

THE MYTH OF INDEX-LINKED BOND DURATION

AN EXAMINATION OF THE INTEREST RATE RISK CHARACTERISTICS OF INDEX RELATED SECURITIES

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1. INTRODUCTION

This paper examines the interest rate risk characteristic of index-related securities. It then proposes an hypothesis that the value of index-related securities are far less sensitive to market fluctuations than fixed coupon bonds. The concept of inflation duration is developed. This has implications for the future issuance, asset liability matching and for the level of solvency reserves.

2. BACKGROUND

In Australia, index related securities first became prominent in 1985 when the Commonwealth Treasury issued both interest indexed bonds and capital indexed bonds with maturities of 1995 and 2005. Interest indexed bonds were defined as those with coupons indexed, whereas capital indexed bonds had indexation applied to both coupons and principal.

This was not the first issuance of indexed linked securities. The State Electricity Commission of Victoria (SECV) issued a capital indexed bond in 1983 with a semi-annual coupon. The Commonwealth issues set the trend for the style of indexed bonds. The SECV issue is an anomaly as the Commonwealth and subsequent issues have quarterly coupons.

The Commonwealth Treasury has issued further amounts and other maturities. Various State and semi-government financing authorities

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have also issued index linked bonds. The latest style of indexed linked securities are indexed annuity bonds. These bonds are similar to an annuity certain but have the periodic payment indexed. Other variations of the indexed securities have been to change the index definition from the CPI (Consumer Price Index) to AWE (Average Weekly Earnings).

A broad discussion of the structure of indexed securities is contained in "Index-Linked Government Securities" by David Knox TIAA 1984.

Total issuance since 1985 has exceeded \$ 3.4 billion.

3. THE DATA

A credible and rigorous analysis of the historic behaviour of index-linked securities is very much constrained by the specific environment which has existed since 1985 when these securities were first issued.

This environment included:

- (a) a reduction in the Commonwealth budget deficits and consequent reduction in the Treasury's bond issuance programme;
- (b) extremely high real rates of return, as a result of the globalisation of world debt markets and bank de-regulation both in Australia and overseas;
- (c) poor liquidity of indexed securities in an immature market;
- (d) extreme volatility of monetary conditions, including a tight money policy pre October 1987, a loose money policy post October 1987 followed a tightening during 1988 and then a loose policy post 1989;
- (e) a decline in the rate of inflation from 8.2% in 1985 to 1.5% in 1991;
- (f) changes in the slope of the yield curve.

These factors mitigate against the development of a robust model of the behaviour of index linked securities in Australia. Consequently any interpretation of the following data needs to be highly qualified.

4. RESULTS OF DATA ANALYSIS

The data in the Appendix which contains nominal yields, real yields and CPI (Consumer Price Index) inflation was analysed to see what sort of relationship exists between these, if any. The yields and inflation were selected at quarterly intervals from November 1987 to February 1992 inclusive. The nominal yields were for a Commonwealth Government Bond with a 12% coupon and maturity of 15/11/2001. The real yields were for a capital indexed Commonwealth Government Bond with a

coupon of 4% and maturity of 20/08/2005. The inflation rates were derived from year on year values of the CPI. There are few long bonds with maturities near the indexed bond so the 15/11/2001 bond was selected as the most comparable bond.

Linear correlations between inflation, real yields and nominal yields were tested. The correlation coefficients are shown below.

	Nominal	Real	Inflation
Nominal	1.000	-0.515	0.893
Real	-0.515	1.000	-0.533
Inflation	0.893	-0.533	1.000

T-statistics were determined based on standard errors of the above coefficients. All *t*-statistics were found to be significant at a 99% confidence interval.

It is not surprising that nominal yields were correlated with historic inflation. The surprising result was the inverse relationship between real yields and nominal yields. This result was predicted by Knox (84) when he observed rising UK yields in a falling inflation environment during the early 1980's. He concluded that investors seeking to maximize returns in long bonds by betting on lower inflation tend to switch out of indexed bonds thus causing real yields to rise. However, during the 1980's real yields may have risen for other special factors eg: globalisation of world capital markets, greater mobility of capital and excess demand for debt. The obvious corollary to the Knox thesis would be that real yields will contract in times of subdued or rising inflation.

5. DURATION OF INDEXED PAYMENTS

The principles of the index-related securities can be illustrated by looking at an example of a single indexed payment. Consider a payment of $g\%$ of an indexed principal at time t . If we assume a constant inflation rate of j per period then the payment at time t would be $g(1+j)^t$.

Assume a nominal interest rate of i per period then the present value of the payment is

$$PV = g \cdot (1+j)^t \cdot v_i^t .$$

If we define r as the real rate of return per period then

$$r = \frac{(1+i)}{(1+j)} - 1 \quad \text{then} \quad PV = g \cdot \nu_r^t .$$

Consider the sensitivity of the PV with respect to i

$$\frac{\partial PV}{\partial i} = -t \cdot \nu_i \cdot PV$$

consider the sensitivity of PV with respect to j

$$\frac{\partial PV}{\partial j} = t \cdot \nu_j \cdot PV$$

consider the sensitivity of PV with respect to r

$$\frac{dPV}{dr} = -t \cdot \nu_r \cdot PV .$$

The above equations are related to the duration of a bond. It is commonly stated that indexed securities have large durations which implies that they can be used for matching liabilities which have corresponding large durations. The previous analysis of the nominal yields, real yields and inflation demonstrated that there is a strong correlation between nominal yields and inflation.

If you are trying to incorporate indexed securities into standard duration analysis of non-indexed securities you can end up with meaningless results. The duration with respect to real yields cannot be compared with duration with respect to nominal yields. If you use the duration with respect to nominal yields you are ignoring the relationship between nominal yields and inflation which causes real yields to be less variable.

The concept of inflation duration is introduced to allow the comparison of nominal durations of indexed securities with non-indexed securities.

$$\text{Modified duration}_i = \frac{-\frac{dPV}{di}}{PV} = t \cdot \nu_i$$

$$\text{Modified duration}_r = \frac{-\frac{dPV}{dr}}{PV} = t \cdot \nu_r$$

$$\text{Inflation duration} = \frac{-\frac{dPV}{dj}}{PV} = -t \cdot \nu_j .$$

Modified duration illustrates the relative change in value for yield change. Inflation duration would do the same for inflation.

Let us make an assumption that nominal yields and inflation move in parallel. Using the above equations we could derive the following relationship using a Taylor Series in two variables.

$$PV(i + \Delta i, j + \Delta j) = PV(i, j) + \Delta i \cdot \frac{\partial PV}{\partial i} + \Delta j \cdot \frac{\partial PV}{\partial j} + \dots$$

assuming $\Delta i = \Delta j$ and ignoring higher order terms then

$$PV(i + \Delta i, j + \Delta j) = PV(i, j) \cdot (1 + \Delta i.t.(-\nu_i + \nu_j)) .$$

The net change in value is $\Delta i.t.(\nu_j - \nu_i)$.

This highlights one of the key issues of the paper. **If inflation and nominal yields move in parallel then the net effect based on yield only overstates the price change for indexed securities.**

6. IMPLICATIONS OF INFLATION DURATION

The capital guaranteed funds for life offices must currently hold an investment reserve for interest bonds equivalent to a 2% move in yields. The Life Insurance Commissioner does not distinguish between a move in the real yield and the nominal yield. Clearly the 2% move itself is somewhat arbitrary, but historical evidence suggests that a 2% move in real yields is far less probable than a 2% move in nominal yields. This is also suggested by the data. Standard deviations of the data are shown below.

Nominal	1.33%
Real	0.43%
Inflation	1.90%
Bank Bills	3.31%

The data indicates that real yields were far less volatile than both nominal yields and bank bill yields. The variability of nominal yields could be associated with the variability of observed inflation over the period.

This data perhaps has equally important implications for conventional fixed coupon bonds. The concept of modified duration has been

frequently applied as the benchmark level of ex ante risk for fixed interest securities and fixed interest portfolios. It is useful as a risk measure because it is objective, is easily calculated, and is also a neat representation. It also fits well with the conventional yield curve model of interest rate behaviour. However, the measure is not robust with respect to non-parallel shifts in the yield curve. This has been well documented and users have been aware of the possible shortcomings of the measure.

The observed term structure of volatility of interest rates suggests a narrowing funnel of doubt with respect to yield volatility.

A fuller understanding of risk for the purposes of testing solvency demands an assessment of the both the riskiness of real yields and the riskiness of underlying maturities of the yield curve.

7. QUANTIFYING INFLATION EXPOSURE

Inflation duration could also be used to match the inflation characteristics of assets and liabilities. Most people are familiar with the Redington style of immunization where interest rate sensitivity is matched. Similar methodology can be applied for inflation. The concept of inflation convexity can also be developed.

Let V_A be the value of assets and V_L be the value of liabilities then the immunization result is to match inflation duration and keep inflation convexity of assets less than the inflation convexity of liabilities.

$$V_A = V_L$$

$$\frac{dV_A}{dj} = \frac{dV_L}{dj} \quad (\text{Inflation duration})$$

$$\frac{d^2V_A}{dj^2} > \frac{d^2V_L}{dj^2} \quad (\text{Inflation convexity}).$$

(Note: The inflation convexity relationship has inflation convexity of assets greater than liabilities. This is the same as the nominal yield relationship even though an increase in inflation increases the value of an indexed bond assuming nominal yields are unchanged. The concavity of the price versus nominal yield and the price versus inflation curves are in the same direction).

This introduces the rationale for why a liability manager should use indexed securities. The basic matching philosophy will allow a quantification of how much of indexed assets to purchase. By quantifying the inflation duration mismatch the inflation exposure can be measured.

The above equations ignore the effect of interest rates. Any matching strategy incorporating the nominal modified duration and convexity as well as inflation risk statistics would have to consider the cross derivative between nominal interest rates and inflation. This is presented in detail in the Appendix.

8. CONCLUSIONS

1. The market has misunderstood the risk characteristic of index-related securities.
2. If inflation and nominal yields move in parallel then the price change for a given nominal yield change is overstated if only the nominal yield modified duration is considered. Inflation duration helps to explain this.
3. Inflation duration is a useful tool which may enable indexed bonds to be incorporated into traditional duration analysis.
4. Asset liability matching of inflation exposures can be quantified using inflation duration and inflation convexity.
5. The application of traditional duration measures to indexed bonds is shown to be naive.
6. The term structure of volatility explains further aspects of fixed interest risk.

9. APPENDIX

Let

P be the price an indexed-related bond.

i be the nominal yield to maturity and ν_i be the present value of one payable in one period.

j be the inflation rate per period.

t be the term in periods.

c_t be the nominal amount payable at time t .

$$P = \sum c_t \cdot (1 + j)^t \cdot \nu_i^t$$

$$\frac{\partial P}{\partial i} = -\nu_i \cdot \sum t \cdot c_t \cdot (1 + j)^t \cdot \nu_i^t$$

$$\frac{\partial P}{\partial j} = -\frac{\nu_j}{\nu_i} \cdot \frac{\partial P}{\partial i}$$

$$\begin{aligned}\frac{\partial^2 P}{\partial i^2} &= \nu_i^2 \cdot \Sigma c_t \cdot t^2 \cdot (1+j)^t \cdot \nu_i^t - \nu_i \cdot \frac{\partial P}{\partial i} \\ \frac{\partial^2 P}{\partial j^2} &= \nu_j^2 \Sigma c_t \cdot t^2 \cdot (1+j)^t \cdot \nu_i^t - \frac{\nu_j^2}{\nu_i} \cdot \frac{\partial P}{\partial i} \\ \frac{\partial^2 P}{\partial i \partial j} &= -\frac{\nu_i}{\nu_j} \cdot \Sigma c_t \cdot t^2 \cdot (1+j)^t \cdot \nu_i^t .\end{aligned}$$

All the above statistics would be needed for an immunization exercise involving nominal yields and inflation. The above statistics can also be used in the Taylor series expansion for function of two variables to identify the combined nominal yield and inflation risk.

$$\begin{aligned}P(i + \Delta i, j + \Delta j) &= P(i, j) + \Delta i \cdot \frac{\partial P}{\partial i} + \Delta j \cdot \frac{\partial P}{\partial j} + \Delta i \cdot \Delta j \cdot \frac{\partial^2 P}{\partial i \partial j} + \\ &+ \frac{\Delta^2 i}{2!} \cdot \frac{\partial^2 P}{\partial i^2} + \frac{\Delta^2 j}{2!} \cdot \frac{\partial^2 P}{\partial j^2} + \dots .\end{aligned}$$

By substituting for all the derivatives it is possible to derive the following:

$$\begin{aligned}P(i + \Delta i, j + \Delta j) &= P(i, j) + \frac{1}{\nu_i} \cdot \frac{\partial P}{\partial i} [\nu_i \cdot \Delta i - \nu_j \cdot \Delta j + \Delta i \cdot \Delta j] \\ &+ \frac{1}{\nu_i^2} \cdot \frac{\partial^2 P}{\partial i^2} \left[\nu_i^2 \cdot \frac{\Delta^2 i}{2!} + \Delta i \cdot \Delta j + \nu_j^2 \cdot \frac{\Delta^2 j}{2!} \right] + \dots .\end{aligned}$$

This indicates that risk including nominal yields and inflation can be derived using conventional risk statistics.

Appendix: data used for investigation

Date	90 Day Bank Bills	Nominal Yields	Real Yields	Historic Inflation
Nov-87	11.3	14.0	5.2	8.3
Feb-88	11.5	13.1	4.8	7.1
May-88	13.2	12.7	4.7	6.9
Aug-88	14.0	12.6	4.5	7.1
Nov-88	15.3	12.5	4.4	7.3
Feb-89	17.5	14.4	4.4	7.7
May-89	18.3	13.8	4.7	6.8
Aug-89	18.4	13.4	4.9	7.6
Nov-89	17.6	13.7	5.0	8.0
Feb-90	15.5	13.6	5.2	7.8
May-90	15.2	14.1	5.4	8.6
Aug-90	13.4	13.9	5.4	7.7
Nov-90	12.1	13.2	5.5	6.0
Feb-91	11.7	11.8	5.6	6.9
May-91	10.6	10.9	5.6	4.9
Aug-91	9.6	10.8	5.6	3.4
Nov-91	7.6	10.1	5.6	3.3
Feb-92	7.5	10.4	5.6	1.5
Mean	13.3	12.7	5.1	6.5

Nominal Yields Based on 12% 15 Nov 2001 CGL
 Real Yields Based on 4% 20 Aug 2005 CGL Capital Indexed Bonds
 Source: SBC-Dominguez Barry
 Bank Bill Yields 90 day bank accepted bill yields mid-rate
 Source: Equinet
 Inflation: 12 month historic CPI year on year prior reported



