

SOME OBSERVATIONS ON THE THEORY OF GAMES

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ABSTRACT

The paper examines how changes in confidence can cause patterns of human behaviour to switch in a discontinuous manner from “normal behaviour” to “survival behaviour”, with the former characterised by vertical indifference curves and the latter by horizontal indifference curves. However, the simplifying assumptions on which the theory of games and utility theory are based imply that no such discontinuities in behaviour pattern can arise. Furthermore, the observed oscillatory nature of many economic and capital market series is inconsistent with the teachings of financial economics, but fully consistent with the postulated switches between “normal behaviour” and “survival behaviour”. It is therefore suggested that significant advances in economic theory will only be possible if the essentially linear frameworks of the theory of games and utility theory are abandoned as being inconsistent with the economic world in which we actually live.

Over the past three years there has been a momentous change in the economic world in that the centrally planned systems of economic management which had their origins in Russia more than seventy years ago have been swept away as a result of complete disillusionment amongst the citizenry of the countries as to their quality of life. Less recognised but as important to the citizens of the UK and many other Western democracies, is the breakdown of many of the economic forecasting models used by the UK Government and British industry. There is, I believe, a common link between these apparently unrelated phenomena, and it is my strongly held view that the same common link will within a few years also lead first of all to the questioning of, and then to the replacement of, many of the currently accepted methods of financial economics.

Everyone in the UK is aware of the depth of the current recession, but few are aware of the ways in which the severity of the downturn can be directly linked to the breakdown of economic forecasting models.

One specific example should explain the linkage. A long-established UK company whose main activity was operating an airline had essentially based its 1991 and 1992 business plans on the “consensus” forecast for the UK economy and on the fact that, in the recent past, airline traffic had grown at around twice the rate of economic activity generally. The “much lower than expected” economic activity combined with the corresponding gearing of airline traffic on the downside led first of all to retrenchment through redundancies and other cost-saving measures, then the financial restructurings, and finally to the demise of the company as an independent enterprise, all at very high cost in human terms.

Keynes (1936) concluded that such harshness was an unavoidable consequence of Western capitalist systems but suggested that, given the likelihood that human behavioural patterns would not be able to be changed sufficiently rapidly to avoid short term overreaction to adverse circumstances, it was desirable to have government intervention to limit the downside in economic activity. Keynes also stressed the importance of “confidence” in investment and economic behaviour, and argued that economists generally had paid far too little attention to this.

Keynes emphasised that his main aim writing his “General Theory” was to have his general principles accepted by other economists; the detailed mathematical and statistical formulations could follow later. Largely because of the difficulty of quantifying confidence, subsequent economic work has progressed in precisely the opposite direction. The lure of immediate numerical results based on more easily understood quantitative techniques (such as the theory of rational expectations and the theory of games) has led to a virtually total neglect of one of the cornerstones of Keynes “General Theory”, namely the central role of confidence. The main theme of this paper is that, while financial economists have failed in this regard, actuaries may well succeed in quantifying confidence and thereby laying the foundations of a better system of economic management.

There was no single “straw that broke the camel’s back” in the case of the centrally-planned or Marxist economic system. The inefficiencies and disappointments of a system which was inflexible, bureaucratic and complex gradually built up to such a degree that the citizenry decided that they wanted a change. One single example from over 20 years ago should again amply demonstrate how these problems arose.

Agriculture in the former Soviet Union had been centrally planned to maximise production by concentrating particular crops in those areas where the climate was thought to be most suitable, on the basis of past

experience. In 1972 the weather patterns did not behave as expected, and the grain crop, which was of crucial economic importance in that very scarce "hard currency" resources had to be used to buy emergency supplies from the West, was a complete disaster. In the regions designated most suitable for wheat and other grain crops, the summer was "unusually" hot, with the result that the yield would be very low. There were then "unexpected" heavy rains before harvest time, which flattened the already poor crop. Finally, there were "unusually early" frosts, which meant that virtually none of the crop could be harvested.

Changes in confidence patterns have, I believe, played the same part in invalidating recent forecasts of the UK economy that changes in weather patterns had done in the case of the 5-year agricultural plan that was in place in the Soviet Union in 1972. In both cases, a "rational expectations" framework based on recent behavioural patterns was totally inadequate as a framework for predicting future behavioural patterns.

Utility theory plays a crucial role as the numerical framework for many microeconomic and macroeconomic models, yet the implicit assumption is that each individual has a unique utility curve that does not vary over time. Utility theory, in turn, is based on an axiomatic system first derived by Von Neumann and Morgenstern (1944) from analogies drawn from the theory of games. I do not believe that the assumptions implicit in the theory of games have any validity in describing the actions of human beings in the economic world in which we actually live.

I also believe that theories of economic behaviour should, as Einstein said should be the case for physical theories, be simple enough "that even a child could understand them". I shall therefore dispense with my extensive and somewhat technical criticisms of the axiom system and the assumptions of linearity and "perfect information" that are essential ingredients of the theory of games and comment instead on the only example that Von Neumann and Morgenstern cite to illustrate how optimal behavioural patterns in the theory of the "two-person zero-sum game" can be translated into patterns of so-called optimal behaviour in the wider world.

In an episode from "The Adventures of Sherlock Holmes" by Sir Arthur Conan Doyle, Holmes, hotly pursued by Professor Moriarty, has boarded a train at Victoria Station, London, in an attempt to reach Dover and escape to the Continent. The only intermediate station is Canterbury. In essence, Holmes and Moriarty have independently to

decide at which station to detrain. If both detrain at the same station, Holmes will be killed by Moriarty. If Holmes detrains at Dover while Moriarty detrains at Canterbury, Holmes survives and can escape to the Continent. This outcome is more advantageous to Holmes than if he detrains at Canterbury while Moriarty detrains at Dover, in which case Holmes escapes for the moment but does not reach the Continent.

The theory of games solution suggested by Von Neumann and Morgenstern for Moriarty is that he should detrain at Canterbury with a 40% probability and detrain at Dover with a 60% probability. If he has, for instance, a pack of playing cards with him, he could pick out an Ace, a Two, a Three, a Four and a Five, shuffle these, and then decide that if the first card he draws is the Ace or the Two he will detrain at Canterbury; otherwise he will continue to Dover.

From Holmes' point of view, on the basis that he deduces that Moriarty will follow this strategy, the situation is "elementary", as he would often say to his faithful Baker Street colleague Dr. Watson. There is a 60% chance that Holmes will survive if he leaves the train at Canterbury but only a 40% chance that he will survive if he leaves the train at Dover. Since the benefit of being at Dover rather than Canterbury should Moriarty leave at the other station pales into total insignificance compared to the much higher probability of survival if he leaves the train at Canterbury, Holmes will of course choose Canterbury, and this is indeed how Conan Doyle's narrative proceeds. Moriarty must have drawn the Three or the Four or the Five, since he proceeded to Dover, thereby allowing Holmes to escape for the moment. This happens to have been the course of action that had the higher probability for Moriarty, namely 60% against 40%.

The theory of games solution, however, is that Holmes should detrain at Canterbury with a 60% probability and detrain at Dover with a 40% probability, which involves a 48% probability that he detrains at the same station as Moriarty and is then killed. We now have a paradox: the "elementary" human behaviour solution differs significantly from the theory of games solution.

This paradox can be explained by considering the situation confronting Holmes in terms of indifference curves with the vertical axis in Figure 1 representing "probability of Holmes being killed" and the horizontal axis representing "advantage to Holmes if he survives".

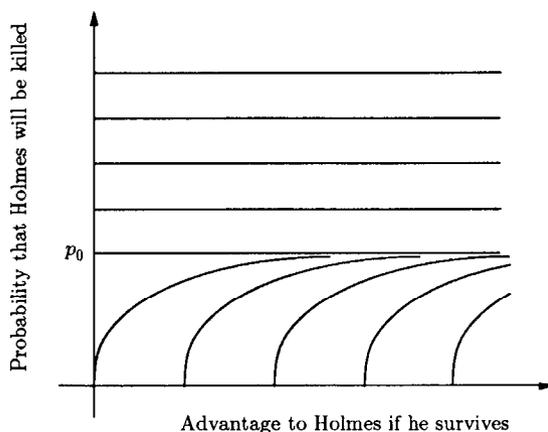


Fig. 1

There is a probability of p_0 , which will be very low in numerical value, above which no advantage to Holmes is sufficiently large to offset the risk of his being killed. Below p_0 , the indifference curves are asymptotic to vertical straight lines for $p = 0$, decrease in gradient as p increases, and are asymptotic to the horizontal line $p = p_0$. Above p_0 , the indifference curves are horizontal straight lines.

In terms of this pattern of indifference curves, Holmes' behaviour is indeed "elementary"; detraining at either Canterbury or Dover implies a probability of his being killed that is clearly in the horizontal indifference curve region, and he therefore opts for the 40% probability of being killed rather than the 60% probability or any intermediate probability that would result from a "mixed strategy" of $x\%$ probability of detraining at Canterbury and $(100-x)\%$ probability of detraining at Dover.

Two analogies with real life behaviour suggest that this pattern of indifference curves is of very general applicability. Consider first a drowning man who is thrown ten thousand straws neatly bound together by twine in the shape of a lifebelt. The buoyancy provided by the air trapped within these straws that comprise the lifebelt will increase his probability of survival very significantly. Suppose alternatively that he sees only one single straw floating on the surface of the water. By grabbing hold of this one straw he will increase his probability of survival, albeit only to an infinitesimal extent. This, of course, is the old adage "A drowning man will clutch at a straw". No matter how small the increase in the probability of survival that may result from pursuing a

particular course of action, someone faced with a high probability of imminent death will prefer that course of action to inaction. This corresponds to horizontal indifference curves above a certain threshold of risk.

The second analogy relates to an individual's choice of means of transport for a long journey for either business or pleasure purposes. The choice of going by car, by aeroplane or by train is normally based on considerations of cost, convenience, comfort and time involved, with the risk of death or injury playing no part whatsoever. In general, however, the risk in terms of deaths per passenger mile is far higher by car than by aeroplane, and also far higher by aeroplane than by train. In all three cases, however, the risk of death or serious injury is so low that it is not normally taken into account in the decision-making process. This corresponds to the indifference curves being asymptotic to vertical straight lines for very small values of risk.

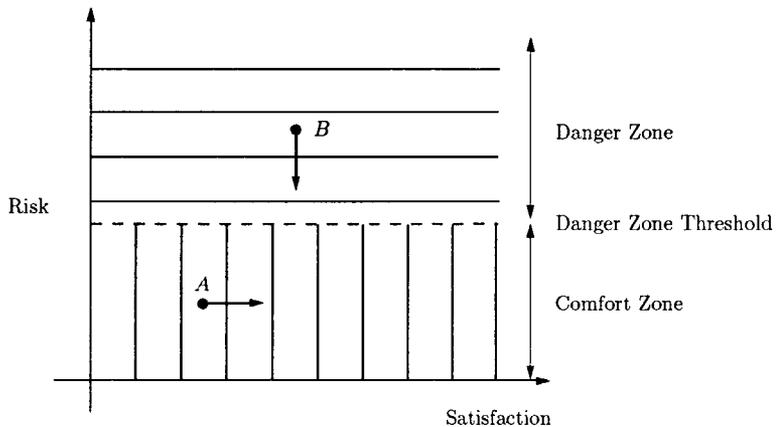


Fig. 2

Since it is generally impossible for an individual to be precise in his assessment of physical or financial risk, we can derive our main conclusions using the very much simplified situation as shown in Figure 2, where the Danger Zone Threshold divides the region of behaviour into a Comfort Zone and a Danger Zone, with the indifference curves being vertical straight lines in the Comfort Zone and horizontal straight lines in the Danger Zone. Behaviour at a general point *A* in the Comfort Zone can be regarded as “normal behaviour” which ignores risk and aims to maximise “satisfaction”, while behaviour at a general point *B* in the

Danger Zone can be regarded as “survival behaviour” which aims to bring the situation back within the Comfort Zone as quickly as possible. In economic terms, normal behaviour relates to an expansionary phase while survival behaviour relates to a contractionary phase.

Keynes discusses confidence mainly from the point of view of the entrepreneur or the stockmarket investor. The particular aspect of confidence that has been of most relevance recently in the UK economy relates to that of homeowners. Whereas a year ago more than 60% of the UK population thought that “housing was a good form of investment”, this proportion has now fallen below 50% according to recent (November 1992) surveys.

Consider an individual who, after a period of very high house price inflation, has committed a high proportion, x , say, of his after-tax income to servicing the mortgage that he has taken out to buy a large house. His Danger Zone Threshold for mortgage costs as a proportion of after-tax income, y , say, can be assumed to be slightly greater than x . Let us suppose that the remainder of his after-tax income is spent either on “Investment” (i.e purchases of expensive consumer durable goods such as cars) or on “Other Spending” (i.e food, drink, clothes, holidays, etc.). This is represented in Figure 3 below. In particular, his mortgage proportion lies in his Comfort Zone.

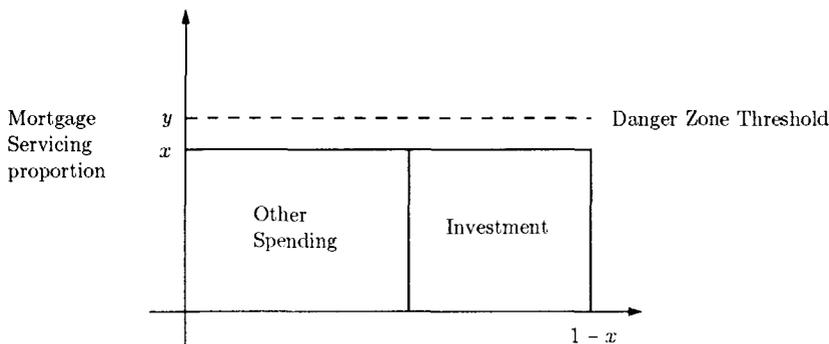


Fig. 3

Suppose now that, as a result of house prices having fallen over the past year rather than having risen as he had expected, he revises his Danger Zone Threshold sharply downwards from y to z where z is significantly lower than x . His mortgage proportion now lies in his Danger Zone, and he switches to “survival behaviour” in that he avoids any spending on “Investment” and also cuts back sharply on non-essential items within “Other Spending”. His reduced spending translates into

a reduction of his net borrowings. The situation is now as shown in Figure 4 below.

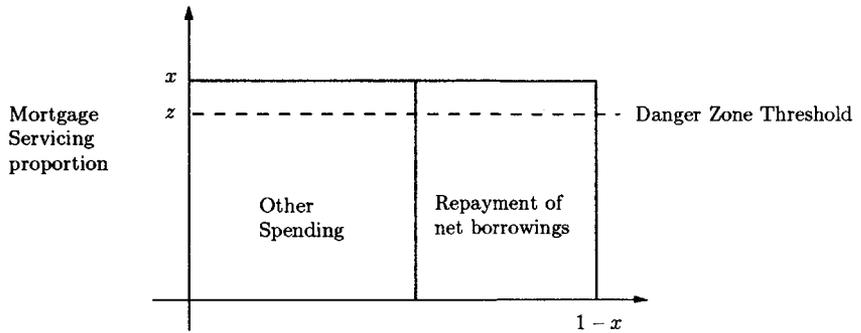


Fig. 4

In particular, he does not consider buying a larger house even although house prices have fallen, since this would increase, not reduce, his mortgage proportion. Also, since other potential house buyers who are still within their Comfort Zone believe it makes no sense to buy a depreciating asset, they generally delay their purchases until they can be confident that house prices have bottomed out. These two changes to the previous short term equilibrium position between buyers and sellers result in house prices falling further.

A similar discontinuity of behavioural patterns occurs amongst industrialists, who, having increased their workforces to cope with the expected increase in demand throughout the economy, realise that their total employee costs are now in the Danger Zone rather than, as previously thought, in the Comfort Zone. Redundancies are announced in an attempt to bring total employee costs back into the new Comfort Zone. For each of the three main components of other disbursements of after-tax revenue, namely dividends to shareholders, "Investment" (i.e. purchase of plant and machinery etc.), and "Other Spending", the industrialists review what cut-backs are possible. Non-essential items of "Other Spending" are generally cut back first, then investment programmes are pruned back, and finally, if necessary, the dividend is cut or "rebased" to a lower level. Again, the reduced spending translates into a reduction in net borrowings.

Unfortunately, all these cut-backs amplify the rate of decline of economic activity generally, with the result that the over-borrowed homeowner again revises downwards his Danger Zone Threshold. The overall

effect is that, once “survival behaviour” is triggered by a group with sufficient economic power, no short term equilibrium position is reached, and economic activity continues to decline at a rapid pace.

Similar arguments suggest that, once the economic contraction finally bottoms out, economic expansion then results from the return of “normal behaviour” and the steady upward revision of the Danger Zone Thresholds.

This process, in terms of appropriate economic ratios, is characterised by oscillatory behaviour between high and low extremes of values rather than a “random walk” about a long term equilibrium position that the teachings of financial economics imply.

Keynes’ rejection of classical economic theory rested largely on his belief, based on extensive practical experience, that economic behaviour is oscillatory in nature rather than determined by equilibrium conditions that are essentially constant over time. Such also, I believe, is the case in capital markets. Clarkson (1978) describes a mathematical model that is based on techniques similar to those used by actuaries in the graduation of mortality data, and provides a control theory framework for assessing which gilt-edged stocks are “cheap” or “dear” relative to the market generally. Clarkson (1981) extends this approach to equities. In both cases, the control theory framework is based on the Mean Absolute Deviation techniques that were first introduced into the UK actuarial methodology by Plymen & Prevett (1972).

Mills (1991) extends this type of analysis to equity market levels relative to gilts in terms of the “confidence” factor, namely the ratio of the gilt-edged yield to the dividend yield on equities. Again oscillatory motion is identified, and control limits are then constructed. The final sentence before the “Conclusions” paragraph is:

“It is perhaps interesting to note that these bounds are strikingly reminiscent of the control limits obtained by Clarkson (1978, 1981) from models analysing individual gilt-edged stocks and equity prices respectively.”

Mills suggests that close monitoring of the behaviour of this confidence factor “may provide useful information concerning possible reversals of the equity price index”. Similarly, I believe that close monitoring of economic ratios such as the proportion of after-tax income committed to mortgage servicing can predict a possible switch from “normal behaviour” to “survival behaviour” which can lead to a collapse of confidence and a reduction in economic activity far more severe than that

predicted by economic models which are essentially based on patterns of "normal behaviour" in the previous expansionary phase.

This switch from "normal behaviour" to "survival behaviour" is consistent with the old adage that stockmarket behaviour alternates between greed and fear, but a discontinuity of this type is inconsistent with the essentially static and continuous patterns of behaviour implied by the assumption underlying financial economics. I believe that the "Achilles' heel" of financial economics is its reliance on utility theory which in turn is based on the theory of games, an essentially linear system that cannot accommodate the discontinuities in behavioural patterns that lead to the observed oscillatory behaviour in the economic world in which we actually live.

Keynes (1936) compares classical economics to a two-dimensional system of geometry when a higher dimension is required to reflect reality:

"The classical theorists resemble Euclidean geometers in a non-Euclidean world who, discovering that in experience straight lines apparently parallel often meet, rebuke the lines for not keeping straight-as the only remedy for the unfortunate collisions which are occurring. Yet, in truth, there is no remedy except to throw over the axiom of parallels and to work out a non-Euclidean geometry. Something similar is required today in economics."

Clarkson (1978) extends to three dimensions the previous highly successful two-dimensional actuarial model for gilt-edged securities described in Pepper (1964). Clarkson (1991) describes a non-linear model of inflation that may be of general applicability in financial and economic time series. Similarly, it seems likely that significant advances in economic theory will only be possible if the essentially linear frameworks of the theory of games and utility theory are abandoned as being inconsistent with the economic system in which we actually live.

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