Market Value of Insurance Contracts with Profit Sharing[1]

Pieter Bouwknegt

Nationale-Nederlanden
Actuarial Dept
PO Box 796
3000 AT Rotterdam
The Netherlands
Tel: (31)10 - 513 1326
Fax: (31)10 - 513 0120
E-mail: pieter.bouwknegt@nn.nl

and

Antoon Pelsser

Nationale-Nederlanden	Erasmus University Rotterdam
Actuarial Dept	Econometric Institute
PO Box 796	PO Box 1738
3000 AT Rotterdam	3000 DR Rotterdam
The Netherlands	The Netherlands
Tel: (31)10 - 513 9485	Tel: (31) 10 - 408 1259
Fax: (31)10 - 513 0120	Fax: (31)10 - 408 9162
E-mail: antoon.pelsser@nn.nl	E-mail: pelsser@few.eur.nl

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Abstract

The valuation of insurance contracts using a market value approach, also known as the *fair value* approach has recently attracted a lot of interest. Seconding this trend, we would like to illustrate in this paper how we can determine the market value of a with-profits insurance policy. The method we use is based on finding a set of financial instruments that can identically replicate the cash flows of the insurance policy. Using this approach we show that the market value of a with-profits insurance policy can be determined objectively.

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[1] This article expresses the personal views and opinions of the authors. Please note that ING Group or Nationale-Nederlanden neither advocates nor endorses the use of the valuation techniques presented here for its external reporting. A Dutch precursor of this paper has been published in *De Actuaris*, the magazine of the "Actuarieel Genootschap", the actuarial society in the Netherlands.

1. Introduction

The valuation of insurance contracts using a market value approach, also known as the *fair value* approach has recently attracted a lot of interest. For an overview of the fair value approach applied to insurance liabilities, see IAA (2000) in their comments to the IASC's Insurance Issues Paper. Furthermore, the Dutch Insurance Supervisory Authority has recently advocated the use of the fair value approach in the discussion paper on New Actuarial Principles. Also from a financial-economic perspective there are many reasons for using market valuation techniques. Seconding this trend, we would like to illustrate in this paper how we can determine the market value of a with-profits insurance policy using a replicating portfolio.

The market value approach is extremely useful for setting up risk management systems where one wants to monitor the joint movements of assets and liabilities. In the classical accounting approach of valuing liabilities with a fixed interest rate, it is difficult to quantify reinvestment risk, also the valuation of return guarantees and profit-sharing is not well defined.

The application of market valuation techniques for nominal insurance policies is still very much under discussion, often with the argument that there are no well-developed markets for this type of policies. In most cases the value of the nominal liabilities is calculated with a fixed interest rate, with a correction for already known discounts on future premiums. These discounts can be profit-sharing or interest-rate discounts. For still unknown discounts on the premium in the future no explicit reservation is made. They are only accounted for at the time the discounts are determined.

In the market value approach, insurance contracts are valued using prices of financial instruments that can be observed in the markets. In this paper we will show how for a simplified form of an insurance contract with an external interest-based profit-sharing rule a market value can be determined. It will be clear from our example that an exact market value can be calculated using contingent claim valuation techniques. The method we use is based on finding a set of financial instruments that can identically replicate the cash flows of the insurance policy. Such a set of financial instruments is known as a *replicating portfolio*.

For expositional clarity, we will ignore mortality in our examples. Under the assumption that mortality rates are independent of interest rates, this is not a serious simplification. Market values can then be calculated on the basis of projected cash flows using the correct mortality table.

The remainder of this paper is organised as follows. We will first recap the valuation of fixed cash flows, then we will discuss the concept of forward interest rates and the valuation of the uncertain future profit-sharing. Finally, draw conclusions on the basis of an example contract we analyse.

2. Valuation of Fixed Cash Flows

In its simplest form, a life insurance contract is an exchange of fixed cash flows. Against the payment of fixed premiums the insurer takes the obligation to make predetermined payments in the future. Examples of this type of contracts are ordinary life insurance, annuities and term insurance.

Let us consider an example. We sell a policy that will pay a guaranteed amount of 124,863.51 in ten years time. This policy is financed by annual premium payments of 10,000.00 at the beginning of each year. The interest basis is 4%. Using this interest basis, the present value of the revenues is equal to the present value of the final payment, both are 84,353.32.

To calculate the *market value* of this set of fixed cash flows, we cannot discount with a fixed interest rate, but we must use the term structure of interest rates which can be observed in financial markets. This approach is equivalent to buying a portfolio of assets which exactly replicates the cash flows of the insurance policy. The assets we buy are fixed-income assets that are valued in the market via the term structure of interest rates.[2]

The term structure of interest rates is represented as a zero-curve: for various maturity dates the return on a zero-coupon bond with that maturity is represented. There returns are known as zero-rates. The advantage of this representation is cash flows at different point in time can be discounted using the corresponding zero-rates. The value of a coupon bearing bond can, for example, be calculated by discounting each cash payments with the corresponding zero-

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[2] For our calculations we will make the assumption that we can buy and sell assets at the same price.

rate. The price of a zero-coupon bond is also known as a *discount factor*. We will denote in this paper the discount factor for maturity j by D_j. Given the zero-rate r_j for maturity j, the discount factor D_j is equal to $(1+r_j)^{-j}$.

Let us consider the market value for our example policy. We will assume a linearly increasing term structure of interest rates, where the one-year zero-rate is 4.1% and the ten-year zero-rate is 5.0%. Hence, a cash payment of 10,000.00 in seven years time is worth $10,000.00 *$ $(1.047)^{-7} = 7,250.59$ today. The market value of our contract is 5,676.29 for the insurer. The positive value is due to the fact that the market interest rates are higher than the interest basis of 4%. In the table below, the complete calculation can be found.

3. Forward Interest Rates

From the term structure of interest rate which we observe today, we can determine the appropriate interest rates for loans we negotiate for future dates. An interest rate which we negotiate today for a loan in the future is called a *forward rate*. Forward interest rates do not only depend on how far in the future the loan will start, but also depend on the type of loan that is considered.

Using a forward interest rate makes it possible to negotiate with a bank a loan starting in x years time, where the interest rate that has to be paid is already fixed today. For a bank such a type of contract can be completely replicated (but this is outside the scope of the current paper) and holds therefore no additional risk for the bank. A forward interest contract has no optionality: none of the parties involved can at a future time withdraw from its obligations in case better investment opportunities have arisen. It is also important to stress that the true market rate for the loan at time x can be different than the forward rate we negotiate today.

As an example for the calculation of forward rates, we take a bond that will be bought at par in 8 years time, pays a coupon c in year 9 and in year 10 the last coupon c plus the principal is repaid. Given the current term structure of interest rates (and using the discount factor notation we introduced earlier) we obtain:

(*) $\qquad D_8 = c\,D_9 + (1+c)D_{10}$

This equation holds, because the present value of the coupon payments and the principal in years 9 and 10 must be equal to the par value for which the bond is bought in year 8. Note that we have scaled the principal (and the par value) of the bond equal to 1 in this example.

The forward rate (or the forward coupon in this particular case) is c in formula (*), and it is clear that c is determined completely by the discount factors D_8, D_9, D_{10} of the current term structure. Solving for c yields:

$$c = (D_8 - D_{10})/(D_9 + D_{10})$$

Hence, given the current term structure of interest rates, we can completely determine the forward interest rates for various maturities. If we substitute the discount factors from the table below, we find a forward coupon of 5.8010%. This forward rate and all other relevant rates for our example can be found in the column "Fwd Cpn" in the table below.

4. Profit Sharing

Since it is also clear to policyholders that a better return can be made in the market than the interest basis of 4%, many insurance contracts have profit-sharing provisions. As an example we use a simplified version of an interest rate based profit sharing which is very common for Dutch insurance contracts.

> Each (net) premium plus interest payments of previous investments is taken to be invested in a bullet bond with maturity date equal to the maturity date of the insurance contract. As profit sharing, the excess return on these investments over 4% is paid out in cash each year.

To determine the market value of such a profit-sharing scheme, we restate it in terms of market rates we can observe. Let us consider an example. Suppose we buy in year 8 a 2-year bond at par. In year 8 we can observe the par coupon rate of this bond. According to the profit-sharing rule we pay the excess coupon return over 4% on this bond in the years 9 and 10. If the coupon rate is below 4% we pay no profit sharing.

This payoff scheme is, in terms of interest rate options, equal to a swaption payoff scheme. More precise, it is the payoff scheme of an 8-year option on a 2-year swap contract with a

fixed rate of 4%. Swaptions are very liquid contract which are traded by banks. The valuation is based on the Black option pricing formula.[3]

Market Value of Profit Sharing

Time	Zero Rate	Cash Flow	DiscFac	Notl	Fwd Cpn	Volatility	Swaption	Projection
0	4.00%	10000.00	1.00000	10000.00	4.9201%		722.04-	722.04-
1	4.10%	10000.00	0.96061	10400.00	5.0345%	12.5%	746.10-	740.93-
2	4.20%	10000.00	0.92101	10816.00	5.1479%	12.5%	758.49-	740.67-
3	4.30%	10000.00	0.88135	11248.64	5.2602%	12.5%	748.43-	720.70-
4	4.40%	10000.00	0.84178	11698.59	5.3712%	12.5%	714.04-	680.54-
5	4.50%	10000.00	0.80245	12166.53	5.4809%	12.5%	655.19-	619.80-
6	4.60%	10000.00	0.76350	12653.19	5.5892%	12.5%	572.06-	538.18-
7	4.70%	10000.00	0.72506	13159.32	5.6959%	12.5%	464.80-	435.48-
8	4.80%	10000.00	0.68724	13685.69	5.8010%	12.5%	333.58-	311.57-
9	4.90%	10000.00	0.65016	14233.12	5.9043%	12.5%	178.58-	166.40-
10	5.00%	124863.51-	0.61391					
Total		5676.29					5893.30-	5676.29-

Based on the data in the table above, we can calculate the market value of the profit-sharing rule and therefore the market value of the complete with-profits insurance contract. The column "Notl" denotes the notional amounts we would invest each year if we would buy bonds with coupons of 4%. The amounts increase because we invest not only the premiums of 10,000, but also the 4% coupons from the bonds we have acquired previously.

Given the market rates we expect to make a better return than 4%. We see that the forward coupon rates (see column "Fwd Cpn") are above 4%. This excess return will be paid back to the policyholder via the profit sharing. Note that the forward coupon rates are implied by the current term structure as we observe it today. Hence, these rates are not subjective predictions, but are observable market values.

If we want to make a projection of the future cash flows for this form of profit-sharing (e.g. for an embedded value calculation), it seems reasonable to use the forward rates implied by the current term structure. On the basis of this scenario market values of the projected profit-sharing payments can be calculated. These values can be found in the column "Projection".

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[3] For a derivation and applications of Black's option pricing formula for pricing swaptions, see Chapter 20 of Hull (2000).

As an example we consider the profit-sharing of the bond we buy in year 8. In this year we invest 13,685.96 in a bond with a maturity of 2 years. The coupon return of the bond is (according to our projection) equal to 5.8010%. Hence, based on this investment, we know that in years 9 and 10 we will pay out the excess return of 1.8010% to the policyholder. The market value of these projected payments is 13,685.96 * 0.018010 * 0.65016 + 13,685.96 * 0.018010 * 0.65016 * 0.61391 = 311.57. As we return the complete excess return to the policyholder, we see that the market value of the projected profit-sharing is equal to the market value of the premium and insurance cash flows. Hence, the total market value of the insurance policy based on these projections is equal to 0. This result is only holds if the market rates do not fall below the interest basis of 4%.

For a correct calculation of the market value of the profit-sharing, we must not only look at what happens with the projected interest rates, but also with the deviations from the projected rates. To calculate the option value with Black's formula, we must not only supply the forward interest rates but also the standard deviation around these forward rates. In option terminology the relative standard deviation is known as *volatility*. In our example we use a volatility of 12.5%, which is a representative level for interest rate volatilities. The market values of the swaptions, calculated with Black's formula, can be found in the column "Swaption" in the table.

The total market value of the profit-sharing rule in our example policy is equal to the sum of the swaption prices and is equal to 5,893.30. This value is larger than the market value of the fixed premium and insurance cash flows. The total market value of the insurance policy is negative for the insurance company with a value of: 5,676.29 - 5,893.30 = -217.01.

5. Some Observations

Due to the principle of put-call parity, we see that the value of a with-profits insurance policy where the excess return over 4% is paid to the policyholder is equivalent to a contract which yields the full investment returns and has also a return guarantee of 4%. We have therefore a clear correspondence between return guarantees and profit-sharing.

The negative market value of our example policy can be explained by the fact that the market value approach explicitly takes the possibility of returns below 4% into account. In case the return on our investments in any year is lower than 4%, we can no longer satisfy the

obligations to the policyholder at the end of the contract as the profit-sharing rule prevents us from creating a buffer in which excess returns can be stored to compensate for bad years. By giving full profit-sharing above 4% without the accompanying "loss-sharing" below 4%, we have implicitly granted the policyholder a free 4% return guarantee. The negative market value of 217.01 is equal to the market value of this return guarantee.

If we want to analyse the profitability of this policy, it is dangerous to consider just one single scenario. A single scenario can never capture the full extend of the (implicit) return guarantees embedded in the contract. Since guarantees are often at a level below the current interest rates, there is a significant danger that they are not recognised in the usual profit-tests. A standard profit-test would indicate for our example a higher value for the policy than the market value approach which takes the full dynamics of the term structure correctly into account.

6. Summary and Conclusion

By creating a replicating portfolio, we can determine the market value of insurance liabilities. This approach can also be applied to determine the value of policies with profit-sharing or return guarantees. For fixed deterministic cash flows the market value can be determined from on the current term structure of interest rates. The valuation is equivalent to finding a portfolio of fixed-income instruments that exactly matches the cash flows. The term structure of interest rates we observe today, also determines the forward interest rates. Forward interest rates are the basis for negotiating today the level of future loans.

The use of interest rate derivatives, like swaptions, makes it possible to determine objectively the market value of profit-sharing and return guarantees. It is very important that actuaries familiarise themselves with these pricing techniques. Determining the value on the basis of a single scenario ignores the inherent dynamic behaviour of guarantees. This can lead to an underestimate of the financial risks involved with guarantees.

References

Bouwknegt, P., and Pelsser A. (2001) "Marktwaarde van Winstdeling" *De Actuaris*, Vol. 8(5), pp. 12-15, Actuarieel Genootschap, Woerden, The Netherlands. (Publication in Dutch)

Hull, J. (2000) *Options, Futures & Other Derivatives* (4th Edition), Prentice Hall, New Jersey.

IAA (2000) "Insurance Liabilities - Valuation & Capital Requirements: General Overview of a Possible Approach", *Comments to the IASC's Insurance Issues Paper: Final Submissions*, downloadable from: http://www.actuaries.org/Public/Documents/submissionse.htm