CAN DIVIDEND YIELDS PREDICT SHARE PRICE CHANGES?

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ABSTRACT

In this paper an investigation is described, in which the performance of a share price index (logged monthly changes) and of a rolled-up (accumulation) index are considered over periods of k months, for $k = 1$ to 120 (ten years). The correlation between performance over k months and the dividend index (defined in two different ways, and either unlogged or logged) at the beginning of the period is considered. The data relates to the United Kingdom from 1923 to 1992.

It is discovered that the correlation coefficient between performance over k months and dividend yield at the start of the period increases with k up to $k = 76$, with the maximum value of the regression coefficient being reached after 79 months. This is for the share price index and the unlogged original dividend yield. The various alternative series investigated show maxima between 70 and 80 months.

The maximum value of the correlation coefficient is a little under 0.7 for one definition of dividend index and about 0.8 for the other definition. This is evidence of very strong predictability of share price performance on the basis of the dividend yield at the beginning of the period. The results are consistent with the author's autoregressive model for share yields, references to which are given in the paper.

1. INTRODUCTION

This paper describes a simple investigation into the question: can the dividend yield on a share price index be used as a predictor of future changes in that price index? If the Efficient Market Hypothesis is to be believed, then it should not be possible to predict future changes in share price indices on the basis of simple publicly available information such as the dividend yield. On the other hand, my own earlier investigations (Wilkie, 1986a, 1986b, 1987 and 1992) suggest that dividend yields follow a clear autoregressive pattern, even though dividends themselves perform something not very different from a pure random walk. In this

paper I investigate the experience in the United Kingdom for a period of nearly 70 years.

2. Data

First, the data: I have available values of a share price index for the United Kingdom, constructed on the basis of a series of different published share indices, mainly the Actuaries Indices (1930 to 1962) and the Financial Times-Actuaries All Share Index (1962 to 1992). I have values at the end of each month from December 1923 to September 1992, a total of 826 months. I also have values of a rolled-up index (alternatively called a total returns index, an accumulation index or a cumulative wealth index), including dividend income gross of tax and reinvested free of expenses. This rolled-up index has been constructed in different ways at different times, most recently being based on the "ex-dividend adjustment".

It is not possible to construct a share index that is both continuous in terms of price and continuous in terms of dividend when the constituents change (except by good luck). With the normal change of constituents in any one index discontinuities in the implied dividend index and hence the dividend yield are normally small, and cannot readily be observed. However, in the series I have available there have been a number of significant jumps in the dividend yield when changing from one index to another. I have therefore constructed an "adjusted dividend yield", ratioing the values of the dividend yield on the earlier series in order to make them more nearly continuous with the later series. The subsequent calculations use both the original dividend yield and the adjusted dividend yield, in order to see the different effects that they may have.

3. Notation

Next, some notation: let $P(t)$ be the value of the share price index at the end of month t.

Let $p(t)$ be the logged change in the share price index from $t-1$ to t ie

$$p(t) = \ln P(t) - \ln P(t-1)\,.$$

Let $Q(t,k)$ be the logged change in the share price index over the k

months between t and $t+k$, ie

$$Q(t,k) = \ln P(t+k) - \ln P(t).$$

Clearly $Q(t,1) = p(t+1)$ and

$$Q(t,k) = \sum_{j=1}^{k} p(t+j).$$

We define the value of the rolled-up index at the end of month t as $R(t)$, and define logged changes in the rolled-up index by $r(t)$ and $S(t,k)$ analogously to $p(t)$ and $Q(t,k)$.

We define the dividend yield at the end of month t as $Y(t)$; the adjusted yield described above as $Z(t)$; and the logarithms of these as $y(t)$ and $z(t)$.

4. Linear regressions

All our calculations are expressed in terms of straightforward linear regression. We first consider the regression of $p(t+k)$ on $Y(t)$. We assume that the relationship is stationary in time. We express the linear relationship as:

$$p(t+k) = a_k + b_k Y(t) + e_k(t)$$

where $e_k(\cdot)$ has zero mean and variance v_k.

Thus for $k=1$ we are considering the extent to which share price index performance in one month can be predicted by the value of the dividend yield at the beginning of that month. For $k=2$ we are using the dividend yield to predict the share price index performance, not in the coming month, but in the following month; and so on.

We now consider the cumulative share price index performance over k months, $Q(\cdot,k)$.

We express the linear regression of $Q(t,k)$ on $Y(t)$ by

$$Q(t,k) = A_k + B_k Y(t) + E_k(t)$$

where $E_k(\cdot)$ has zero mean and variance V_k.

Then, since

$$Q(t,k) = \sum_{j=1}^{k} p(t+j)$$
$$= \sum_{j=1}^{k} \{a_j + b_j Y(t) + e_j(t)\}$$

we get

$$A_k = \sum_{j=1}^{k} a_j$$
$$B_k = \sum_{j=1}^{k} b_j$$

and

$$E_k(t) = \sum_{j=1}^{k} e_j(t)\,.$$

It is not necessarily the case, however, that

$$V_k = \sum_{j=1}^{k} v_j\,.$$

This depends on whether the residuals $e_j(\cdot)$ from the successive regressions are themselves uncorrelated. Part of the investigation is to discover whether this is so.

The above formulae express theoretical relationships. However, we are able only to estimate the various parameters from the given data. In doing this we run into a small problem. If we have N observations of $P(t)$ - in this case $N = 826$ - we have no more than $N-k$ observations to estimate the values of a_k, b_k, A_k and B_k. But if we actually use $N-k$ observations to estimate these values, we shall not find that A_k and B_k are exactly the sums of the preceding a_j and b_j, for $j = 1, \ldots, k$. That is, if we use 825 observations to estimate a_1 and b_1, 824 observations to estimate a_2 and b_2 and 823 observations to estimate a_3 and b_3, we shall then find that A_3 and B_3, which can only be estimated on 823 observations, do not equal $a_1 + a_2 + a_3$ and $b_1 + b_2 + b_3$ respectively. However, if we estimate a_1, b_1, a_2, b_2, a_3 and b_3 on the first 823 observations, we shall then find that the estimated values of the a's and b's add up to A_3 and B_3 respectively.

We therefore choose a value K, and estimate all the parameters using the first $N-K$ observations. This "wastes" the last $K-1$ possible observations for $k = 1$, the last $K-2$ possible observations for $k = 2$, and so on, and it is worth investigating the effect of this.

But what value should we use for K? Preliminary investigations suggest that we should take $K = 120$, ie a period of ten years. The reason for this will become apparent as the results of the investigations are displayed.

5. Share price index and dividend yield

The first analysis, on which I shall report in some detail, is of the regression of share price index changes, $p(t+k)$ and $Q(t,k)$, against the original dividend yield, $Y(t)$. Having explained this first investigation, I need only summarise the investigations relating to the other series.

We start with the regression of $p(t+1)$ against $Y(t)$, that is the logged change in the share price index in each month against the dividend yield at the beginning of that month. The maximum number of observations is 825. The mean value of $p(\cdot)$ for the 825 observations is 0.0046, and the standard deviation is 0.0508. The regression equation is:

$$p(t+1) = -0.0136 + 0.0036Y(t) + e_1(t),$$

where $e_1(\cdot)$ has zero mean and standard deviation 0.0507. The correlation coefficient is 0.0768, corresponding to a reduction in variance (R^2) of 0.0059, a rather small value. The standard error for a correlation coefficient, assuming 825 normally distributed values, is 0.0348, so the correlation coefficient is 2.2 times the standard error, significant at a 2.5%, but not at a 1% level. The regression coefficient (b_1), whose value is 0.0036, is also 2.2 times its standard error, also significantly different from zero at a 2.5%, but not at a 1% level.

The evidence so far is that the share price index performance in any month has a small positive correlation with the dividend yield at the beginning of the month, but this is hardly enough to be worth paying much attention to.

We then look at the regression of $p(t+k)$ against $Y(t)$, for $k = 2, 3, \ldots$. The results in each case are rather similar to those for $k = 1$. But the correlation coefficient is in each case positive, as far as $k = 79$. Within these first 79 values the correlation coefficient ranges from 0.0052 ($k = 79$) to 0.1171 ($k = 29$). Now, although each regression in itself

shows a small correlation coefficient hardly significantly different from zero, if the correlation coefficients were really distributed around zero, the chances of 79 of them in sequence being positive are, outside the pages of Tom Stoppard (1967), remote.

The first negative correlation coefficient occurs with $k = 80$. The next 32 values, up to $k = 111$, are negative. It is not necessary to carry out even a simple runs test to see that something significant is going on.

From month 112 onwards the signs of the correlation coefficients fluctuate. It is therefore reasonable to consider the first 120 months in detail, so the value of K is chosen as 120.

The first set of calculations was done with reducing numbers of observations for each value of k, that is 825 observations for $k = 1$, 824 for $k = 2$, etc. The next set is done with a constant number of observations for each value of k, namely $N - K = 826 - 120 = 706$.

The first question is: what effect does reducing the number of observations have on the results for low values of k? Answer: it makes a numerical difference, but it is not obvious that the difference is very significant. The mean value of $p(\cdot)$ for the first 706 observations is 0.0037 (as compared with 0.0046 for all 825), and the standard deviation is 0.0499 (as compared with 0.0508). The regression equation for $k = 1$ is:

$$p(t+1) = -0.0148 + 0.0036Y(t) + e_1(t)$$

and $e_1(\cdot)$ has zero mean and standard deviation 0.0498. The correlation coefficient is 0.0810 (compared with 0.0768). The regression coefficient b_1 is unchanged. The correlation coefficients for the constant number of 706 observations are generally a little higher than those for the reducing numbers of observations, but not consistently so.

The general pattern remains the same. The first 79 correlation coefficients are positive (79 previously), and the next 33 (32 previously) are negative. The positive correlation coefficients range from 0.0014 ($k = 79$) to 0.1383 ($k = 28$). The lowest correlation coefficient for the first 36 values of k is 0.0768 ($k = 36$). For the next 35 values the correlation coefficients range from 0.0428 ($k = 58$) to 0.0823 ($k = 52$). The next eight correlation coefficients ($k = 72$ to 79) are less than 0.04, and the largest negative correlation coefficient is $-0.0923(k = 91)$.

We now consider the regressions of $Q(t, k)$ on $Y(t)$ for $k = 1, 2, \ldots$. In each case the first 706 values of $Y(t)$ are used. $Q(t, 1)$ is the same as $p(t+1)$, and the results are therefore the same. The correlation coefficient is 0.0810.

As the value of k is increased, so the correlation coefficient between $Q(t,k)$ and $Y(t)$ increases, reaching 0.3020 by $k = 12, 0.4326$ by $k = 24$, and a maximum of 0.6864 for $k = 76$. Changes from month to month are generally upwards, but there are a couple of months when there is a very small decrease in the correlation coefficient.

The values of A_k and B_k are necessarily equal to the sums of the corresponding values of a_k and b_k. Since the regression coefficients b_k have the same signs as the corresponding correlation coefficients, they are positive up to $k = 79$, and negative for the next 33 months. Thus the value of B_k increases steadily up to month 79, for which the regression equation is:

$$Q(t, 79) = -1.0354 + 0.2648Y(t) + E_{79}(t)$$

where $E_{79}(\cdot)$ has zero mean and standard deviation 0.3197.

Thereafter the values of B_k decrease to a local minimum of 0.1833 at month 112, for which the correlation coefficient is 0.4165 and the standard deviation of the residuals is 0.4500.

At around month 79 the standard error of B is about 0.01, so there is not the slightest doubt about the significance of the regression coefficient.

The practical effect of the regression coefficient is that a 1% difference in the dividend yield at the time of purchase of the share makes a difference of 0.2648 in the logged performance over 79 months, or just over 30% in the nominal performance (exp (0.2648) = 1.3032). This is equivalent to about 4.1% a year compound for about six and a half years.

The value of V_k, the variance of the residuals $E_k(\cdot)$, also increases with k. By month 79 the variance has reached 0.1022 (corresponding to a standard deviation of 0.3197), and it carries on increasing up to month 120.

However, V_k is far from equalling the sum of the corresponding values of v_j, ie summed from $j = 1$ to k. By month 79, V_k is almost exactly one half of the sum of the v_j's. We consider the step from k to $k+1$. $Q(\cdot, k)$ has residuals $E_k(\cdot)$ with residual variance V_k. $Q(t, k+1)$ is equal to $Q(t,k) + p(t+k+1)$. $p(\cdot + k + 1)$ has residuals $e_{k+1}(\cdot)$ with variance v_{k+1}. V_{k+1} is therefore given by

$$V_{k+1} = V_k + 2r_{k+1}\sqrt{V_k v_k} + v_k,$$

where r_{k+1} is the correlation coefficient between $e_{k+1}(\cdot)$ and $E_k(\cdot)$.

All these "incremental" correlation coefficients of the residuals have been calculated. They are all small. However, for the first 14 months they are positive, and for the next 71 months (up to $k = 85$) they are negative. For the remaining 34 months (from $k = 86$ to $k = 119$) they are all positive.

Positive incremental correlation coefficients mean that V_k increases faster than the sum of the v_j's, and negative correlation coefficients mean that it increases more slowly. A practical interpretation is that, for the first 14 months, if the share price index has increased more than was expected according to the dividend yield at the beginning of the period, it is likely in the next following month to go up more than average, whereas for the next 71 months, up to month 85, if it has gone up more than expected according to the initial dividend yield, then it is likely to go down relative to the average increase. One might associate the first 14 months with a "bull" or "bear" market, and the next six years as a period of reversion to an average increase. However, it would require further investigation of correlations between returns over k_1 and k_2 months to be more precise.

The results so far show conclusive evidence (a) that the future performance of the share price index is strongly correlated with the dividend yield at the beginning of any period, with the regression coefficient reaching its maximum after 79 months, and (b) that the generally negative autocorrelation in the residuals means that the variance of residuals for the k-month performance is considerably less than the sum of the residuals for the individual k months.

Values of the regression coefficients A_k and B_k, the standard deviation $\sqrt{V_k}$ the correlation coefficient between $Q(t,k)$ and $Y(t)$, R_k, and the ratio of V_k to $\sum_{j=1}^{k} v_j, w_k$, are shown for $k = 6, 12, 18, \ldots, 120$ in Table 1.

6. Rolled-up index and dividend yield

The rolled-up index, including gross dividend income reinvested, can be treated in just the same way as the share price index. The logged change in the index per month, $r(\cdot)$, has a higher mean than the corresponding value for the share price index, $p(\cdot)$, 0.0089 for the 825 observations as compared with 0.0046, but the standard deviation is very little different at 0.0507 as compared with 0.0508.

We consider only calculations using the first 706 observations. For the first 706 monthly logged changes in the rolled-up index the mean is 0.0081 (as compared with 0.0037 for the share price index) and the standard deviation is 0.0498 (0.0499 for the share price index). The regression equation for $k = 1$ is:

$$r(t+1) = -0.0142 + 0.0043Y(t) + e_1(t)$$

and $e_1(\cdot)$ has zero mean and standard deviation 0.0496. The correlation coefficient is 0.0977. The value of the regression coefficient, b_1, is rather higher than that for the share price index, and the correlation coefficient is also a little higher.

The general pattern for the rolled-up index is similar to that for the share price index, but the correlation coefficients for the individual months are greater than those for the share price index for the first 32 months, and smaller for the next 70 months, up to $k = 102$, being irregular thereafter. It is not surprising that there is a higher correlation between the rolled-up index and the dividend yield than between the share price index and the dividend yield, because part of the return on the rolled-up index comes from the dividend itself. But it is not obvious why this effect should last for only 32 months, with an effect in the other direction for nearly six years thereafter.

The cumulative logged change in the rolled-up index over k months is denoted $S(t,k)$. We consider the regression of $S(t,k)$ on $Y(t)$ for $k = 1, 2, \ldots$. It is not surprising that the correlation coefficients between $S(t,k)$ and $Y(t)$ increase in the same way as do the correlation coefficients between $Q(t,k)$ and $Y(t)$. Indeed the correlation coefficient for $S(\cdot,k)$ is greater than that for $Q(\cdot,k)$ for the first 69 values of k, dropping to lower values thereafter.

The peak value of the correlation coefficient for $S(\cdot,k)$ is 0.6834 for $k = 71$, as compared with 0.6864 for $Q(\cdot,k)$ for $k = 76$ for the share price index. The correlation coefficients for individual months are positive up to month 77 (as compared with month 79 for the share price index), and the value of B_k reaches a maximum in month 77 of 0.2681 (as compared with 0.2648 in month 79 for the share price index). The standard deviation of the residuals for $S(\cdot,77)$ is 0.3269, compared with a value of 0.3197 for the residuals of $Q(\cdot,79)$.

The incremental correlation coefficients of the residuals are positive for the first 13 months (as compared with 14 months for the share price index) and negative thereafter up to month 80 (85 months for the share

price index). Whereas the value of V_k for the share price index fell to just one half of the sum of the corresponding values of v_j, for the rolled-up index the ratio drops only to 0.54.

The results for the rolled-up index are similar to, but not identical with, those for the share price index, allowing necessarily for the higher mean return. Some of the differences in pattern are explicable, but others may need further investigation.

7. Adjusted dividend yield

It is a pity that we have to consider the adjusted dividend yield, $Z(t)$, which was described earlier. However, a continuous and consistent share price index for the whole of the period does not exist, so it is necessary to make the adjustments. The mean value of the adjusted dividend yield for the first 706 observations is 4.16%, with a standard deviation of 1.17, as compared with a mean of 5.13% and a standard deviation of 1.12 for the original dividend yield.

We consider first the regression of the share price index on the adjusted dividend yield. The results are very similar to those for the original dividend yield. The correlation coefficients for individual months are sometimes higher, sometimes lower than for the original dividend yield. The first 80 (cf 79) are positive. For the cumulative months, the maximum correlation coefficient is 0.8004 in month 79 (cf 0.6864 in month 76). This is a substantially higher value than for the original dividend yield. The maximum value of B_k is 0.2997, reached in month 80 (cf 0.2648 in month 79). The ratio of V_k to the sum of the corresponding v_j's is as low as 0.34 in month 80 (cf 0.50 in month 79).

The adjusted dividend yield has therefore been a better predictor than the original dividend yield in the past. Their values for the last 30 years have been the same, so it is not clear which would be the better predictor in future.

When regressions of the rolled-up index with the adjusted dividend yield are considered, we find that the correlation coefficients for individual months are again positive up to month 80 (cf 77 for the original dividend yield and the rolled-up index). For the cumulative months, the maximum correlation coefficient is 0.7909 in month 77 (cf 0.6834 in month 71). The maximum value of B_k is 0.3034, reached in month 80 (cf 0.2681 in month 77). This series of regressions falls consistently into pattern, with a higher maximum correlation coefficient than that for the

rolled-up index and the original dividend yield, but lower than that for the share price index and the adjusted dividend yield.

8. Logged dividend yield and adjusted dividend yield

The same calculations have been carried out for changes in the share price index and the rolled-up index regressed on the natural logarithms of the dividend yield and the adjusted dividend yield, $y(t)$ and $z(t)$. The results, in terms of correlation coefficients, peak months and ratios of variances, are very similar. The regression coefficients, B_k, are of course different. The most important results are shown in Table 2.

Since the logged dividend yield and logged adjusted dividend yield are more nearly symmetrical than the original dividend yield and adjusted dividend yield, it may well be desirable to use the logged versions in preference to the unlogged ones, but the results have a less immediate intuitive meaning.

9. Conclusion

Similar calculations to these have been carried out by Fama and French (1988), who discuss United States data. Unfortunately, Fama and French quote results only up to four years ahead. They find that the correlation coefficients between the cumulative performance and the starting dividend yield increase up to that point, but they do not go on to see whether the correlation coefficients beyond four years continue to increase, and at what point they reach a maximum. Such calculations for the United States, or for any other country for which sufficient data exists, would be interesting.

The investigations described in this paper show that there is a very strong correlation between the performance both of a share price index and of a rolled-up index with the dividend yield at the beginning of any period, whether that dividend yield is taken as the original dividend yield, the adjusted dividend yield, or the logarithms of either of these values. The correlation coefficients reach a peak between six and seven years ahead, and the regression coefficients generally reach a peak a small number of months later than the peak of the correlation coefficients.

It is insufficient just to use these linear regression structures. I prefer the more detailed analysis using time-series methodology, described in my own papers already referred to (Wilkie, 1986a, 1986b, 1987 and

1992). In those papers I show that dividend yields follow an autoregressive pattern, the consequences of which are similar to the results shown in this paper. Further analysis, however, would be required to show whether they are numerically equivalent. But that is another story.

Table 1 - Results of regressions of share price index performance on dividend yield

k	A_k	B_k	$\sqrt{V_k}$	R_k	w_k
6	−0.1080	0.0254	0.1301	0.2145	1.14
12	−0.2256	0.0528	0.1875	0.3020	1.18
18	−0.3445	0.0803	0.2256	0.3718	1.14
24	−0.4565	0.1065	0.2497	0.4326	1.05
30	−0.6138	0.1417	0.2609	0.5214	0.91
36	−0.7237	0.1678	0.2648	0.5803	0.78
42	−0.7690	0.1814	0.2702	0.6027	0.70
48	−0.8202	0.1964	0.2767	0.6239	0.64
54	−0.8959	0.2161	0.2844	0.6497	0.60
60	−0.9412	0.2303	0.2989	0.6548	0.59
66	−0.9933	0.2453	0.3049	0.6708	0.55
72	−1.0395	0.2593	0.3094	0.6859	0.52
78	−1.0400	0.2648	0.3175	0.6841	0.50
84	−0.9563	0.2543	0.3363	0.6479	0.52
90	−0.8337	0.2366	0.3602	0.5942	0.55
96	−0.6929	0.2154	0.3860	0.5316	0.59
102	−0.5812	0.1999	0.4095	0.4814	0.63
108	−0.4695	0.1846	0.4352	0.4306	0.67
114	−0.4323	0.1836	0.4571	0.4117	0.70
120	−0.4218	0.1876	0.4765	0.4049	0.72

Table 2 - Summary of results of regression of $Q(\cdot, k)$ or $S(\cdot, k)$

		Maximum R_k		Maximum B_k		Minimum w_k	
Variables		Value	k	Value	k	Value	k
$Q(\cdot, k)$	$Y(\cdot)$	0.6864	76	0.2648	79	0.50	79
$Q(\cdot, k)$	$Z(\cdot)$	0.8004	79	0.2997	80	0.34	80
$Q(\cdot, k)$	$y(\cdot)$	0.6984	72	1.4196	78	0.49	78
$Q(\cdot, k)$	$z(\cdot)$	0.8099	78	1.3775	80	0.32	80
$S(\cdot, k)$	$Y(\cdot)$	0.6834	71	0.2681	77	0.54	77
$S(\cdot, k)$	$Z(\cdot)$	0.7909	77	0.3034	80	0.37	79
$S(\cdot, k)$	$y(\cdot)$	0.6949	70	1.4340	77	0.53	76
$S(\cdot, k)$	$z(\cdot)$	0.7984	74	1.3892	79	0.36	78

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