Asset-Liability Modeling for Insurers: Incorporating a Regime-Switching Process for Equity Returns into a Dynamic Financial Analysis Model

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Abstract

This paper discusses a framework for asset-liability modeling of property-liability insurers. In particular, the paper has a dual purpose: (1) to describe a *dynamic financial analysis* (DFA) framework for insurer analysis which combines an underwriting model reflecting pricing, regulatory, and catastrophe risk with an economic and investment performance module, and which incorporates the interrelationships among these various insurance and financial processes; and (2) to evaluate the impact on projected insurer risk of incorporating into this framework a regime-switching model for the equity return process. The results suggest that, compared with a more common and simpler regression-based model of equity markets, introducing a regime-switching approach within a DFA framework can better reflect the overall risks facing an insurer. Such a model can provide a stronger basis for asset-liability management, capital allocation, and general operational and strategic planning by insurer management, as well as for regulatory supervision of the insurance industry.

1. Introduction

Measuring the financial strength of property-liability (P-L) insurers can be a significant challenge. As the external economic environment changes, so does the value of a P-L insurer, due to a variety of possible influences[1]. Regulators look favorably upon an insurer whose financial strength can be positively demonstrated under a variety of possible future economic scenarios. Similarly, insurer management desires to protect the company's surplus from unusual or unnecessary fluctuations.

In order to evaluate the financial condition of an insurer, or to assess the future impact of management's alternative strategic decisions, a thorough understanding of the risk environment in which the insurer operates is required. This environment includes economic, financial, and regulatory components, as well as the condition of the insurance industry itself[2]. Since financial analysis typically involves a multi-year horizon, it is necessary, from a modeling standpoint, to represent a variety of financial and insurance processes mathematically over a period of several years. By modeling the potential future movements of these processes, the sensitivity of the company's value to those movements can be measured and assessed.

A number of papers have described approaches to modeling economic and insurance time series, for the purpose of evaluating possible future financial conditions of insurers[3]. One of the more difficult processes to model – for general investment purposes, not to mention insurance applications – is equity returns. Effective modeling of equity returns is critical for measuring insurer value, since a significant proportion of a P-L insurer's assets is invested in equities[4].

This paper has a dual purpose. First, we introduce a "dynamic financial analysis" (DFA) framework for analysis of P-L insurers. The overall goal of the DFA framework is to develop a thorough, representative, and robust process for modeling real-world financial variables, in order to provide a strong foundation for the simulation of future insurer financial conditions. Second, we evaluate the impact and test the significance of two alternative modeling assumptions for the equity return generating process. More specifically, we incorporate into an existing, public-access DFA framework a regime-switching model for equity returns. We find that the regime-switching approach may yield important information related to the uncertainty in an insurer's operations; other assumptions for equity returns (e.g., a simple linear regression relationship between equity returns and interest rate movements) may distort measures of risk, and may not adequately capture the risk profile of the insurer.

The remainder of this paper is structured as follows. Section 2 describes approaches for measuring insurer risk, given the uncertainty in the economy and the financial markets. Section 3 summarizes the dynamic financial analysis model used in this research and how key risks faced by insurers are each represented by the DFA model[5]. Section 4 discusses two alternative procedures used to model equity returns, including a simple linear relationship used in a public-

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[1] For example, changes in interest rates impact market values of assets and liabilities (and thus of insurer net worth), inflation affects future loss payout values, and economic and business cycles can impact industry competitiveness and profitability.

[2] The P-L insurance industry has an empirically well-established "underwriting cycle," which varies over time from "hard" (associated with relatively high premiums and profitability) to "soft" (associated with relatively low premiums and profitability).

[3] See, for example, Ahlgrim, D'Arcy, and Gorvett (1999), Browne and Hoyt (1995), and Browne, Carson, and Hoyt (2001).

[4] Common stock holdings accounted for approximately 16% of U. S. P-L insurers' invested assets as of the end of 2001 (according to A.M. Best Company and the Insurance Information Institute).

[5] While this model is intended for use by a P-L insurer with exposures in the United States, the general DFA framework and approach are also valid in a global setting, with appropriate adjustments.

access DFA model, and the regime-switching equity return model incorporated here. Stochastic simulation techniques are used to illustrate and compare these two alternative approaches. Section 5 presents a test of the two equity models in the context of projecting future insurer operations, focusing on the impact of the equity return models on the evaluation of insurer risk. Section 6 concludes.

2. Risk Analysis of Property-Liability Insurance Companies

Actuaries attempt to determine an insurance company's surplus[6] by projecting future experience using appropriate assumptions. Historically, there has been something of a dichotomy between the liability and asset sides of an insurer's balance sheet – or, said another way, between an insurer's underwriting and investment operations[7]. The integration of the two aspects of the insurance process – underwriting and investment – is a relatively recent, but critical, analytical development. Appropriate financial risk management – in particular, asset-liability management – depends upon a thorough understanding of the risks underlying both the investment and liability processes, and how those risks are correlated with each other.

The general framework for the risk analysis of insurers often takes the form of scenario testing or dynamic financial analysis. Scenario analysis measures the impact on an insurer's financial condition and performance of alternative assumptions regarding the economic and/or insurance environment. For example, different "scenarios" might include (1) changes in interest rates, (2) an increase in loss frequency, and/or (3) a decrease in policyholder persistency[8]. The hypothetical future financial results of the insurer are projected under each specified scenario. Depending on the level of detail desired, pro forma financial statements may be generated to understand how various items are individually affected.

Unfortunately, scenario analysis has a number of drawbacks. First, the number and form of scenarios are at the discretion of the modeler. This manual process may lead to internally inconsistent assumptions for a specific economic scenario. A second drawback is that scenarios are too dispersed to be of any use in decision-making. The fragmented scenarios do not provide an accurate depiction of the actual range of potential outcomes and, more importantly, the probability of those outcomes.

The static nature of scenario analysis has led to the development of dynamic financial analysis models of insurance. DFA is a modern, integrated approach to modeling operations of an insurer within a risky economic and insurance environment.
DFA investigates the financial condition of P-L insurers by evaluating the impact of *any* modeled economic or financial factor that might affect an insurer's underwriting or investment activities. It also accounts for the *correlations* among those factors[9], overcoming any

[6] "Surplus," or "policyholders' surplus," is the difference between assets and liabilities; technically, it is measured according to insurance statutory accounting principles, which tend to be somewhat more conservative than GAAP accounting. However, one can also refer to surplus more loosely as an economic, or "net worth," concept.

[7] Basically, when a P-L insurance policy is written, the insured pays a premium to the insurer. From this premium, the insurer pays expenses, and sets aside liabilities for future payments on claims arising from the exposure insured under the policy. Because there is a time lag – often significant, depending upon the type of insurance – between the receipt of premiums and the payout of claims, the investment of those dollars between receipt and payout is a critical insurance operating function, and is an essential component of the insurer's potential for profitability.

[8] "Persistency" refers to the proportion of policyholders who renew their insurance coverage.

[9] Some of the risk factors are interest rates, inflation, loss experience, investment performance (both fixed-income and equity returns), claim payment patterns, and tax effects. D'Arcy, Gorvett, et. al. (1997 and 1998) discuss many of the issues to consider when constructing a DFA model for use in the property-liability insurance industry.

inconsistencies associated with hand-constructed scenarios. This approach is implemented by means of stochastic simulation[10].

The major benefit of DFA is that it allows insurers (and regulators) to assess the likelihood of certain outcomes. Using DFA, an insurer can generate hundreds or thousands of stochastic simulations and project future financial results to evaluate the adequacy of surplus to protect against future uncertainty. The resulting distribution of future surplus can assist insurers in determining the amount of surplus that is necessary to protect policyholders against adverse circumstances. Insurance regulators may also use these projected distributions when promulgating capital requirements. For example, a regulator may require that the insurer set aside a level of capital such that at least 99% of the time, the insurer will remain solvent over the next year. This approach is similar to the value-at-risk (VaR) methodology that is often used in the banking industry. The insurer may also test the sensitivity of its performance to variations in operating strategies and make changes to its asset portfolio or underwriting strategy if financial performance is not acceptable.

This paper uses a DFA framework to measure insurer risk. In the following section, we describe this DFA framework, through reference to a specific, publicly-available model which the authors helped to design, build, and apply. Later, in Section 4, we go beyond the simple approach to modeling equity markets in this public-access model, and discuss an alternative equity modeling framework: regime-switching.

3. A Public-Access DFA Model

In developing a DFA model, there are two primary problems facing a property-liability insurer. First, insurance operations are affected by an almost overwhelming number of factors, many of which deserve considerable attention. Second, the proprietary nature of most existing models (and of the analyses underlying their parameterizations) limits the amount of information that has been shared and made publicly available regarding the modeling process. The model described in this paper has the long-term objective of addressing both of these problems.

In recognition of the fact that no model can successfully consider every potential source of risk, this model focuses on the *key* variables that affect the financial results and condition of a typical property-liability insurer. Addressing only the more important quantifiable financial risks to which property-liability insurers are exposed facilitates model comprehension and communication.

Our response to the second problem mentioned above is *public accessibility*: the model used in this research is based upon a public-access model available from Pinnacle Actuarial Resources[11]. at http://www.pinnacleactuaries.com/pages/services/dfa.asp . All assumptions, techniques and calculations are explained in enough detail (through accompanying software documentation and several published articles) that other researchers and practitioners, with an appropriate understanding of the basic concepts and issues, will be able to use the model. The publication of this public-access model and the underlying theory and documentation is meant to foster peer review and lead to improvements in the overall methodology. Thus, the model

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[10] Some authors use the term "DFA" generally, to cover both scenario analysis (or scenario testing) and stochastic simulation. We have chosen to use a narrower definition, one specifically involving a stochastic simulation framework.

[11] The model can be downloaded from the following website: http://www.pinnacleactuaries.com/pages/services/dfa.asp

provides a valuable learning tool for individuals wanting to understand DFA for property-liability insurers. It should also help the profession deal with the issue of developing standards of practice in this emerging and important area[12].

The public-access DFA model is a useful start to modeling an insurer's operations. The model is simple enough to be easily understood, yet sufficiently sophisticated to integrate the various aspects of insurer operations within a unified environment. In some cases, the assumptions of the public-access model are quite simple, reflecting the educational objectives of the model, and/or the lack of sufficient data to emulate the variable more precisely. One powerful aspect of the model is that it is simple and accessible enough to allow for changes in such assumptions by an informed user. In general, the model assumptions were not meant to replicate the intricacies of historical movements, but only serve to indicate the inherent randomness of potential outcomes.

The model simultaneously considers both sides of the insurer's balance sheet and contemplates many of the key risks that insurers face, including interest rate risk, investment risk, underwriting risk, catastrophe risk, and pricing risk. (Each of these risks is described later in this section). By encompassing all the insurer's operations in one model, internal consistency across different functional areas of the company is enhanced.

As mentioned above, each assumption of the DFA model has been chosen with an eye toward simplicity and ease of comprehension. However, the choice of a specific approach to modeling key financial variables may very well have an impact on projected results. For example, incorporating a model of interest rates seems an important step in evaluating insurer risk given the significant investment of insurers in fixed income securities. However, the degree to which alternative interest rate frameworks would impact the financial projections emerging from the DFA model is an open (but important and testable) question[13]. Likewise, catastrophe modeling is advancing rapidly, and we do not suggest that the public-access model's catastrophe module is the best. For this paper, we have generally used the default assumptions in the DFA model (described below), and have focused on testing one important piece of this model – the equity return generating process. The question we will seek to answer in Sections 4 and 5 is: what is the impact on an insurer's projected performance if a more realistic model of equity returns is employed in this DFA framework?

In this section, we discuss some of the key risks that need to be recognized by a property-liability insurance DFA model, and how those risks are incorporated into the public-access model. Many of these risks are common to other financial service companies, including life insurers; others are unique to the operations of property-liability insurers and the P-L insurance industry. Some of the specific risks faced by P-L insurers, several of which are highlighted and discussed below, include[14]:

[12] The public-access model and its development and application are documented in several articles: e.g., D'Arcy, Gorvett, et al (1997a, 1997b, and 1998).

[13] The results of Ahlgrim (2001) suggest that relatively simpler models may be adequate to capture a significant amount of the interest rate risk faced by many P-L insurers.

[14] It is important to keep in mind that these risks are not stand-alone items, but can be significantly correlated and interconnected. One of the key tasks associated with a dynamic financial analysis is the specification of these interrelationships – conceptually, the specification of a potentially huge correlation matrix.

Economic / Financial	Insurance / Underwriting
Interest rates	Loss frequency and severity
Inflation	Catastrophic losses
Fixed-income yields	Loss reserving and development
Equity returns	Loss payout patterns
Asset default rates	Underwriting cycle
Mortgage prepayment patterns	Aging phenomenon

Interest Rates

The primary driver of the public-access DFA model is the interest rate generator, as interest rates seem to be, both theoretically and empirically, correlated with so many other economic, financial, and insurance industry processes. Extensive work has been done in finance to develop sophisticated interest rate models[15]. In this DFA model, a relatively simple single-factor interest rate model is used, one derived by Cox, Ingersoll, and Ross (1985) (hereafter referred to as CIR). This simpler process was selected primarily to enhance understanding of the DFA model by actuaries; other interest rate frameworks can be incorporated into the DFA model as desired. The CIR model describes the short-term interest rate as a mean-reverting stochastic process, and a process whose volatility is related to the level of the process. In a continuous-time framework, the process *dr* for the instantaneous change in the level of the short-term risk-free interest rate is characterized by the equation

$$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}\,dz$$

where θ = the long-run mean to which the interest rate reverts,
κ = the speed of reversion of the interest rate to its long-run mean,
r = the current (instantaneous) short-term interest rate,
σ = the volatility of the interest rate process (as expressed by the standard deviation), and
dz = a standard Wiener process (essentially, a random walk).

For purposes of this DFA model, a discrete-time version of this model is required. According to Cox, Ingersoll, and Ross (1985), the short-term interest rate, in discrete-time, follows a (non-central) chi-square distribution with non-centrality parameters being a function of the κ, θ, and σ parameters above. For simplicity, in this DFA model we approximate the discrete-time form of the CIR model using the following formula:

$$\Delta r = a(b - r)\Delta t + s\sqrt{r}\,\varepsilon$$

where Δr = the discrete-time (annual) change in the short-term interest rate,
Δt = the discrete time interval (one year), and
ε = a random sampling from a standard normal distribution.

In this interest rate model, the current interest rate is the actual short-term interest rate in the economy at the time the model is run. As of mid-March, 2003, 3-month Treasury bills, a

[15] The interested reader is referred to Chan, et al (1992) and Hull (2000) for detailed descriptions of some of these models.

common proxy for short-term rates, were yielding 1.12%. Thus, in this model, *r(0)* is set to 1.12%. The long-run mean, *b*, is set at 5%. This is a variable that can, and should, be altered by the user to reflect individual views of interest rate movements, and to test the sensitivity of results to this variable. Once selected, the short-term interest rate is used to generate the term structure of interest rates. Based on the interest rate model parameters selected, and upon the simulated short-term interest rate, rates on zero-coupon Treasury bonds are generated for each annual duration up to thirty years. This Treasury term structure is used to determine the market value of the company's bond holdings. The specific equations used to generate the term structure are taken from Cox, Ingersoll, and Ross (1985):

$$R(r,t,T) = \frac{rB(t,T) - \ln A(t,T)}{T - t}$$

where *R* is the yield-to-maturity at time t on a discount bond that matures at time *T*, and

$$A(t,T) = \left[\frac{2\gamma e^{[(\kappa+\gamma)(T-t)]/2}}{(\kappa+\gamma)(e^{\gamma(T-t)} - 1) + 2\gamma}\right]^{2\kappa\theta/\sigma^2}$$

$$B(t,T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\kappa+\gamma)(e^{\gamma(T-t)} - 1) + 2\gamma}$$

$$\gamma \equiv (\kappa^2 + 2\sigma^2)^{1/2}$$

Investment Risks

In addition to (and possibly correlated with) interest rate movements, investment risk is an important source of uncertainty for insurers. Two key aspects of investment risk involve the potential for default of the issuer, and the performance of equity investments.

- ***Default:*** Fixed-income securities pose the risk of default on interest and/or principal. Default rates are a function of both the underlying security (in line with the ratings assigned to the debt) and economic conditions (more volatile interest rates engender a higher level of defaults). The model allows for the simulation of corporate bond defaults in accordance with default probabilities input by the user.
- ***Equity Returns:*** To the extent an insurer is invested in common stocks, the return on its asset portfolio is subject to the risk associated with future movements in the equities markets. The public-access DFA model utilizes a simple linear relationship (with a standard-normal stochastic error term) between interest rate movements and stock portfolio returns – the latter are simulated based upon the simulation of the former for a given projected future period. This relationship was parameterized by performing a one-factor linear regression between historical values of the two processes. This, along with the regime-switching alternative, is discussed further in Section 4.

Underwriting results

There are several factors involved in projecting an insurer's future loss experience, including the frequency and severity of policyholder losses, catastrophic losses, loss development, payout patterns, and the aging phenomenon.

- ***Non-catastrophe loss experience:*** The amount of losses which an insurer experiences is a two-dimensional risk: the *frequency* of claims per unit of exposure insured (which leads to the number of claims experienced), and the *severity* distribution of those claims that do occur (the size of each claim):

  $$Losses = \{Frequency\ per\ exposure\ unit \times Exposures\} \times Severity$$

  The public-access DFA model uses historical data from the insurer to simulate both the frequency (per unit of exposure) and the average severity (per claim) stochastically; total losses are then determined for each simulation by combining these two dimensions. The model allows these frequency and severity parameters, as well as the underlying distribution of each, to differ according to the type, or line, of insurance business written.
- ***Catastrophes:*** Property-liability companies face significant risk of a catastrophic incident since hurricanes, earthquakes, winter storms, and fires all have the potential to quickly affect the financial condition of an insurer[16]. In this model, catastrophes are handled as follows, for each simulated year:
  - (i) The number of catastrophes (by our definition, events of any type causing industry-wide insured losses in excess of $25 million) during the year is simulated based on a Poisson distribution, with the parameter based on historical experience.
  - (ii) Each catastrophe is assigned to a specific geographical area, or "focal point," again based on historical probabilities.
  - (iii) Once assigned to a focal point, the industry-wide size of each catastrophe is simulated, based on a lognormal distribution. The size of the event is correlated with the location, as both the type of loss and the amount of insured property exposed to a loss is a function of where the catastrophe occurs. The parameters of the lognormal distribution are based on historical industry experience, appropriately adjusted to future cost levels.
  - (iv) The geographical distribution of the loss from the event, by state, is determined based on a state-by-state frequency correlation matrix developed from historical patterns. We refer to this as a "contagion" effect, with damage from the event spreading out geographically from the focal-point state.
  - (v) The loss is allocated to the company based on the firm's market share in each affected state, for the lines of business exposed to catastrophic risk.
- ***Loss Reserving and Development:*** The starting value used for the loss reserve – the insurer's liability for future claim payment obligations on all losses that have occurred to-date – should be the value indicated by an actuarial analysis of the company's historical experience. However, due to the inherently stochastic nature of the insurance loss process, there is likely to be some ultimate variation relative to that best estimate. Another complication is the correlation between interest rates and loss reserve development and payout, since both are correlated with inflation. However, whereas the relationship between inflation and interest rates is well recognized and has been extensively documented[17], the relationship between inflation and loss development is

[16] In this analysis, we consider only natural catastrophes. The events of September 11, 2001, also indicate the potential significance of man-made catastrophes, which are extremely challenging to model.
[17] See, for example, Fama (1984 and 1990) and Fama and Bliss (1987).

much harder to quantify. Since loss reserving techniques traditionally assume that past inflation rates will continue, if there is any deviation from historical (or other forecasted) rates, then future loss payments may differ from the amount reserved. Using a normal distribution, the public-access DFA model allows for either favorable or adverse loss reserve development that is implied from historical patterns. The selected volatility parameter is also based on the company's size, and can incorporate line-of-business-specific considerations that affect the entire industry.

- ***Inflation:*** The inflation rate applicable for both non-catastrophic and catastrophe losses is determined after the interest rate has been simulated. In the public-access DFA model, the general rate of inflation is determined by looking at the historical relationship between short-term interest rates and inflation. In the model, inflation is calculated by multiplying the simulated risk-free interest rate by a regression coefficient, and adding in an independent stochastic (random walk) term. This approach recognizes the correlation between interest rates and inflation but still allows for variability around the standard inflation-interest rate relationship. Once chosen, the inflation rate affects loss experience on the current book of business, on policies to be written or renewed in the future, and the loss development patterns for current reserves. It also affects the indicated rate level changes for future years.
- ***Aging phenomenon:*** Woll (1987) and D'Arcy and Doherty (1989 and 1990) document the aging phenomenon in insurance. Experience shows that loss ratios gradually decline with the length of time the policies have been in force with the same insurer. The overall result is that new business should have the highest loss ratio, first renewal business should have a slightly lower loss ratio, and the remainder (second and subsequent renewals) should have the lowest loss ratio. Based on data published in D'Arcy and Doherty (1990), the loss ratio on new business ranged from 8 to 42 percentage points above the loss ratio on second and subsequent renewals. The public-access DFA model reflects the aging phenomenon by separately modeling the writings for each line of business into new business, first renewals, and then second and subsequent renewals.

Premium Levels (Pricing)

The risk associated with P-L insurance pricing is that, since most insurance premiums are set prior to the effective date of the policy, the insurer may incorrectly estimate future experience, causing the price to be either inadequate or excessive. In addition, the regulatory freedom of insurers to set premium levels varies by state, with some states allowing relatively unrestricted pricing and other states having extensive restrictions. Thus, there are several components to insurance pricing risk as addressed in the public-access DFA model: the risk inherent in the insurance loss process, underwriting risk, and jurisdictional risk.

- ***Loss process risk:*** As mentioned in the last sub-section, insurance losses (whether catastrophic or non-catastrophic) are a stochastic process. In general, from an actuarial standpoint, historical losses will form the basis for estimates of future losses[18]. Thus, in

[18] The specific actuarial techniques by which loss experience is analyzed to estimate premium requirements depend upon the line of business, type and size of insured, availability of data, and a variety of other factors. For example, a (generally large) insured might have a policy whose premium is determined on a *retrospective* basis, meaning that

our model, in order to represent the insurance pricing process, the emerging simulated (stochastic) loss experience during the projection period is used as the rolling basis for determining the subsequent rate changes which the modeled insurer will require. To the extent that the simulated losses reflect risk and uncertainty, and also to the extent that historical losses might not be fully representative of anticipated future loss experience, the pricing process for future exposures is risky and uncertain.

- ***Underwriting cycle:*** The premium level at which policies are written depends on the insurer's targeted growth rate and the position in the underwriting cycle. The property-liability insurance industry underwriting cycle has been the subject of extensive study and is recognized as being quite complex. In line with the goal of keeping this model as straightforward as possible, especially for this early version, the underwriting cycle is simplified. However, it still reflects the different relationships of growth rates and price levels depending on the position of the cycle. In this model, the underwriting cycle, which can vary by line of business, is characterized as being in one of four conditions: mature hard, mature soft, immature hard and immature soft. In a hard market, rates can generally be increased somewhat and growth may still be attainable. In a soft market, rates generally have to be reduced in order to grow. For each of the four cycle conditions, the probability of moving to another condition in the cycle (e.g., from mature soft to immature hard) is specified as an input[19]. Thus, over the course of the multi-year simulation, the company moves through different phases in the underwriting cycle.
- ***Jurisdictional risk:*** Because the U.S. insurance industry is primarily regulated at the individual state level, the jurisdictions in which a company operates can differ significantly with respect to regulatory environment, imposing additional risks on insurers. Residual market subsidies, retroactive premium rebates, and benefit changes on workers compensation policies already written, are all examples of jurisdictional burdens on insurers that increase the financial risk of the company. Thus, the model contains a two-pronged jurisdictional risk factor, which depends upon the geographical distribution of insurance writings.

  First, each jurisdiction has a range of "acceptable" rate changes – that is, there is associated with each state a range of rate changes that can be implemented without extraordinary company cost (in terms of time or money) and/or additional insurance department scrutiny. Generally, these ranges limit rate increases more than they do rate decreases, and the ranges are narrower in states with more restrictive regulation. The obvious effect of strict rate regulation is to prevent insurers from increasing rates to the degree they feel necessary. However, a side effect of capping rate increases is to make companies more reluctant to lower rates as much as would be otherwise indicated if pure premiums were improving.

  The second component of jurisdictional risk involves a lag in implementing indicated rate changes. This lag, shown in the model in terms of years, is longer in states with restrictive rate regulation. The jurisdictional risk parameters are based on industry information regarding relative regulatory restrictiveness among states. States considered

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the ultimate premium is determined after the fact, based on actual losses incurred during the actual policy period. In *prospective* rating, historical data for an insured is used to estimate future losses. *Exposure* rating uses historical experience from "similar" risks to estimate future losses for an insured (appropriately adjusted, as necessary and feasible, for the specific characteristics of the insured).

[19] Essentially, these represent transition probabilities in a four-state Markov chain.

to be most restrictive were assigned the lowest acceptable rate ranges and the longest lags. The actual values were selected primarily based on the judgment of individuals with experience with rate filings in those states.

4. Modeling Equity Returns

The creators of the public-access DFA model recognized that more sophisticated models and techniques for certain processes may have provided more accurate representations of economic uncertainty. However, greater sophistication often leads to models that are less tractable and more difficult to communicate and understand (an especially important issue, given the model's educational and training objectives). Thus, it was decided, in designing the public-access model, to maintain relative simplicity and transparency, but also to structure the model so that it could easily accommodate substitutions for various default assumptions and techniques.

In this section, we discuss the default approach to modeling equity returns in the public-access DFA model. We then describe a different approach: a regime-switching model of equity market performance. This provides background and preparation for Section 5, in which we describe the results from running the DFA model with each assumption, and evaluate their impact and significance.

Public-Access DFA Model Approach to Equity Returns

The public-access DFA model derives equity returns by considering the historical risk premium earned on equity investments in excess of the risk-free rate as well as incorporating the effects of changing interest rates on the business cycle. In each simulation, or trial, of the model, the underlying interest rate generator (the CIR approach described in the previous section) determines the risk-free rate ($r_{f,t}$) for each period. Then the DFA model simulates equity returns using a three-step process.

(1) The "expected" market return [$E(r_{M,t})$] is determined by combining the model risk-free rate with the historical market risk premium (*MRP*):

$$E(r_{M,t}) = r_{f,t} + MRP$$

(2) Next, the expected market return is adjusted for recent changes in the (simulated) short-term interest rate. For example, when interest rates change, stock market participants (in theory) apply different discount rates to a stock's future cash flows, altering the present value of those expected flows. In addition, changes in interest rates often affect credit purchases, and eventually the macro-economy. Therefore, the stream of future profits of corporations is often significantly impacted by interest rate changes. Specifically, when interest rates increase (decrease), the economy slows down (picks up) and stock prices tend to fall (increase). Using regression analysis, the model estimates this sensitivity coefficient (*h*) of equity returns to interest-rate changes to determine "adjusted" market returns [$E^*(r_{M,t})$]:

$$E^*(r_{M,t}) = E(r_{M,t}) + \{h \times (r_{f,t} - r_{f,t-1})\}$$

(3) Finally, a stochastic component (ε_t), where $\varepsilon_t \sim N(0, \sigma_e)$, is added. Here, the standard deviation of the distribution of the random component is derived from the standard error

of the estimate from the adjusted market return regression. This then yields the simulated return on the "market portfolio," i.e., the return on the average stock:

$$r_{M,t} = E^*(r_{M,t}) + \varepsilon_t$$

The DFA model recognizes that not all insurer's asset portfolios will have the same sensitivity to overall market returns. Some insurers may be more aggressive in their portfolios and opt to increase their risk. The user inputs an estimate of the insurer's relative sensitivity to the market (β_i) and the DFA model estimates the return on the insurer's stock portfolio ($r_{i,t}$) using the capital asset pricing model (CAPM):

$$r_{i,t} = r_{f,t} + \beta_i (r_{M,t} - r_{f,t})$$

This process is repeated for each simulated trial, for each year of the DFA model projection.

Regime Switching Model of Equity Returns

Researchers have noted that the distribution of historical stock returns exhibit "fatter tails" than is implied by a normal distribution (see Campbell, Lo, and MacKinlay, 1997). This makes the normality component of the approach described above inappropriate. Two examples may help to illustrate the non-normality of equity returns. First, consider the experience of October 1987. Hopefully, this single monthly observation is an unlikely event. In fact, given the extreme nature of the single day's loss on October 29th, 1987, the normal distribution may suggest this single return is nearly impossible.

As a second example, consider the growth of Nasdaq stocks prior to and after March 2000. The losses that occurred in subsequent months were not single day events of the type encountered in October 1987. Instead, it appears that the stocks had entered into a different return-generating environment – an environment characterized by higher volatility and/or lower average returns. These Nasdaq and October 1987 illustrations suggest that perhaps an alternative process is more appropriate than one characterized by the simple default relationship in the public-access DFA model.

In order to test this possibility, we introduced an alternative model for equity returns based on Hardy (2001). In her model, the return on stocks is selected from one of two normal distributions. Each distribution represents a different economic environment, or "regime," for stocks. [20]

Formally, let ρ_t denote the regime at time *t*. Each month, the regime-dependent return generating process is:

$$r_{M,t} \mid \rho_t \sim N(\mu_{\rho_t}, \sigma_{\rho_t})$$

The conditional probability of switching to the other regime from the current regime ρ_t is dictated by a matrix of transition probabilities ***P***.

$$P = \begin{pmatrix} 1 - p_{12} & p_{12} \\ p_{21} & 1 - p_{21} \end{pmatrix}$$

[20] While regime-switching models can incorporate more than two regimes, Hardy claims that no significant improvement in historical fit is achieved by using more than two regimes.

where p_{ij} is defined as the probability of switching from regime *i* to regime *j*. For example,

$$p_{12} = \Pr\{\rho_{t+1} = 2 \mid \rho_t = 1\}.$$

The model is completely determined by the parameter set $\theta = \{\mu_1, \mu_2, \sigma_1, \sigma_2, p_{12}, p_{21}\}$. We use the maximum likelihood approach described in Hardy (2001) to fit data on large stocks for the period 1926-2001, based on the returns provided in Ibbotson (2002). The likelihood function $L(\theta)$ shows the probability of observing the historical series of returns $r^h = (r_1^h, r_2^h, \ldots, r_n^h)$, given θ:

$$L(\theta) = \prod_{t=1}^{n} f\left(r_t^h \mid \theta, r_1^h, \ldots, r_{t-1}^h\right)$$

where $f(\cdot)$ denotes the density function of the historical returns r^h. To determine $f(\cdot)$, Hamiton and Susmel (1994) discuss a recursive calculation that accounts for four distinct combinations of regime movements over the last two periods (stay in regime 1, switch from 1 to 2, switch from 2 to 1, and stay in regime 2). The contribution of the t^{th} return to the likelihood is:

$$f(r_t^h \mid \theta, r_1^h, \ldots, r_{t-1}^h) = \sum_{i=1}^{2} \Pr\{\rho_{t-1} = i \mid \theta, r_1^h, \ldots, r_{t-1}^h\} \sum_{j=1}^{2} p_{ij} \times \phi\left[\frac{r_t^h - \mu_{\rho_t}}{\sigma_{\rho_t}}\right]$$

where ϕ represents the standard normal density function. The (monthly) parameters derived from maximizing the resulting likelihood function which were used in our regime-switching equity projections are shown below:

	Low Volatility Regime	***High Volatility Regime***
Mean	0.8%	-1.1%
Volatility	3.9%	11.3%
Probability of Switching	1.1%	5.9%

It should be noted that the parameter estimates of the regime-switching equity return model are quite sensitive to outliers in the data. In particular, depending on the length of the time series of equity returns and whether October 1987 data is included, parameter estimates can fluctuate fairly significantly.

In modeling stock returns for an insurer, the monthly returns generated from the regime switching model are compounded to determine the annual return on the market. A similar CAPM adjustment to that described above allows one to model insurers which accept either more or less risk than that implied by the market return.

Comparison of Equity Models

Table 1 shows statistics for projections under both equity return modeling approaches: the original linear regression approach that serves as the default approach in the public-access DFA model (occasionally referred to below as the "linear" approach), versus the same DFA model with the default approach replaced by the regime-switching approach to equity returns just described. As indicated by the statistics, the linear approach has lower mean and median returns in the first two projection years. However, the center of the distribution (the median) does not vary significantly between the two assumptions in later projection years. The lower average return in the early years of the linear projection can be explained by considering the assumed interest rate environment. In this environment, it is assumed that interest rates tend toward a mean reversion level of 5.0%. In 2003, the yield curve existing in the U.S. is steeply sloped and the initial short-term rate used in the projections is about 1%. Since, under the linear approach, stock returns are adjusted (with opposite sign) based on changes in interest rates, and since interest rates are mean reverting, they have a tendency to increase in the early years of the projection, on average. Stock returns are consequently negatively affected by the increasing interest rate environment.

Figures 1 and 2 show the frequency histogram of stock returns in the third projection year (identified as year 2005, or 05, in the figures in Table 1). The early years of the projection were not selected due to the impact of interest rates on subsequent stock returns mentioned above. But in 2005, the means of the two equity return approaches are equivalent. This "unbiasedness" allows us to focus more directly on the dispersion of equity returns.

As a benchmark, Figure 3 shows historical annual S&P 500 returns from years 1926 to 2002. The mean historical return in this period was 11.7%, and a standard deviation of returns was 19.6%. The worst year for the S&P 500 was 1931, when the market declined by 42.5%, while the best year for the market was 1933, with a positive return of 55.3%. Based on Figures 1 through 3, it initially appears that the DFA model may understate volatility, while the regime-switching model slightly overstates volatility.

A casual inspection of the equity return distributions in Figures 1 and 2 also shows the relative flatness of the distribution for the regime-switching model vs. the original linear approach. For comparison purposes, the excess kurtosis (peakedness) of the annual return distribution for the S&P 500 is -0.05. The linear approach is markedly peaked, indicating that the flatter regime-switching distribution is statistically more consistent with the historical return distribution.

The linear approach is significantly more peaked in the early projection years. This peakedness is the result of the dominating effects of interest rate mean reversion, mentioned above, and its impact on stock returns. As discussed earlier, the mean stock return under the linear approach is lower than the regime-switching approach in the early projection years (see Table 1). While it may appear that the interest rate mean reversion effects die out, they persist in the later projection years as evidenced by the increasing mean stock return throughout the projection period. In fact, the "steady state" stock market return is 13.5%, which is equal to the long run mean reversion level interest rate of 5% plus the historical market risk premium of 8.5%. Under the linear assumption, the effects of interest rate mean reversion, and its related effects on stock returns, appear to dominate any underlying randomness. As a result, the equity return distribution is more peaked in the early projection years. As interest rates increase during the projection period, the decaying mean reversion effects have less impact on the resulting stock return distribution.

The impact of these differences on the projections of insurer results is clear: since the current public-access DFA model incorporates (as its default assumption) the linear approach to equity returns, and thus understates the standard deviation of returns and reduces the likelihood of extreme events, total insurer risk may be significantly higher than predicted by the DFA model.

5. The Effects of Alternative Equity Modeling Assumptions on Insurers

The ultimate aim of the public-access DFA model is to provide a reasonable tool for understanding the interaction of both sides of an insurer's balance sheet. In its original form, the DFA model uses simple, easy-to-communicate assumptions to attempt to capture the uncertainty inherent in many variables. Alternative assumptions can be easily substituted into the model. To test the ultimate effects of alternative assumptions, we continue the use, from the prior section, of the public-access DFA model under two approaches for modeling equity returns: the original linear regression approach, and the regime-switching approach. It is clear, based on looking at the comparison of equity returns from the prior section, that the regime-switching model should lead to greater volatility of projected future insurer results. Our goal in this section is to understand the significance of these two different approaches.

Of course, the magnitude of the difference implied by the two alternative equity modeling assumptions is at least partly dependent upon the asset allocation of the insurer. Specifically, the greater the portion of the asset portfolio that is dedicated to stocks, the greater the comparative effects on projected future financial performance. Therefore, the choice of an illustrative insurance company to be used in the comparisons is an important matter. In our tests, we used two representative insurers: a "low equity exposure insurer" (LEE) and a "high equity exposure insurer" (HEE).

Different individual P-L insurers can have vastly different asset allocations. International insurers, in particular, appear to allocate a greater portion of their assets to stocks than do those domiciled in the U.S. The Organization for Economic Cooperation and Development (OECD) publishes the distribution of investments of life and property-liability insurance companies for many countries. The data shows that U.S. P-L companies allocate approximately 14% of their portfolios to equity investments. Globally, there is a good deal of variation in asset allocation as shown in the table below.

Country	***% Invested in Stocks***
Australia	38%
Denmark	37%
Italy	23%
Korea	13%
Sweden	42%
United Kingdom	34%

We have chosen a hypothetical proxy company for HEE insurers that invests 45% of total assets in equities, perhaps showing an upper limit based on the OECD data. The LEE insurer proxy assumes a 14% allocation to equities, equivalent to the above-mentioned U.S. figure.[21]

We ran the public-access DFA model for the two insurers (LEE and HEE) under both equity return modeling assumptions. For each projection, 1,000 sample paths of interest rates, stock returns, and underwriting results were simulated. Tables 1 and 2 show summary statistics

[21] We recognize that the percentage of an insurer's portfolio allocated to stocks is a function of many variables including mix of business, regulation, accounting standards, market performance, etc. We provide these statistics only to motivate our use of an alternate asset allocation in our tests.

for some key variables from the projections, which are common to both insurer proxies. Table 1 shows statistics for the alternative assumptions for stock returns and was discussed in the previous section. Table 2 shows statistics for the projection of interest rates and underwriting results that apply to all the simulations. The short-term interest rate represents the short end of the curve from projections of the CIR term structure model. Underwriting gain or loss (UW G/L) represents the profit from the underwriting side of business, expressed as a percent of earned premium.[22] The important thing is that UW G/L ignores any return from investment activity of the insurer:

$$\text{UW G/L} = 100\% - \frac{\text{Calendar Year Losses} + \text{Expenses}}{\text{Earned Premium}}$$

Low Equity Exposure Insurer Results

Tables 3 and 4 show results for the projections of the insurer with Low Equity Exposure that has invested 14% of its assets in stocks. Table 3 looks at the operating ratio of the insurer, which is a key measure of annual performance used in the insurance industry. The operating ratio (OR) adjusts the UW G/L by adding the return from investments held by the company. It should be noted that the OR restates part of UW G/L by relating expenses to written, rather than earned, premiums, to more accurately account for the effects of timing of premiums received, the expenses directly associated with premium collection, and investment earnings.

$$\text{OR} = \frac{\text{Calendar Year Losses}}{\text{Earned Premium}} + \frac{\text{Expenses}}{\text{Written Premium}} - \frac{\text{Net Investment Income}}{\text{Earned Premium}}$$

The top third of Table 3 indicates performance without recognizing the capital gains (CG) earned from the investment in stocks; the bottom two-thirds of Table 3 include the effects of equity returns, first under the original linear equity assumption, then under the regime-switching equity model.

First, note that in the first projection year, the mean (and median) OR with CG is slightly lower under the regime-switching model. This improved performance measure follows from the average stock return, which was higher under the regime-switching assumption (see Table 1). In later years, the improved average performance pattern under the regime-switching model disappears since the average stock return is similar to the linear approach assumption. With respect to volatility, the insurer's performance is more uncertain for all projection years under the regime-switching equity assumption, as expected. The standard deviation of OR with CG is higher under each projection year, and the range of outcomes is wider. Consider the 5th percentile in the third projection year (identified by the year 2005): under the linear regression assumption, the 5th percentile for OR is 83.3% while under the regime-switching assumption the OR is 82.2%. At the other end of the distribution (where the insurer's performance is not as good), the 95th percentile under the linear assumption is 103.4% vs. 108.8% under the regime-switching assumption. Thus, while the LEE insurer has the opportunity for higher performance measures under the regime-switching assumption, it also appears that the DFA model with the linear assumption may understate the insurer's risk when projecting performance.

Table 4 displays the projection of surplus for the LEE insurer. Surplus is a closely watched variable by regulators since it represents the cushion available to absorb adverse

[22] A related ratio, called the "trade ratio," identifies the proportion of premiums which is expended upon losses and expenses; however, in determining this ratio, losses are related to earned premiums, and expenses are related to written premiums.

experience in either losses or investment returns. Consistent with Table 3, the regime-switching equity assumption indicates the potential accumulation of larger surplus (especially in the early years of the projection) as indicated by the higher mean and median values of projected policyholder surplus (PHS) under the regime-switching equity returns. However, the uncertainty in the projection, as represented by the standard deviation of PHS, is also elevated under the regime-switching assumption and the range of outcomes is much higher under the more volatile equity assumption. In the third projection year, the 95th percentile under regime-switching is 64,986. (Note: the numbers shown are in thousands.) Under the linear equity return approach, the 95th percentile is slightly lower, at 61,570. The projections also show the greater likelihood of decreasing PHS under the regime-switching model: the 5th percentile of policyholder surplus under the regime-switching model in the third projection year is 41,267, while under the linear assumption it is approximately 43,400. Finally, the last line in the left-hand side of Table 4 shows the probability of surplus being impaired by more than 10% from the initial PHS level of 40,000. Under each projection year, the probability of surplus impairment is higher under the regime-switching equity return assumption. The result is that the current insurer strategy appears more risky than initially projected by the public-access DFA model with the default linear approach to modeling equity returns.

The right hand side of Table 4 shows projections of the premium-to-surplus (P/S) ratio, a common measure of leverage in the insurance industry. Given that surplus is a cushion against adverse or unexpected experience of the insurer, the P/S ratio is a common gauge used by regulators to measure the amount of business that can be supported for each dollar of surplus. In the comparison between the two equity modeling assumptions, the standard deviations do not appear dramatically different. In fact, the statistics for standard deviation appear to be heavily influenced by significant outliers under both equity return assumptions. By concentrating on the percentiles of the distributions, the regime-switching model appears to project more uncertainty of projected leverage for the insurer. For example, the 5th percentile under the linear assumption shows a P/S ratio in the third projection year of 1.13 vs. the regime-switching model's 5th percentile of 1.08. At the other end of the distribution (the 95th percentile), the comparison is 1.72 vs. 1.78. These results indicate the wider range of results using the regime-switching model and, therefore, greater uncertainty.

Taken together, these results suggest that even though the LEE insurer allocates only 14% of its asset portfolio to stocks, differences in an insurer's operations under two alternative equity return assumptions are notable. In particular, the regime-switching equity return model shows greater uncertainty in insurer performance than is assumed in the public-access DFA model's linear default assumption. While the probability of surplus impairment and the likelihood of increased regulatory scrutiny may indicate little difference between the competing equity return generators, those insurers that allocate more than the average insurer in the U.S. may dramatically understate their future risks. In the next section, we look at an insurer which has invested more than three times the amount allocated by the LEE insurer.

High Equity Exposure Insurer Results

Tables 5 and 6 present similar statistics for the projections of an insurer that invests 45% of its assets in stocks. Table 5 shows the operating ratio and Table 6 shows the effects on PHS. As expected, the results from the HEE insurer signal similar concerns about the modeling approach under the original linear equity return assumption; the major difference between the projected results of the two insurers is the size of the comparisons.

In Table 5, the insurer performance, as measured by OR, shows no predictable pattern in the mean or median between the different equity return assumptions. However, any measure of risk clearly highlights significant differences between the two approaches. The standard deviations for OR with CG are all significantly higher under the regime-switching equity assumption and the range of outcomes is much more dramatic. While the result indicates that the linear equity assumption understates operating performance in good states of the world, of more concern is the significant risks that are not captured under difficult financial scenarios. Consider the right-hand tail for the OR with CG (bad outcomes) in the third projection year. At the 90th percentile of the distribution, the linear equity assumption projects an OR of 114.6% vs. 122.9% under the regime-switching model. Things can get much worse than is projected under the linear equity return assumption.

In Table 6, the projections of PHS for the HEE insurer show similar dramatic differences between the two equity assumptions. Even though the mean and median policyholder surplus projections indicate the available cushion against adverse experience may be higher when allocating a higher percentage of insurer's assets in equities, the risks of this strategy cannot be ignored. The standard deviation for all projection years is close to 50% higher than under the less volatile linear equity assumption. The percentiles again show the wide range of potential outcomes. Most striking, however, is the value showing the probability of surplus impairment. In later years, the DFA model equity assumption indicates that the current strategy eventually leads to decreasing the risk of impairment. However, the regime-switching equity approach correctly illustrates that uncertainty increases over time.

The P/S ratios on the right-hand side of Table 6 similarly show the additional uncertainty predicted by the regime-switching approach. In particular, the linear approach dramatically understates the risk of regulatory inquiries.

All of these results indicate that insurers need to be careful about which equity return assumption they might use in a DFA projection. In particular, the original linear approach in the public-access DFA model seems to imply a lower level of risk than indicated by historical returns.

6. Conclusion

By using an integrated asset-liability model – one that dynamically models the interrelationships between and among investment and underwriting processes, an approach which we have referred to as “dynamic financial analysis” throughout this paper – insurers and regulators can better manage the solvency risk that insurance companies face resulting from financial and underwriting volatility. If an insurer is found to face an unacceptable level of risk, regulators can require that the insurer increase its capitalization, reduce its underwriting capacity, or make other operating changes. Asset-liability and DFA modeling also provides management with a tool to better identify optimal operational decisions involving, for example, investment allocation strategies, reinsurance programs, growth rates, geographical underwriting distributions, and other strategies. By focusing on the results of such stochastic simulation models, including percentile distributions, users can identify potentially unacceptable results, and test alternative strategies and assumptions in an attempt to increase the likelihood of acceptable financial and operating performance.

The public-access DFA model described in this paper is an important step in the integrative modeling of insurers. The underlying assumptions of the model are intuitive and easy to understand for managers from different areas of an insurer, so the resulting model is quite simple to communicate. The assumptions are internally consistent in the model and represent a good starting point for analysis of an insurer. However, the appropriateness and precision of modeling a specific variable under alternative approaches is an important theoretical and empirical question. In this research, we have tested an alternative approach (relative to the simple linear regression approach included as a default assumption in the public-access DFA model) for the modeling of equity returns. Specifically, we looked at a regime-switching equity model, which is better able to capture the "fat tails" associated with historical stock returns. We find that the use of this alternative model for equity returns does affect projections of future insurer performance. In particular, and especially given the significant use of equities in some (including non-U.S.-) insurer asset portfolios, the results can be dramatic, particularly for relatively high equity allocations. Thus, the use of a DFA or asset-liability model that does not adequately reflect investment and market risk may severely understate the future uncertainty and risk profile of the insurer, possibly leading to sub-optimal strategic and operational decisions.

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FIGURE 1
Original Linear Equity Return Assumption - Histogram

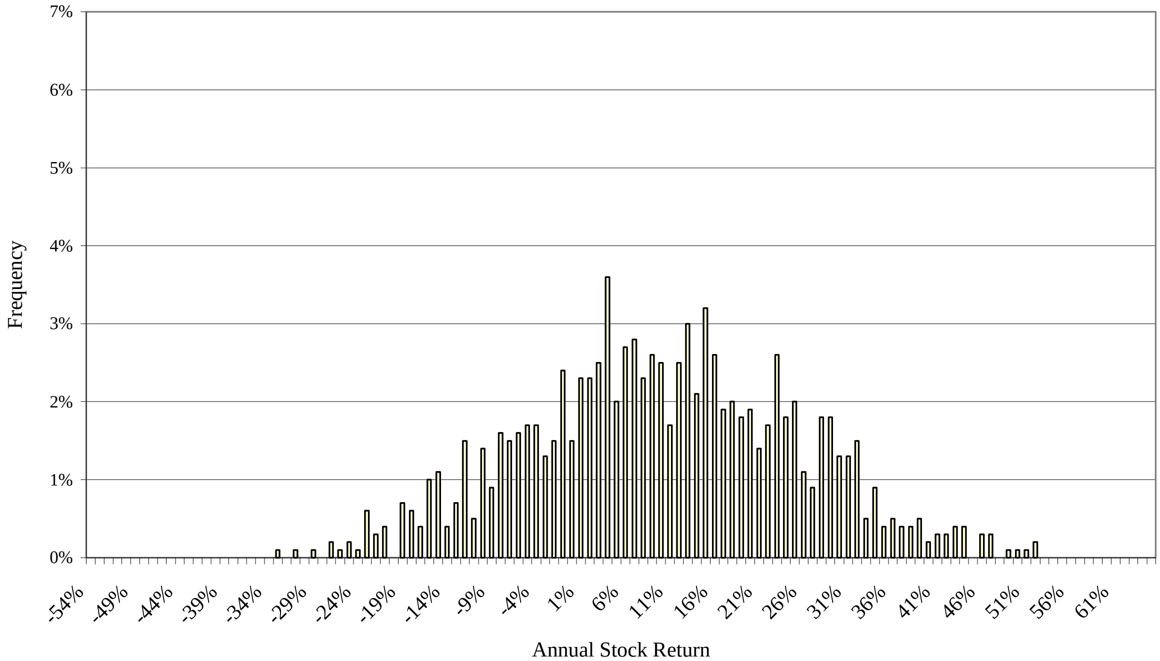

FIGURE 2
Regime Switching Equity Return Assumption - Histogram

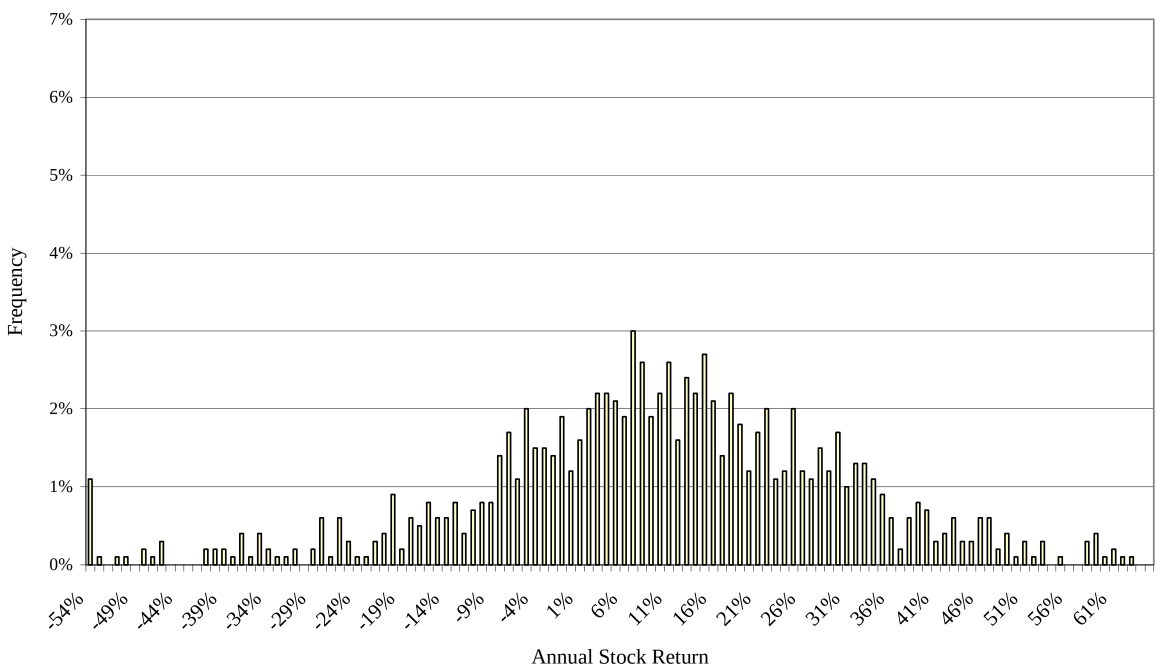

FIGURE 3
S&P 500 Historical Stock Returns - Histogram

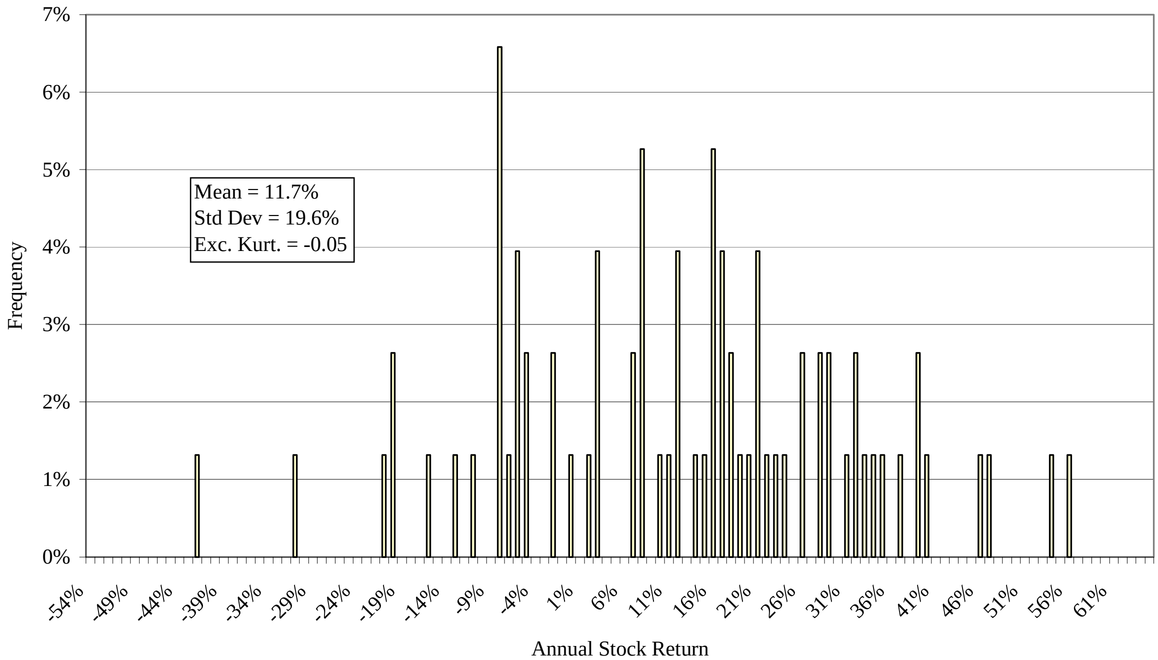

TABLE 1

Equity Return Statistics

Original Linear Equity Assumption

	Stock Ret 03	*Stock Ret 04*	*Stock Ret 05*	*Stock Ret 06*	*Stock Ret 07*
Mean	7.0%	8.2%	9.8%	10.6%	12.1%
Stdev	15.1%	15.5%	15.3%	14.7%	15.3%
CV	216.9%	187.8%	156.5%	139.3%	125.6%
Excess Kurtosis	4.564	3.701	1.910	1.652	1.015
Min	-51.1%	-37.4%	-38.6%	-34.1%	-43.8%
1%	-29.1%	-25.4%	-24.4%	-23.5%	-26.1%
5%	-18.3%	-15.8%	-16.0%	-13.0%	-12.3%
10%	-11.7%	-10.6%	-10.2%	-8.3%	-6.4%
25%	-3.2%	-2.9%	0.5%	0.9%	1.9%
50%	7.5%	7.4%	10.0%	10.5%	11.9%
75%	16.9%	18.5%	19.6%	20.9%	22.3%
90%	25.7%	29.3%	28.8%	29.7%	32.2%
95%	31.5%	34.3%	35.5%	34.9%	37.0%
99%	43.3%	45.3%	43.6%	43.7%	49.7%
Max	59.1%	57.9%	62.8%	54.9%	58.9%

Regime Switching Equity Assumption

	Stock Ret 03	*Stock Ret 04*	*Stock Ret 05*	*Stock Ret 06*	*Stock Ret 07*
Mean	9.6%	10.7%	9.8%	11.3%	11.4%
Stdev	22.6%	22.3%	21.9%	21.9%	21.5%
CV	233.7%	207.4%	223.3%	193.2%	188.5%
Excess Kurtosis	-0.024	-0.111	-0.277	-0.173	-0.179
Min	-67.9%	-72.4%	-67.2%	-72.5%	-70.4%
1%	-53.5%	-49.4%	-54.1%	-54.6%	-48.0%
5%	-28.8%	-26.3%	-27.2%	-30.9%	-25.9%
10%	-16.0%	-12.8%	-15.7%	-13.3%	-15.3%
25%	-2.4%	-0.8%	-2.1%	-1.1%	-0.4%
50%	10.5%	10.8%	10.0%	12.8%	11.9%
75%	22.4%	22.4%	22.9%	24.7%	24.7%
90%	34.0%	34.7%	34.5%	36.0%	37.5%
95%	42.9%	42.1%	43.9%	42.0%	44.8%
99%	61.7%	68.7%	61.0%	64.2%	59.5%
Max	160.5%	131.3%	120.0%	98.0%	106.7%

TABLE 2

Projection Output Statistics

	Short Term Interest Rate				
	ST Int 03	*ST Int 04*	*ST Int 05*	*ST Int 06*	*ST Int 07*
Mean	2.1%	2.8%	3.4%	3.9%	4.2%
Stdev	0.5%	0.8%	1.1%	1.2%	1.4%
CV	25.7%	29.8%	31.3%	32.1%	32.9%
Min	0.5%	0.6%	0.4%	0.4%	0.6%
1%	0.8%	1.0%	1.2%	1.4%	1.5%
5%	1.2%	1.5%	1.7%	2.0%	2.2%
10%	1.4%	1.7%	2.0%	2.4%	2.5%
50%	2.1%	2.7%	3.3%	3.8%	4.1%
90%	2.8%	3.9%	4.7%	5.5%	6.0%
95%	3.0%	4.2%	5.2%	6.1%	6.7%
99%	3.4%	4.7%	6.2%	7.3%	8.1%
Max	3.8%	5.3%	7.1%	9.0%	9.1%

	Underwriting Gain/(Loss)				
	UWG/L 03	*UWG/L 04*	*UWG/L 05*	*UWG/L 06*	*UWG/L 07*
Mean	(1.8%)	(6.5%)	(4.0%)	(4.6%)	(4.7%)
Stdev	6.3%	4.8%	3.7%	4.3%	4.9%
CV	(353.9%)	(73.5%)	(91.9%)	(92.6%)	(104.0%)
Min	(131.6%)	(87.7%)	(35.3%)	(42.8%)	(53.9%)
1%	(17.5%)	(17.9%)	(12.7%)	(14.4%)	(18.9%)
5%	(9.9%)	(11.2%)	(8.0%)	(9.5%)	(10.8%)
10%	(6.3%)	(9.6%)	(7.0%)	(8.0%)	(8.4%)
50%	(0.9%)	(6.6%)	(4.8%)	(5.6%)	(5.5%)
90%	2.7%	(2.0%)	0.5%	0.9%	1.3%
95%	3.7%	(0.5%)	2.2%	2.7%	2.9%
99%	6.2%	3.4%	5.6%	6.4%	6.3%
Max	10.0%	6.8%	11.2%	10.0%	14.1%

NOTE: These statistics are invariant to insurer type (High vs. Low Equity Exposure) and the equity assumption (linear or regime switching).

TABLE 3

Operating Ratios - Low Equity Exposure Insurer (14%)

Based on Net Investment Income

	OR 03	OR 04	OR 05	OR 06	OR 07
Mean	89.9%	95.6%	93.9%	95.1%	95.6%
Stdev	6.3%	4.8%	3.8%	4.3%	4.8%
CV	0.07	0.05	0.04	0.04	0.05
Min	78.1%	82.1%	78.1%	80.7%	77.9%
1%	82.0%	85.8%	84.2%	84.6%	84.6%
5%	84.5%	89.5%	87.6%	87.9%	87.9%
10%	85.4%	91.1%	89.4%	89.7%	89.8%
50%	89.1%	95.7%	94.6%	96.0%	96.3%
90%	94.4%	98.7%	97.0%	98.6%	99.3%
95%	98.1%	100.2%	98.2%	100.0%	101.7%
99%	105.7%	107.2%	103.2%	107.1%	109.3%
Max	219.8%	176.8%	130.1%	133.8%	144.1%

Original Linear Equity Assumption
Based on Net Investment Income and All Capital Gains

	OR 03	OR 04	OR 05	OR 06	OR 07
Mean	88.9%	95.0%	93.4%	94.6%	94.7%
Stdev	7.2%	6.8%	6.3%	6.9%	7.4%
CV	0.08	0.07	0.07	0.07	0.08
Min	74.2%	74.9%	73.6%	74.5%	71.2%
1%	77.3%	81.1%	79.5%	79.3%	76.5%
5%	80.6%	85.1%	83.3%	84.1%	83.5%
10%	82.0%	87.1%	85.6%	86.4%	86.1%
50%	88.1%	95.0%	93.3%	94.5%	94.8%
90%	95.7%	102.3%	100.9%	103.2%	103.4%
95%	98.4%	104.7%	103.4%	105.9%	106.7%
99%	105.8%	110.8%	110.1%	111.6%	112.7%
Max	216.0%	174.1%	128.3%	131.4%	141.6%

Regime Switching Equity Assumption
Based on Net Investment Income and All Capital Gains

	OR 03	OR 04	OR 05	OR 06	OR 07
Mean	88.3%	94.9%	93.9%	94.1%	95.1%
Stdev	8.3%	8.5%	8.5%	8.8%	9.8%
CV	0.09	0.09	0.09	0.09	0.10
Min	52.5%	58.5%	63.9%	63.5%	55.2%
1%	71.7%	76.1%	75.0%	74.4%	71.8%
5%	78.7%	82.7%	82.2%	80.3%	80.2%
10%	80.7%	85.6%	84.7%	83.2%	84.1%
50%	87.5%	94.4%	93.3%	93.9%	95.0%
90%	96.7%	104.2%	103.2%	104.5%	106.3%
95%	100.0%	107.5%	108.8%	107.8%	111.1%
99%	107.8%	116.8%	118.4%	120.3%	122.6%
Max	217.7%	173.4%	152.1%	134.9%	144.8%

TABLE 4

Surplus Projections - Low Equity Exposure Insurer (14% in Equities)

Original Linear Equity Assumption

	PHS 03	PHS 04	PHS 05	PHS 06	PHS 07		P/S 03	P/S 04	P/S 05	P/S 06	P/S 07
Mean	45,929	48,235	52,050	56,057	59,480	*Mean*	1.27	1.38	1.41	1.45	1.49
Stdev	3,511	5,426	6,891	8,269	10,415	*Stdev*	0.16	0.48	0.38	0.72	0.64
CV	0.08	0.11	0.13	0.15	0.18	*CV*	0.13	0.35	0.27	0.50	0.43
Min	(20,377)	(42,027)	(57,398)	(66,426)	(62,297)	*Min*	(2.86)	(1.53)	(1.29)	(1.28)	(1.58)
1%	37,772	35,816	36,777	38,106	30,607	*1%*	1.12	1.08	1.04	0.99	0.98
5%	41,486	41,286	43,399	44,985	45,585	*5%*	1.17	1.16	1.13	1.11	1.09
10%	42,667	43,520	45,705	48,114	48,550	*10%*	1.18	1.20	1.18	1.16	1.15
50%	46,212	48,223	51,830	55,975	59,607	*50%*	1.26	1.36	1.39	1.40	1.42
90%	49,244	53,705	59,582	65,132	71,253	*90%*	1.36	1.52	1.61	1.68	1.81
95%	49,879	55,190	61,570	67,681	75,308	*95%*	1.40	1.61	1.72	1.83	1.94
99%	51,654	58,590	65,446	72,898	81,959	*99%*	1.53	1.85	1.98	2.14	2.86
Max	53,078	62,278	71,967	83,191	87,736	*Max*	2.50	15.42	9.86	21.88	16.27
Prob. Of 0% Impairment	0.6%	1.2%	1.0%	0.8%	1.4%	*Prob. of Reg Scrutiny*	0.0%	0.3%	0.3%	0.3%	0.8%

Regime Switching Equity Assumption

	PHS 03	PHS 04	PHS 05	PHS 06	PHS 07		P/S 03	P/S 04	P/S 05	P/S 06	P/S 07
Mean	46,277	49,035	52,925	57,357	60,880	*Mean*	1.26	1.37	1.34	1.40	1.45
Stdev	4,174	6,613	8,630	10,986	13,521	*Stdev*	0.18	0.86	1.88	0.60	0.64
CV	0.09	0.13	0.16	0.19	0.22	*CV*	0.14	0.63	1.41	0.43	0.44
Min	(21,344)	(46,689)	(66,003)	(72,822)	(70,534)	*Min*	(2.73)	(1.38)	(57.41)	(13.81)	(13.58)
1%	36,954	34,737	34,044	33,686	29,112	*1%*	1.06	1.01	0.96	0.88	0.85
5%	40,430	40,095	41,267	41,500	42,110	*5%*	1.14	1.10	1.08	1.02	0.99
10%	42,237	42,615	43,793	45,691	45,595	*10%*	1.16	1.16	1.14	1.11	1.08
50%	46,605	49,313	53,045	57,640	60,772	*50%*	1.25	1.33	1.36	1.37	1.40
90%	50,071	55,696	61,823	69,497	76,791	*90%*	1.38	1.54	1.68	1.75	1.89
95%	51,087	57,685	64,986	74,236	82,423	*95%*	1.44	1.64	1.78	1.97	2.09
99%	54,577	64,020	71,435	83,589	94,992	*99%*	1.57	1.89	2.10	2.36	2.87
Max	65,775	78,582	96,427	118,771	123,720	*Max*	3.69	27.45	6.53	6.61	6.52
Prob of 10%	0.7%	1.8%	1.5%	1.8%	1.7%	*Prob of Reg.*	0.1%	0.3%	0.3%	0.3%	0.8%

Impairment	Scrutiny

TABLE 5

Operating Ratios - High Equity Exposure Insurer (45%)

	Based on Net Investment Income				
	OR 03	***OR 04***	***OR 05***	***OR 06***	***OR 07***
Mean	92.2%	97.6%	95.8%	96.8%	97.1%
Stdev	6.3%	4.8%	3.8%	4.3%	4.9%
CV	0.07	0.05	0.04	0.04	0.05
Min	80.4%	84.1%	79.9%	82.1%	79.1%
1%	84.2%	87.8%	86.0%	86.0%	86.0%
5%	86.7%	91.4%	89.3%	89.6%	89.4%
10%	87.7%	93.2%	91.2%	91.3%	91.1%
50%	91.3%	97.7%	96.4%	97.6%	97.7%
90%	96.6%	100.7%	98.8%	100.3%	100.8%
95%	100.3%	102.2%	100.0%	101.6%	103.1%
99%	107.9%	109.1%	104.9%	108.4%	111.3%
Max	222.0%	178.7%	130.6%	135.3%	146.0%

	Original Linear Equity Assumption Based on Net Investment Income and All Capital Gains				
	OR 03	***OR 04***	***OR 05***	***OR 06***	***OR 07***
Mean	88.8%	96.1%	94.1%	95.4%	94.7%
Stdev	13.1%	15.9%	16.4%	16.8%	17.9%
CV	0.15	0.17	0.17	0.18	0.19
Min	48.2%	53.1%	48.1%	47.1%	32.5%
1%	61.3%	61.3%	61.3%	61.1%	50.8%
5%	68.9%	71.2%	68.5%	68.9%	66.6%
10%	72.9%	75.5%	73.2%	75.1%	72.5%
50%	88.4%	95.9%	93.5%	94.4%	94.6%
90%	104.4%	115.9%	114.6%	117.1%	117.8%
95%	109.1%	122.1%	122.9%	124.5%	125.5%
99%	118.7%	133.4%	136.2%	140.5%	138.7%
Max	209.7%	170.5%	170.0%	155.0%	159.0%

	Regime Switching Equity Assumption Based on Net Investment Income and All Capital Gains				
	OR 03	***OR 04***	***OR 05***	***OR 06***	***OR 07***
Mean	86.7%	95.6%	95.8%	93.8%	95.9%
Stdev	18.3%	22.8%	23.9%	23.5%	26.0%
CV	0.21	0.24	0.25	0.25	0.27
Min	-27.3%	-6.8%	1.3%	7.1%	-6.1%
1%	44.5%	44.2%	47.6%	39.1%	34.7%
5%	60.9%	63.2%	61.0%	56.3%	55.5%
10%	67.6%	70.4%	69.5%	67.1%	66.9%
50%	85.7%	94.5%	94.3%	92.9%	94.9%
90%	108.3%	121.2%	122.9%	121.7%	126.4%
95%	117.8%	131.5%	136.0%	129.7%	140.8%
99%	136.1%	157.1%	167.2%	160.3%	169.1%
Max	215.4%	255.3%	274.8%	216.2%	239.9%

TABLE 6

Surplus Projections - High Equity Exposure Insurer (45% in Equities)

Original Linear Equity Assumption

	PHS 03	PHS 04	PHS 05	PHS 06	PHS 07		P/S 03	P/S 04	P/S 05	P/S 06	P/S 07
Mean	46,110	49,067	54,216	60,176	67,309	*Mean*	1.28	1.40	1.45	1.45	1.47
Stdev	6,947	11,115	14,590	18,253	23,269	*Stdev*	0.25	0.40	0.67	0.72	0.92
CV	0.15	0.23	0.27	0.30	0.35	*CV*	0.20	0.29	0.46	0.50	0.62
Min	(16,867)	(30,876)	(36,196)	(48,056)	(38,365)	*Min*	(3.46)	(2.09)	(2.05)	(1.76)	(2.56)
1%	30,262	25,246	25,799	25,340	20,661	*1%*	0.95	0.82	0.77	0.71	0.64
5%	35,025	32,460	32,053	32,924	32,041	*5%*	1.02	0.96	0.90	0.84	0.77
10%	37,740	36,698	37,108	38,396	39,152	*10%*	1.06	1.03	0.97	0.93	0.86
50%	46,186	48,486	53,475	58,761	66,128	*50%*	1.26	1.35	1.35	1.33	1.29
90%	54,721	63,048	73,656	82,869	97,232	*90%*	1.54	1.79	1.97	2.08	2.18
95%	56,881	66,897	78,648	92,092	107,448	*95%*	1.66	2.00	2.26	2.37	2.70
99%	60,889	78,281	89,409	109,708	130,638	*99%*	1.91	2.49	2.82	3.16	4.04
Max	68,370	92,231	109,495	126,576	153,359	*Max*	2.59	6.24	17.16	17.33	14.16
Prob. Of 10% Impairment	6.2%	8.8%	9.0%	7.5%	7.2%	*Prob. Of Reg. Scrutiny*	0.0%	0.4%	0.6%	1.6%	3.0%

Regime Switching Equity Assumption

	PHS 03	PHS 04	PHS 05	PHS 06	PHS 07		P/S 03	P/S 04	P/S 05	P/S 06	P/S 07
Mean	47,141	51,438	56,710	63,873	71,067	*Mean*	1.30	1.44	1.49	2.44	2.00
Stdev	9,934	16,051	21,451	28,508	36,033	*Stdev*	0.40	1.14	3.42	25.86	10.87
CV	0.21	0.31	0.38	0.45	0.51	*CV*	0.31	0.80	2.30	10.60	5.43
Min	(19,551)	(44,774)	(61,983)	(67,555)	(63,776)	*Min*	(2.98)	(18.51)	(94.85)	(166.22)	(166.18)
1%	20,118	12,205	8,076	2,333	1,105	*1%*	0.82	0.65	0.62	0.52	0.37
5%	30,795	26,444	22,777	18,192	17,749	*5%*	0.94	0.85	0.77	0.68	0.61
10%	35,583	32,929	31,176	27,443	26,988	*10%*	1.01	0.93	0.87	0.78	0.70
50%	47,637	51,434	56,443	63,355	69,160	*50%*	1.22	1.27	1.28	1.23	1.22
90%	57,373	69,609	81,739	97,085	117,328	*90%*	1.63	1.97	2.30	2.78	3.00
95%	61,388	75,870	90,448	113,297	132,735	*95%*	1.87	2.48	3.14	3.99	4.23
99%	69,496	95,999	113,706	137,653	168,550	*99%*	2.85	4.68	6.32	15.91	17.53
Max	110,170	142,827	194,170	258,007	283,386	*Max*	5.46	14.81	23.97	794.13	253.51
Prob. Of 10% Impairment	10.6%	13.1%	14.4%	15.1%	15.1%	*Prob. Of Reg. Scrutiny*	0.8%	3.3%	5.7%	8.6%	10.0%