A STOCHASTIC MODEL FOR CONSUMER PRICE INDICES AND EXCHANGE RATES IN SEVERAL COUNTRIES

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SUMMARY

In this paper an investigation is described into a suitable stochastic model for representing the behaviour of consumer price indices and currency exchange rates in a number of countries. The model is based on the author's earlier work in this field based on data from the United Kingdom (in what has become known there as 'the Wilkie model') and also on his paper for the 24th International Congress of Actuaries in Montreal in 1992.

The data analysed consists of monthly values of the consumer price index for 23 countries from 1969 to 1993, and of the exchange rate, versus U.S. dollars, for 22 of those countries from 1972 to 1993.

The author's original representation of rates of consumer price inflation was by a first order autoregressive time-series model for the annual inflation rate. This same model fits the data for all 23 countries, with somewhat different parameters. With monthly data available there are twelve different possible annual series for each country, and it is also possible to analyse series with differencing intervals other than yearly. These complexities are discussed, and the original conclusion is substantiated.

The author has previously suggested that exchange rates between two currencies could be modelled by an appeal to 'purchasing power parity', i.e. that the exchange rate responds in an appropriate way to changes in the consumer price indices in those two countries. Discrepancies between the model exchange rate and the actual exchange rate can be represented also by a first order autoregressive time series model. In this case the models for values at monthly intervals and at yearly or other intervals can be reconciled, and have the same mathematical form with different parameter values. This too is investigated, and the general form of such a model is confirmed.

Certain aspects of the mathematical models are considered, methods of simulating future scenarios are discussed, and the paper concludes by indicating ways forward for further research.

Un modèle stochastique pour les indices des prix à la consommation et les taux du change dans plusieurs pays

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Résumé

Le présent exposé décrit une recherche sous forme de modèle stochastique adéquat visant à représenter le comportement des indices des prix à la consommation et des taux de change des devises dans un certain nombre de pays. Le modèle est basé sur un travail précédent de l'auteur dans ce domaine et fondé sur des données provenant du Royaume-Uni (dans ce qui est maintenant appelé « le modèle Wilkie ») et également sur la présentation qu'il a faite au 24ème Congrès international des actuaires à Montréal en 1992.

Les données analysées consistent en valeurs mensuelles de l'indice des prix à la consommation pour 23 pays de 1969 à 1993 et du taux de change, comparé au dollar américain, pour 22 de ces pays de 1972 à 1993.

La représentation originale de l'auteur des taux d'inflation des prix à la consommation avait été effectuée par un modèle auto-régressif de série temporelle de premier ordre. Ce même modèle convient aux données provenant de tous les 23 pays, bien que les paramètres soient quelque peu différents. Dans les données mensuelles disponibles, il existe douze différentes séries annuelles possibles pour chacun des pays, et il est également possible d'analyser les séries à des intervalles de différenciation autres qu'annuels. Ces complexités sont traitées ici et des preuves sont apportées à l'appui de la conclusion initiale.

L'auteur a précédemment suggéré que les taux de change entre deux devises pouvaient faire l'objet d'un modèle en faisant appel à la « parité du pouvoir d'achat», c'est-à-dire que le taux de change répond en fonction de changements dans les indices des prix à la consommation dans ces deux pays. Des divergences entre le taux de change du modèle et le taux de change réel peuvent être représentées également par un modèle auto-régressif de série temporelle de premier ordre. Dans ce cas, les modèles pour les valeurs à intervalles mensuels et annuels ou autres peuvent être réconciliés et ont la même forme mathématique avec différentes valeurs de paramètres. Cet aspect est également étudié, et la forme générale d'un tel modèle est confirmée.

Il est tenu compte de certains aspects des modèles mathématiques et il est traité de méthodes de simulation de scénarios à venir. L'exposé conclut en suggérant plusieurs moyens de recherche supplémentaire.

1. Introduction

This paper describes an investigation into the behaviour of consumer price indices and currency exchange rates in a number of countries. I first described a variety of time-series models for the United Kingdom Retail Prices Index in Wilkie (1981). In that paper I found that, at least for the period since the First World War, the annual rate of inflation could be represented statistically by a first order autoregressive model. This model was incorporated into my fuller stochastic investment model, first presented in Wilkie (1986), and most recently in Wilkie (1992), in which I extended the analysis to the United States and France. In this last paper I also looked at the cross-correlations between the different consumer price indices studied, and I also began an investigation into a statistical model to represent exchange rates between the different currencies.

In this paper I extend this investigation further. I now have available data for the consumer price indices for 23 countries, from 1969 to 1993, and also data for the same 23 countries for exchange rates from 1972 to 1993. In both cases these data are at monthly intervals.

In Section 2 of this paper I describe and comment on the data. In Sections 3 and 4 I discuss a model for representing movements in consumer price indices, and in Sections 5 and 6 I extend the analysis to exchange rates.

2. Data

The data available are values of the Consumer Price Index (CPI) for 23 countries (whose names are listed in the Tables), in general at monthly intervals from January 1969 to May 1993, together with exchange rates at the end of each month for 22 countries versus the US dollar (the 23rd country being the United States), from September 1972 to May 1993.

The values of the consumer price indices have been taken, for the most part, from the OECD publication of "*Main Economic Indicators*", using the Historical Review, 1969-1988, and subsequent monthly volumes. Exceptions are the United Kingdom, for which I have used the General Index of Retail Prices, and South Africa, data for which has been kindly supplied by Mr Rob Thomson. According to the OECD publications, the indices used are generally the total "Consumer Price Index" but in some cases this is subject to qualifications. For the United States the index given is for "wage and salary earners only". I have extended the data for Japan back through 1969 by splicing on the published index for Tokyo, and I have extended the Swedish data for recent months by using the OECD published Consumer Price Index "excluding indirect taxes".

Values of the CPI for Australia, Ireland, New Zealand and South Africa are available only quarterly, and I have attributed the quarterly figure to the middle month of each quarter (February, May, August and November) and then interpolated linearly to give figures for intervening months.

Sometimes the published data appears to get revised. I cannot be sure that I have used the latest published version in each case. Nor can I be sure that the Consumer Price Index in the source used is in fact the most satisfactory one for the country concerned. Indeed I realise that for the UK the OECD figure is not the standard Retail Prices Index, which is why I have used this index instead.

Generally the indices are based on a 1985 average of 100.0, and include one decimal place. Values of the CPI have almost always been rising, so that recent values show numbers around 150.0, and early values around 20.0, though these ranges vary from country to country. In a few cases early values are only given in whole numbers. This diminishes the accuracy with which changes can be measured. The most extreme example of this is seen in the figures for Finland, where the early months all show a value of exactly 22. When the value changes to 23, this is equivalent to a 4.5% change in one month, but really this should be spread over some of the preceding or succeeding period.

Values of the exchange rate, measured in units of foreign currency per US dollar, have been supplied to my firm by Quantec Investment Technology Ltd, whose assistance I hereby acknowledge. These exchange rates are given to four decimal places, which is ample. The only gap in this source is in the data for South Africa, for which suitable exchange rates were missing from March 1983 to September 1985; these have been filled in from another source.

The number of countries in the exchange rate investigation should perhaps be diminished by one, because Belgium and Luxembourg are shown separately, although they share the same currency. In fact the given exchange rates for these two countries differ trivially in a small number of months.

Exchange rates between any two currencies other than the US dollar are not independently available, but on the assumption that cross-rates between countries i and j correctly reflect the rates of each with country k, exchange rates versus any chosen currency can readily be calculated.

3. Consumer Price Indices: Preliminary Investigations

We start with monthly values of the Consumer Price Index from January 1969 to May 1993 for all of the 23 countries. For a number of countries values for a few more recent months are also available, but these have not been used.

It is convenient to introduce some notation. The value of the Consumer Price Index in month t ($t = 1, \ldots, 293$, with January 1969 = 1 and May 1993 = 293) for country c ($c = 1, \ldots, 23$) is denoted by $Q(t, c)$. Where there is no ambiguity the parameter c will be omitted.

We must first decide whether to analyse the data at monthly intervals, or at some lesser frequency. We could, for example, pick out a series at annual intervals, and for each country there are 12 different series we might choose, those for January, February, ... We shall denote the series picking values every m months, starting with month h ($1 \leq h \leq m$), by $Q_{m/h}(u)$ where

$$Q_{m/h}(u) = Q(h+m(u-1)).$$

So, for example, if $m/h = 3/2$, ie picking at three-month intervals, starting with month 2 (February 1969), we pick $Q(2)$, $Q(5)$, $Q(8)$, ..., ie values for February 1969, May 1969, August 1969, ...

Let $n_{m/h}$ be the number of values of Q observed for the m/h series. The value of $n_{m/h}$ is roughly $293/m$, but where this is not an integer the number of values is the just higher integer for low values of h and the just lower integer for high values of h.

It is natural to use values of m that divide neatly into a year, ie $m = 1, 2, 3, 4, 6$ and 12, but it is possible to use other values of m too.

My previous investigations had used values at annual intervals, sometimes using values for June, sometimes for December, and for years in the more distant past whatever numbers were available. In Britain at least there is much to be said for considering values at twelve-monthly intervals, for a number of reasons. The popularly quoted figure is the rate of inflation for the preceding twelve months. Many wage negotiations take place at annual intervals, with the rate of inflation over the preceding year being one of the elements taken into account; hence certain prices are increased also at annual intervals. There is some seasonal variation, with certain foodstuffs (fresh fruit and vegetables) being cheaper in the summer months. There has been a convention (until 1993) of having the Government's Budget Statement annually in March, with changes in taxes on goods applying shortly thereafter, and hence affecting the April or May values of the Retail Prices Index.

My earlier investigations showed that there was a substantial correlation between values of the rate of inflation in one year and the rate of inflation in preceding years, with a correlation coefficient of about 0.6. For the purposes of the rest of my stochastic model, I required only to forecast values at annual intervals ahead. For this purpose I was not interested in seasonal variations, even though these might well be of interest for other purposes.

It was nevertheless useful to start by investigating what type of time-series model seemed likely to fit data picked at other intervals than twelve-monthly, ie for values of m other than 12.

A little more notation is needed. Let the force of inflation be defined as

$$I_{m/h}(u) = \ln Q_{m/h}(u) - \ln Q_{m/h}(u - 1)$$

for $u = 2, \ldots, n_{m/h}$. Thus the number of observations is reduced by one.

We first calculate the autocorrelation function and partial autocorrelation function for the series $I_{m/h}(u)$. See Box and Jenkins (1970) or any other standard textbook on time-series analysis for a definition of these two functions. The autocorrelation function for lag k is the usual (Pearson product-moment) correlation coefficient between $I_{m/h}(u)$ and $I_{m/h}(u-k)$ for all available pairs of observations. The partial autocorrelation function is the residual correlation between these series, after taking account of the correlation at lags less than k. The shapes of the autocorrelation function and partial autocorrelation function give an indication of whether an autoregressive (AR) or a moving average (MA) or a mixed (ARMA) model is likely to fit the data. For an autoregressive model of order one, AR(1), the theoretical partial autocorrelation function has a non-zero value at lag 1, dropping away to zero

immediately thereafter, whereas the theoretical autocorrelation function starts with the same non-zero value and declines exponentially thereafter. The observed functions are likely to be more irregular.

The first analysis was for the 292 values of $I(.)$ at monthly intervals for the United Kingdom, ie $I_{1/1}(u)$ for $u = 2$ to 293. The autocorrelation function showed significantly high values for eleven out of the first thirteen lags and a further four significantly high values up to lag 60 (five years). The partial autocorrelation function showed rather fewer significant values, but there were still seven within the first thirteen lags and a further seven thereafter.

Conspicuously high values of the autocorrelation function were observed at the values shows below:

lag	acf
1	0.44
2	0.31
3	0.29
5	0.24
6	0.38
12	0.54
24	0.39
36	0.32
48	0.33
60	0.32

It is clear that there is a strong seasonal effect, with high values of the autocorrelation function at lags 12, 24, 36, ..., which do not die away very quickly, together with considerable short-term autocorrelation. A first order autoregressive model would not fit, and we would at least have to go to a seasonal model such as recommended by Box and Jenkins (1970), taking into account autoregressive effects at lags 1 and 12, and their interaction at lag 13. Even this is unlikely to be sufficient, because of the strong effects at lags 3 and 6.

I therefore stepped up the differencing interval to two months, ie the value of m is increased to 2. There are now two series, with $m/h = 2/1$ and $2/2$, with 147 and 146 observations of $I(u)$ respectively. Selected values of the autocorrelation function for these two series are shown below.

m/h:	2/1	2/2
lag	acf	acf
1	0.42	0.53
2	0.32	0.32
3	0.45	0.38
4	0.20	0.25
5	0.24	0.25
6	0.54	0.51
7	0.16	0.29
12	0.37	0.35

Although the values are not the same for the two series, they are reasonably similar, with conspicuously high values at lag 6, which now represents a yearly interval, but rather high values at many other lags.

A similar pattern is found when we step up to three-month intervals ($m = 3$), with high values of the autocorrelation function at lags 1, 2, 3, 4 and 8 for each of the three possible series, as shown below:

m/h:	3/1	3/2	3/3
lag	acf	acf	acf
1	0.47	0.45	0.52
2	0.42	0.45	0.43
3	0.29	0.26	0.32
4	0.49	0.52	0.52
8	0.32	0.34	0.35

For data taken every four months the first three autocorrelation coefficients are significantly high:

m/h:	4/1	4/2	4/3	4/4
lag	acf	acf	acf	acf
1	0.47	0.53	0.61	0.48
2	0.32	0.38	0.45	0.36
3	0.53	0.48	0.51	0.53

When we step up to six-monthly intervals only the first two autocorrelation functions are conspicuously high, as shown in the table below. They do not, however, die away exponentially, and indeed the autocorrelation function at lag 2 is in some cases higher than that at lag 1.

m/h:	lag 1	lag 2
6/1	0.55	0.50
6/2	0.63	0.51
6/3	0.70	0.53
6/4	0.53	0.53
6/5	0.48	0.54
6/6	0.42	0.52

As might have been expected we are pushed towards taking observations at annual intervals. There are now twelve different series, some with 24, some with 23, values of $I(u)$. As the series gets shorter, it is perhaps not surprising that we do not find that values of the autocorrelation function for higher lags are significantly different from zero, even if they are as high say as 0.4. But other investigations with much longer series of the Retail Prices Index for the United Kingdom have shown a substantial autocorrelation at lag 1, which indeed dies away exponentially, with relatively low values of the partial autocorrelation function for higher lags. This is all consistent with a first order autoregressive or AR(1) series. The formula for this can be expressed as

$$I(u) = QMU + QA.(I(u-1) - QMU) + QSD.QZ(u),$$

where each term in this formula has the subscript $12/h$. *QMU* is the mean annual force of inflation. *QA* is the autoregressive parameter. *QSD* is the standard deviation of the residuals, $QZ(u)$, and the residuals have zero mean, unit standard deviation, and are approximately normally distributed. Whether or not the residuals can in fact be assumed to be normally distributed is to be tested. Values of *QMU*, *QA* and *QSD* are shown for each of the twelve separate series below. Note that the series for $h = 1$ to 5 (January to May) contain one extra observation as compared with the series for the other months (24 values of $I(u)$ against 23).

h	*QMU*	*QA*	*QSD*
1	0.0871	0.666	0.0352
2	0.0871	0.665	0.0350
3	0.0871	0.666	0.0353
4	0.0870	0.605	0.0398
5	0.0873	0.544	0.0437
6	0.0903	0.497	0.0451
7	0.0901	0.529	0.0431
8	0.0903	0.563	0.0418
9	0.0903	0.587	0.0406
10	0.0901	0.616	0.0386
11	0.0900	0.659	0.0364
12	0.0895	0.670	0.0360

Note that the mean is about 0.087 when values from 1969 to 1993 are taken, and 0.090 when the series stops in 1992. In fact the equivalent annual rate over the whole period is 0.0868, lower than any value shown in the table. This is because the earliest months in 1969 and the latest months in 1993 showed low values, and each of the annual series omits some of these.

Values of *QA* range from 0.497 to 0.670, with a mean of 0.605. There is a curious seasonal fluctuation in these value. It seems as though inflation is rather more predictable on a December to December basis than on a June to June one. It is not obvious why this should be the case. Values of *QSD* range from 0.0350 to 0.0451, with a mean of 0.0392. When the value of *QA* is low, the value of *QSD* is high and vice versa. This is to be expected; when variations are well explained by a high value of *QA*, the residual variance is less.

This rather lengthy preliminary investigation makes it clear that, at least for one country, annual differencing is appropriate. Substantial statistical tests show that the residuals after fitting the relevant autoregressive model appear to be independently distributed, and are approximately normally distributed. They are generally rather positively skewed, but in no case conspicuously fat-tailed. This was not true, however, for the values taken at monthly intervals, which were strongly positively skewed and conspicuously fat-tailed; this was before fitting an appropriate model, but the existence of certain extreme values suggested that the corresponding residuals would also be extreme even after fitting a suitable model.

4. Consumer Price Indices: 23 Countries

We now turn to investigating the data for all 23 countries for the same period, from January 1969 to May 1993. In each case we use data picked at annual intervals, and for each country we therefore have 12 different series.

For each series the mean and the first few autocorrelation coefficients of the values of $I(u)$ were calculated. Then the first autocorrelation coefficient was used as the autoregressive parameter in an AR(1) model, and the residuals under this model were calculated. This is exactly equivalent to a linear regression of the values of $I(u)$ for $u = 2$ to n on the values of $I(u)$ for $u = 1$ to $n - 1$.

For almost all of the 276 series the first autocorrelation coefficient was high, subsequent autocorrelation coefficients dropped away roughly exponentially, and the residuals after fitting an AR(1) model appeared to be random; the autocorrelation coefficients of the residuals were not significantly high.

The results are summarised in Table 1. This shows, for each country, certain values of *QMU*, the mean value of the $I(u)$ series, *QA*, the autoregressive parameter and *QSD*, the standard deviation of the residuals. For each of these three parameters are shown the lowest value observed for any of the twelve months, a mean value (explained further below) and the highest value for any of the twelve months. In the case of *QA* and *QSD* the mean value shown is the mean of the values for the twelve separate series. For *QMU* the overall mean is shown, which is 12 times the mean of the total monthly series from January 1969 to May 1993. For almost every country this overall mean is lower than the mean of the twelve separate yearly series, as was found for the United Kingdom and for the same reasons.

The overall mean is below 0.05 for four countries: Germany (0.0373), Switzerland (0.0405), Netherlands (0.0447) and Austria (0.0456). These countries are not only geographical and linguistic neighbours, but have pursued similar monetary policies. Four countries have overall means in excess of 0.10: Portugal (0.1481), Greece (0.1462), South Africa (0.1145) and Spain (0.1036). The overall means for the remaining 15 countries are distributed reasonably uniformly between 0.05 and 0.10.

The range of values of *QA* or of *QSD* gives an indication of the variability that can be found in such an investigation when the number of observations is not large. The range is typically of the same order as one standard error of the parameter estimate. For most countries the range of values of *QA* includes some part of the interval from 0.6 to 0.7; that is, either the lowest value is greater than 0.6 or the highest value is less than 0.7. There are six exceptions, three that lie wholly below this interval and three that lie wholly above. Those that lie wholly below are Sweden (0.174 to 0.391), New Zealand (0.394 to 0.573) and Greece (0.305 to 0.562). Those that lie wholly above are Ireland (0.717 to 0.823), Germany (0.729 to 0.786) and Netherlands (0.774 to 0.841). Norway is another country that shows unusually low values, but has a very wide range, from 0.175 to 0.638.

The particularly low values might indicate unusual peculiarities of the data, or simple errors, perhaps of transcription. But inspection of the particular values

does not suggest that these latter reasons could explain these particular values. There may, however, be some local reasons for these oddities, for which I am not aware.

All but eight of the countries have a range of values of *QSD* that spans the interval 0.02 to 0.03. Three lie wholly below this range: Germany (0.0115 to 0.0137), Austria (0.0132 to 0.0167) and Netherlands (0.0159 to 0.0184). Five lie wholly above this interval: Portugal (0.0448 to 0.0633), Greece (0.0393 to 0.0541), New Zealand (0.0362 to 0.0434), United Kingdom (0.0350 to 0.0451) and Japan (0.0314 to 0.0388). There is necessarily some connection between high values of *QA* and low values of *QSD* and vice versa, but this is not a uniform connection.

There is also some tendency for countries with low mean inflation rates to have low standard deviations of the original observations, and low values of *QSD*, and vice versa. This lends some support to the suggestion that an ARCH type of model might be appropriate, possibly even for one country. In an ARCH model the residual standard variation in the model is a function of time, and depends on the size of the preceding observation. High inflation rates become associated with high 'volatilities' of inflation, and vice versa.

The skewness and kurtosis coefficients of the values both of the original observations and of the residuals for each of the 276 series were calculated. Generally these indicated no marked departure from normality, as was already the case for the United Kingdom. But for certain countries, Belgium, Finland, Japan and Spain in particular, the distribution of residuals seemed particularly fat-tailed. The reasons for this have not been investigated.

The evidence so far is that an AR(1) model, with an autoregressive parameter typically in the range 0.6 to 0.7, gives an adequate representation of the annual forces of inflation in each of the 23 countries considered. Although there were some exceptional series for particular months for particular countries, in no country were there not several series for which an AR(1) model was not satisfactory.

The next step was to calculate the correlation coefficients between the residuals of the different series, both simultaneously and with appropriate lags.

Some notation will make this precise. Let $QE_{m/h}(u,c)$ for $u = 3, \ldots, n_{m/h}$ be the residuals for series m/h for country c after fitting the relevant parameters. The residuals are calculated as:

$$QE(u) = I(u) - QMU - QA.(I(u-1) - QMU),$$

where all the terms have subscripts m/h and parameter c. Residuals for $u = 1$ and $u = 2$ are not available.

Then for countries $c1$ and $c2$, the simultaneous correlation coefficient between the residuals is the correlation coefficient between the series $QE(u,c1)$ and $QE(u,c2)$ for $u = 3$ to n, for each of the m/h series. The cross-correlation coefficient of lag k between countries $c1$ and $c2$, with $c1$ following $c2$, is the correlation coefficient between the series $QE(u,c1)$ for $u = 3+k$ to n, and $QE(u,c2)$ for $u = 3$ to $n - k$. For any lag other than $k = 0$ (simultaneous correlation coefficient) there are two cross-correlation coefficients for any two countries, one with $c1$ following $c2$ and the other with $c2$ following $c1$. If $c1 = c2$ the formula produces the autocorrelation coefficients already calculated.

Simultaneous and lagged cross-correlation coefficients were calculated for low values of k for each of the twelve sets of series, each including the 23 countries. Tables 2a and 2b show the simultaneous correlation coefficients for the June (upper triangle) and December (lower triangle) series. Values of 0.6 or greater are shown in bold type. With a few exceptions the values are positive, with many exceeding 0.5, and a few exceeding 0.8. There is considerable similarity in the patterns of the two triangles.

It is possible to pick out clusters of countries with high correlations: Germany and Switzerland form one such, Belgium and Luxembourg another; and there is a cluster including Belgium, Denmark, France, Ireland, Italy and Japan. Some countries are more independent: South Africa has no correlation coefficient exceeding 0.5; Greece, New Zealand, Norway and Portugal have each no more than one or two values exceeding 0.5.

The lagged correlation coefficients are not shown. Even for lag 1 they are much smaller than the simultaneous correlation coefficients. But it is clear that they are not symmetrically distributed around zero, which they should be if there were in fact no lagged cross-correlations.

This is an area worth further study. It would be possible to treat the 23 separate series for the 23 countries as a series of observations of a single vector of 23 terms. This series of vectors could then be investigated in just the same way as the series for each individual country, by taking logarithms, differencing with suitable intervals, and so on. The vector series could then be analysed using a vector autoregressive (VAR) model. The equivalent of *QMU* would be a vector of 23 means, the same as the already calculated values. The equivalent of *QA* would be a 23×23 matrix, with autoregressive terms connecting the value of inflation in each country in each year with the values of inflation in each of the 23 countries in the previous year. And the standard deviations of the residuals would be replaced by a covariance matrix of residuals, containing implicitly all 23 standard deviations and all the simultaneous cross-correlations.

Such an investigation is well worth carrying out, but I have not done this at this stage.

Although a VAR model might be statistically satisfying, it has practical disadvantages. If one is wishing to make a short-term forecast of inflation in any one country, it may not be convincing to explain that one needs to know the rate of inflation in the preceding year in 22 other countries as well. The relevant values may not be obtainable timeously; indeed it is clear from the OECD data that not all countries do report their data so that it can appear simultaneously.

Further, if one is wishing to carry out a simulation exercise for only one country, the results from using a univariate model for that country may be equally as satisfactory as using a multivariate model for all 23 countries. Whether or not this would be the case needs investigation, but it is not intuitively obvious that using a VAR model would make a substantial difference when simulating prospective futures for any one country, though it might well make a difference when simulating futures for several countries together. There is plenty still to investigate in this area.

5. Exchange Rates: Preliminary Observations

We can now turn to an analysis of exchange rates. There are available monthly values of the exchange rate for each country, versus the US dollar, from September 1972 to May 1993. In fact that data is available for a few further months, but I wish to limit the investigation to the period for which values of the Consumer Price Index are also available.

Some more notation is needed. Let $X(t,c/b)$ denote the exchange rate in month t (t=45, ..., 293, with September 1972 = 45 and May 1993 = 293; this keeps the month subscript in step with that for the Consumer Price Indices), between the currencies for countries c and b, measured in units of currency c per one unit of currency b. In the initial investigation b represents US dollars, but the no-arbitrage feature of exchange cross-rates means that we can derive the exchange rate between any two countries, $c1$ and $c2$, from their exchange rates with the base currency, according to the rule:

$$X(t,c1/c2) = X(t,c1/b)/X(t,c2/b).$$

Note that

$$X(t,c2/c1) = 1/X(t,c1/c2).$$

As with the Consumer Price Index, if we wish to pick data at intervals other than monthly, we can denote the extracted series as $X_{m/h}(u)$.

One hypothesis commonly put forward by economists is that exchange rates conform to 'purchasing power parity', that is, they reflect changes in the purchasing power of the respective currencies. If we use the given Consumer Price Indices to represent purchasing power this would mean that

$$X(t,c1/c2).Q(t,c2)/Q(t,c1) = \text{constant}.$$

It is immediately obvious, by calculating values of the left hand side of this equation, that these are not constant. However, plots of this function suggest that the left hand side is reasonably stationary.

We therefore define the function:

$$P(t,c1/c2) = X(t,c1/c2).Q(t,c2)/Q(t,c1).$$

It is convenient, and justifiable for a number of reasons, to work in logarithms, and put

$$L(t,c1/c2) = \ln P(t,c1/c2) = \ln X(t,c1/c2) + \ln Q(t,c2) - \ln Q(t,c1).$$

The reasons for using logarithms are that all these values, the X's and the Q's, are simply ratios. Proportionate changes in any of them have an equivalent effect, whereas absolute changes do not. Further, there is no particular reason to analyse exchange rates expressed in $c1/c2$ form rather than their reciprocal expressed in $c2/c1$ form. Finally, using logarithms transforms the range of P, which is essentially positive, onto the whole real line.

As with the Consumer Price Index, the first question is whether to analyse the data at monthly intervals, or with some other differencing interval. In contrast with the consumer price indices, there is no particular reason to assume, in advance, any conspicuous seasonality in the exchange rates, and it is reasonable to assume that the

L series might itself be stationary, without any differencing, as was done for the Consumer Price Indices.

In my earlier analysis of exchange rates in Wilkie (1992) I used annual values, and found that a first order autoregressive model for L (which was not denoted thus) was indeed a first order autoregressive one. Would a first order autoregressive model also suit the monthly data? This requires a little more algebra.

Consider any series $Z(t)$, observed at monthly intervals, which has zero mean and is first order autoregressive. This means that

$$Z(t+1) = ZA.Z(t) + ZE(t+1).$$

Recursive application of this formula gives

$$Z(t+m) = ZA^m.Z(t) + ZA^{m-1}.ZE(t+1) + ZA^{m-2}.ZE(t+2) + \ldots + ZE(t+m).$$

This can be re-expressed as

$$Z_m(u+1) = ZA_m.Z_m(u) + ZE_m(u),$$

where $Z_m(u)$ is a series observed at $m$thly intervals, $ZA_m = ZA^m$, and $ZE_m(u)$ is distributed with mean zero and standard deviation ZSD_m, which is a function of ZA and ZSD; in fact $ZSD_m^2 = ZSD^2.(1 - ZA^{2m})/(1 - ZA^2)$.

Thus a first order autoregressive series with one differencing interval is equivalent to a first order autoregressive series with a longer differencing interval, with a suitable change in the parameters. The continuous equivalent is an Ornstein-Uhlenbeck process, just as the continuous equivalent of a pure random walk is a Weiner process.

If the original Z series conforms to a higher order autoregressive model, then the results are more complicated. It suffices to state, without going into the algebra, that, provided the principal root of the relevant characteristic equation is large in absolute value relative to the other roots, then the values of the series taken at $m$thly intervals, for m sufficiently large, approximate to a first order autoregressive series, with the principal root taking the place of ZA. The additional terms over and above the corresponding first order autoregressive model account for a short-term 'flutter', which dies away more quickly than the first order effect.

It is therefore convenient to analyse a series that may have an autoregressive model in the first place at monthly intervals, and then at say yearly intervals, and then to compare the values of ZA_1^{12} with ZA_{12}. If these are roughly equal, then it is reasonable to assume that the monthly series is itself first order autoregressive; if they are not, then an appropriate model for the monthly series may require additional terms, but these possibly die away quite quickly.

We are now ready to consider the analysis of different values of $L(t,c1/c2)$. A first order autoregressive series for the monthly data would mean that:

$$L(t,c1/c2) = XMU + XA(L(t-1,c1/c2) - XMU) + XSD.XZ(t,c1/c2)$$

and a corresponding first order autoregressive model for each of the possible yearly series is given by a similar formula, where each term is subscripted by $12/h$, and t is replaced by u.

6. Exchange Rates: 23 Countries

The exchange rates available were those quoted against the US dollar, and the first analysis was done using the US dollar as the base currency. The results, both for monthly and for yearly series, are shown in Table 3.

The first block in Table 3 shows estimates of *XMU*, the mean of each $L(t,c/b)$ series. In each case *XMU* is calculated as the mean of the values of $\ln X$ plus the difference between the mean values of $\ln Q$, for the appropriate countries. The value of *XMU* therefore depends on the 'scale' of the two currencies, and also on the base values of the Consumer Price Indices. Since all the Consumer Price Indices, except that for South Africa, were based on 1985 = 100, the value of *XMU* broadly represents the scale of the two currencies at 1985 exchange rates. The exponential of this, exp(*XMU*), indicates the relevant rate. One can see that Italy, Japan, Greece, Portugal and Spain have currencies whose units are small, whereas Ireland and the United Kingdom have currencies whose units are large, indicated by the negative signs for *XMU*. The values of *XMU* for South Africa are also negative, but this is affected in addition by the base value of the South African price Index used, which had a value of around 150 in 1985.

The three values in the *XMU* column show, respectively, the lowest value for any of the yearly series, the value for the one monthly series, and the highest value for any of the yearly series. For any one country these are quite close together.

The next column shows the value of the monthly *XA* to the power of 12. These range from 0.635 for Switzerland to 0.820 for Sweden. A low value means that the actual exchange rate converges more quickly to the purchasing power parity exchange rate than is the case when the value is high. There is not, however, a very great range in these values.

The next column shows values of *XA* from the yearly series, giving the lowest value for any month, the mean of the 12 values calculated, and the highest value for any month. Comparison of the values of *XA* for the yearly and the monthly series shows no very great difference between the values, though the monthly XA^{12} is larger than the typical yearly *XA* for more countries than it is smaller.

The range of values of the yearly *XA* overlaps the range 0.65 to 0.75 for all but two countries: Switzerland (0.533 to 0.606) is below and Sweden (0.770 to 0.809 is above).

The next two columns show values for the monthly standard deviation, *XSD*, and the low, mean and high for the yearly *XSD*. These are not quickly comparable one with another, but application of the formula for the yearly standard deviation in terms of the monthly values for *XA* and *XSD* shows that the yearly *XSD* should be roughly three times the monthly one. The monthly *XSD* lies between 0.03 and 0.04 for all countries but one, Canada, for which the value is 0.0134. This indicates the not unexpected result that the Canadian dollar diverges from the US dollar by less than do other currencies. The value of the yearly *XSD* for Canada is also relatively small, the range being 0.0419 to 0.0491. Other than for Canada the range of values of *XSD* overlaps the range 0.11 to 0.12 for all but three countries: Australia (0.0753

to 0.1039) and Norway (0.0856 to 0.1049) lie below this range and Belgium (0.1222 to 0.1440) lies above this range.

Simultaneous correlation coefficients between the residuals, that is between values of $XE(t,c1/b)$ and $XE(t,c2/b)$ were calculated for the monthly series and for each of the yearly series. Tables 4a and 4b show values for the December yearly series in the lower triangle, and the June yearly series in the upper triangle. Values of 0.6 or greater are shown in bold type. These triangles are reasonably similar, and both resemble the corresponding monthly triangle which is not shown.

Note that the near identity of the currencies of Belgium and Luxembourg is demonstrated by the almost perfect correlation between their residuals, rounding to 1.00 in each case. Note also how a group of currencies in Europe clusters together, including Belgium, Denmark, France, Germany, Ireland, Italy, Luxembourg and Netherlands. These are the countries that have adhered most closely to the European Exchange Rate Mechanism. The statistics show that Austria also comes into this group. Typical correlation coefficients between these countries exceed 0.9.

Another group of countries is closely associated with this European core, but with rather lower correlation coefficients: Finland, Greece, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom. Only Australia, Canada (which is closely associated with the United States), Japan, New Zealand and South Africa have rather lower correlations, though even these are generally significantly larger than zero.

The clustering within Europe suggests that it might be interesting also to analyse this data against the German mark, and this has also been done. Table 5 shows results analogous with those in Table 3. The overall values of XMU, derived from the monthly series, differ from those in Table 3 only by a constant factor, as do the values of *XMU* calculated from the series for each individual month. This is necessarily the case.

The values of *XSD* in Table 5 for both the monthly and the yearly series are generally quite a lot smaller than those in Table 3. This indicates that most of these currencies are more closely associated with the German mark than with the US dollar, and that the actual rate fluctuates much less.

The values of *XA*, measured both on a monthly and on a yearly basis, are also generally rather lower than in Table 3. This indicates a stronger tendency for the actual exchange rate to revert to the purchasing power parity rate.

Simultaneous cross-correlations between the series for different countries related to the German mark are shown in Tables 6a and 6b, with values for December being in the lower triangle and values for June in the upper triangle. The pattern is now rather different from that when exchange rates versus the US dollar are taken. Note that with only 20 or 21 observations available, a correlation coefficient needs to be bigger than about 0.45 in absolute value in order to be considered significantly different from zero. Many of the correlation coefficients in this table are smaller than this. Belgium and Luxembourg remain closely connected, and the close connection between Canada and the United States is apparent. Australia and New Zealand also adhere to this group, and Finland, Greece, Norway and Sweden also have highish

correlation coefficients with these countries. There are more negative correlation coefficients and even one rather large negative value of −0.68 (Denmark and Netherland, December).

Lagged correlation coefficients with small values of k were also calculated, for both the monthly and the yearly series, and versus the US dollar and against the German mark. While some values were large enough to be worth further investigation, the general pattern was of reasonable independence of all the series for all lags beyond simultaneity.

In Wilkie (1992) I suggested that exchange rates for three countries (in this case the United Kingdom, France and the United States) could be represented by three 'hidden' independent series, one for each country, with the exchange rates being represented as the ratios of these hidden series. In each case the hidden series depended on the Consumer Price Index in that country, together with a scale factor to represent the size of the unit of currency. Deviations between the hidden series and the CPI were represented by a first order autoregressive series with a yearly parameter of 0.7 for all countries.

Unfortunately this method of representation does not extend readily to more than three countries. It is still possible to hypothesise hidden series, but these cannot all be independent in the general case. However, it is still possible to investigate the effect of fitting a first order autoregressive series with a parameter, *XA*, for the yearly series, of 0.7 for each exchange rate, rather than the value of *XA* found so far. The monthly equivalent is close to 0.97. Both these parameters were used, both for the monthly series and for each of the twelve yearly series, for each exchange rate, both against the US dollar and against the German mark.

Using a less than optimal value for *XA* must increase the value of the observed standard deviation of the residuals, *XSD*, and indeed it does so, but only by a very small amount. For the monthly series versus the US dollar, the values of *XSD* are increased by no more than 0.0002, and versus the German mark by no more than 0.0004; for a majority of countries there is no observable change. For the yearly series the effect is larger, but versus the US dollar the effect is still small, no larger than 0.0022 for the mean value of *XSD*. Versus the German mark the effect is larger, since the optimal values of *XA* for some countries, e.g. France and Sweden, are much smaller than 0.7, but the overall effect is such that one would hesitate to assert that 0.7 per annum was definitely not an acceptable value for all countries.

The effect on the simultaneous correlation coefficients is observable, but not large. Even versus the German mark the differences are not such that it is worth showing the equivalents of Tables 6a and 6b.

An alternative approach might be to use the statistical method called principal components analysis to derive a model in which the simultaneous correlations can be represented in an economical way.

7. Simulations

On the basis of the models described above it is not difficult to set up a computer program to carry out random simulations of alternative futures with annual intervals for any selected group of countries. It is convenient to decompose each relevant matrix of covariances (calculated from the standard deviations and the correlation coefficients) into the equivalent triangular matrices, the Choleski decomposition of the matrix, as shown in Wilkie (1992). Then a suitable number of random unit normal variables are generated and the formulae applied mechanically.

An alternative approach is to use an analytical method to project the means and variances of the s step ahead forecasts. Since the entire model described in this paper is linear in the logarithms of all the variables it would be possible, using the methods described by Hürlimann (1992), to calculate the forecast means and variances and their covariances. I have not yet done this.

8. Conclusions

The investigations described in this paper support the author's earlier work. In each of the 23 countries investigated the CPI can be modelled by a first order autoregressive series for the annual logged changes in the value of the CPI, with reasonably similar parameters for each country and with simultaneous but not lagged correlations between countries. Exchange rates can be modelled first by 'purchasing power parity' and then by a first order autoregressive model, corresponding to an Ornstein-Uhlenbeck process, for the logged deviation between the actual rate and the purchasing power parity rate, again with certain of the parameters being similar for all countries, and with simultaneous but not lagged correlation.

Further investigations worth carrying out include: looking at the significant lagged correlation coefficients; trying VAR models for both sets of series; and finding parsimonious ways of characterising the strong correlations between certain groups of currencies, either by principal components analysis or otherwise. But not in this paper.

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Table 1. Analysis of Consumer Price Index for 23 countries from 1/69 to 5/93: 12 series with 23 or 24 yearly steps: (i) Parameters of AR(1) model

	QMU	*QA*	*QSD*
	Low - overall mean - high	Low - mean - high	Low - mean - high
Australia	0.0779 - 0.0768 - 0.0804	0.513 - 0.564 - 0.616	0.0249 - 0.0267 - 0.0297
Austria	0.0458 - 0.0456 - 0.0470	0.667 - 0.734 - 0.768	0.0132 - 0.0144 - 0.0167
Belgium	0.0530 - 0.0527 - 0.0542	0.631 - 0.712 - 0.776	0.0187 - 0.0211 - 0.0243
Canada	0.0613 - 0.0611 - 0.0630	0.631 - 0.674 - 0.728	0.0185 - 0.0200 - 0.0213
Denmark	0.0683 - 0.0677 - 0.0706	0.456 - 0.620 - 0.735	0.0215 - 0.0268 - 0.0335
Finland	0.0768 - 0.0759 - 0.0798	0.614 - 0.702 - 0.758	0.0228 - 0.0268 - 0.0304
France	0.0695 - 0.0690 - 0.0715	0.677 - 0.786 - 0.817	0.0188 - 0.0208 - 0.0267
Germany	0.0373 - 0.0373 - 0.0380	0.729 - 0.755 - 0.786	0.0115 - 0.0127 - 0.0137
Greece	0.1464 - 0.1462 - 0.1518	0.305 - 0.435 - 0.562	0.0393 - 0.0486 - 0.0541
Ireland	0.0907 - 0.0899 - 0.0941	0.717 - 0.765 - 0.823	0.0292 - 0.0348 - 0.0397
Italy	0.1006 - 0.0996 - 0.1033	0.566 - 0.698 - 0.800	0.0269 - 0.0341 - 0.0433
Japan	0.0501 - 0.0503 - 0.0520	0.587 - 0.650 - 0.714	0.0314 - 0.0355 - 0.0388
Luxembourg	0.0505 - 0.0501 - 0.0516	0.664 - 0.738 - 0.782	0.0173 - 0.0195 - 0.0231
Netherlands	0.0445 - 0.0447 - 0.0463	0.774 - 0.820 - 0.841	0.0159 - 0.0166 - 0.0184
New Zealand	0.0959 - 0.0950 - 0.0995	0.394 - 0.492 - 0.573	0.0362 - 0.0397 - 0.0434
Norway	0.0715 - 0.0709 - 0.0744	0.175 - 0.480 - 0.638	0.0196 - 0.0232 - 0.0290
Portugal	0.1494 - 0.1481 - 0.1554	0.337 - 0.446 - 0.627	0.0448 - 0.0567 - 0.0633
South Africa	0.1147 - 0.1145 - 0.1182	0.548 - 0.615 - 0.660	0.0168 - 0.0189 - 0.0209
Spain	0.1050 - 0.1036 - 0.1086	0.693 - 0.765 - 0.808	0.0238 - 0.0282 - 0.0350
Sweden	0.0768 - 0.0761 - 0.0781	0.174 - 0.304 - 0.391	0.0242 - 0.0255 - 0.0284
Switzerland	0.0406 - 0.0405 - 0.0415	0.551 - 0.601 - 0.652	0.0176 - 0.0196 - 0.0217
UK	0.0870 - 0.0865 - 0.0903	0.497 - 0.605 - 0.670	0.0350 - 0.0392 - 0.0451
USA	0.0565 - 0.0564 - 0.0575	0.623 - 0.663 - 0.692	0.0207 - 0.0224 - 0.0247

Table 2a.
Analysis of Consumer Price Index for 23 countries from 1/69 to 5/93:
12 series with 23 or 24 yearly steps: (ii) Correlation coefficients of residuals.
Lower triangle for December series, upper triangle for June series

	Aus	Ost	Bel	Can	Den	Fin	Fra	Ger	Gre	Ire	Ita
Australia	**1.0**	.11	.52	.43	.40	.52	.50	.07	.26	.48	**.60**
Austria (Ost)	.09	**1.0**	**.73**	.39	.57	.59	**.69**	.57	.36	**.70**	.53
Belgium	.43	**.60**	**1.0**	.53	.50	**.66**	**.82**	.58	.29	**.76**	**.63**
Canada	.55	.24	**.61**	**1.0**	.56	.49	**.65**	.42	.33	.52	.51
Denmark	.53	.47	.48	.51	**1.0**	.43	**.80**	.42	.40	.44	**.67**
Finland	.59	.39	.57	.52	.35	**1.0**	.54	.54	-.06	**.69**	**.79**
France	.42	**.65**	**.77**	**.61**	**.71**	.48	**1.0**	**.60**	.44	**.70**	**.69**
Germany	.13	.41	.30	.47	.25	.48	.37	**1.0**	-.06	**.65**	.52
Greece	.31	-.04	-.07	.21	.11	.26	.01	.28	**1.0**	.25	.28
Ireland	.58	**.65**	**.60**	.47	**.68**	.54	**.84**	.42	.18	**1.0**	**.68**
Italy	.53	**.62**	**.63**	.33	**.80**	.40	**.78**	.15	.13	**.78**	**1.0**
Japan	.53	**.63**	.54	.43	**.72**	.56	**.67**	.29	.35	**.67**	**.72**
Luxembourg	.51	.45	**.79**	.38	.24	.52	.55	.39	.02	.53	.56
Netherlands	.22	**.60**	**.62**	.44	.31	.55	**.65**	.55	-.08	**.66**	.41
New Zealand	.54	-.05	.11	.52	.34	.16	.05	.01	.26	.26	.23
Norway	.46	.17	.34	.49	.35	.40	.33	-.16	.21	.35	.39
Portugal	.46	.09	.32	.15	.49	.18	.53	-.08	.11	.47	.48
South Africa	.26	-.03	.17	.31	.12	.00	.17	-.03	.24	.24	.25
Spain	.51	.06	.38	.52	**.72**	.40	.36	.09	.11	.30	.44
Sweden	.18	.21	.54	.50	.40	.35	.52	.11	.17	.29	.49
Switzerland	.29	.40	.33	.53	.33	.42	.32	**.83**	.51	.33	.20
UK	.48	.25	.51	.54	.14	.57	.45	.35	.20	.53	.27
USA	.35	.42	.47	**.62**	.47	.41	**.71**	.40	.41	.53	.53

Table 2b.
Analysis of Consumer Price Index for 23 countries from 1/69 to 5/93:
12 series with 23 or 24 yearly steps: (ii) Correlation coefficients of residuals.
Lower triangle for December series, upper triangle for June series

	Jap	Lux	Net	NZ	Nor	Por	SA	Spa	Swe	Swi	UK	US
Australia	.50	.31	.05	.56	.37	.30	.27	**.44**	.26	.15	.58	.39
Austria (Ost)	**.60**	**.62**	**.62**	.04	.28	.34	.01	.31	.27	.36	.47	.52
Belgium	**.64**	**.78**	.53	.22	.21	.51	.23	.37	.28	**.60**	.58	.53
Canada	.45	.36	.16	.36	.50	.23	.35	.32	.46	.48	.36	**.61**
Denmark	**.77**	.32	.28	.11	.35	.44	-.01	.56	.41	.33	.50	**.68**
Finland	.45	**.77**	.53	.29	.55	.09	.06	.18	.45	.36	**.69**	.45
France	**.75**	.57	.42	.33	.25	.53	.23	.46	.33	.52	**.62**	**.83**
Germany	.43	**.63**	**.75**	.09	.09	.01	.24	.35	.12	**.64**	.50	.53
Greece	.56	.05	-.14	.09	.22	.22	.34	.12	.10	.01	.05	.41
Ireland	.45	**.72**	**.67**	.33	.30	.30	.39	.36	.22	.47	**.77**	**.64**
Italy	**.72**	**.70**	.40	.18	.44	.29	.15	.38	.53	.34	**.74**	**.66**
Japan	**1.0**	.42	.29	.04	.12	.34	.24	.45	.20	.39	.47	**.68**
Luxembourg	.29	**1.0**	**.66**	.06	.32	.25	.06	.23	.39	.45	.53	.35
Netherlands	.44	.47	**1.0**	.10	.06	-.02	.00	.30	.23	.59	.56	.31
New Zealand	.10	.05	-.05	**1.0**	.48	.06	.33	.19	.25	-.01	.43	.35
Norway	.28	.20	.09	.45	**1.0**	-.13	-.05	.15	.52	-.17	.24	.15
Portugal	.43	.29	.15	-.02	.05	**1.0**	.09	.46	.27	.22	.38	.36
South Africa	.17	.04	.26	.45	.27	-.11	**1.0**	.05	-.18	.31	.26	.36
Spain	.48	.15	.14	.35	.31	.40	-.05	**1.0**	.28	.16	.51	.30
Sweden	.28	.36	.37	.19	.45	.26	.26	.49	**1.0**	.12	.44	.27
Switzerland	.53	.32	.41	.13	-.08	.02	.15	.17	.09	**1.0**	.37	.36
UK	.24	.52	.46	.41	.13	.19	.14	.19	.37	.22	**1.0**	**.68**
USA	**.64**	.32	.29	.17	.18	.31	.14	.27	.43	.45	.55	**1.0**

Table 3. Analysis of Exchange Rate for 22 countries versus US dollar from 9/72 to 5/93: 12 series with 20 or 21 yearly values: (i) Parameters of AR(1) model

	XMU Low - overall mean - high	Monthly XA^{12}	Yearly *XA* low - mean - high	Monthly *XSD*	Yearly *XSD* Low - mean - high
Australia	0.0714 - 0.0790 - 0.0834	0.685	0.561 - 0.718 - 0.802	0.0302	0.0753 - 0.0880 - 0.1039
Austria	2.6634 - 2.6779 - 2.6910	0.706	0.616 - 0.658 - 0.696	0.0349	0.1103 - 0.1205 - 0.1315
Belgium	3.6721 - 3.6894 - 3.7037	0.813	0.734 - 0.763 - 0.789	0.0365	0.1222 - 0.1336 - 0.1440
Canada	0.1830 - 0.1862 - 0.1876	0.802	0.746 - 0.787 - 0.810	0.0134	0.0419 - 0.0454 - 0.0491
Denmark	1.9745 - 1.9902 - 2.0034	0.747	0.684 - 0.712 - 0.749	0.0346	0.1133 - 0.1216 - 0.1302
Finland	1.5515 - 1.5653 - 1.5789	0.688	0.634 - 0.684 - 0.726	0.0367	0.0907 - 0.1056 - 0.1299
France	1.8311 - 1.8462 - 1.8568	0.751	0.624 - 0.683 - 0.725	0.0337	0.1055 - 0.1184 - 0.1291
Germany	0.6735 - 0.6894 - 0.7042	0.744	0.665 - 0.699 - 0.726	0.0358	0.1116 - 0.1215 - 0.1302
Greece	4.5408 - 4.5683 - 4.6010	0.693	0.679 - 0.716 - 0.749	0.0357	0.0996 - 0.1057 - 0.1133
Ireland	−0.2963 - −0.2797 - −0.2677	0.685	0.475 - 0.592 - 0.694	0.0326	0.0950 - 0.1043 - 0.1139
Italy	7.2204 - 7.2429 - 7.2546	0.761	0.617 - 0.671 - 0.742	0.0347	0.1038 - 0.1186 - 0.1343
Japan	5.2393 - 5.2585 - 5.2703	0.717	0.652 - 0.680 - 0.709	0.0348	0.1173 - 0.1311 - 0.1435
Luxembourg	3.6716 - 3.6905 - 3.7064	0.814	0.743 - 0.771 - 0.798	0.0363	0.1178 - 0.1299 - 0.1391
Netherlands	0.8020 - 0.8207 - 0.8366	0.737	0.651 - 0.695 - 0.728	0.0356	0.1108 - 0.1217 - 0.1317
New Zealand	0.3885 - 0.3993 - 0.4091	0.693	0.533 - 0.597 - 0.695	0.0342	0.1015 - 0.1149 - 0.1272
Norway	1.8445 - 1.8579 - 1.8679	0.666	0.619 - 0.680 - 0.760	0.0314	0.0856 - 0.0932 - 0.1049
Portugal	4.7423 - 4.7715 - 4.7951	0.751	0.670 - 0.742 - 0.803	0.0364	0.1083 - 0.1160 - 0.1282
South Africa	−0.0910 - −0.0713 - −0.0535	0.784	0.582 - 0.628 - 0.689	0.0348	0.1182 - 0.1356 - 0.1462
Spain	4.7646 - 4.7770 - 4.7908	0.817	0.675 - 0.708 - 0.725	0.0312	0.1142 - 0.1272 - 0.1437
Sweden	1.7792 - 1.7874 - 1.7945	0.820	0.770 - 0.787 - 0.809	0.0323	0.1022 - 0.1118 - 0.1245
Switzerland	0.5560 - 0.5755 - 0.5848	0.635	0.533 - 0.568 - 0.606	0.0388	0.1164 - 0.1300 - 0.1451
UK	−0.4593 - −0.4430 - −0.4275	0.705	0.544 - 0.620 - 0.664	0.0347	0.0987 - 0.1150 - 0.1287

Table 4a.
Analysis of Exchange Rate for 22 countries versus US dollar from 9/72 to 5/93: 12 series with 20 or 21 yearly values: (ii) Correlation coefficients of residuals. Lower triangle for December series, upper triangle for June series

	Aus	Ost	Bel	Can	Den	Fin	Fra	Ger	Gre	Ire	Ita
Australia	1.0	.43	.34	.63	.41	.47	.42	.46	.60	.43	.37
Austria (Ost)	.05	1.0	.93	.27	.98	.71	.93	.99	.83	.89	.87
Belgium	.05	.96	1.0	.14	.95	.70	.88	.93	.73	.77	.80
Canada	.55	-.04	-.05	1.0	.24	.42	.24	.27	.38	.33	.34
Denmark	.09	.98	.96	-.06	1.0	.73	.91	.98	.83	.85	.85
Finland	.24	.87	.85	.20	.87	1.0	.76	.68	.59	.74	.83
France	.03	.95	.94	-.17	.96	.84	1.0	.93	.79	.91	.86
Germany	.02	.99	.96	-.08	.98	.84	.95	1.0	.86	.85	.81
Greece	.20	.87	.84	.01	.89	.89	.81	.87	1.0	.71	.67
Ireland	.01	.92	.83	-.11	.90	.85	.89	.90	.87	1.0	.94
Italy	-.03	.94	.89	-.04	.92	.85	.92	.92	.84	.95	1.0
Japan	.06	.69	.62	.03	.67	.43	.63	.65	.42	.55	.65
Luxembourg	.02	.97	1.00	-.05	.96	.84	.94	.97	.83	.83	.89
Netherlands	.03	.98	.97	-.08	.96	.86	.94	.99	.88	.87	.89
New Zealand	.58	.35	.31	.33	.37	.46	.31	.34	.45	.40	.28
Norway	.09	.95	.93	.01	.92	.89	.91	.93	.86	.87	.89
Portugal	.05	.84	.81	.01	.85	.88	.84	.80	.85	.84	.88
South Africa	.41	.49	.49	.15	.46	.45	.52	.51	.54	.48	.52
Spain	.08	.89	.81	-.08	.92	.85	.89	.88	.86	.91	.91
Sweden	.08	.92	.87	-.05	.93	.87	.93	.90	.81	.87	.88
Switzerland	.10	.93	.87	-.03	.94	.77	.92	.94	.80	.87	.88
UK	.09	.76	.69	-.17	.76	.74	.82	.76	.78	.90	.82

Table 4b.
Analysis of Exchange Rate for 22 countries versus US dollar from 9/72 to 5/93: 12 series with 20 or 21 yearly values: (ii) Correlation coefficients of residuals. Lower triangle for December series, upper triangle for June series

	Jap	Lux	Net	NZ	Nor	Por	SA	Spa	Swe	Swi	UK
Australia	.33	.35	.45	.74	.59	.37	.68	.35	.45	.48	.63
Austria (Ost)	.61	.93	.98	.78	.91	.84	.65	.87	.76	.93	.71
Belgium	.59	1.00	.93	.67	.83	.76	.57	.77	.80	.82	.61
Canada	.07	.15	.24	.50	.34	.36	.22	.22	.24	.31	.37
Denmark	.63	.95	.97	.79	.90	.81	.62	.86	.79	.91	.66
Finland	.33	.69	.72	.61	.85	.70	.46	.77	.91	.68	.74
France	.52	.88	.93	.73	.88	.86	.63	.87	.78	.90	.78
Germany	.56	.94	.99	.79	.89	.82	.67	.83	.74	.91	.67
Greece	.47	.74	.82	.80	.83	.81	.70	.80	.73	.79	.69
Ireland	.56	.77	.84	.77	.85	.81	.60	.83	.73	.91	.84
Italy	.47	.79	.81	.70	.83	.77	.54	.85	.83	.84	.82
Japan	1.0	.60	.54	.62	.57	.42	.36	.51	.43	.61	.42
Luxembourg	.61	1.0	.94	.67	.83	.76	.58	.77	.79	.81	.60
Netherlands	.61	.97	1.0	.76	.90	.81	.68	.83	.76	.91	.67
New Zealand	.24	.29	.33	1.0	.81	.60	.75	.67	.61	.80	.73
Norway	.57	.93	.93	.40	1.0	.86	.72	.87	.88	.85	.80
Portugal	.51	.81	.80	.21	.83	1.0	.52	.82	.77	.77	.69
South Africa	.23	.48	.50	.32	.52	.38	1.0	.52	.55	.60	.69
Spain	.58	.80	.85	.37	.80	.87	.39	1.0	.84	.79	.75
Sweden	.63	.87	.88	.30	.88	.92	.36	.93	1.0	.67	.78
Switzerland	.74	.87	.91	.38	.84	.75	.50	.89	.89	1.0	.75
UK	.29	.70	.74	.45	.75	.71	.52	.83	.77	.74	1.0

Table 5. Analysis of Exchange Rate for 22 countries versus German mark from 9/72 to 5/93: 12 series with 20 or 21 yearly values: (i) Parameters of AR(1) model

	XMU Low - overall mean - high	Monthly XA^{12}	Yearly *XA* low - mean - high	Monthly *XSD*	Yearly *XSD* Low - mean - high
Australia	−0.6244 - −0.6104 - −0.5979	0.578	0.398 - 0.475 - 0.567	0.0405	0.1081 - 0.1282 - 0.1519
Austria	1.9796 - 1.9885 - 1.9932	0.826	0.736 - 0.803 - 0.839	0.0070	0.0140 - 0.0168 - 0.0196
Belgium	2.9954 - 3.0000 - 3.0079	0.653	0.437 - 0.542 - 0.577	0.0144	0.0365 - 0.0459 - 0.0603
Canada	−0.5181 - −0.5032 - −0.4865	0.659	0.487 - 0.572 - 0.618	0.0356	0.1069 - 0.1240 - 0.1365
Denmark	1.2944 - 1.3008 - 1.3089	0.644	0.535 - 0.674 - 0.751	0.0114	0.0238 - 0.0283 - 0.0322
Finland	0.8505 - 0.8759 - 0.8983	0.699	0.513 - 0.631 - 0.688	0.0300	0.0676 - 0.0924 - 0.1103
France	1.1506 - 1.1568 - 1.1622	0.526	0.097 - 0.187 - 0.364	0.0139	0.0360 - 0.0426 - 0.0512
Greece	3.8673 - 3.8789 - 3.8997	0.269	0.162 - 0.337 - 0.553	0.0303	0.0488 - 0.0590 - 0.0696
Ireland	−0.9748 - −0.9691 - −0.9626	0.882	0.789 - 0.848 - 0.890	0.0191	0.0461 - 0.0622 - 0.0864
Italy	6.5437 - 6.5534 - 6.5632	0.709	0.646 - 0.764 - 0.862	0.0263	0.0426 - 0.0688 - 0.0803
Japan	4.5456 - 4.5691 - 4.5793	0.792	0.664 - 0.748 - 0.795	0.0314	0.1012 - 0.1116 - 0.1206
Luxembourg	2.9960 - 3.0011 - 3.0086	0.621	0.455 - 0.574 - 0.698	0.0142	0.0347 - 0.0433 - 0.0569
Netherlands	0.1219 - 0.1313 - 0.1387	0.551	0.365 - 0.579 - 0.695	0.0088	0.0127 - 0.0179 - 0.0212
New Zealand	−0.3067 - −0.2902 - −0.2720	0.440	0.290 - 0.551 - 0.663	0.0395	0.0743 - 0.0947 - 0.1312
Norway	1.1584 - 1.1685 - 1.1856	0.653	0.522 - 0.583 - 0.653	0.0186	0.0518 - 0.0571 - 0.0611
Portugal	4.0611 - 4.0821 - 4.0972	0.641	0.330 - 0.445 - 0.507	0.0250	0.0728 - 0.0820 - 0.0893
South Africa	−0.7694 - −0.7608 - −0.7455	0.677	0.580 - 0.649 - 0.740	0.0372	0.0918 - 0.1158 - 0.1340
Spain	4.0770 - 4.0876 - 4.0945	0.767	0.583 - 0.662 - 0.713	0.0247	0.0635 - 0.0749 - 0.0840
Sweden	1.0894 - 1.0980 - 1.1112	0.427	0.098 - 0.222 - 0.302	0.0226	0.0508 - 0.0668 - 0.0817
Switzerland	−0.1252 - −0.1140 - −0.1066	0.723	0.573 - 0.685 - 0.735	0.0176	0.0453 - 0.0531 - 0.0638
UK	−1.1515 - −1.1325 - −1.1139	0.779	0.617 - 0.700 - 0.787	0.0290	0.0752 - 0.0994 - 0.1249
USA	−0.7042 - −0.6894 - −0.6735	0.744	0.665 - 0.699 - 0.726	0.0358	0.1116 - 0.1215 - 0.1302

Table 6a. Analysis of Exchange Rate for 22 countries versus German mark from 9/72 to 5/93:
12 series with 20 or 21 yearly values: (ii) Correlation coefficients of residuals.
Lower triangle for December series, upper triangle for June series

	Aus	Ost	Bel	Can	Den	Fin	Fra	Gre	Ire	Ita	Jap
Australia	**1.0**	-.18	-.21	**.86**	-.05	.45	.22	**.71**	.41	.15	.37
Austria (Ost)	.05	**1.0**	.20	.01	.51	.51	.38	-.17	.27	.55	.10
Belgium	-.00	.19	**1.0**	-.24	.46	.16	-.01	-.31	-.07	.19	.10
Canada	**.91**	.14	-.15	**1.0**	.09	.47	.32	**.70**	.35	.22	.39
Denmark	.09	.37	.14	.06	**1.0**	.50	.15	-.07	.05	.39	.38
Finland	**.62**	.52	-.04	**.72**	.41	**1.0**	.59	.19	.58	**.70**	.11
France	.15	.28	.05	.20	.43	.39	**1.0**	.22	**.75**	.56	.23
Greece	.58	.08	-.20	**.61**	.10	**.60**	.05	**1.0**	.26	-.03	.25
Ireland	.48	.26	-.23	.43	.26	.55	.45	**.60**	**1.0**	**.75**	.14
Italy	.04	.45	-.08	.17	.40	.43	.51	.28	.56	**1.0**	.00
Japan	.27	.38	-.13	.39	.21	.12	.19	-.12	-.02	.22	**1.0**
Luxembourg	.08	.20	**.96**	-.02	.08	.04	.07	-.09	-.21	-.07	-.16
Netherlands	.10	-.18	.15	-.04	**-.68**	-.21	-.12	-.14	-.05	-.46	-.20
New Zealand	**.70**	.16	-.22	**.74**	.26	**.70**	.15	.43	.55	.18	.30
Norway	**.68**	.38	-.07	**.78**	.13	**.78**	.39	.57	.48	.26	.27
Portugal	.18	.44	.15	.29	.43	.55	.36	**.61**	.27	.34	.05
South Africa	.54	-.11	.06	.43	-.12	.16	.11	.37	.33	.24	.02
Spain	-.00	.19	-.25	.01	**.73**	.37	.41	.26	.47	.52	.04
Sweden	.41	.17	-.06	.44	.36	.53	.56	.52	.39	.15	.12
Switzerland	-.09	.14	-.34	-.06	.33	-.06	.24	-.13	-.02	.10	.46
UK	.41	-.04	-.13	.34	.32	.44	.54	.46	**.84**	.45	-.19
USA	**.89**	.05	-.20	**.95**	.02	**.61**	.30	**.60**	.57	.21	.37

Table 6b. Analysis of Exchange Rate for 22 countries versus German mark from 9/72 to 5/93:
12 series with 20 or 21 yearly values: (ii) Correlation coefficients of residuals.
Lower triangle for December series, upper triangle for June series

	Lux	Net	NZ	Nor	Por	SA	Spa	Swe	Swi	UK	US
Australia	-.12	-.07	**.61**	**.63**	.30	**.63**	.16	.44	-.01	.50	**.82**
Austria (Ost)	.14	.38	-.10	.51	.31	-.03	.30	.32	.24	.27	.02
Belgium	**.98**	.27	-.31	-.11	-.03	-.09	.04	.32	-.21	.07	-.16
Canada	-.16	-.13	.48	**.63**	.47	.36	.24	.38	-.01	.34	**.93**
Denmark	.41	.28	.19	.37	.08	.01	.39	.35	.17	.05	.13
Finland	.15	.43	.31	**.79**	.33	.22	.55	**.74**	.20	**.64**	.40
France	-.01	.21	.17	.51	.51	.18	.49	.40	.25	.58	.38
Greece	-.25	-.20	.35	.50	**.66**	.54	.33	.50	-.02	.32	**.69**
Ireland	-.11	.20	.37	.53	.25	.35	.36	.42	.45	**.77**	.44
Italy	.10	.26	.19	.44	.13	.16	.50	.49	.28	**.67**	.24
Japan	.14	-.12	.51	.44	.14	.29	.18	.17	.18	.23	.47
Luxembourg	**1.0**	.24	-.29	-.08	-.01	-.02	.00	.32	-.32	.04	-.09
Netherlands	.13	**1.0**	-.26	.23	-.02	-.01	.01	.35	.21	.03	-.05
New Zealand	-.20	-.20	**1.0**	.51	-.06	**.60**	.15	.14	.19	.49	.44
Norway	.07	-.10	**.67**	**1.0**	.50	.46	.53	**.72**	.16	**.62**	**.61**
Portugal	.19	-.32	.08	.45	**1.0**	.13	.41	.57	.03	.27	.39
South Africa	.09	-.00	.31	.35	.04	**1.0**	.16	.37	-.17	.53	.43
Spain	-.33	-.57	.20	.02	.50	-.13	**1.0**	.57	-.05	.52	.29
Sweden	-.02	-.13	.23	.53	**.79**	.11	.52	**1.0**	-.02	**.62**	.37
Switzerland	-.39	-.34	.10	-.17	-.05	-.19	.30	.01	**1.0**	.24	-.02
UK	-.09	-.16	.51	.43	.21	.35	.47	.41	-.14	**1.0**	.36
USA	-.09	.06	**.68**	**.76**	.21	.46	.04	.45	-.10	.47	**1.0**