THEORETICAL PRICES OF CURRENCY-BASKET BONDS

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1. INTRODUCTION

Currency-basket bonds are not a new presence in financial markets. Classical examples include Special Drawing Rights (SDR), European Unit of Account (EUA) and European Currency Unit (ECU) denominated bonds, where the composition of the basket is linked to an officially recognized benchmark, but financial contracts denominated in a cocktail of currencies are a general tool available in financial markets. However, in recent years, ECU denominated bonds have been playing an increasingly important role in European financial markets. Due to its basket nature, the ECU has traditionally been considered a derivative currency. However, it has recently been observed that the arbitrage relationship with its component currencies does not seem satisfied for some financial instruments. This may be attributed to the absence of perfect equivalence between the ECU and the basket of currencies. Without touching on this issue, we note that, although no arbitrage relationships may be easy to verify in some cases (e.g. short-term interest rate), they may be more complex in others. In particular, the construction of the ECU basket and the derivation of the theoretical interest rate is usually based on the assumption of a "complete bond market", i.e. that zero coupon bonds for all maturities are traded (see Steinherr and Girard, 1992).

In this paper, we examine carefully the arbitrage relationships between the ECU basket and the component currencies relaxing the assumption of a complete bond market. We show, using a methodology based on linear programming, that, under more general and realistic conditions, ECU bonds may not always be considered derivative instruments. Therefore, greater care may be needed in assessing violations of

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theoretical pricing.

We derive the theoretical price and yield of an ECU bond, based on the component-currency bond markets. This may be of use when it is necessary to construct a theoretical benchmark yield based on the component currencies' bonds of a given credit standing.

In section 1 we introduce the basics of the ECU basket, then, in section 2, we derive relationships between the ECU term structure and those of the component currencies. In section 3 we develop linear programming techniques to provide bounds to the theoretical price of an ECU bond in the general case in which, due to the market incompleteness of component currencies, an ECU bond cannot be exactly replicated. It is noted that, although the pricing relationships have been derived having in mind the particular example of the ECU, they apply to any basket of currencies. Section 4 summarizes the main conclusions.

2. ECU

The ECU is a basket of fixed quantities of the twelve EEC currencies that participate in the European Monetary System (EMS). In the operation of the EMS, it has an official status as a means of settlement between member countries' central banks[1]. A basket of currencies with the same composition is widely used as a numeraire in financial markets (private ECU). The fixed quantities allow, in principle, the use of a replicating portfolio, that is a synthetic asset that contains quantities of component currencies equal to those in the ECU, to price or hedge an ECU denominated financial instrument. The quantities that define the official ECU may vary in prespecified cases called redefinitions. Every five years since September 1979, as a normal routine, the quantities of the currencies are subject to re-examination. This may or may not result in a redefinition. In addition, the quantities may be re-examined if, due to cumulated exchange rate variations, the weight of a currency in the ECU changes by more than 25% with respect to the 1979 starting weight. Up to now, this second type of redefinition has never occurred. The possibility of a redefinition complicates the analysis of those financial contracts where the ECU is treated as an "open basket" that must

[1] An extensive treatment of the role of the ECU in the EMS and of the EMS itself is beyond the scope of this note. See Van Ypersele and Koeune [1985], Allen [1986], and Levich [1987].

be delivered according to its official composition[2]. In general, the possibility of a redefinition is an element that could make the value of ECU denominated instruments diverge from the arbitrage relationship with the component currencies. However, ECU redefinitions are rare, and in the everyday problem of evaluating ECU denominated instruments redefinitions are often disregarded. In the following simplified analysis they will be assumed away.

2.1. ECU EXCHANGE RATE

The theoretical exchange rate of the ECU in terms of any currency is a linear combination of the exchange rates of the n component currencies, and this applies to both the ECU spot and forward exchange rate. We denote e_{ij} the spot exchange rate of the ith component currency expressed as units of currency j per unit of currency i, F_{ij} its forward exchange rate and Q_i the quantity of currency i in the ECU. The ECU spot and forward exchange rate in terms of currrency j, $e_{\varepsilon j}$ and $F_{\varepsilon j}$ respectively, can be expressed as:

$$e_{\varepsilon j} = \sum_{i=1}^{n} e_{ij} Q_i = E_j' Q, \qquad F_{\varepsilon j} = \sum_{i=1}^{n} F_{ij} Q_i = F_j' Q,$$

where E_j is the (column) vector of component currencies' spot exchange rates against currency j, F_j the vector of component currencies' forward exchange rates, E_j' and F_j' their transpose, and Q is the vector of component currencies' quantities. When exchange rates in terms of a generic currency are considered, the subscript j will often be omitted. The theoretical Spot and Forward exchange rate can be calculated for both bid and ask quotes. However, ECU exchange rates are quoted by the market directly (as in the case of many ECU denominated financial instruments), without explicit reference to the price of the "synthetic" instrument. Also, the principle for the calculation of the ECU theoretical spot exchange rate applies to the forward exchange rate only if the possibility of a redefinition occurring before the contract settlement date is excluded. Apart from this possibility, the usual spot/forward relationship applies to the ECU exchange rate in terms of both non-EMS currencies and EMS currencies.

[2]This is not always the case for currency-basket bonds; EUA bonds, for example, are linked to a historical benchmark.

2.2. ECU WEIGHTS

To find the weights of the component currencies in the ECU the quantities need to be evaluated using a common numeraire, which can be any currency. The quantities are converted into an "external currency", as the US dollar, or an EMS currency, at the current exchange rate, so that percentage value weights can be calculated. Since the exchange rates used to evaluate the ECU component currencies vary over time, the value weights vary continuously as well.

The weights obtained using the forward exchange rates of the component currencies to convert the quantities into a common numeraire (Forward Weights) are different from the weights calculated using the spot rate, since the forward exchange rates of the component currencies in terms of the numeraire contain a different premium/discount. Denoting W_{si} the spot weight and W_{fi} the forward weight of currency i in the ECU, we have:

$$W_{si} = e_{ij} Q_i [E'Q]^{-1}, \qquad \text{while} \quad W_{fi} = F_{ij} Q_i [F'Q]^{-1}.$$

It is straightforward to verify, using spot/forward parity, that a sufficient condition for the spot weight W_{si} to be equal to the forward weight W_{fi} is that all the component currencies' interest rates are equal[(3)]. The spot weight will be henceforth simply denoted W_i.

It will be convenient to define the (column) vector of component currencies' spot weights, W, the vector of forward weights, W_f , and a (row) vector V obtained as the element by element product of Q and E: $V = [Q_1 e_1, Q_2 e_2, \ldots Q_n e_n]$. It follows, $W' = V[E'Q]^{-1}$.

3. ECU INTEREST RATE, ECU TERM STRUCTURE, ECU PAR YIELD

We define here a risk-free asset as a claim to the payment of 1 unit of money, say 1 ECU, at a certain date in the future, with certainty. A market in which it is possible to trade a set of such claims (buy and sell short) which include all possible maturities is called a complete bond market. It is assumed that the market is perfect, that there are no taxes, transaction costs or other constraints. Although it is possible, and in many cases extremely useful, to consider a continuum of maturities for a

(3) Forward weights are useful for the calculation of ECU interest rate.

risk-free claim, in the present context a discrete time framework will be adopted where the future is subdivided into T periods or maturities. We denote by $P_j(t)$, $1 \leq t \leq T$, the price in period 0 of 1 unit of currency j to be delivered t periods ahead, and we express the interest rate:

$$R_j(t) \equiv P_j(t)^{-1/t} - 1.$$

The term structure will have here the precise meaning of a relationship between the time to maturity of a pure claim, the price of which is $P_j(t)$, and the interest earned on it, compounded on a one period basis. Equivalently, we can identify the term structure for currency j as the vector of discount factors $T_j' = [P_j(1), P_j(2), \ldots P_j(T)]$.

If we assume that the bond market of each of the ECU component currencies is complete, the ECU bond market turns out to be complete as well and the ECU term structure can be computed. The derivation uses the spot/forward or covered exchange rate parity relationship. Writing $P_\varepsilon(t)$ the price of an ECU pure discount bond with maturity t, the spot/forward parity relationship of the ECU with respect to a generic foreign currency j can be written:

$$e_{\varepsilon j} P_\varepsilon(t) = F_{\varepsilon j}(t), \quad \text{or} \quad P_\varepsilon(t) = \frac{F_{\varepsilon j}}{e_{\varepsilon j}} P_j(t)\,.$$

Since every ECU component currency must satisfy the spot/forward parity as well, and

$$e_{\varepsilon j} = E'Q, \qquad F_{\varepsilon j} = F'Q = [P_j(t)]^{-1} V P(t),$$

where $P(t)$ is the column vector of the prices of component currencies' pure discount bonds with maturity t, we have:

$$P_\varepsilon(t) = V[E'Q]^{-1} P(t) = W'P(t). \tag{1}$$

It follows that the theoretical price of an ECU denominated t periods pure discount bond is equal to the weighted sum, with ECU *spot* weights, of the prices of pure discount bonds denominated in the component currencies with the same maturity as the ECU bond. The derivation can be modified easily to include bid-ask spreads and transaction costs.

If [1] is not satisfied, in the complete market described an arbitrage gain can be picked up. In the case in which $P_\varepsilon(t) > W'P(t)$, the unit face value ECU bond can be sold short while quantities Q of the component currencies' bonds are purchased, locking in a risk-free gain. Note that the quantities of the component currencies' bonds that must be purchased are equal to the ECU quantities, which are different from the ECU spot weights W_i that appear in the no-arbitrage condition (1). The opposite strategy of buying the ECU bond and short selling the vector Q of component currencies' pure discount bonds with maturity t must be followed if $P_\varepsilon(t) < W'P(t)$.

In our complete bond market it is possible to replicate the cash flows of ECU pure discount bonds of any maturity. The prices of ECU pure discount bonds of different maturities give the ECU term structure, $T'_\varepsilon = [P_\varepsilon(1),\ P_\varepsilon(2), \ldots P_\varepsilon(T)]$; this allows the computation of the theoretical price, $P_\varepsilon(T, C)$, of a generic ECU bond with maturity T and cash flow profile in the various periods given by the $(T \times 1)$ vector C as:

$$P_\varepsilon(T, C) = T'_\varepsilon C, \tag{2}$$

where a constant coupon bond of unit face value will have the familiar cash flow profile $C' = [c, c, c, \ldots 1 + c]$.

The ECU theoretical interest rate, which can be deduced from the discount factor $P_\varepsilon(T)$, is instead the weighted sum of component currencies' interest rates with *forward* weights. We derive the latter relationship in Appendix A.

It is clear that, if the ECU term structure is observed and the term structure the of $n - 1$ component is known as well, the term structure for the nth currency is determined. This has again the nature of a no-arbitrage condition.

Once the ECU term structure is determined, the price of an ECU bond is given by [2]. However, by making use of [1], we can rewrite [2] as:

$$P_\varepsilon(T, C) = C'\underline{T}W, \tag{3}$$

where $\underline{T}$ is a $(T \times n)$ matrix with the ith column given by the term structure, T_i , of currency i. The price of an ECU bond is the sum of the present values of the "spot weighted" ECU flows discounted at the component currencies' spot rates, or to the price of a particular portfolio of pure discount bonds denominated in the component currencies.

The complete market assumption ensures in fact replicability of *every* ECU bond by means of a portfolio of component-currency pure discount bonds.

It may be possible to replicate some ECU bonds even if bond markets are incomplete. For example, the price of the same ECU bond in [3] can be seen as a weighted combination of the prices, $P_i(T, C)$, of coupon bonds denominated in the component currencies with the same face value, cash flow profile, and maturity as the ECU bond, since such bonds, if they exist, allow replication of the ECU bond, regardless of market completeness.

$$P_\varepsilon(T, C) = \sum_{i=1}^{n} W_i P_i(T, C). \tag{4}$$

However, this may not be possible for an arbitrary cash flow profile C. If [4] holds as a no-arbitrage condition, the component-currency term structures (the spot rates $R_i(t)$) may be replaced with the yields, y_i , of the component-currency bonds with prices $P_i(T, C)$. For a constant coupon bond we have:

$$P_\varepsilon(T, c) = \sum_{i=1}^{n} W_i \left[c \sum_{t=1}^{T} (1 + y_i)^{-t} + (1 + y_i)^{-T} \right]. \tag{5}$$

In general, whenever the ECU bond can be replicated, a precise theoretical value with respect to the replicating securities will exist; if the resulting price for the ECU instrument does not obtain in the market, an arbitrage possibility will emerge.

A concept of operational importance is the coupon that makes an ECU bond with maturity T sell at par (ECU theoretical par yield). Although it might be tempting to solve [5] for the size of this coupon given the component currencies' bond yields estimated from bonds market prices [Banque Paribas, 1986]:

$$c = \frac{1 - \sum_{i=1}^{n} W_i (1 + y_i)^{-T}}{\sum_{t=1}^{T} \sum_{i=1}^{n} W_i (1 + y_i)^{-t}}, \tag{6}$$

this is clearly inaccurate as the yields y_i used to discount component-currency cash flows are not generally unique, but depend on the size

of the coupon of the component-currency bond from which they are estimated [see Buse (1970), Khang (1975), Caks (1977), Schaefer (1977)]. The correct approach would be to recognize the interaction between the yield and the size of the coupon in [6] rewriting it as

$$[7] \qquad c = \frac{1 - \sum_{i=1}^{n} W_i (1 + y_i(c))^{-T}}{\sum_{t=1}^{T} \sum_{i=1}^{n} W_i (1 + y_i(c))^{-t}} = 0 .$$

The correct determination of the ECU theoretical par yield would then be the solution to equation [7], which implies the estimation of n functions, $y_i(c)$, i describing the effect of the change of the coupon on the yields of the bonds denominated in the various component currencies. Many analythical properties of such functions are known, and solving the problem of the ECU theoretical par yield through this route is certainly feasible. We can here note, following Schaefer (1977), that only in the special case in which the term structures of all component currencies intersect the annuity yield curve at maturity T, or are flat, and the yields of all the component currencies bonds are equal to the T periods spot interest rates, will the size of the coupon not affect the bond yields, and [6] will trivially hold.

On the other hand, a straightforward route to derive the ECU theoretical par yield is to estimate the component currencies term structures, T_i, and infer from them the ECU theoretical par yield. We suggest also here (and prove in Appendix B) a simple relationship between the ECU theoretical par yield and the component currencies par yields, c_i , for the same maturity:

$$[8] \qquad c = \sum_{i=1}^{n} \left[W_i T_i' 1 (W' \underline{T} 1)^{-1} \right] c_i = \sum_{i=1}^{n} W_i^* c_i ,$$

which shows the ECU par yield as a weighted average of the component currencies' par yields, with weights W_i^*.

4. Incomplete markets

All the relationships of the previous sections hold as general no-arbitrage conditions only if markets are complete ([4] may be an exact

no-arbitrage condition outside complete markets only in special cases). However, even if the ECU bond cannot be exactly replicated, some no-arbitrage conditions on its price can still be derived. In fact, portfolios that dominate or are dominated by the ECU bond can still be identified, and their prices provide bounds on the price of the ECU bond. We employ in this section an extension of linear programming techniques developed in Hodges and Schaefer (1977), Ronn (1987), and Ehrhardt (1989) for the purpose of selecting optimal bond portfolios for investors in different tax brackets(4).

The component currencies' flows implicit in an ECU bond with cash flow $C, Q_i C$, must be replicated separately with the securities available in the ith component currency market. Moreover, each of the component currency flows must be replicated in the most efficient way.

Let $\underline{A}_i$ be the $(K \times T)$ matrix of the cash flows produced in the various periods by the K available securities denominated in the ith currency, P_i the $(K \times 1)$ vector of their prices. Consider the set of $(K \times 1)$ portfolios $X_i = [X_i^b - X_i^s]$, $i = 1, 2, \ldots n$, which solve:

$$\text{Min } X_i' P_i$$

$$\text{Sub } X_i' \underline{A}_i \geq Q_i C$$

$$X_i^b, X_i^s \geq 0\,.$$

The vectors X_i give the quantities of the securities denominated in currency i that are either purchased (if the corresponding element of X_i^b is positive while the corresponding element of X_i^s equals zero), or sold (if instead the corresponding element of X_i^s is positive while the corresponding element of X_i^b equals zero). No-shortselling constraints may also be incorporated.

The prices (expressed in the component currency i) of the portfolios that solve the above problems are equal to the minimized values of the objective function, $P_u = [X_1' P_1, X_2' P_2, \ldots X_n' P_n]$. The price of the ECU bond with cash flow C, $P_\varepsilon(T, C)$, must be compared with the price of the replicating portfolio using a common numeraire, which we choose to be the ECU. Let

$$P_\varepsilon(T, C) - E_\varepsilon' P_u = g.$$

(4) The approach developed by Ronn (1987) and Ehrhardt (1989) is centred on the existence of bid-ask spreads, which we assumed away.

If $g \geq 0$, the simple strategy of short selling the ECU bond and buying the optimized portfolios of securities, X_i , will yield a riskless gain[5]. It turns out, however, that $E'_\varepsilon P_u$ is only an upper bound on the price of an ECU bond: if $g < 0$, in fact, an arbitrage gain is not ensured, since the feasibility of the reverse arbitrage strategy must be checked by solving the problems:

$$\text{Max } X'_i P_i$$

$$\text{Sub } X'_i \underline{A}_i \leq Q_i C$$

$$X^b_i, X^s_i \geq 0 .$$

$i = 1, 2, \ldots n$.

This will yield n maximized values of the objective function , P_1. The price of the ECU bond must then be compared with the price of the new replicating portfolio. We have

$$P_\varepsilon(T, C) - E'_\varepsilon P_1 = g' .$$

If $g' \leq 0$, the strategy of buying an ECU bond and short selling optimal vectors, X_i, of the component-currency bonds will yield a riskless gain[6]. The value $E'_\varepsilon P_1$ can be considered a lower bound on the price of the ECU bond. Therefore,

$$E'_\varepsilon P_u \geq P_\varepsilon(T, C) \geq E'_\varepsilon P_1 .$$

If this relationship holds as an equality, the equilibrium price of the ECU bond with respect to the component-currency bonds is unique, and so will be the theoretical yield. However, this result will be obtained for every possible coupon and maturity only in complete markets. In general, the theoretical price of the ECU bond, (and consequently its yield), is not unique and may lie anywhere between the prices of the two portfolios, without implying the possibility of a riskless arbitrage against the existing securities.

[5] If $g = 0$, at least one element of $X'_i \underline{A}_i$ must be strictly greater than the corresponding element $Q_i C$ for some i.

[6] As in the previous case, if $g' = 0$, at least one element of $X'_i \underline{A}_i$ must be strictly smaller than the corresponding element of $Q_i C$ for some i.

To the matrices $\underline{A}_i$ describing the cash flows available in the ith security market from the K existing securities, a $(T \times T)$ partition, Y, may be added, as shown below, which allows excessive cash flows in a period to be carried over to the next one. This represents a series of forward borrowing/lending transactions at a hypothetical "one plus" rate ρ , which may differ across time periods.

$$\underline{A}_i = \left[\begin{array}{cccc} a_{11} & a_{12} & & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & \cdots & \cdots & a_{mn} \\ \hline & & Y & \end{array}\right]$$

The matrix Y has the form

$$Y = \begin{bmatrix} +\rho & & & & & \\ -1 & +\rho & & & & 0 \\ & -1 & +\rho & & & \\ & & -1 & +\rho & & \\ & & & -1 & +\rho & \\ & 0 & & & -1 & +\rho \\ & & & & -1 & +\rho \end{bmatrix}$$

The prices at time 0 of the forward agreements represented by the matrix Y are all 0 but the first one, which equals 1.

Note that the conditions presented are simple linear programming problems that, for practical purposes, can easily be handled by standard computer programs.

4.1. An example of the incomplete market case

A simple example will be presented which illustrates the mechanics of the ECU bond price arbitrage bounds. Assume that the ECU is made up of two currencies with quantities $Q_1 = 0.719$, $Q_2 = 1.131$. The spot exchange rates against ECU of the two component currencies are $E_1 = 0.778$, $E_2 = 0.389$. In the market for each component currency there is only one bond with the following cash flows:

	Period 0	Period 1	Period 2	Period 3
	Price	Coupon	Coupon	Coupon+Principal
Currency 1	89.75	4	4	104
Currency 2	94.45	9	9	109

The bounds that can be put on the price, P_ε , of an ECU bond with the same maturity and coupon $c = 5$, result from the solution of the following linear programming problems, where we know a priori that no component currency bond will be sold short:

1) Solve, for each component currency, the minimization problem, that is,

$$\begin{aligned} \text{Min } & X_1\ 89.75 \\ \text{Sub } & 4X_1 \geq 5Q_1 \\ & 104X_1 \geq 105Q_1 \\ & X_1 \geq 0 \end{aligned} \qquad \text{and} \qquad \begin{aligned} \text{Min } & X_2\ 94.45 \\ \text{Sub } & 9X_2 \geq 5Q_2 \\ & 109X_2 \geq 105Q_2 \\ & X_2 \geq 0 \end{aligned}$$

It is easy to verify that the optimal solutions are $X_1 = 0.898$, $X_2 = 1.089$. This yields the vector $P_u = [80.66\ 102.9]$.

2) Solve the maximization problems

$$\begin{aligned} \text{Max } & X_1\ 89.75 \\ \text{Sub } & 4X_1 \leq 5Q_1 \\ & 104X_1 \leq 105Q_1 \\ & X_1 \geq 0 \end{aligned} \qquad \text{and} \qquad \begin{aligned} \text{Max } & X_2\ 94.45 \\ \text{Sub } & 9X_2 \leq 5Q_2 \\ & 109X_2 \leq 105Q_2 \\ & X_2 \geq 0 \end{aligned}$$

This gives optimal solutions $X_1 = 0.725$ and $X_2 = 0.628$. Therefore, $P_l = [65.14\ 59.34]$.

The upper bound on the price of the ECU bond will be $E'P_u = 102.84$, and the lower bound will be $E'P_l = 73.8$, or

$$73.8 < P_\varepsilon < 102.84\,.$$

The problem could have been more efficiently set allowing an excessive cash flow in a period to be carried over to the next. However, it is clear that there will not exist a single theoretical yield for the ECU bond, but rather an admissible interval whithin which it must lie.

5. Conclusion

In this paper we have presented some theoretical relationships between the price of currency-basket bonds and the price of bonds denominated in the component currencies. Although many simplifying assumptions have been made, including in particular frictionless markets and the absence of taxes, the derivative nature of currency-basket bonds has been somewhat clarified. In particular, it is shown that, if the component currencies' bond markets are complete, the price of a currency-basket bond (and consequently its yield) may be determined by arbitrage considerations. We illustrate a number of useful formulae relating the currency-basket price to the term structure of the component currencies. We show, however, that if some of the component currencies markets are incomplete (which may be taken as a more realistic representation in the case of the ECU medium-long term bond market), the component currencies' bond markets may provide only bounds on the price of some currency-basket bonds, which we identify by using a linear programming technique. In such cases, the market for currency-basket bonds may not be considered a derivative market, but rather an independent market providing new investment opportunities. This is a fundamental issue for the understanding of the role that currency-basket bonds play in financial markets, and may help to explain their development.

Appendix A

We derive here, using spot/forward parity, the relationship between the ECU and the component-currency interest rates. Let R_ε , R_i , R_j, be the interest rates capitalized on a T periods basis relative to the ECU, to the component currency i and to a third currency:

$$R_k = \left[P_k(T)\right]^{-1} - 1.$$

Applying spot/forward parity, where the forward exchange rates relate to the same maturity:

$$R_\varepsilon = \left[(1+R_j)E'Q\right](F'Q)^{-1} - 1 = \left[(1+R_j)E'Q - F'Q\right](F'Q)^{-1} =$$

$$= \frac{\sum_i^n Q_i\left[(1+R_j)e_{ij} - F_{ij}\right]}{F_{\varepsilon j}}$$

since every component currency must conform to spot/forward parity,

$$(1 + R_j)e_{ij} - F_{ij} = R_i F_{ij}\,.$$

And it follows

$$R_\varepsilon = \sum_i^n \frac{Q_i F_{ij}}{F_{\varepsilon j}} R_i = \sum_i^n W_{fi} R_i\,,$$

which is the well-known rule for calculating the ECU interest rate as a weighted combination of the component currencies' interest rates [see Dewes, 1986, Levich, 1987].

Appendix B

We derive here formula [8] relating the ECU and the component-currency par yields for the same maturity.

The price of an ECU bond of unit face value trading at par satisfies:

$$1 = cT_\varepsilon' 1 + P_\varepsilon(T) = W'P(T, c) \tag{8.1}$$

where the elements of $P(T, c)$ are the prices of bonds denominated in the component currencies with the characteristic of having coupons equal to c, the ECU par yield, and unit face values. It must be stressed that all these bonds will not necessarily trade at par. If they do, it is easy to see that the above equation [8.1] will be satisfied since $W'1 = 1$; so this is a sufficient but not a necessary condition. To see that the ECU theoretical par yield can be expressed in terms of the yields of component currencies' bonds that trade at par, which may have coupons that differ from c, let us define again the ECU par yield, c:

$$c = \left[1 - P_\varepsilon(T)\right]\left[T_\varepsilon' 1\right]^{-1}. \tag{8.2}$$

This coupon will also imply

$$W'P(T, c) - 1 = 0, \tag{8.3}$$

where, as defined above, the elements of $P(T, c)$ are the prices, $P_i(T, c)$, of i bonds denominated in the component currencies, with unit face

value and coupon equal to c, the ECU par yield. Their price can also be written:

$$P_i(T,c) = cT_i'1 + P_i(T). \tag{8.4}$$

Note that [8.3] represents a no arbitrage condition. Define now c_i to be the par yield on a bond denominated in component currency i and with maturity T. As mentioned earlier, the par yields c_i need not be equal to c or equal to each other. The par yields c_i satisfy:

$$1 = c_i T_i'1 + P_i(T). \tag{8.5}$$

Using [8.4] and [8.5],

$$1 - P_i(T,c) = (c_i - c)T_i'1. \tag{8.6}$$

Using [8.3],

$$\sum_{i=1}^{n} W_i T_i'1(c_i - c) = 0. \tag{8.7}$$

Rearranging [8.7], we obtain

$$c = \sum_{i=1}^{n} \left[W_i T_i'1(W'\underline{T}1)^{-1}\right] c_i = \sum_{i=1}^{n} W_i^* c_i. \tag{8}$$

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