ENHANCED IMMUNIZATION TECHNIQUE APPLIED TO THE FRENCH BOND MARKET

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ABSTRACT

Duration hedging of a bond portfolio often proves to be insufficient specially in case of the ample swings of the yield curve European markets experienced over the last three years. An enhanced immunization technique was thus developed with the objective of being more efficient at a tolerable cost. A careful statistical analysis of the price and yield variation leads us to the conclusion that most of the yield curve movements could be decomposed in two basic movements: translation and torsion, that happen to be time–stable. This paper aims at presenting the immunization technique which rests on this observation and shows several tests of its usefulness. The analysis is performed on the OAT bond market and on the strips and produces in both cases a significant improvement over current practice.

1. INTRODUCTION

In recent years European bond market participants witnessed how difficult it is to trade or invest upside down. Owing to the economic situation and also to the monetary policies of the ERM phase yield curves were more often inverted than historical long versus short term spread would have suggested – some 120 basis point in France and in Germany. Traders found it more difficult to finance their bond position being exposed to a positive cost of carry. Investors had to rethink both their strategies and interest rate risk control, because of the large swings in the slope of the yield curve. However some events like the Kuwait invasion or the September '92 turmoil in European currency markets triggered in some cases back steps toward "normal" ascending yield curves. Unfortunately in such situations duration measurement and control, for example in an immunization scheme, proved to be inadequate. Moreover how puzzling was it to observe in October '92 that Matif went up whereas interest rates rose! Does it mean that one

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should forget about mathematics and financial theory and that Matif is not reliable as an hedging instrument? Of course not!

In order to improve the technical grasp on those troubles whose consequences are far from negligible, this paper presents an analysis of the risk borne when holding OAT (Obligation Assimilable du Trésor) in the spirit of multifactor risk models that flourish in the area of equities. Observations lead us to keep in the end two major factors that happened to be quite stable. Thus at the expense of adding a new factor called "torsion" we can set a more efficient risk control technique. Its use will be illustrated in an example pertaining to portfolio management.

To start with, a short market review of the French Government bond market and its recent development will allow us to introduce most of the yield curve concepts and estimations upon which subsequent risk measurement and analysis are made.

2. The French market

a) The OAT

The "Obligations Assimilable du Trésor" (OAT) are the main instrument used by the French Treasury for long term funding. These government bonds pay an annual coupon, are bullet and their maturity can reach 30 years (today, the longest one is the 8.5% 2023, which ends the 25th of April 2023). The denomination of an OAT is 2,000 FRF.

The 19 market makers are called "Spécialistes en Valeurs du Trésor" (SVT). They are the following: Banque Nationale de Paris, Caisse Centrale des Banques Populaires, Caisse des Dépôts et Consignations, Caisse Nationale du Crédit Agricole, Caisse de Gestion Mobilière Intermédiation, Crédit Commercial de France, Crédit Lyonnais, Banque d'Escompte, Banque Indosuez, Banque JP Morgan, Nomura, Banque Paribas, Société Générale, Union des Banques Suisses France, Warburg, Deutsche Bank, Morgan Stanley, Louis Dreyfus, Banque de l'Union Européenne.

b) The zero coupon bonds

In May 1991, the French Treasury decided to launch the zero coupon bond market, based on the OAT market. The SVT were allowed to strip the OAT which had the longest maturity at that time: the 8.5% 2019 (maturity date: 25th of October 2019). At this date, this OAT had still 29 coupons to come.

Stripping the OAT 2019 consists of creating 30 independent securities: the 29 former coupons and the former principal. Each one is considered as a zero coupon bond, as it delivers a single cash flow on its maturity date. For each of the 29 former coupons, the amount of this cash flow is 170 FRF (i.e. 2,000 × 8.5%); for the former principal, the amount is 2,000 FRF.

The following figure 1 shows the mechanism of stripping the OAT 2019.

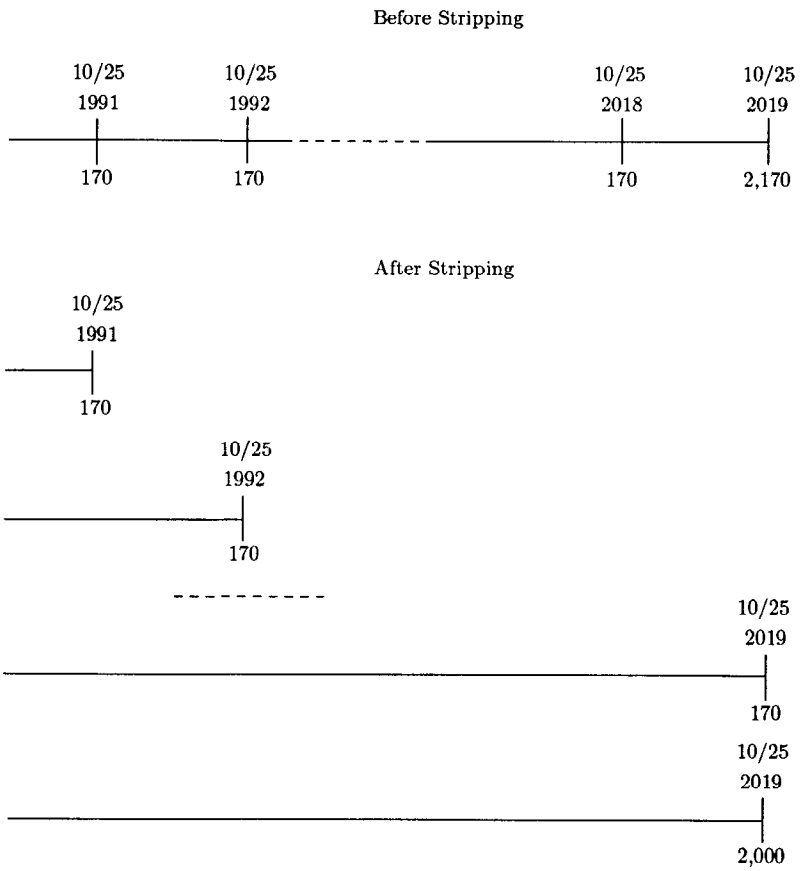

Fig. 1

The zero coupon bonds market, launched in May 1991, developed rapidly, and the volume of deals on these securities increased considerably. Nine months after the start of the market, the total amount of zero coupon bonds topped FRF 20 Billion FRF.

Following the OAT 2019 stripping, the OAT 8.5% 2023 (25th April 2023) was stripped so that today strips are made by 6–months maturities, up to April 2020 (every 25th of October and 25th of April).

3. Yield curves

The price of a bond, for instance an OAT, can be linked to a yield, the Internal Rate of Return. This rate is defined by equality between the price of the bond and the actual value of each cash flow to come, as shown in the following equation:

$$P = \sum_{i=1}^{N} \frac{C}{(1+y)^i} + \frac{100}{(1+y)^N} \tag{1}$$

P: Price of the bond
y: Yield of the bond
N: Number of years remaining (assuming a whole number)
C: Coupon

A zero coupon rate is used to calculate the actual value of a single cash flow to come in the future. Such a rate can be calculated for a zero coupon bond:

$$P = \frac{100}{(1+yz)^N} \tag{2}$$

P: Price of the zero coupon bond
yz: Rate of the zero coupon bond
N: Number of years remaining (assuming a whole number).

Let us consider the OAT bond market. Starting with the price of these bonds, and their yield, it is possible to calculate a theoretical zero coupon yield curve. This curve is determined by a constraint: for any OAT, if we calculate the present value of its coupons and its principal, the sum of these present values must be equal to the price of this OAT.

$$P = \sum_{i=1}^{N} \frac{C}{(1+yzi)^i} + \frac{100}{(1+yzn)^N} \,. \tag{3}$$

P: Price of the bond
yzi: Zero coupon rate i–years
N: Number of years remaining (assuming a whole number)
C: Coupon

This equation must be true for every bond on the market.

In fact, drawing such a zero coupon yield curve, which would make this equality true for every bond, is generally impossible, and the constraint will be softened: the sum of the present value of the coupons and principal will have to be as near as possible to the actual price of the bond.

Several methods have been elaborated to build a theoretical zero coupon yield curve. One of the most efficient was proposed by Vasicek and Fong.

Independently from this theoretical yield curve, the development of the zero coupon bond market gives us a zero coupon rate's market value of each strip quoted. Due to the stripping of OAT 2019 (25th of October) and OAT 2023 (25th of April), these securities are spread every six months up to April 2020 (and beyond, every year up to April 2023). This curve is named from now on "the market yield curve".

The following figure 2 represents both curves on the 18th of December 1992. They perceptibly differ, and the spreads we can see are structural: they have been quite the same for several months.

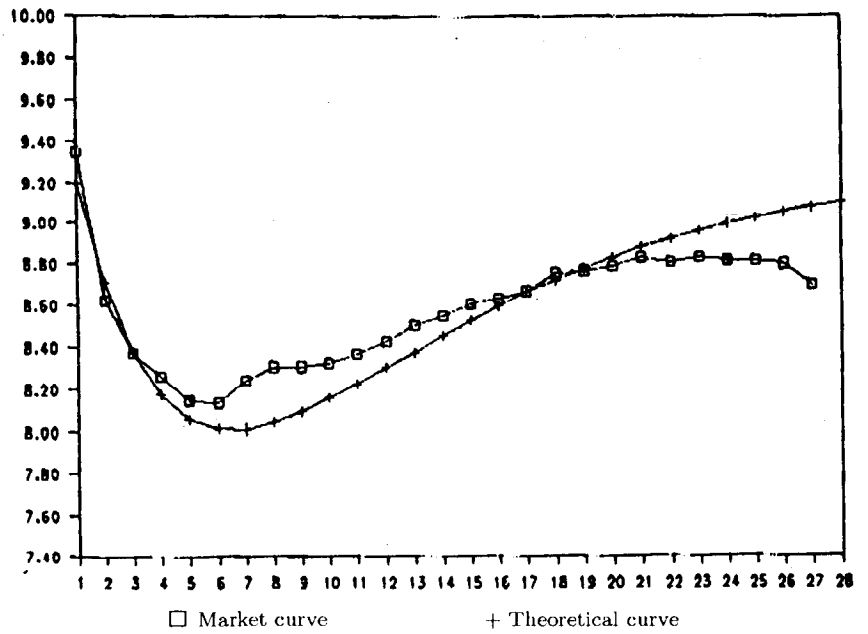

Fig. 2 Zero yield curves (18th december 1992)

Let us divide the yield curve into three parts: short term, medium term and long term. On the long term part, the market yield curve is under the theoretical curve, it is above on the medium term part, and the two curves are very similar on the short term curve.

The spread on the long term part means that the long term strips are more expensive than they should be according to the theoretical curve. They are expensive because their characteristics are very attractive. Among the zero coupon bonds, the longest represent the main innovation because of their long duration.

The longest maturity for an OAT is about 30 years, which leads to an average 10–years duration. The duration of a zero coupon bond is equal to its maturity, thus the duration of such a security can reach 30 years. Such a duration is much higher than what was previously available on the French market. That is why long term zero coupon bonds are very attractive for investors who are looking for a high sensitivity to the yield variations. It can be the case of foreign investors who seek to purchase securities with a high leverage. The principal is particularly appreciated because of its better liquidity.

The similarity between the two curves on the short term part is due to the little difference between bond yield and zero coupon rate for such maturities. In fact, short term zero coupon bonds behave like other bonds with the same maturity. The zero coupon bonds market is thus tightly linked to the OAT market. Therefore, the market curve cannot be very different from the theoretical curve, calculated from the OAT market.

The spread on the medium term part of the curve can be explained by the arbitrage between an OAT and its coupons after stripping. As it is always possible (for any SVT) to strip or to reconstitute, the price of a strippable OAT (i.e. OAT 2019 or OAT 2023) must be equal to the total price of all its coupons and its principal. Hence, if some of its coupons are expensive (long term ones), others must be cheap. As the medium term strips are not as attractive as long term ones, they are cheaper than they theoretically should be.

4. Risk analysis

The questions addressed in this section are: how to cope with the changes in the slope of the yield curve? What are the magnitude of different risks like translation of the curve or changes in its slope?

Are the conclusions dependent on the underlying instruments OAT's or strips?

a) Data Sets

Two data sets are selected in order to examine the above questions:

Sample n° 1. A set of 14 most liquid bonds was chosen (see below table 1). We considered closing weekly mid spread prices from January 1988 to September 1990. This sample was representative of the market at that time. All yield curves were computed with the Vasicek–Fong (1983) method and the "theoretical" prices were obtained by assuming cash flows of a given OAT are discounted with zero coupon yields.

Table 1. Sample 1

OAT	9.900%	Sept 1991
OAT	8.500%	May 1994
OAT	9.900%	Sept 1994
OAT	8.700%	May 1995
OAT	9.800%	Jan 1996
OAT	8.500%	June 1997
OAT	9.700%	Dec 1997
OAT	9.500%	June 1998
OAT	8.125%	May 1999
OAT	10.000%	May 2000
OAT	8.500%	Nov 2002
OAT	8.500%	Feb 2004
OAT	8.500%	Dec 2012
OAT	8.500%	Oct 2019

Data January 1988 to September 1990

Sample n° 2. Strips of the OAT 8.5% maturing in October 25. 2019 were analyzed through daily closing prices starting December 2, 1991 finishing May 15, 1992. It should be noted that for sake of comparison we also interpolate linearly prices corresponding to maturities from 1 to 28. Yield curve both actual or fitted by means of the Vasicek–Fong (1983) methodology were added to market prices.

b) Methods

We perform essentially principal comportment analysis (PCA) in order to break down the variances into independent pieces whose magnitudes are de facto obtained through the analysis. The mathematical technique rests on the construction of linear combinations of the input data which are mutually independent from a statistical point of view.

Variances are used to rank the component and add themselves to the sum of the input data variances.

The advantages of the method are clear and were used in many studies (Knez and al. 1989, Boulier and al. 1992). However one must check that the given sample effectively represents the market segment under review.

c) Results

PCA on OAT prices

Two separated PCA are applied to 1) market or 2) theoretical prices relative variations. This was done in order to check if interest rate risk is similarly captured from the yield curve movements and from actual prices and second, to assess the relative importance of the interest rate risk in market prices. The resulting first two principal components are shown in figure 3 where the linear coefficients (read on the y-axes) are plotted for each bond in data set 1. This comparison demonstrates that interest rate risk can be almost identically captured from either market or theoretical prices. Nevertheless percentages of total variance due to the first and second components slightly differ as depicted in table 2.

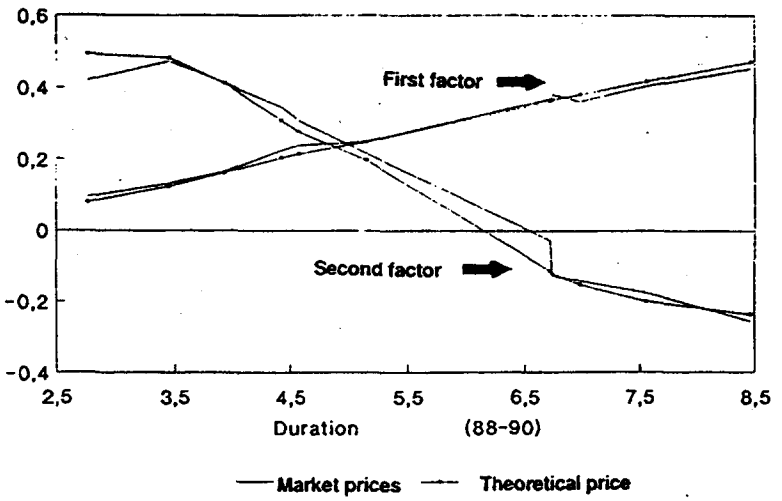

Fig. 3 First principal components - comparison of market and theoretical prices

Table 2. Contribution to total variance (%)

prices/component	n° 1	n° 2
market	87	7
theoretical	90	8

Whereas 98% of interest rate risk is related to only two factors, the same two factors account for only 94% price variance. This means that apart from the "idiosysmatic risk" there is a fairly important amount of specific risk pertaining to other market factors not directly related to interest rate risk, such as coupon rate (for tax reasons) or possible delivery at the expiry of Matif Futures contracts.

In the case of another time period similar results were obtained (Sikorav and Fitoussi 1991). The first component is indeed directly related to the translation of the yield curve (thus producing a first component proportional to duration as far as relative prices changes are concerned) whereas the second factor is a torsion of the yield curve.

To put it shortly, when breaking down the variance of relative price changes of a given OAT, one could expect the following orders of magnitude: 80 to 90% of the variance is due to prevailing mean interest rate conditions, 5 to 15 to tie to the effects of change of torsion of the yield curve on prices and the remaining part is mostly specific to that particular bond.

4.1. PCA ON THEORETICAL AND ON MARKET ZERO YIELD CURVE

If our portfolio contains both zero coupon bonds and other bonds, we are interested in both market and theoretical curves. The first one will determine the price variation of zero coupon bonds, the second one the price variation of other bonds.

For each of the two curves, the PCA gives us two main day–to–day curve variations. The data we used went from datset 2. Figure 4 shows these two main movements for the theoretical curve, figure 5 for the market curve.

Curves in figure 4 are much smoother than in figure 5. It is quite logical: the theoretical curve is analytic, hence smooth: so are its variations. The market curve is defined with 29 prices, and each one varies in its own way: the variation of such a curve is therefore more irregular.

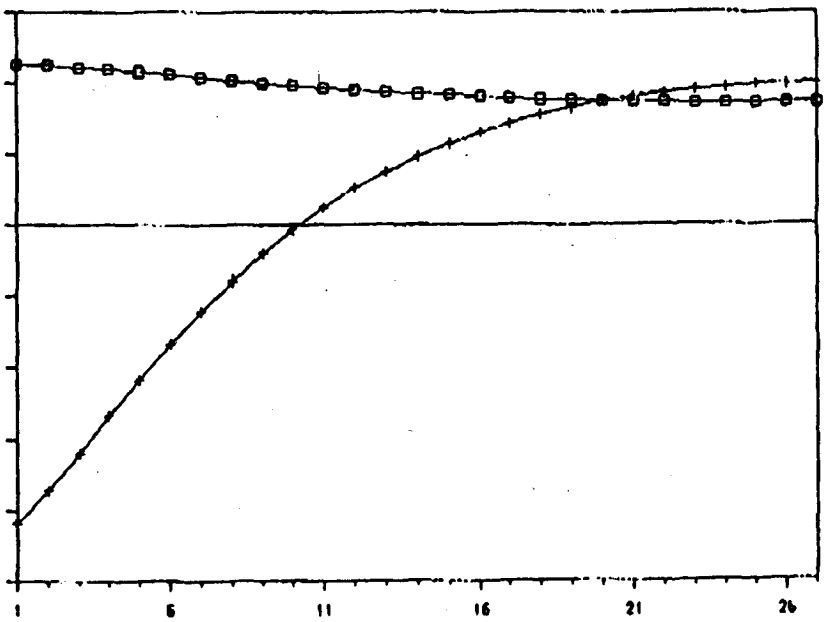

Fig. 4 Principal component analysis (Market yields) □ n° 1 + n° 2

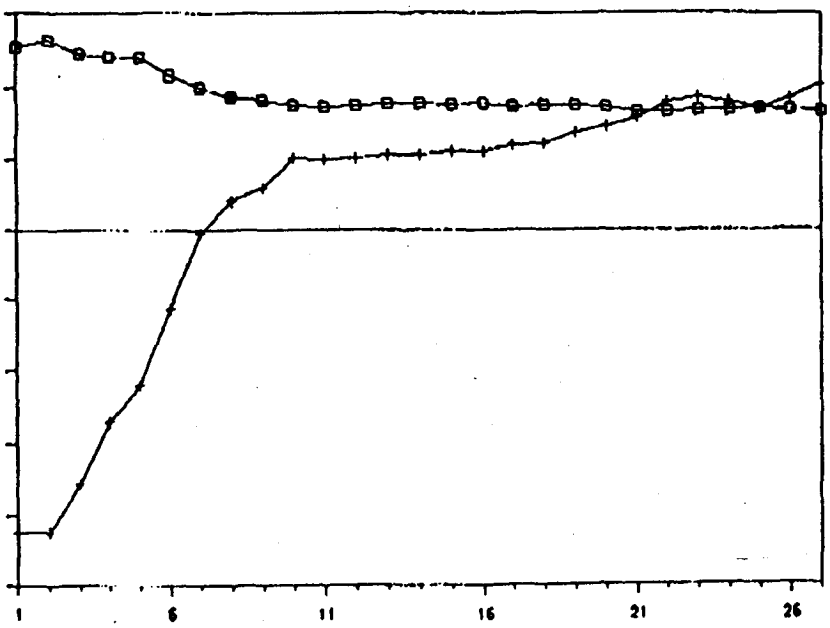

Fig. 5 Principal component analysis (Theoretical analysis) □ n° 1 + n° 2

Aside from this general difference aspect, the similarity between both figures is quite impressive. The first main movement is almost a translation, with a variation slightly more important for short term yields. The second main movement is a twisting: the short term rates vary strongly in one way, the long term ones in the other way, but much more weakly.

Also similar is the variation's percentage explained by these two main movements. The following chart shows these results for both curves.

We can note that the percentage explained by each of the two main movements is almost the same for both curves. Moreover, the percentage explained by the two of them is in both cases very near 100%, and therefore the study of the third main movement would not be very relevant.

This conclusion is very important because it allows us to be hopeful in trying to hedge a zero coupon bond portfolio with other bonds, or using zero coupon bonds to hedge other bond portfolios.

5. Immunization technique

The above statistical analysis was performed in order to enchance the risk control of a bond portfolio. As a typical example of such risk management we emphasize now the principle of an immunization technique that takes into account the main conclusions of the interest rate analysis.

Let P denote a portfolio invested in constant amount of bonds whose prices B can be split into two parts:

$$B = B_T + B_R$$

when B_T denotes the present value of the bond's cash flows corresponding to the theoretical zero coupon yield curve, and therefore B_R denotes the residual.

At a given date and for a given time horizon, the variations of P can be decomposed into a deterministic part (due to the interest accruals and the slight move of the cash flows along the yield curve) and a stochastic part due to yield curve fluctuations and residual price changes.

The immunization objective is at best to eliminate the uncertainties related to the stochastic component.

According to the classical duration hedging technique (see Bierwag 1985) immunization is achieved when equalling the portfolio duration with the investment horizon duration of the liabilities. Because we know from the preceding risk analysis that the underlying hypothesis made to obtain this well-known result is not met we propose a new method.

Several alternative techniques have already been proposed in order to cope with the main drawbacks encountered with the classical approach. Among the most recent attempts Chambers method (1988) use a duration vector whose coordinates generalize the usual duration and represent the portfolio sensitivity to changes of the yield curve corresponding to a polynomial function of the maturity. Reitano (1990) considers a set of sensitivities to changes of precise zero coupon maturities. Our method aims at more efficiently focusing on the most important risks and therefore concentrates on the two first components revealed by the risk analysis.

Following the same lines, the portfolio value is split into a theoretical value and a residual value. Moreover the theoretical value can be considered as the present value of the whole set of cash flows stemming from the actual bonds. For each cash flow paid at date m is a zero coupon of the same maturity m and yield y_m, its stochastic component is easy to derive:

$$\Delta z/z = -m\Delta y_m/(1+y_m) + \frac{1}{2}m(m+1)/(1+y_m)^2\Delta y_m^2 + \varepsilon_m \ .$$

Resting on the results of Section 4 we may approximate the variations of all the interest rates y_m as follows:

$$\Delta y_m = c_1(m)\Delta f_1 + c_2(m)\Delta f_2 + \varepsilon'_m$$

where C_1 and C_2 are the principal component already obtained, and Δf_1 and Δf_2 are the stochastic variations of the first two factors over the same time horizon.

Combining the two equations with the relevant proportion in wealth of the cash flows we obtain:

$$\begin{aligned}\Delta P/P = &-(\Sigma, C_1(m)m/(1+y_m))\Delta f_1 - (\Sigma, C_2(m)m/(1+y_m))\Delta f_2 + \\ &+ \frac{1}{2}(\Sigma C_1(m)m(m+1)/(1+ym)^2\Delta f_1^2) + E\end{aligned}$$

where E is an error term mainly due to:

1) SRP $= \Sigma w B_R$, the weighted Sum of the Residual Prices
2) ε_m, that is to say the high orders in the Taylor expansion of bond price as a function of its yield
3) the torsion effect (Δf_2) in the convexity term
4) the remaining principal components beyond the first two.

From our experience of the French bond market, one should only need to worry about the first point. Occasionally the second order term related to the convexity could also be disregarded because its order of magnitude in case of bond portfolio remains much lower than Δf_1, Δf_2, and even SRP terms.

In the end a similar decomposition should be done for the objective when it is necessary (stream of liabilities) and the hedging technique is simply implemented by equalling each sensitivity term.

Note that when the yield curve is flat and yield curve movement reduced to parallel shifts, torsion no longer exists and the first component is a constant. In such a case, our immunization technique coincides with the classical duration hedging. Namely the sensitivities to Δf_1 and Δf_1^2 may be respectively identified as the traditional modified duration and convexities.

6. Tests

The efficiency of any risk management technique relies on two basic ingredients:

- precise knowledge of the risk on a given universe of securities,
- smart adaptation of management principles to a given situation, especially by analyzing which factors to concentrate on and which errors to neglect.

For sake of illustration, we show in this section how efficient the above immunization technique would have been when applied to a very special bond replication exercise. The objective is to track a given OAT with a portfolio of two other OATs. For each bond in the first data set and for each weekly we built portfolios in such a way that they track as close as possible the target bond. Equating the first sensitivity (the durations) of the target and the possible tracking barbells will set the weights of the two OATs of the barbell. Among the possible barbells we will therefore select the ones which respectively minimize and maximize the sensitivity to the factor, thus reducing and amplifying the torsion effect of the yield curve.

Results of this tracking exercise are shown in table 3 below, where annualized tracking errors (standard deviation of relative price changes) in percentage are reported against the duration of the target bond. The second and third columns represent the tracking errors of the theoretical prices (present values computed with the zero coupon the yield curve). Reduction of the torsion effect in this exercise has a clear benefit; it results in a ratio of the tracking errors between 4 (low durations) and 5 (high durations). Unfortunately the true advantage is not that visible when looking at columns 4 and 5 where the market prices were used in order to compute the tracking errors. Indeed, the residual term accounts for a large proportion of the variance because we consider positions of three bonds.

Table 3.

	Replication market prices		Replication theoretical prices	
D	$R - R_0$ min	$R - R_0$ max	$R - R_0$ min	$R - R_0$ max
7	0.25%	1.24%	1.12%	1.63%
6	0.21%	1.19%	1.21%	1.49%
5	0.23%	1.15%	1.28%	1.36%
4	0.20%	0.65%	1.20%	1.40%
3	0.17%	0.68%	1.48%	1.42%

This is also shown on the four charts below in figure 6 which depict the replication errors (difference in values between the target and the best tracking portfolio) both in case of theoretical and market prices. Namely the differences in value at the end of the two and a half years may reach almost 1% (40 basis point a year) just because residuals were not taken into account. Nevertheless the volatility reduction is demonstrated in both cases.

Convexity is often invoked when making precise immunization adjustments. We therefore carry out a simple test of its usefulness in the preceding exercise. For each week the difference of performances between the minimizing and the maximizing barbells were compared to the convexity effect. Results are reported in figure 7 below and show that the yield curve torsion is indeed the major effect. This is confirmed by the ratio of explained variance in the regressions which are in the 65% to 75% range (depending on the target bond) in case of the torsion and only 0.3% in case of convexity.

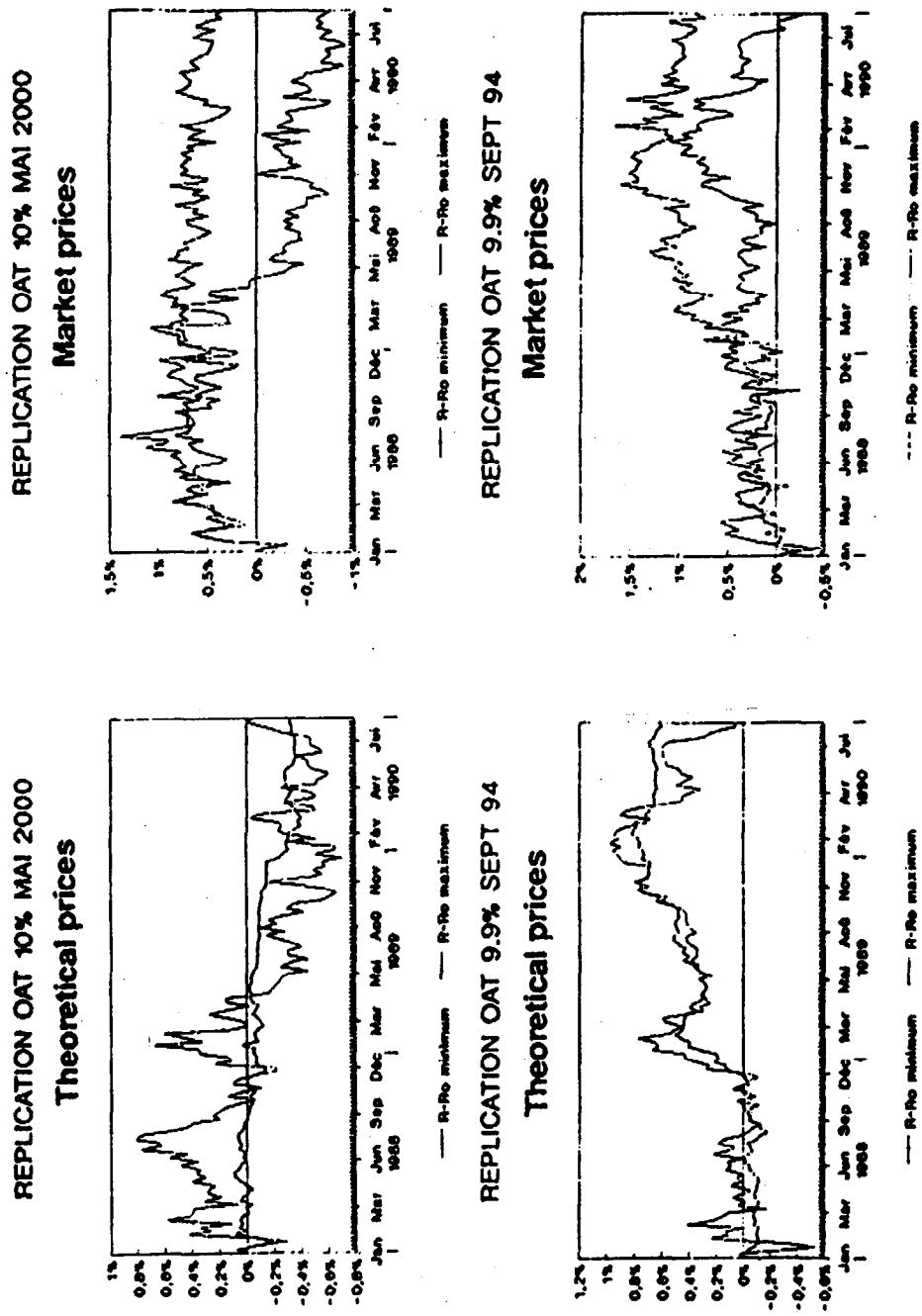
REPLICATION OAT 10% MAI 2000
Market prices
REPLICATION OAT 9.9% SEPT 94
Market prices
REPLICATION OAT 10% MAI 2000
Theoretical prices
REPLICATION OAT 9.9% SEPT 94
Theoretical prices
R-Ro minimum
R-Ro maximum

Fig. 6

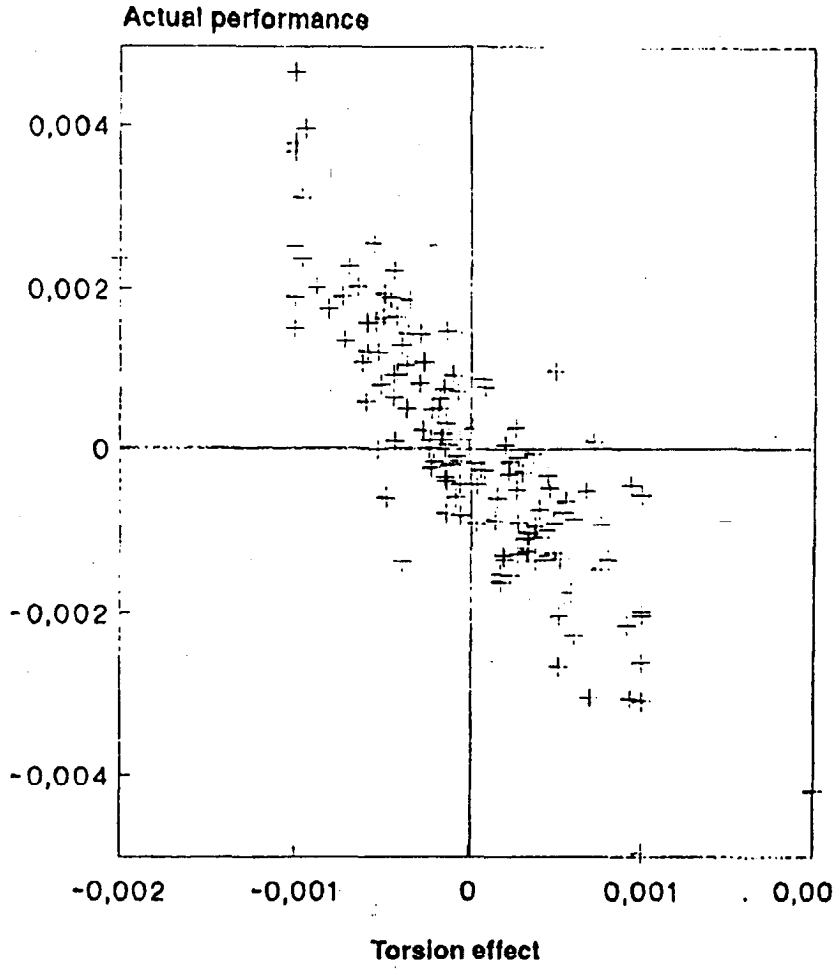

Fig. 7 Comparison of predicted versus actual – Portfolio performances

Practical implementation of this immunization technique proved also to be a useful enhancement over the traditional method whose main drawbacks are first, a too restrictive hypothesis on the yield curve variations and second, too little attention paid to the specific risk taken when hedging or embedded in the hedged position. In reality, those two drawbacks are easily overcome even with a less stringent discipline than the direct cash flow matching which clearly avoids the two difficulties. As an example, the actual risk management of a portfolio composed of

several strips all over the yield curve (1 to 29 years) was simulated with several hedging methods.

The absolute risk was a significantly reduced when controlling the torsion sensitivity, but this reduction was even more pronounced with a ad–hoc diversification of the hedge composition. Overall diversification and torsion monitoring produced the best results.

7. Conclusions

Bond markets in Europe turn out to be more sophisticated as a result of the technological evolution of customers and participants. With the deregulation process, customers have recently realized that interest rate risk is a major concern whose management should be a priority. Market participants have faced a steadily growing demand from investors for quality in execution and for ever increasing volumes of transactions. High margins and high risk positions could certainly not have been achieved without a strong commitment to technology. That is why we, at CCF, have steadily tried to improve our understanding of our domestic and of other European bond markets in order to trade more efficiently and to provide our client base with adequate information. CCF enjoys a fairly high share of the very competitive OAT market and was among the first to set the stripping business. The risk management technology also helped us to create in France mutual funds with a guarantee for the holder of a minimum interest rate, which happened to be a marketing success. The techniques presented here in the context of immunization on the OAT market can of course be adapted to fixed income management on other markets, for which our studies show similar patterns far as risk is concerned. Mixing this kind of method with return enhancement techniques either by means of good forecasts or by means of appropriate bond selection process, should certainly turn out to be very profitable.

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