Fit	Exp Power	EL ^{1/ρ}	% Diffs Quartic
1	0.489%	0.489%	0.489%	0.00%
1.1	0.79%	0.79%	0.79%	0.66%
1.2	1.17%	1.18%	1.19%	1.11%
1.3	1.65%	1.65%	1.67%	1.42%
1.4	2.20%	2.21%	2.24%	1.63%
1.5	2.83%	2.84%	2.88%	1.77%
1.6	3.53%	3.54%	3.59%	1.87%
1.7	4.29%	4.31%	4.37%	1.93%
1.8	5.10%	5.12%	5.20%	1.96%
1.9	5.96%	5.99%	6.08%	1.97%
2	6.85%	6.88%	6.99%	1.98%

This example supports the assertion that the simple pricing formula is more than adequate for pricing ILS's and for comparing price level differences between securities and over time. We note also that the simple formula actually over-estimates the spreads slightly, which is the preferable form of bias in these circumstances.

APPENDIX 4

Validating the use of Generalised Pareto Distribution

The following table shows the data used in the example taken from Sanders et al (2002).

Hurricane	Year	Cost (2000 \$b)	Hurricane	Year	Cost (2000 \$b)
Andrew	1992	34.3	Donna	1960	2.4
Hugo	1989	10.9	Iniki	1992	2.3
Agnes	1965	8.4	Celia	1970	2.1
Betsy	1965	8.4	Elena	1985	2
Camille	1969	6.8	Bob	1991	2
Dianne	1955	5.4	Hazel	1954	1.9
Frederic	1979	4.9	Miami	1926	1.7
New England	1938	4.7	Marilyn	1995	1.7
Floyd	1999	4.6	Galveston, Texas	1915	1.5
Fran	1996	3.6	Dora	1964	1.5
Opal	1995	3.5	Eloise	1975	1.5
Alicia	1983	3.4	Gloria	1985	1.5
Carol	1954	3.1	Puerto Rico	1928	1.3
Carla	1961	2.5	Northeast US	1944	1.2
Juan	1985	2.4	Beulah	1967	1.1

The Mean Exceedance Plot is shown below. The trend line fit confirms choice as reasonable.

