

RISK-LOADING TO MATCH COST OF EQUALISATION RESERVE FOR BANKS

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ABSTRACT

The aim of the present study is to evaluate the cost of an Equalisation reserve which protects a financial firm against the risk of ruin. In section one we shall introduce the problem of capital adequacy for financial intermediaries. We shall describe the features and characteristics of a system of Equalisation reserves meant to improve the solvency of financial intermediaries. In section two we shall recall the principles and the algebra involved in this system of protection. In section three we shall concentrate on the costs related to this method. In section four and five we shall define a risk-loading suitable to cover such costs.

KEY WORDS: Capital adequacy, risk of ruin, equalisation reserve, financial cost, risk-loading.

1. INTRODUCTION

Risk is one of the main aspects of financial intermediaries; banks and other financial institutions are dealing constantly with a systematic level of risk. Financial intermediaries are subject to various types of regulations intended to ensure their solvency. Control authorities often define a minimum level of capital or set constraints on Assets and Liabilities composition. These requirements are structured to ensure capital adequacy, even if the concept of capital adequacy is generally left undefined and the effectiveness of these measures is left to the prudent appraisal of regulators. The insurance sector, on the other hand, has developed more sophisticated criteria of solvency and capital adequacy.

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Risk is the fundamental characteristic of insurance products and the management is constantly engaged to control and minimise it. In this view insurance companies themselves need to minimise the probability of going bankrupt. The simplest way is to set up a reserve of capital to be used in case of unfavourable events. There are several methods to calculate such reserves, but the most comprehensive of them is the Finnish-System developed by Peintikainen, Rantala and others in the early 1980s (7) (9).

In a previous paper (4) we proposed a model for the minimisation of bankruptcy crisis of financial intermediaries which is derived from the Finnish System. In this paper we wish to study the costs connected with an Equalisation reserve and to evaluate the financial strain for the firm, comparing it with the level of confidence obtained. This particular subject seems not to have been analysed yet. Actuarial research has long taken care of finding the best way to minimise the risk of ruin to very small levels, performing simulations to verify if a system gave the requested level of confidence. But the involved costs have not been examined yet with comparable attention. The management of an insurance company, or of a financial firm, is certainly sensitive to the risk of going bankrupt, but also to the cost of the proposed instruments of protection. For them it is important to compare not only the level of reliability that different systems are able to give, but also their costs.

According to Santomero and Vinso (10) "the last century has seen a steady decline in bank capital ratios". In present days some financial institutions fail to reach the amount of capital which should be required for the risk they accept. "Recent figures show that during the 1980s the Federal Deposit Insurance Corporation (FDIC) closed 1039 banks, more than during the previous 46 years. Only in 1991 the FDIC closed 24 banks with combined assets of \$ 63 billion. At the end of June 1992 about 60 banks would have failed to meet the new capital standards FDIC is going to set at the end of 1992. Independent figures suggest a larger total of 123 banks with combined assets of \$ 78 billion" [13]. In this view the main problem for banks wishing to set up adequate reserves is to find the capital. This situation is not only typical of the US, even in Europe most of the financial intermediaries are not likely to find the required capital autonomously and need to raise additional capital. To achieve this there are two solutions: either to borrow additional capital or to raise it from shareholders. In both cases there is a cost to pay. In the following sections we shall study a way to cover "this cost" with minimum effort.

2. THE EQUALISATION RESERVE

In this section we recall briefly the main features of the Equalisation reserves for financial firms, focusing on the points which differ from the insurance model. For an extensive treatment see (4).

The target of the model is the minimisation of the risk of ruin for a bank as a consequence of an excessive concentration of credit claims. A reserve is set up to smooth the variability of annual results. The reserve, put up during the years of good profitability, is made out of pre-tax profits, provided that such procedure is provided by law. During the years of bad results, technical losses are covered with adequate withdrawals from the reserve. The reserve acts like a clearing system going beyond the annual balance target, redistributing credit claims fluctuations over time and balancing profits in the long run. The system considers two levels for the reserve: a minimum level, usually called R_{\min} and a maximum called R_{\max} . These two values define a dynamic double barrier and the reserve must be kept within it. The minimum and the maximum value of the reserve are calculated requiring that the probability of ruin, respectively for one year and five years, is equal to a certain ε with $\varepsilon > 0$. Therefore R_{\min} and R_{\max} are obtained solving the two following inequalities with respect to r .

$$[1] \quad P\left\{u_R r + u_R^{1/2}(p_1 - S_1) \geq 0\right\} = 1 - \varepsilon$$

$$[2] \quad P\left\{u_R^5 r + \sum_{t=1}^5 u_R^{5-t+1/2}(p_t - S_t) \geq 0\right\} = 1 - \varepsilon$$

Where $t = 1, 2, 3, 4, 5$

$u_R = (1 + i_R)$ capitalisation factor at the rate i_R earned on the reserve

$p_t =$ Risk premium i.e. quota of interest to match the credit risk of the exercise between $t - 1$ and t

$S_t =$ Global amount of credit claims accrued in the exercise between $t - 1$ and t

$\varepsilon =$ Probability of going bankrupt. Usually ε is set equal to 3% or 1%.

This model requires some specific hypothesis about the financial risk that differs from the case of an Equalisation reserve for insurance companies. These peculiarities involve the amount p_t and the distribution function of the random variables S_t .

p_t is the global amount of the risk premiums. Risk premium is defined as the quota of the interest earned by the bank on the capital lent which has to cover the risk of credit. p_t must be equal, as it is in the insurance version, to the mean of credit claims $E[S_t]$. We assume that the risk premium can be expressed as the spread between the rate at which the capital is lent and the prime rate. The width of this spread is dependent on the risk of non-return of the capital, in the sense that the more the risk the wider the spread.

In creating a system dedicated to the financial sector we can not any more deal with stochastic independence among events. It is commonly accepted that financial events react because of some exogenous results or information. In this view, for the description of the new arrivals we dropped the Poisson process as it is not suitable to represent financial operations. We adopted the Polya process which is able to take into account the alteration during time of the basic probabilities and to represent those phenomena of contagion typical of credit claims.

Knowing the first three moments of the process at time t , we can solve inequalities [1] and [2] with the use of the Edgeworth Expansion and obtain a fair approximation for the values of R_{\min} and R_{\max} . In the case of an ε equal to 0,01, a u_R equal to 1,03 and with Z_5 defined as $Z_5 = \sum_{t=1}^5 S_t u_R^{1-t}$ we obtain

$$[3] \quad R_{\min} = u_R^{-1/2} \sigma[S_1] \left\{ 2.326 + 0.735 \gamma[S_1] \right\}$$

$$[4] \quad R_{\max} = u_R^{-1/2} \sigma[Z_5] \left\{ 2.326 + 0.735 \gamma[Z_5] \right\}$$

where $\sigma[*]$ is the standard deviation and $\gamma[*]$ the coefficient of skewness of the random variable $[*]$.

The management, with the aid of these values, is now able to define the correct dimension for the reserve depending on the level of protection required against the risk. If a maximum level of protection is needed they shall set a reserve equal or close to R_{\max} , on the contrary if they think the risk is well under control they shall adopt a level equal or near R_{\min} . During the years the value of the reserve fluctuate as a consequence of negative and positive economic results. If the value of the reserve at time t becomes lower than R_{\min} , the management has to readjust the reserve at least to a minimum value to maintain the desired level of protection.

3. COST OF AN EQUALISATION RESERVE

Using this method the management is able to compute the range of values of the reserve needed to protect the bank from its specific level of risk. The problem is how expensive setting up this system of protection could be and whether the costs the bank is going to bear are reasonable compared with the level of protection obtained.

We assume that the bank has to raise new capital to set up this reserve. Two are the solutions: borrowing additional capital from another institution or asking shareholders to increase the capitalisation of their firm. In any case there is a cost to pay, maybe the cheaper - or more flexible - solution is the latter.

Following this logic we suppose that the bank asks its shareholders additional capital for the establishment of an Equalisation reserve.

We assume that, for control purposes, the shareholders check the economic strength of the bank every N years. If they find the firm in healthy conditions, they will leave their capital invested for another period of N years. If they do not like how the bank has been managed, they will require the payment of the capital and the interest accrued computed at a certain agreed rate i_C .

How can the management face this debt and make this system efficient, considering not only the level of protection from the risk of ruin but even its cost? It is obvious that if the bank is not able to give back the amount of capital obtained, the system of protection would appear too expensive and scarcely feasible. In this view we start analysing the different type of costs that could arise in setting up and keeping this reserve.

There are two types of debts. The first one is represented by the capitalised value of the funds raised and its magnitude depends on the value of the rate of interest granted to shareholders. The second one is an additional cost due to the readjustments of the level of the reserve when it falls below the minimum level admissible. In this case the bank is supposed to ask additional funds from its shareholders at the same rate i_C increasing the amount of the debt at time N . In formula the debt at time t is:

$$[5] \quad r_0 u_C^N + \sum_{t: R_t < R_{\min}} (R_{\min} - R_t) u_C^{N-t}$$

where r_0 is the amount of funds raised to set up the reserve and the second term is the summation, with interest, of readjustments whenever

R_t is lower than R_{\min} . The first type of debt is certain, easy and direct to compute. The second one is a random variable, it depends on the nature of risk process and, due to the complexity because of the stochastic dependence among events, it is practically impossible to compute even the principal moments. These moments can be calculated analytically in very simple cases on the basis of strict assumptions, like the constancy of the value of each readjustment. But such hypothesis would prove too unrealistic, introducing a great bias in the subsequent analysis. Therefore we shall take into consideration the first term of [5] while some information on the dimension of the second term could be obtained by means of simulations.

At the end of the period N the management has the amount of reserve R_N available to cover the debt. If R_N is greater or equal to the amount due at time N then the system has worked in equilibrium giving an extra protection to the bank with no cost. On the contrary, if R_N is not high enough to cover the debt, there is an unbalance which should be considered as the cost of this system. Its cost is

$$[6] \quad C_N = r_0 u_C^N - R_N.$$

Using these assumptions, the amount of the cost becomes a random variable as it depends only on the value of the final reserve which is itself a random variable. To have a measure of the size of [6] we can consider the moments of C_N .

To compute the expected value of [6] we need to compute first the expected value of R_N . Let us consider the value of the reserve at time t . R_t is equal to the capitalised value of the reserve at time $t - 1$ plus the capitalised value of the difference between the risk premiums and the amount of credit claims occurred at time t . In formula

$$[7] \quad R_t = u_R R_{t-1} + u_R^{1/2} (p_t - S_t)$$

where u_R is the capitalisation factor computed at the rate i_R earned on the reserve.

We take now the expectation and obtain

$$[8] \quad \begin{aligned} E[R_t] &= E \left[u_R R_{t-1} + u_R^{1/2} (p_t - S_t) \right] = \\ &= u_R E[R_{t-1}] + u_R^{1/2} \left\{ p_t - E[S_t] \right\}. \end{aligned}$$

Since p_t is equal to $E[S_t]$ we have that the second term of the right member is 0 and it remains just the first one. We obtain

$$[9] \quad E[R_t] = u_R E[R_{t-1}]$$

and therefore taking back to time 0 we obtain

$$[10] \quad E[R_t] = u_R^t E[R_0] = u_R^t r_0.$$

We are now able to compute the expectation of the cost at the end of the period N , C_N

$$[11] \quad E[C_N] = E[u_C^N r_0 - R_n] = r_0(u_C^N - u_R^N).$$

From [11] it is evident that the cost depends on the spread between the two interest rates. Since we must consider that the rate i_C is greater than the rate i_R earned on the reserve, we have a positive value for C_N .

4. LOADING COEFFICIENT AS A RANDOM VARIABLE

From [11] it is almost sure that the bank will not meet its commitment adopting the model for the Equalisation reserves exactly as it was described in the second section. That model in fact is based on the hypothesis that the firm is able to find autonomously adequate capital to set up the reserve. If we drop this assumption so that the bank has to raise additional funds, as it is ever more likely to be in the present financial market, we need to adjust the hypothesis of the system.

The easiest way to cover the amount due in N , is to apply a loading coefficient θ to the risk premium, transferring part or the entire cost of the system on the money-borrowers. So, instead of having just p_t to face the credit claims of year t , we have $(1 + \theta)p_t$.

Given the final value R_N of the reserve it is easy to compute an ex post θ as that value which covers the debt due at the time N . In formula

$$[12] \quad \theta \sum_{t=1}^N p_t u_R^{N+1/2-t} = r_0 u_C^N - R_N.$$

Since the increments of a Polya process are stationary and particularly we have that $E[S_t] = E[S_1]$ for all t , it is clear that p_t is constant over

all t . So writing p instead of p_t the explicit value for θ is

$$[13] \quad \theta = \frac{r_0 u_C^N - R_N}{p u_R^{1/2} S_{N,i_R}}$$

where S_{N,i_R} is the capitalised value of a postponed annuity for the interval $(0, N)$ at the i_R rate.

Now θ is itself a random variable as it depends only on the value of the reserve at time N . If we want to have an ex ante value of θ to be applied in the model we have to consider the moments of θ . We take the expectation of θ and we obtain

$$[14] \quad E[\theta] = E \left[\frac{r_0 u_C^N - R_N}{p u_R^{1/2} S_{N,i_R}} \right]$$

and

$$[15] \quad E[\theta] = \frac{r_0 u_C^N - E[R_N]}{p u_R^{1/2} S_{N,i_R}}.$$

Therefore

$$[16] \quad E[\theta] = \frac{r_0 (u_C^N - u_R^N)}{p u_R^{1/2} S_{N,i_R}}.$$

If the process we consider has a small variance, the mean of θ can be considered as a good approximation of an adequate value for the loading coefficient. If we adopt a Polya process which has a greater variance due to the dependence among events, this value could be of little help for the management. [16] in fact does not take into consideration the high probability of levels of R_N significantly different from the mean value $E[R_N]$. In this case we need to compute the variance of θ . From the [13] we obtain

$$[17] \quad \text{Var}[\theta] = \frac{1}{\{p u_R^{1/2} S_{N,i_R}\}^2} \text{Var}[R_N].$$

We recall that R_N is

$$[18] \quad R_N = r_0 u_R^N + (p - S_1) u_R^{1/2+N-1} + \dots + (p - S_N) u_R^{1/2}$$

and that $\text{Var}[R_N]$ is equal to the summation of the variance of each S_t for all t plus twice the covariance between two different exercises. In formula we have

$$[19] \text{Var}[R_N] = \sum_{i=1}^N \text{Var}[S_i]u_R^{1+2(N-i)} + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \text{Cov}[S_i, S_j]u_R^{1+2N-i-j}$$

but as long as $\text{Var}[S_i]$ is equal to $\text{Var}[S_1]$ for all i and $\text{Cov}[S_i, S_j]$ is equal to $\text{Cov}[S_1, S_2]$ for all i and j we can simplify [19] into [20]

$$[20] \text{Var}[R_N] = \text{Var}[S_1]u_R \frac{(u_R^2)^N - 1}{u_R^2 - 1} + \text{Cov}[S_1, S_2]u_R \left[\left(\frac{u_R^N - 1}{i_R} \right)^2 - \frac{u_R^{2N} - 1}{u_R^2 - 1} \right].$$

As $\text{Var}[S_1]$ and $\text{Cov}[S_1, S_2]$ are known and easy to compute, after some straightforward calculations we can use [20] to compute [17] and to obtain the value for the variance of the loading coefficient.

5. PROBABILISTIC APPROACH

We shall now describe a method to compute a value for θ which is based upon a logic similar to that of formula [1] and [2]. Let us consider a bank wishing to establish an Equalisation reserve r_0 to protect itself against insolvency risk. For the sake of simplicity we suppose at first that it requires protection just for one year. Then we shall extend this period.

The optimal loading coefficient is that value of θ which reduces to 0 the cost of setting up an Equalisation reserve. In probabilistic terms we can express this condition requiring that the probability that the reserve shall match the debt is very close to 1. If we set this probability equal to $1 - \epsilon$, with $\epsilon > 0$, we have that the optimal θ_ϵ (we write θ_ϵ as now θ is a function of ϵ), is the value which makes true the following inequality

$$[21] \quad P \left\{ u_R r_0 + u_R^{1/2} \left[(1 + \theta_\epsilon)p - S_1 \right] \geq u_C r_0 \right\} = 1 - \epsilon$$

where all the variables have the same meaning of the previous formulas. We can now rearrange terms so as to put in the right-hand member only

the terms which are random. We have

$$[22] \quad P \left\{ r_0 \frac{(u_R - u_C)}{u_R^{1/2}} + (1 + \theta_\varepsilon)p \geq S_1 \right\} = 1 - \varepsilon.$$

If we knew the distribution function $F(\ast)$ of S_1 we would be able to compute the S_ε fractile i.e. the value of the random variable such that $F(S_\varepsilon) = 1 - \varepsilon$. From this value we could find immediately the required θ_ε .

As a matter of fact, it is more likely to know just a few moments of the random variables S_1 . If we know the first three moments of S_1 we can apply the Edgeworth Expansion again and find an approximate value for S_ε . In formula

$$[23] \quad S_\varepsilon = E[S_1] + \sigma[S_1] \left\{ y_\varepsilon + \frac{1}{6}(y_\varepsilon^2 - 1)\gamma[S_1] \right\}$$

where y_ε is the ε fractile of the Normal distribution. So we have

$$[24] \quad r_0 \frac{(u_R - u_C)}{u_R^{1/2}} + (1 + \theta_\varepsilon)p = E[S_1] + \sigma[S_1] \left\{ y_\varepsilon + \frac{1}{6}(y_\varepsilon^2 - 1)\gamma[S_1] \right\}$$

and after simple passages we obtain

$$[25] \quad \theta_\varepsilon = \frac{\sigma[S_1]}{p} \left\{ y_\varepsilon + \frac{1}{6}(y_\varepsilon^2 - 1)\gamma[S_1] \right\} - \frac{r_0(u_R - u_C)}{pu_R^{1/2}}.$$

This is the value of the loading coefficient θ_ε that guarantees with probability $1 - \varepsilon$ that the bank, at the end of the year, will meet its commitment with the shareholders and so will be able to pay back the capital and the interest in case it was requested.

Note that the value of θ_ε computed in [25] can be split into two components. The first component is due to the variability of S_t and is represented by the first term of the right-hand side of [25]. The higher the level of risk, i.e. the variance of S_t , the higher the value of the optimal θ_ε . The second component depends only on the spread between the two interest rates. If we set u_C equal to u_R then θ_ε is only a function of the risk of the process.

We can now extend this method over a longer period of time of, for example, five years and obtain

$$[26] \quad P \left\{ u_R^q r_0 + \sum_{t=1}^q u^{q-t+1/2} [(1 + \theta_\varepsilon)p_t - S_t] \geq u_C^q r_0 \right\} = 1 - \varepsilon.$$

The optimal value of θ_ε over the five years of time is the maximum value computed by [26] where q varies from 1 to 5. We have, in fact, to consider that, by introducing the loading coefficient, we created a positive drift in the process of the accumulated reserve. It is well known that in this case the maximum level of risk is not necessarily connected with the sup of the time interval. In this sense we have to compute θ_ε for every maturity from 1 to 5 years.

Following the same logic we adopted before, we have that

$$[27] \quad P \left\{ r_0(u_R^q - u_C^q) + (1 + \theta_\varepsilon)p \sum_{t=1}^q u^{q-t+1/2} \geq \sum_{t=1}^q u^{q-t+1/2} S_t \right\} = 1 - \varepsilon$$

or

$$[28] \quad P \left\{ r_0 \frac{u_R^q - u_C^q}{u_R^{q-1/2}} + (1 + \theta_\varepsilon)p \sum_{t=1}^q u^{1-t} \geq \sum_{t=1}^q u^{1-t} S_t \right\} = 1 - \varepsilon.$$

Knowing the first three moments of Z_q where $Z_q = \sum_{t=1}^q S_t u_R^{1-t}$, we could use the Edgeworth expansion again. After simple passages we obtain

$$[29] \quad \theta_\varepsilon = \frac{\sigma[Z_q] \left\{ y_\varepsilon + \frac{1}{6}(y_\varepsilon^2 - 1)\gamma[Z_q] \right\} - r_0 \frac{u_R^q - u_C^q}{u_R^{q-1/2}}}{p \sum_{t=1}^q u^{1-t}}.$$

Where $\sigma[Z_q]$ and $\gamma[Z_q]$ are the standard deviation and the coefficient of skewness of the random variable Z_q .

6. CONCLUSIONS

In this paper we started considering a system of protection against the risk of ruin suitable for financial firms. Financial intermediaries,

in fact, are facing a fierce competition which increases the level of risk and the probability of bankruptcy crisis. In this situation they need to set up adequate reserves to ensure their solvency in the medium and the long run. Especially in present times, due to the reduction of profit margins, banks are not likely to find autonomously the capital they need as a protection against the risk of ruin. In this way they are forced to raise additional funds paying interest on them.

In this paper we supposed that the bank asks its shareholders additional capital for the establishment of an Equalisation reserve. The bank will then pay back this capital plus the accrued interests at the end of a period of N year. Using these assumptions and considering the value of the accumulated reserve at the end of the period N , we computed the cost of this system of protection and we introduced a risk-loading coefficient θ as a way to cover it. This coefficient is a mark-up of the rate at which the bank lends the capital, and acts transferring part or the entire cost of this system of protection on the money borrowers. According to an early projection, this θ seems to have reasonable size and to be comparable to other fees currently included in the lending rate.

In section four and five we presented two methods to compute a value of this loading coefficient following two different approaches.

In the first one we considered the loading coefficient as a random variable depending on the cost of the Equalisation reserve which is itself a random variable. This method involves a simple algebra and it is easy and direct to apply. The management must consider the principal moments of θ . In section four we computed the mean value and the variance of θ . The mean is that value which covers on average the cost of the Equalisation reserve. If we have a Poisson process representing the risk phenomena the mean value can be considered as a good approximation for θ . But as long as we adopted a Polya process to represent the dependence among the events, this value alone could be of little help. In this view, for an appropriate setting of the coefficient, it is necessary to consider the variability of the process evaluating the dimension of the variance of θ .

As we can see, this method gives some suggestions to the management on how to define a value of θ , but can not compute an exact and unique value. In this sense the effectiveness of this method is left to the prudent estimates of the managers.

The second approach is based on a much more refined logic. The loading coefficient θ is computed on the basis of a probabilistic procedure

and it is defined as that value which reduces in probability to 0 the cost of the reserve.

The great advantage of this method is that the management is free to apply directly this value without the need for any further appraisal. This value of θ in fact takes into consideration all the features of the distribution function of the process, while the first method covered just the first two moments. In this sense this approach has a higher reliability and can be better fitted to the specific characteristics of risk of each bank.

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