

THE EQUITY SELECTION PROCEDURE MODEL

Robert S. Clarkson
Visiting Professor of Actuarial Science
City University, London
Cherrybank
Manse Brae
Dalsersf
Lanarkshire ML9 3BN
U.K.
Telephone: +44 (0)1698 882 451

ABSTRACT

The Capital Asset Pricing Model (CAPM), a crucial cornerstone of many of the methodologies of modern finance theory, is based on a number of highly questionable simplifying assumptions, such as that risk is equivalent to a symmetric measure of variability of return (e.g. variance or standard deviation of return), that investors behave rationally in comparing expected return with risk, and that the capital market is in equilibrium. However, over recent years the intuitively much more plausible downside approach to risk has become increasingly popular, particularly with investment practitioners; empirical findings from the rapidly expanding new field of behavioural finance invalidate the rationality assumption; and the acute instability of stockmarkets world-wide makes the equilibrium assumption untenable. Furthermore, the CAPM, being wholly theoretical in nature, fails to answer many of the difficult practical questions faced on a day-to-day basis by investment professionals.

The objective of this paper is to apply the principles of the downside risk approach described in Clarkson (2000) to the construction of an alternative model which will be of direct practical assistance in equity portfolio management. A simplified model taking only short-term earnings per share growth into account is first of all constructed using the equity selection procedures described in Clarkson (1998a). This model is then extended in two stages, first of all to accommodate the more difficult area of long-term earnings growth estimates and then to accommodate the even more difficult area of identifying, and if possible anticipating, periodic performance-damaging swings in aggregate investor sentiment such as those classified within behavioural finance as over-confidence, over-reaction bias, and myopic loss aversion. The resulting Equity Selection Procedure Model (ESPM) is therefore a mathematical statement to the effect that the achievement of superior long-term equity portfolio performance is a skill-dependent double increment process that is subject to understandable, but often unpredictable, temporary downward aberrations. The main practical conclusion of the paper is that the portfolio guidelines followed by the vast majority of professional investors in a misguided attempt to “manage risk” are totally counterproductive in that they not only reduce the performance achieved but also increase the risk of failing to achieve a benchmark or target return. The main theoretical conclusions are that systematic irrationality of the nature identified by behavioural finance studies is closer to being the rule than the exception within equity markets and that the dynamics of equity share price movements are not, as is assumed for instance in Black & Scholes (1973) and Markowitz (1959), invariant over time.

1. Introduction

An absolutely crucial simplifying assumption of the Capital Asset Pricing Model (CAPM) is that a symmetric measure of variability of return, such as variance or standard deviation of return, gives a satisfactory numerical proxy for risk. However, the downside approach to risk described in detail in Clarkson (2000) is shown to be far more satisfactory in both theoretical and practical terms. It is therefore highly likely that a completely different, and much more realistic, capital market model can be developed by incorporating the principles of this downside approach to risk.

Suppose we have a variable t of financial outcome at some specified time horizon, with a high value of t being “good”. Then the essence of the conceptual approach set out in Clarkson (2000) is that a highly satisfactory measure of risk can be obtained by taking the probability-weighted integral or summation of a continuous risk weighting function $w(t)$. Suppose we identify the value L_0 of t above which outcomes are deemed to be satisfactory and the value L_1 below which financial ruin (e.g. insolvency for a financial institution or bankruptcy for an individual) occurs. Then $w(t)$ is 0 above L_0 , 1 below L_1 , and concave upwards between L_0 and L_1 . A highly desirable property of this measure of risk is that it has the same range as probability, namely 0 to 1, with 0 corresponding to no possibility of an unsatisfactory outcome and 1 corresponding to financial ruin with certainty.

For portfolio investment in limited liability securities L_0 can in general be taken as the value corresponding to a target or benchmark return, L_1 can be taken as 0, and a highly satisfactory formulation for $w(t)$ is the square of the proportionate shortfall below L_0 .

A simplified example involving portfolio investment may be useful here. Consider an initial investment of 100 and a target rate of return that equates to an end-period value of 200. Suppose that over 10 different periods in the past outcomes have been (in increasing order of magnitude) 140, 180, 200, 200, 215, 245, 260, 265, 280 and 340. The outcomes of 140 and 180 correspond to proportionate shortfalls of 0.3 and 0.1 respectively, squared proportionate shortfalls of 0.09 and 0.01, a risk summation of 0.1 over all 10 outcomes, and hence an expected (or mean) value of risk of 0.01.

The empirical findings of behavioural finance will be discussed in more detail later, but it is important to note at this point that when investors are questioned about how they regard risk, virtually none mention variance of return or standard deviation of return. Instead, the vast majority tend to equate risk with the possibility of the return they receive being unsatisfactory in some regard. This alone is fairly strong evidence that the CAPM should be regarded as tentative in the extreme in the extent to which it encapsulates the realities of capital market behaviour.

Even stronger evidence about the inadequacy of the CAPM comes from detailed numerical studies. One of the earliest was Mandelbrot (1963), in which it was pointed out that the dynamics of share price changes were far more complex than previously thought in that small price changes tend to be followed by small changes, while large price changes tend to be followed by large changes. A much more recent study, Mandelbrot (1999), the cover story in the February 1999 issue of “Scientific American”, warns that modern portfolio theory poses a very grave danger to those who believe in it too strongly:

“A cornerstone of finance is modern portfolio theory, which tries to maximise returns for a given level of risk. The mathematics underlying portfolio theory handles extreme situations with benign neglect: it regards large market shifts as too unlikely to matter or as impossible to take into account. It is true that portfolio theory may account for what occurs 95 percent of the time in the market. But the picture it presents does not reflect reality, if one agrees that major events are part of the remaining 5 percent. An inescapable analogy is that of a sailor at sea. If the weather is moderate 95% of the time, can a mariner afford to ignore the possibility of a typhoon?”

Other anomalous evidence was the mean absolute deviation multiplier value of 1.6 that Plymen & Preveit (1972) found to be optimal for UK investment trusts; the multiplier would be 2 if the price formation process was consistent with the classical finance models. Clarkson (1978, 1981) shows that the 1.6 value for the multiplier also works exceptionally well for British Government securities and for UK equities in general. Furthermore, it is noted in Clarkson & Plymen (1988) that for a security price exhibiting simple harmonic motion (i.e. a pure sine wave) the theoretical value of the multiplier is 1.57.

Numerous other studies also show that real world capital market behaviour departs markedly from that predicted by the classical finance models. In particular, Shiller (1989) documents how stockmarkets exhibit “excess volatility”; the general magnitude of share price changes is vastly in excess of what could be attributed to “rational” investor behaviour in response to changes in fundamental factors such as profits or dividends. Also, Peters (1991) describes how the Hurst exponent, a very powerful diagnostic statistic, generally has a value of around 0.7 in all major equity markets as against the value of 0.5 that would be consistent with the classical finance models. Finally, Mills (1991) reports conclusions similar in nature to those in Clarkson (1978, 1981) and Clarkson & Plymen (1988), namely that simple “confidence” limits have a far higher predictive power than would be expected under the classical finance models.

Perhaps the strongest simplifying assumption behind the CAPM is that the dynamics of capital market price changes are invariant over time, in the sense that, no matter what time period is investigated, the general statistical properties of price changes are constant. To see how unrealistic this assumption is, consider what Rudd & Clasing (1982) describe as the most common form of the CAPM:

$$E_j - i_F = B_j (E_M - i_F) \text{ for all risky securities } j,$$

where E_j is the expected return on asset j ,
 i_F is the “risk-free” rate of interest,
 E_M is the expected return on the market portfolio,
and B_j is a constant, namely the “beta” of asset j defined as:

$$B_j = \text{Cov} (r_M, r_j) / \text{Var} (r_M),$$

where r_M is the random rate of return on the market portfolio
and r_j is the random rate of return on security j .

In practice, the value of beta is usually estimated as the gradient of the straight line best fit of plots of r_j against i_F . But consider now the dichotomy of share price behaviour during

periodic bouts of general investor fearfulness as against the behaviour during what might be called “normal” or benign market conditions. A classic example here is of UK bank shares during the short, but very sharp, market setback in the third quarter of 1998. In response first of all to worries of worldwide economic slowdown as a result of economic dislocation in the Far East, then to Russia’s default on its debt payments, then to acute anxiety over the possible collapse of the Long-Term Capital Management hedge fund, UK bank shares typically fell by around 45% as against a fall in the FTSE All-Share Index of close to 25%. Since the betas of these shares were close to 1, the CAPM would have predicted share price falls only of the order of 25%.

This one counterexample alone is sufficient to demonstrate not only that the CAPM can lead to potentially catastrophic underestimates of investment risk but also that the applicability of elementary probability theory to capital market behaviour has to be challenged. As a first step towards the construction of a potential successor to the CAPM, these general probability considerations are discussed in Section 2.

2. Probability Considerations

It has often been observed that the probabilistic foundations of many attempted applications of econometrics are unsound, in that the samples of past behaviour from which probabilistic statistics are derived are generally far too few in number to give any reliable indication of likely future behaviour. Very extensive repetitions of highly standardised trials are a necessary prerequisite of any attempt to use the past to obtain estimates of future behaviour. This, of course, is very difficult in the context of capital market behaviour. Gray (1967), for instance, points out that it is meaningless to talk of the probability that War Loan, an undated British Government stock, will be above a certain price on a given future date. However, the same might have been said four hundred years ago about the probability that a man now aged 30 will live to age 60. This suggests that a promising line of attack will be to examine first of all the conceptual foundations of the exceptionally successful application by actuaries of probability theory to life insurance.

The first stage is to screen out those lives who propose for insurance that are unsuitable for acceptance at “normal” rates. This is accomplished through an underwriting process which will involve as a minimum the completion of a proposal form to elicit information on the proposer’s medical history and on other relevant factors such as occupation, and may also involve a medical examination. Those lives who are accepted at normal rates form a select (in the actuarial sense) subgroup of the group of all lives who propose for life insurance.

The second stage is to obtain reliable estimates of the probability of survival from any one given age to any other. This involves the construction of a mortality table; crude estimates of the probability of dying within one year from any given attained age are obtained from the mortality of assured lives (generally on an industry, rather than a company, basis) as the ratio of the number of deaths to the total of those exposed to risk. These crude rates for different ages are then smoothed, or graduated, and summarised in a life table. Extensive experience has shown that the derived mortality rates provide a very reliable estimate of the average rates experienced by a large group of assured lives of the same age. However, these mortality rates cannot be regarded as a reliable estimate of the mortality expected to be experienced by any one particular assured life.

Since the derived mortality rates are mean values around which random fluctuations will occur, the third stage is to identify the maximum sum assured that it is prudent for a company to accept for any one life. The basic statistical result in this connection is that, for n lives of a given age, each with unit sum assured, and one-year survivorship and mortality rates of p and q respectively, the variance of the annual amount of the death claims is npq .

A parallel application of probability to equity portfolio selection already exists, in that the “group of 70 shares”, the growth/rating utility ranking measure, and the “model portfolio” of 30 shares described in Clarkson (1998a) correspond exceptionally closely to the above three stages in the application of probability theory to life assurance. The three corresponding stages in the application of probability theory to equity portfolio selection are discussed in turn in Sections 3, 4 and 5.

3. Identifying a Select Subgroup

We begin with the shares of all 100 of the constituents of the FTSE-100 Index, essentially the 100 largest UK companies in terms of market capitalisation, as the universe of shares from which an equity portfolio has to be constructed. Since the general approach will be to assess shares in terms of expected short-term earnings growth, we exclude on grounds of heterogeneity real estate companies and investment trust companies, since net asset value rather than earnings growth is the primary valuation criterion in these cases.

For the remaining shares, we wish to introduce some screening process that corresponds to the underwriting process in life assurance which screens out the weakest lives in terms of expected future mortality. Bearing in mind the objective, as discussed in Section 1, of incorporating a downside approach to risk, the obvious line of attack here is to screen out those companies perceived as most likely to experience serious setbacks in earnings per share in the future.

Suppose that a company has just declared earnings per share E_0 immediately after the end of its financial year and that the earnings per share for the previous financial year and for the one before that were E_{-1} and E_{-2} respectively. Suppose also that we have earnings per share estimates E_1 and E_2 for the current and subsequent financial years respectively. We now wish to construct a “quality of earnings” measure Q as a function of E_{-2} , E_{-1} , E_0 , E_1 and E_2 .

Consider four different shares A, B, C, and D with earnings per share profiles (in pence) as shown below:

	E_{-2}	E_{-1}	E_0	E_1	E_2
A	10	11	13	16	20
B	10	15	12	14	16
C	10	8	6	10	11
D	10	10	8	6	10

For share A, there is an increase in earnings each year, with the smallest increase being 10%. In this case Q is taken as 0.1, namely the smallest proportionate increase.

For share B, the poorest outcome is a decrease over one single year of 20%. In this case Q is taken as -0.2.

For share C, there are two successive annual falls of first 20% and then 25%. In this case Q is taken as -0.4, the proportionate fall over the period of two years.

For share D, there is a fall in earnings of 0.4 over two years as with share C. However, part of the fall relates to an estimate for the current financial year rather than to historic information. On the basis that many investors may be unaware of the earnings fall that is expected to be announced in a year's time, it is prudent to regard this expected forthcoming fall of 25% as being far more serious than one that has occurred in some period in the past. Accordingly, a further contribution to quality of earnings is calculated as four times the expected proportionate fall to a year hence, namely -0.25×4 , i.e. -1. The resulting quality of earnings value is now $-0.4 - 1$, i.e. -1.4.

The measure Q can now be defined as below:

Case 1 – no annual fall in earnings

The minimum annual proportionate increase in earnings

Case 2 – one or more annual falls in earnings

The maximum proportionate fall in earnings from a local maximum to a local minimum plus four times any expected proportionate fall over the forthcoming twelve months.

These examples assume that the situation is being analysed immediately after the end of the financial year and the announcement of annual results, whereas in general assessments have to be made part of the way through the financial year. This is done by using “real-time” earnings interpolated on a linear basis for any fall over the forthcoming twelve months. If, for example, we were three months into the current financial year for share D, current earnings would be $0.75 \times 8 + 0.25 \times 6$, i.e. 7.5, and earnings a year hence would be $0.75 \times 6 + 0.25 \times 10$, i.e. 7, giving a proportionate fall of 0.067 over the next year and a value of Q of $-0.4 - 4 \times 0.067$, i.e. -0.668.

There is an implicit assumption above that all five values of earnings per share are positive. If any values of earnings are negative, then Q is taken as minus infinity.

The required select subgroup of shares, the “group of 70”, consists of the 70 with the highest values of Q .

4. Constructing a Complete Ranking of Attractiveness

The objective is to rank the “group of 70” shares by attractiveness in terms of expected short-term earnings growth. Let the proportionate change in earnings over the next twelve months, using real-time earnings as described above, be G , and let the ratio of earnings twelve months hence to the current share price be R . Then for a given value of G the attractiveness of a share in terms of expected future share price performance will decrease as R increases. Also, on the basis that the short-term growth rate G is the only measure of future earnings growth that can be taken into account, for a given value of R the attractiveness of a share in terms of expected future price performance will increase as G increases. We therefore wish to find an

appropriate utility measure $U(G, R)$ as a function of G and R where the first partial derivative with respect to G is positive and the first partial derivative with respect to R is negative.

Consider the situation in mortality studies where, for other than very low ages, the annual rate of mortality increases with age. The utility ranking measure formulation of this single predictor variable, attained age, can be expressed as

$$\text{attained age} = \text{current year} - \text{year of birth.}$$

This suggests the following parallel formulation for the share selection utility ranking measure:

$$U(G, R) = G - cR,$$

where c is a positive constant.

It might be thought by some that the obvious next step is to evaluate c as the value which, when applied to current data, obtains the best fit on some “least squares” criterion. However, this tacitly assumes an equilibrium position, whereas, as discussed in Section 1, equilibrium cannot be taken for granted. Moreover, the objective is to find the most powerful ranking measure in terms of future predictive power. As c increases from zero to infinity, the ranking measure $U(G, R)$ changes gradually from attaching zero weight to R to attaching zero weight to G , and it seems obvious on intuitive grounds that some particular value of c will result in a unique maximum in terms of predictive power. As described in Clarkson (1998a), when tests were carried out using publicly available data for both historic and expected future earnings, there did indeed exist an optimal value of c that differed from the equilibrium value. It is interesting to note that this was consistent with empirical studies of the U.S. equity market over a period of more than 40 years as described in O’Shaughnessy (1996).

5. Establishing Prudential Portfolio Guidelines

Consider first of all the corresponding situation in life assurance. An insurance company will often, for administrative convenience, specify some maximum sum assured for any one life that is the same for all entry ages. The parallel approach for equity portfolio selection would be to specify the maximum proportion to be invested in any one share and then to hold the required number of the most attractive shares as reflected by the utility ranking measure. This might be 5% in each of the 20 shares with the highest utility value, or 2½% in each of the top 40, and is the approach suggested by Williams (1938).

These crude approaches, however, are not optimal in theoretical terms. In the life assurance case, the npq basic value for the variance of death claims shows that the higher the entry age the higher will be the value of npq and hence of the variance of death claims. The maximum sum assured for any one life should therefore decrease as entry age increases. In the share selection situation, the downside philosophy of risk that we wish to embed in the procedures means that, assuming the utility ranking measure does indeed have significant predictive power, the “most attractive” shares will involve significantly lower risk (assuming similar variability about mean returns) than the “least attractive” shares within any equally weighted portfolio. The weightings should therefore taper downwards from the “most attractive” shares to the “least attractive” shares.

A degree of pragmatism is appropriate here. For, say, the ten “most attractive” shares there are likely to be other significant factors not taken into account in the highly simplistic share selection process described in Section 4. This suggests that equal weightings for these shares might be appropriate, with tapering below these. On similar pragmatic grounds we can argue that the computational advantages of having equal, rather than tapered, weightings for each subsequent group of ten shares outweigh any slight theoretical benefits from having strictly tapered weightings.

In the light of the above considerations, the “model portfolio” of 30 shares whose performance will be monitored in detail is taken as consisting of 4% in each of the ten most attractive shares in terms of the utility ranking measure, 3½% in each of the next ten, and 2½% in each of the subsequent ten.

Three general observations are appropriate at this point. First, the equity selection procedure described in this and the preceding two sections, and discussed in more detail in Clarkson (1998a), is only one of many possible approaches. However, this paper would have been of very little theoretical or practical value had it not been possible to show that its conclusions are supported by at least one equity selection procedure that has been fully documented in a refereed journal. Second, the earnings data used to obtain the performance results described later in the paper come from a monthly investment service that is available by annual subscription to any interested person and hence comes into the category of “publicly available information” for the purposes of tests of the validity of the “Efficient Market Hypothesis”. Third, the fully documented nature of the approach, together with the ready availability of earnings data to implement it, means that, probably for the first time, a transparent algorithmic procedure exists whereby even someone with no practical experience of investment can construct an equity portfolio likely to produce superior long-term performance using nothing more elaborate by way of mathematical techniques than primary school arithmetic.

6. An Efficiency Identity

In the world of engineering, the term “efficiency” relates to the relative effectiveness with which potential energy is translated into useful output energy. For example, the statement that a certain type of coal-burning steam engine has an efficiency of 12% means that 12% of the calorific energy of the coal burned is translated into useful output energy. Consider now the observation by Benjamin Graham, the founding father of security analysis, to the effect that “the market price of a security only rarely coincides with the true value around which it tends to fluctuate”. Then, regarding equity portfolio management as a “financial engineering” process which endeavours to translate price changes within a particular subgroup of equity shares into superior investment performance, we can obtain a simple but highly powerful and practical efficiency measure for any particular equity selection procedure as the ratio of the actual increment in return above an appropriate benchmark to the potential increment that could have been achieved by investing in those shares that, with hindsight, showed the highest returns over the period in question. Yet again a simple may be useful.

Over the twelve months from 2nd June 1997, the model portfolio of 30 U.K. equity shares resulting from the equity selection process described in Sections 3, 4 and 5 showed a capital return of 40.5% as against a capital return of 31.5% for the “group of 70” select subgroup and a capital return of 62.8% for the corresponding “hindsight portfolio” consisting of 4% in each

of the 10 top performing shares, 3½% in each of the next 10 in terms of performance, and 2½% in each of the subsequent 10. The efficiency ratio is then:

$$\begin{aligned} E &= (40.5 - 31.5) / (62.8 - 31.5) \\ &= 0.288, \\ &\text{i.e. 28.8\%.} \end{aligned}$$

In an obvious notation, the general identity for the efficiency ratio E that has been achieved from an equity selection procedure that takes only short-term earnings growth into account can be written as:

$$E = (R^P - R_M) / (R_H^P - R_M),$$

where P means “subject to the relevant portfolio guidelines”
 H means “highest on hindsight over the period”
and R_M is the “group of 70” benchmark or “market” return.

Suppose for the moment that for the particular equity selection procedure in question the efficiency ratio is reasonably stable at a mean value of E_I , with fluctuations about this value being random in nature. Then we can regard any achieved value of E as this mean value E_I plus a small residual amount E_R that is random in nature. Re-arranging the above equation and substituting for E gives:

$$R^P - R_M = (E_I + E_R)(R_H^P - R_M).$$

The resemblance to the most common form of the CAPM as described in Section 1, namely:

$$E_j - i_F = B_j (E_M - i_F),$$

is striking.

7. Incorporating Long-Term Earnings Growth

The equity selection procedure described in Sections 3, 4 and 5 can be generalised to accommodate long-term earnings growth by replacing the utility ranking measure:

$$U(G, R) = G - cR$$

by the utility ranking measure:

$$U(G, G_L, R) = G + kG_L - cR,$$

where G_L is a measure of the expected rate of earnings growth beyond the horizon to which short-term earnings growth is assessed and k is the positive constant which maximises the predictive power of this revised utility ranking measure.

Although the precise formulation of G_L is unimportant, a simple but powerful approach here is to use the basis described in Clarkson (1981). Each company is rated on a scale of, say, 1 to 10 for each of four essentially independent factors – growth of turnover, size and stability

of profit margins, financial strength, and a residual factor which can be described as “operational flexibility”. The long-term earnings growth measure G_L is then the sum of the scores for each of these four factors. The enhanced mean efficiency ratio of the equity selection procedure can now be expressed as $E_1 + E_2$, where E_2 , like E_1 , is positive, and the revised form of the new model is now:

$$R^P - R_M = (E_1 + E_2 + E_R)(R_H^P - R_M).$$

This provisional model is a general mathematical statement to the effect that the achievement of superior long-term equity portfolio performance is a double-increment process subject to small random variations, with the increments corresponding the predictive powers of suitably formulated measures of expected short-term earnings growth and of expected long-term earnings growth.

8. Accommodating Behavioural Finance

It had been expected that the equity selection procedure described in Section 3, 4 and 5 and first implemented in June 1997 would be at least modestly successful in identifying undervalued shares and that any divergence in performance between “cheap” and “dear” shares would emerge fairly smoothly over time. However, not only was the long-term power of the relatively crude selection process far greater than expected, but the short-term and medium-term fluctuations in relative performance were an order of magnitude larger than what had been expected. It is intended that a further paper setting out the detailed results for the first four years of operation from June 1997 will be written later this year, but brief mention will be made here to three periods so far of very different performance characteristics.

The first, which is described in detail in Clarkson (1998a) is the period of twelve months from the beginning of June 1997, when the UK equity market showed a strongly rising trend and could be described as having been in a benign or “normal” mode. After 3, 6, 9 and 12 months, the capital performances of the model portfolio relative to the FTSE All-Share Index were 103.6, 108.0, 106.0 and 109.8 respectively, while the corresponding performances of the ten least attractive shares were 95.3, 95.6, 90.0 and 92.6 respectively. It seemed highly likely that such strong and regular divergences in performance could not continue forever.

The second period was the twelve months from the end of June 1998, when the UK equity market fell 25% in the three months and then staged a fairly steady recovery over the following nine months. The capital performances of the model portfolio as at 29th June 1998 relative to the FTSE All-Share Index after 3, 6, 9 and 12 months from the end of June 1998 were 90.6, 93.4, 98.8 and 102.4 respectively, while the corresponding relative performances of the ten least attractive shares as at 29th June were 92.3, 103.4, 101.6 and 98.7 respectively. The main downward thrusts in the market collapse were characterised by three quite distinct phases: first, sharp price falls in a few companies directly affected by economic weakness in the Far East, by Russia’s debt default, or by the critical situation of the Long-Term Capital Management hedge fund; second, sharp price falls in many other companies that might just possibly suffer from these problem areas; and third, indiscriminate selling of highly liquid shares, regardless of their relative immunity to these temporary economic storms, in a classic “flight to liquidity”, namely the unthinking selling of equity shares at any price and reinvestment in supposedly “safe” investment classes such as high quality government bonds

and cash, regardless of the overwhelming empirical evidence that, over the long term, equities have by far the best performance record. A more detailed commentary on this period appears in Clarkson (1999).

The third period was the fifteen months from the end of December 1999 to the end of March 2001. “New economy” shares, which consist mainly of those in the TMT (technology, media and telecommunication) sectors, having already been very strong over the second half of 1999, soared in price over the first ten weeks of 2000 on general investor euphoria over their perceived long-term growth prospects. Thereafter prices fell, slowly and spasmodically at first, then at a faster pace on a more general front, and finally in the first quarter of 2001, at a very dramatic rate. The performance of “old economy” shares, the remainder of them market, was the exact mirror image. The table below shows, at various durations, the performance of the FTSE All-Share Index and the absolute and relative performances of the model portfolio as at 30 December 1999 (all of which were “old economy” shares) and of the “group VII” ten least attractive “group of 70” shares at that date (all but one of which were “new economy” shares).

Duration in months	Index	Absolute		Relative	
		Model Portfolio	Group VII	Model Portfolio	Group VII
1	91.8	85.7	91.8	93.4	100.0
2	90.6	81.0	111.0	89.4	122.5
3	94.8	92.6	91.4	97.7	96.4
6	95.4	94.8	71.7	99.4	75.2
9	93.4	90.3	75.8	96.7	81.2
12	92.0	104.9	61.2	114.0	66.5
15	83.6	102.8	50.6	123.0	60.5

A brief description, in the language – where appropriate – of behavioural finance, of the markedly different phases of performance to date of the model portfolio and of Group VII is given below.

Over the twelve months from the beginning of June 1997, the magnitudes of both the rise in the FTSE All-Share Index (namely 28.0%) and the outperformance of the model portfolio (namely 9.8%) suggested “forecast feedback” over-reaction in the latter part of the period with regard to the medium-term prospects for both economic growth and for earnings per share growth over the whole market. The initial setbacks in the middle of 1998 then triggered over-reaction on the downside, namely sharp mark-downs in the share prices of companies not materially affected by the adverse economic factors that were visible at that time. This further market fall then led to a “flight to liquidity” that constituted myopic loss aversion – panic selling both of the model portfolio shares (where investors suddenly began to question whether the expected earnings growth would materialise) and of group VII shares (which were mainly highly liquid “blue chip” shares, the only ones that could be sold in any size by investors wishing to take cash out of the market). For most of the recovery phase, investors once again exhibited myopic loss aversion by favouring group VII shares which, being mostly companies with little exposure to overseas economics or cyclical industries, had fallen very little during the “market collapse”. Much if not all of the strong upward re-rating of “new economy” shares in the second half of 1999 could be regarded as a rational response to their undoubtedly superior growth prospects over the very long term. However, the quite extraordinary “new millennium” euphoria surrounding “new economy” shares early in 2000

is a classic example of reckless overconfidence – what US Federal Reserve chairman Alan Greenspan has described as “irrational exuberance” arising out of “lottery mentality”. This lottery metaphor mirrors Adam Smith’s very same use of the lottery analogy more than two hundred years ago to support his conclusion that “the chance of gain is by every man more or less over-valued, and the chance of loss is by most men under-valued.” The corresponding equal and opposite investor aversion to “old economy” shares was yet another instance of myopic loss aversion. Once the downward price trend in “new economy” shares became established, a highly dangerous example of systemic risk came into play as a direct consequence of the earlier reckless overconfidence. Many unsophisticated private individual investors, having bought highly speculative “new economy” shares on margin at the height of the “irrational exuberance”, were faced with margin calls from their stockbrokers. Some of these unfortunate investors had their shares sold out by their stockbrokers, triggering further price falls, which in turn triggered further margin calls and yet further price falls, and so on in a self-feeding downward spiral reminiscent of the one that economist John Kenneth Galbraith identified as the main reason for the extreme severity of the “Wall Street Crash” of 1929 and of the length and severity of the consequential economic depression in the United States.

In the provisional form of the Equity Selection Procedure Model as set out at the end of Section 8 it is assumed that the residual efficiency term E_R is fairly small in magnitude and random in nature, which would be consistent with the rationality assumptions underlying the classical finance models. However, the above commentary shows that almost the entire period of nearly four years from June 1997 to date consists of distinct subperiods during which E_R was either increasing or decreasing fairly regularly at a rate which was higher in absolute terms – often by an order of magnitude – than the rate of change in price attributable to the $E_1 + E_2$ trend rate of efficiency. Furthermore, the high rate of change of E_R within each subperiod could be explained by one of the behavioural finance classifications of investor behaviour that is “irrational” in the context of classical finance theory. Accordingly, we introduce a new term E_3 which corresponds to the underlying change in E as a result of “systematic irrationality” (in the context of classical finance theory) on the part of investors. E_R can now be regarded as being both random and small in magnitude. The final form of the ESPM is thus:

$$R^P - R_M = (E_1 + E_2 + E_3 + E_R)(R_H^P - R_M).$$

9. The January Effect

Even after only the first twelve months’ experience of the equity selection procedure described by Sections 3, 4 and 5 it was obvious that the dynamics of price changes within equity markets were far more complex, and subject to far greater variability over time, than had previously been thought to be the case. This led the present author to formulate in Clarkson (1998b, 1998c) the outline of an alternative to the time-invariant “rational expectations” models of classical finance theory. This “fundamental preferences model” explicitly recognises that investors’ preferences in the aggregate for different fundamental attributes such as “quality”, “liquidity”, “momentum”, “size”, “new concept”, “ethicality” and “familiarity” varies very significantly over time, often in a “short-termism” extrapolation of the recent past into the foreseeable future. The over-reaction, over-confidence, irrational exuberance and myopic loss aversion examples of such changes in fundamental preferences as described in Section 8 are essentially unpredictable as regards their precise timing, even though these behavioural patterns are readily understandable – at least to investment

practitioners – with the benefit of hindsight. An example of a change of fundamental preferences in the aggregate that is highly predictable in timing is described below.

Classical finance theory implicitly assumes that investors constitute what actuaries call a “stable population” in that their aggregate preferences remain invariant over time. Consider, however, the heightened attention that many people devote to equity investment strategy around the turn of each year. Private individual investors, financial journalists, pension fund trustees, and directors of financial companies all tend to devote far more attention around the turn of the year, as against any other time throughout the year, to what their general investment strategy should be. As compared to investment professionals such as investment analysts and fund managers who are day-to-day market practitioners all year long, these private individuals, trustees and directors are far less aware of the short-term and medium-term outlook for company profits and earnings per share. The latter economic agents will accordingly pay far more attention to perceived long-term earnings prospects than to the short-term and medium-term outlook, with the result that the aggregate investor preference for long-term growth will increase around the turn of each year. Since the model portfolio constructed in line with the procedures of Sections 3, 4 and 5 is strongly biased towards short-term earnings growth, we expect the model portfolio and group VII to underperform and outperform respectively over January but for this pattern to reverse by, say, the end of March, by which time the short-term estimates of earnings to the end of the previous calendar year have crystallised into known information. For each of the years 1998, 1999, 2000 and 2001, each of these expected relative performances had the expected sign, i.e. either positive or negative. The probability of this happening by mere random chance is 1 in 256 or 0.004.

Similar arguments can be applied to equity market levels, in that the turn of the year is when many investors decide on their general asset allocation strategy for the year ahead. Since for both the UK and the US capital markets the long-term returns are far higher for equities than for bonds or cash, these turn of the year strategic decisions will tend to focus more on reasoned long-term considerations as against the tactical short-term considerations that day-to-day investment professionals tend to follow – as vividly documented in Chapter 12 of Keynes (1936) – in their endeavour for personal and commercial success. These arguments suggest that, although negative economic factors may of course prove to be stronger on occasions, January is likely to be the month that shows the highest relative frequency of rises in equity markets. Such does indeed prove to be the case, and this “January effect” is often cited as being one of the very few generally accepted instances of capital market inefficiency.

10. Skill and Judgement

In precisely the same way as coal-burning steam engines of different designs can be expected to differ in efficiency, different processes for obtaining a select subgroup of “quality” shares can be expected to lead to different values of E_1 and E_2 in the context of equity portfolio selection. An informative demonstration of this over calendar year 2000 is described below.

The HS REFS investment service used for earnings data incorporates a very much stricter screening process than the one described in Section 3. The author’s judgement was that many perfectly sound companies were excluded unnecessarily, thereby significantly reducing the enhancement in performance that could be achieved.

At 2nd June 1997 it was necessary to go as far down as the share rated at number 44 in attractiveness within the “group of 70” (selected in terms of the Section 3 rules) before a total

of 30 shares admissible under the HS REFS screening procedure was reached. The alternative model portfolio then consists of 4% in each of the 10 “HS REFS admissible” shares that are most attractive in terms of the Section 4 utility ranking measure, 3½% in each of the next 10 such shares, and 2½% in each of the subsequent 10. Over the twelve months to 1st June 1998, this alternative model portfolio outperformed the FTSE All-Share Index in capital terms by 7.6%, more than 2% below the corresponding outperformance of 9.8% by the “Section 3” model portfolio. It is interesting to note that many “blue chip” companies such as Bank of Scotland, BOC, Boots, General Electric, Glaxo Wellcome, Grand Metropolitan, Great Universal Stores, Lloyds TSB, Royal Bank of Scotland, and Shell are admissible as at 2nd June 1997 under the Section 3 selection rules but are screened out by the HS REFS selection rules.

At 30th December 1999 it was necessary to go even further down the “group of 70” shares, namely to the one ranked at number 65, before the alternative model portfolio could be completed. In the twelve months to 29th December 2000 this alternative model portfolio outperformed the FTSE All-Share by 11.4%, as against 14.0% for the “Section 3” model portfolio. By 30th March 2001, the divergence in performance was remarkably high; an outperformance of only 12.5% for the alternative model portfolio as against 23.0% for the “Section 3” model portfolio. Again there are many “blue chip” companies amongst those excluded, notably Barclays, Diageo, Glaxo Wellcome, HSBC, Halifax, Lloyds TSB, National Westminster Bank, Prudential, Rio Tinto, and Unilever.

It is patently obvious that, in the estimation of long-term earnings growth rates, the predictive power of the process will increase with the skill and practical experience of the individual. Accordingly, both in the design and implementation of a three-stage equity selection procedure such as that defined in Sections 3, 4, 5 and 7 there can be no “absolutes” in terms of what degree of superior long-term performance can be achieved. Instead, as with other competitive areas of human behaviour, such as tennis or chess, innate levels of relevant skills, and judgemental abilities resulting from practical experience will be crucial determinants of the degree of success that can be achieved.

11. Bond Markets and Long-Term Capital Management

Part of the equity market commentary in Section 8 suggests a very general cyclical pattern comprising six distinct phases: recovery from oversold levels, further price rises on improving fundamentals, still further rises to overbought levels driven by forecast feedback and the entry for the first time in the cycle of relatively unsophisticated investors, falls in response to isolated instances of economic or corporate problems, further falls on more general worries of problems ahead, and finally further very sharp falls in a “flight to liquidity” when many institutional investors attempt to reduce their equity exposure in the light of the now established downtrend and many totally disillusioned private individual investors “throw in the towel” and sell out at any price.

Similar investor psychology is readily visible in bond markets. Early in the upswing of a “confidence cycle” there is a string preference for high quality government securities. As memories of previous default problems fade, the recovery in lower quality bonds triggers a “forecast feedback” switch in their favour which takes them to low yield spends that are vulnerable to the first hint of future default worries.

Suppose we have an upswing in confidence over three years taking yields on undated high quality bonds from 5% p.a. to 4% p.a. and taking yields on low quality bonds from 7% p.a. to 5% p.a. a reduction from 200 to 100 basis points in the “spread” over the period. Ignoring reinvestment of dividends, an investment of 100 would have given end-period values of 137.0 and 161.0 for high quality and low quality bonds respectively. Suppose that serious default worries appear over the ensuing year, causing a “flight to quality” that leaves the high quality and low quality bonds on yields of 3% p.a. and 8% p.a. respectively. Then a value of 100 at the beginning of this final year gives 137.3 at the end of the year for high quality binds but only 67.5 for low quality bonds.

Consider now the decline and fall of Long-Term Capital Management during 1998. The key investment strategy had been to have highly leveraged long positions in low quality bonds and highly leveraged short positions in very high quality bonds such as US Treasuries. For several years this gave a dramatically high fund default worries in 1998 led to a “flight to quality”, the result was an even more dramatic collapse in performance which, together with the equally suicidal policy of betting on a massive scale that volatility in equity markets would decline at a time of increasing investor fearfulness, proved terminal. A full account of the rise and fall of LTCM is given in Lowenstein (2001).

12. General Implications of the Equity Selection Procedure Model

It is apparent from the commentary in Section 8 that the “systematic irrationality” term E_3 in the Equity Selection Procedure Model is often far more powerful than the two “rational behaviour” terms E_1 and E_2 . The first and most fundamental implication is therefore that the use of the Equity Selection Procedure Model as a measurement framework for equity share price dynamics shows that “systematic irrationality” tends to be the rule rather than the exception, but these departures from rational behaviour cannot be observed using classical finance models.

The second implication is that any attempt to use the CAPM or any other time-invariant methodology of classical finance theory is unsound in the extreme. The dynamics of equity price changes can vary greatly from one period to another, and in particular the general degree of turbulence can, without warning, rise far above what has been observed over the previous few years.

The most important practical implication is that, over the long term, a structural and systematic approach to equity portfolio construction is capable of achieving far higher “efficiency” (in the sense used in Section 6) than is generally thought to be the case. More accurate estimates will be possible when the detailed investigation of the first four years’ performance has been completed, but it seems likely that an average annual outperformance of at least 5 % for the model portfolio, corresponding to an average “efficiency ratio” of at least 20%, should be achievable using only short-term earnings estimates.

Clearly the degree of outperformance will vary with the precise nature of the screening, ranking and diversification procedures used. An alternative to the screening procedure of Section 3 has already been mentioned. Alternative procedures for ranking shares and for portfolio construction are discussed briefly in the following two sections.

13. A Classification of Equity Selection Processes

A necessary preliminary stage before further useful scientific investigation can be carried out is often to “divide difficulties” by partitioning the objects of the investigation into distinct classes. As regards equity selection processes, of which the process described in Section 4 is an example, no such classification seems to have been attempted before, presumably because most financial economists believe that stockmarkets are so nearly “efficient” in terms of all knowable information being already encapsulated in prices that it is pointless to attempt to identify “undervalued” securities. In this section we describe a numerical classification system similar in concept to that used for autoregressive integrated moving average (ARIMA) time series, which gives classifications such as ARIMA (0,1,1) and ARIMA (1,2,0). The classification basis is $ESP(w, x, y, z)$, where ESP stands for equity selection process and w, x, y and z are defined below:

w is zero if it is not assumed that equilibrium exists, and 1 if equilibrium is assumed.

x is the length in years of the horizon at which the projected price-earnings ratio (or, equivalently, price-dividend ratio on the basis of a constant dividend payout ratio) is expressed as some function of predictor variables.

y is the number of factors that are allowed to vary in determining a “best fit” specification of the underlying price model.

z is the number of factors that are allowed to vary to determine the optimal specification of the process in terms of predicting future price performance.

Then the process described in Section 4 is $ESP(0,1,0,1)$, and when it is generalised as described in Section 7 to incorporate long-term earnings growth it becomes $ESP(0,1,0,2)$. Clearly the latter should be the more powerful by at least a modest margin, but this has not yet been verified empirically. The equity model described in Weaver & Hall (1967) is $ESP(1,0,5,0)$. This multiple regression model is a standard example of what is generally known in the finance literature as a factor model. The Clarkson (1981) model, which is similar in certain conceptual respects to the Weaver & Hall model, is $ESP(1,1,3,0)$. Although there are many different versions of the dividend discount model, the most commonly used versions involve specific estimates of dividends to a horizon of, say, ten years, and then the discounting of estimated dividends at a rate of interest equal to or slightly higher than the redemption yield on long-dated government bonds. Such a model is $ESP(0,10,0,0)$.

In principle it should be possible to back-test all five of the above models using consistent input data to rank them in order of predictive power, but this would involve a vast amount of computational effort and even then might not lead to statistically meaningful conclusions. However, based mainly on the author’s practical experience of the $ESP(1,1,3,0)$ and $ESP(0,1,0,1)$ models, some useful conclusions can be drawn with a reasonable degree of confidence.

The most important, and quite unexpected, conclusion in this context is that the predictive power of short-term earnings growth alone (as shown by the results of the $ESP(0, 1, 0, 1)$ model) is extremely high. Accordingly, the dividend discount model, which attaches very little weight to short-term dividend growth, and the Weaver & Hall model, which treats short-term dividend growth as one of five factors taken into account in the regression process, will be very far from optimal in terms of predictive power. Secondly, the assumption of equilibrium and the use of variable parameters to obtain some “best fit” specification will

result in a model that is too flexible for predictive purposes in the sense that it will tend to accommodate extremes of mispricing rather than highlighting them as exploitable anomalies. The tentative overall conclusion is therefore that, of the five equity selection processes discussed above, the ESP (0,1,0,2) approach incorporating the methodologies of Section 4 and Section 7 offers the most promising framework in terms of a disciplined and systematic approach to equity share selection.

14. Equity Portfolio Selection Revisited

The vast majority of institutional investors attempt to reduce risk within their equity portfolios by putting in place a set of guidelines which normally includes a fixed maximum amount, such as 1%, 2% or 3%, by which the weighting in a particular share is allowed to vary from its market weighting, as measured against a broadly based index such as, in the UK, the FTSE All-Share Index. On the assumption that satisfactory processes have been identified for the initial screening of shares and for ranking the screened shares in order of attractiveness, common sense suggests that this attempt to reduce risk by constraining portfolio weightings to be close to market weightings will be counterproductive in the medium to long term as compared to the model portfolio constructed in accordance with the approach set out in Sections 3, 4 and 5. Less will be invested in shares that are ranked as most attractive, and significant investment will have to be made in large companies which are either ranked as relatively unattractive or would otherwise be excluded entirely by the initial screening process.

Consider, by way of illustration, the model portfolio of 30 shares as at 30th December 1999 and the corresponding constrained portfolio resulting from a 2% maximum deviation in weighting being superimposed on the ranking system. The latter portfolio has 14.8% in shares that are either ranked as relatively unattractive or would otherwise have been excluded by the screening process described in detail in Section 3, with the remainder consisting of 2% in excess the market weighting for as many of the most attractive shares as are required to total 85.2%. For an investment of 100 as at 30th December 1999, the capital values of the model portfolio and the constrained portfolio as at 29th December 2000 were 104.9 and 98.4 respectively, with the FTSE All-Share Index value being 92.0. In other words, the unthinking implementation of the most widely used “risk minimisation” teaching of modern finance theory reduced the investment return by the staggering amount of more than 6%. By 30th March 2001, after a quarter in which most “new economy” shares fell sharply and the Index fell further to 83.6, the superiority of the model portfolio selected as at 30th December 1999 was even more marked. Quite remarkably, the model portfolio outturn remained in positive territory at 104.9 despite the market fall of more than 16% since 30th December 1999, while the constrained portfolio fell further to 94.7.

An additional and supposedly more sophisticated risk control methodology that many institutional investors follow, often at the direction of trustees or others with overall responsibility for the investment process, is to constrain the portfolio profile to be close to the market average in terms of fundamental characteristics such as price-earnings ratio, dividend cover, balance sheet gearing, and exposure to “third world” economies. Since investors in the aggregate tend to overreact wildly from time to time in their attitudes to these characteristics, these well intentioned constraints on the portfolio balance will tend to prevent the buying of shares that are ranked as very attractive, thereby reducing the expected long-term return and increasing the risk of underperforming an appropriate benchmark.

The behavioural finance expression “myopic risk aversion” is most commonly applied to the tendency of many investors to prefer cash or bonds to equities for long-term investment despite a wealth of empirical evidence, such as described in a US context in Thaler & Williamson (1994) and in a UK context in Clarkson (2000), showing that equities have by far the best long-term performance record. The above discussion suggests that myopic loss aversion is also a highly appropriate label for the unthinking implementation of portfolio constraints by most investment managers and trustees in a misguided endeavour to “reduce risk”.

15. Stockmarket Efficiency Revisited

The ability of the equity selection procedure described in Sections 3,4 and 5 to achieve above average long-term equity portfolio returns contradicts what has become known as the “semi-strong version” of the Efficient Market Hypothesis, namely the conjecture that equity share prices react so quickly, and so accurately, to new information that it is pointless to attempt to achieve superior performance using information that is publicly available. Paul Samuelson, the first winner of the Nobel Memorial Prize for Economic Science, described the situation as follows in a 1994 article:

“A few stars with superior information and clever ability to squeeze excess risk-corrected returns from those data could earn high rents; but investors will find it hard to identify the stars from the merely lucky; and after competition makes you pay top dollar for superior advice, most investors can eke out for themselves only skimpy excess returns.”

Although the key methodologies of Modern Portfolio Theory (MPT) depend crucially on the belief that the market is reasonably efficient, MPT textbooks tend not to cite any scientific evidence in support of efficiency. For example, Rudd & Clasing (1982) give only the following heuristic justification:

“Rather than stress the statistics and discuss in detail the various tests, we feel it more important to develop the intuition that the inefficiency of the market is inherently improbable ... Our feeling is that only a few investors have a competitive edge ... and that once these investors have acted, few ‘pickings’ are left for the remaining investors. This implies that the market is exceedingly efficient in both the weak and the semi-strong form.”

The study that is often presented as being the most rigorous proof of semi-strong efficiency is Fama, Fisher, Jensen & Roll (1969), which shows that US share prices adjust rapidly to the informational implications of stock split announcements. However, as philosopher of science Karl Popper stresses, induction is not valid scientific proof:

“No matter how many white swans we see, we can never conclude that all swans are white.”

Moreover, most investment practitioners would expect widely known historic information to be of very little predictive value; expectations of what might happen in short-term earnings growth, as discussed in Section 4, are of far greater importance in the price formation process. It is interesting to note that, in the three years since the equity selection procedure described in Sections 3, 4 and 5 was first presented at an actuarial investment conference, no

researchers have attempted to test whether statistically significant breaches of semi-strong efficiency have indeed been identified.

The frequently quoted Jensen (1968) investigation into strong level efficiency is also fatally flawed in terms of scientific rigour. Jensen bases his efficiency conclusion on a chi-squared analysis, which is only valid if price changes follow a normal distribution, even though Fama, a leading researcher, had drawn attention some years earlier to how dangerously unsound such a normality assumption appeared to be:

“If the population of price changes is strictly normal, on average for any stock ... an observation more than five standard deviations from the mean should be observed about once every 7,000 years. In fact such observations seem to occur about once every three to four years.”

There seems little doubt that some, but by no means all, of the predictive power of a disciplined and systematic approach to share selection is due to the widespread (but, in the present author's view, erroneous) belief that it is futile even to attempt to identify undervalued securities. The supreme irony is therefore that this belief in efficiency has resulted in stockmarkets being far more inefficient than would otherwise have been the case, as Warren Buffett has quipped in the context of his own company's searches for undervalued investments:

“From a selfish point of view, perhaps we should endow chairs to ensure the perpetual teaching of Efficient Market Theory.”

16. Conclusions

Using the downside approach to risk first described in Clarkson (1989, 1990) and then developed in much more detail in Clarkson (2000), and guided by the way that probability theory has been used with great success in life assurance, the Equity Selection Procedure Model has been constructed as a possible successor to the CAPM to guide logical thinking in the very important practical area of equity portfolio management.

The main practical conclusion of the paper is that the portfolio guidelines followed by the vast majority of professional investors in a misguided attempt to “manage risk” are totally counterproductive in that they not only reduce the performance achieved but also increase the risk of failing to achieve a benchmark or target return. The portfolio selection procedure developed in the paper represents a far more satisfactory approach and is similar in concept to the one advocated by Williams (1938) and – as described in Hagstrom (1994, 1999) – used with outstanding success by Warren Buffett.

The main theoretical conclusions are that systematic irrationality of the nature identified by behavioural finances studies is closer to being the rule than the exception within equity markets, and that the dynamics of equity share price movements are not, as is assumed for instance in Black & Scholes (1973) and Markowitz (1959), invariant over time. These findings support the views expressed in Clarkson (1996), namely the many of the teachings and methodologies of classical finance theory and unsound in principle and thereby fall far short of the very high standards of prudent financial management required for the continued success of the actuarial profession worldwide.

References

- BLACK, F. & SCHOLES, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, **81**, 637-654.
- CLARKSON, R. S. (1978). A mathematical model for the gilt-edged market, *T.F.A.* **36**, 85-106 and *J.I.A.* **106**, 85-148.
- CLARKSON, R. S. (1981). A market equilibrium model for the management of ordinary share portfolios. *T.F.A.* **37**, 349-607 and *J.I.A.* **110**, 17-134.
- CLARKSON, R. S. (1989). The measurement of investment risk, *T.F.A.* **41**, 677-750 and *J.I.A.* **116**, 127-178.
- CLARKSON, R. S. (1990). The assessment of financial risk, *Transactions of the 1st AFIR International Colloquium*, Paris, **2**, 171-194.
- CLARKSON, R. S. (1996). Financial economics – an investment actuary's viewpoint. *B.A.J.*, **2**, 809-973
- CLARKSON, R. S. (1997). An actuarial theory of option prices. *B.A.J.*, **3**, 321-380.
- CLARKSON, R. S. (1998a). A continuous performance investigation of selected UK equities, *1998 Investment Conference of the Faculty and Institute of Actuaries*, and *B.A.J.*, **5**, 801-822.
- CLARKSON, R. S. (1998b). A fundamental preferences model of equity share returns. *Proceedings of the 8th AFIR International Colloquium*, Cambridge.
- CLARKSON, R. S. (1998c). A fundamental preferences model of common stock returns. *The Journal of Portfolio Management*, Fall 1998, **25**, 33-42.
- CLARKSON, R. S. (1999). The dynamics and psychology of the 1998 bear market in UK equities. *1999 Investment Conference of the Faculty and Institute of Actuaries*.
- CLARKSON, R. S. (2000). A general theory of financial risk, *Proceedings of the 10th AFIR International Colloquium*, Tromso, 179-209.
- CLARKSON, R. S. & PLYMEN, J. (1988). Improving the performance of equity portfolios. *J.I.A.* **115**, 631-691 and *T.F.A.* **41**, 631-675.
- FAMA, E. F. (1970). Efficient capital markets: a review of theory and empirical work. *Journal of Finance*, **25**, 383-417.
- FAMA, E. F., FISHER, L., JENSEN, M. & ROLL, R. (1969). The adjustment of stock prices to new information. *International Economic Review*, No.1 (February 1969), 1.
- GRAHAM, B. & DODD, D. (1951). *Security analysis*. McGraw-Hill, New York.
- GRAY, J. R. (1967). *Probability*. Actuarial Tuition Service, London.
- HAGSTROM, R. G. (1994). *The Warren Buffett way*. Wiley, New York.
- HAGSTROM, R. G. (1999). *The Warren Buffett portfolio*. Wiley, New York.
- JENSEN, M. C. (1968). The performance of mutual funds in the period 1945-64. *Journal of Finance*, **23**, 389-416.
- KEYNES, J. M. (1936). *A general theory of employment, interest and money*. Macmillan London.
- LOWENSTEIN, R. (2001). *When genius failed: the rise and fall of Long-Term Capital Management*. Fourth Estate, London.
- MANDELBROT, B. B. (1963). The variation of certain speculative prices. *Journal of Business*, **36**, 394-419.
- MANDELBROT, B. B. (1999). A fractal walk down Wall Street. *Scientific American*, February 1999, 70-73.
- MARKOWITZ, H. M. (1959). *Portfolio Selection*. Second edition (1991), Blackwell, Oxford.
- MILLS, T. C. (1991). Equity prices, dividends and gilt yields in the U.K.: cointegration, error correction and 'confidence'. *Scottish Journal of Political Economy*, **38**, 242-255.

- O'SHAUGHNESSY, J. P. (1996). *What works on Wall Street*. McGraw-Hill.
- PETERS, E. E. (1991). *Chaos and order in the capital markets*. Wiley, New York.
- PLYMEN, J. & PREVETT, R. M. (1972). The computer for investment research. *T.F.A.* **33**, 143-186.
- RUDD, A. & CLASING, H. K. (1982). *Modern portfolio theory*. Dow Jones-Irwin, Homewood, Illinois.
- SAMUELSON, P.A. (1994). The long term case for equities. *The Journal of Portfolio Management*. Fall 1994, 15-24.
- SHILLER, R. J. (1989). *Market volatility*. Massachusetts Institute of Technology.
- SMITH, A. (1978). *An enquiry into the nature and causes of the wealth of the nations. Volume II of the Glasgow Edition of the Works and Correspondence of Adam Smith*. Oxford University Press.
- THALER, R. H. & WILLIAMSON, J. P. (1994). College and university funds: why not 100% equities? *The Journal of Portfolio Management*. Fall 1994, 27-37.
- WEAVER, D. & HALL, M. G. (1967). The evaluation of ordinary shares using a computer. *J.I.A.* **93**, 165-227.
- WILLIAMS, J. B. (1938). *The theory of investment value*. Harvard University Press.