

Long-Term Investment Strategy Using Stock  
As A Hedge Against Inflation

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**Summary**

This paper suggests an investment strategy to formulate a long-term stock portfolio. It allows the investor to specify the desired return on investment to be equal to the expected rate of inflation plus a certain premium rate. The model then helps the investor select those stocks which will provide the greatest chance of meeting that specified long-term investment goal. The formulated portfolio would generate a return exceeding the rate of inflation. Thus, the suggested model can become an effective vehicle for using stock as a hedge against inflation over the specified period of investment.

A term structure model is derived to estimate long-term returns on stock investments based on short-term market information. The projected rates of return and their associated risks are included in a portfolio formulation framework to derive the optimal stock investment strategy. This approach permits the investor to set up the appropriate long-term investment strategy without involving the assessment of his risk preference. Furthermore, the decision maker is able to adjust the structure of the portfolio from time to time during the decision time frame without altering the established long-term investment objectives.

An empirical testing of the proposed model is included in the paper to illustrate the effectiveness of the suggested investment formulation strategy. It is also used to compare against the strategy suggested by many practitioners where the portfolio is suggested to consist of equal portion of ten highest-yield stocks in the Dow Jones industrial average.

## **Stratégie d'investissement à long terme utilisant les actions comme couverture contre l'inflation**

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### **Résumé**

Nous suggérons dans cet exposé une stratégie d'investissement permettant la formulation d'un portefeuille de titres à long terme. Elle permet à l'investisseur de spécifier un rendement souhaité de l'investissement qui soit égal au taux d'inflation anticipé plus une certaine prime. Le modèle permet alors à l'investisseur de sélectionner les titres offrant la meilleure chance de satisfaire à l'objectif d'investissement à long terme ainsi spécifié. Le portefeuille formulé produirait un taux de rendement actuariel dépassant le taux de l'inflation. Le modèle suggéré peut ainsi donc constituer un moyen efficace qui permettrait d'utiliser les titres comme couverture contre l'inflation au cours d'une période d'investissement spécifiée.

Un modèle de structure à terme est dérivé qui permet d'estimer les rendements à long terme d'un portefeuille de titres en se basant sur les informations du marché à court terme. Les taux de rendement escomptés, ainsi que les risques leur étant associés, sont inclus dans le cadre de la formulation du portefeuille pour en dériver la meilleure stratégie possible d'investissement. Cette méthode permet à l'investisseur de mettre en place la stratégie appropriée d'investissement à long terme sans avoir à évaluer sa préférence de risque. De plus, le décideur peut ajuster la structure du portefeuille de temps à autre au cours de la période de décision sans altérer les objectifs d'investissement établis à long terme.

Une mise à l'essai empirique du modèle proposé est incluse dans notre exposé pour illustrer l'efficacité de la stratégie de formulation d'investissement suggérée. Elle est également utilisée pour comparer celle-ci à la stratégie suggérée par un grand nombre de conseillers financiers qui recommandent un portefeuille constitué à parts égales des dix actions offrant les rendements les plus élevés dans l'indice « Dow Jones ».

LONG-TERM INVESTMENT STRATEGY USING STOCK  
AS A HEDGE AGAINST INFLATION

I. INTRODUCTION

Although the incentive for holding stock investments over a long period may vary from one investor to another, such investments could become an effective hedge against inflation if stocks have been carefully selected. The feasibility of using stocks as an inflation hedge has been the focus of several previous studies (for example, see Oudet 1973; Branch 1974; Fama and MacBeth 1974; Bodie 1976) which define a stock as an inflation hedge only if its real return is independent of the rate of inflation. This implies that the beta estimate for the relationship between the real return from a stock and the inflation rate should be one.

A recent experiment conducted by analysts at Merrill Lynch (See Money, January 1990), however, suggests that from a portfolio management practitioner's point of view, a simple rule could be followed to formulate a long-term stock portfolio that could yield a return well above the rate of inflation. The report indicated that over the past 20 years (1968-1988) an annually adjusted portfolio composed of the 10 highest-yield stocks in the Dow Jones industrial average provided a total return of 1,557%. During the same period, inflation increased by only about 353%.

The results reported by the practitioners suggest that a carefully formulated stock portfolio could have yielded a real return by as much as 1,200% over 20 years. According to the practitioners' report, an alternative definition of the term

"inflation hedge" as applied to common stocks may be stated as follows: a stock is an inflation hedge as long as it will yield a return exceeding the rate of inflation. This definition suggests that an investor might be able to formulate a long-term stock portfolio by specifying the desired return on investment as equal to the expected rate of inflation plus a certain premium rate, and then select, according to some rules, those stocks which will provide the greatest chance of meeting that specified investment goal. In so doing, this formulated stock portfolio would be qualified as an inflation hedge.

The strategy suggested by the portfolio management practitioners is simple. Buy an equal number of shares of each of the ten highest-yielding stocks in the Dow Jones industrial average. Once a year, in January for instance, replace those stocks that no longer rank among the top ten. While this approach did yield phenomenal return, it would be interesting to examine if this strategy can continue to work in the future and if it will always yield the highest possible returns. From an academic viewpoint, it would be of value to study whether stocks can become an effective tool to hedge inflation in the long run. In addition, one would also like to know if any model could be devised to systematically analyze the stock return expectations, and thereby allow the selection of the optimal long-term stock investment portfolio. The purpose of this paper is to develop such a model, hereafter called the Inflation-Hedge Stock Investment Portfolio Model, or simply the IHSI model.

The implementation of the IHSI model requires the use of information about the expected long-term return on stocks.

Several prior studies have addressed the question of estimating stock returns (for example, see Sharpe 1964; Nelson 1976; Kwan 1984; and Flannery and James 1984), but only in the short-run, which makes none of the methods suggested by these studies suitable for a model with the specific goal of providing long-term protection against inflation via investment in stocks. What is needed then is a new methodology that will take stock return estimates beyond the immediate short-term basis. This methodology will be explained in Section II of this paper.

Historical data on Dow-Jones industrial average stocks will be used in Section III to illustrate how the newly developed methodology can be used to provide long-term estimates of stock returns. The IHSI model will be presented in Section IV. In Section V an example will be given to show how the information generated in Section III can be used in conjunction with the IHSI Model to compose an inflation-hedge portfolio. Empirical testing of the model will occur in Section VI. The paper will conclude with some remarks and suggestions for future research.

## II. TERM STRUCTURE OF STOCK RETURNS

The task now is to develop a model for estimating long-term stock returns. Long-term stock returns can be related to short-term returns in a manner similar to the way long-term interest rates are related to short-term interest rates (Chambers, Carleton, and Walsman 1984; Hilliard 1984; and Campbell 1987). In the interest-rate case the relationship is referred to as the term structure of interest rates. Similarly, the relationship between long-term stock returns and its relevant short-term

determinants will be referred to as the term structure of stock returns. To examine the term structure of stock returns, a two-factor model similar to that suggested by Stone (1974) is used. The model, hereafter called the TSSR Model, is specified as follows:

$$R_{j,t+1} = \alpha_0 + \alpha_1 R_{i,t+1} + \alpha_2 R_{m,t+1} + v_{j,t+1} \quad (1)$$

where

$t$  = the time when the investment decision is to be made for a holding period ending at  $t+1$ . Therefore,  $t$  counts the prior investment periods. When  $t = 3$ , for example, the TSSR model will project stock returns during the fourth investment period,

$R_{j,t+1}$  = expected return on the  $j$ th stock for the next (i.e.,  $t+1$ ) holding period,

$R_{i,t+1}$  = the interest rate factor index for the same period,

$R_{m,t+1}$  = The selected equity market index for the same period,

$v_{j,t+1}$  = the random disturbance.

To formulate a portfolio using equation (1), the user has to project the  $R_m$  and  $R_i$  over a specified period ending, say, at time  $t+1$ . Unless  $R_m$  and  $R_i$  can be obtained from outside sources, and are constantly available, it would be necessary to follow a systematic procedure to determine them before  $R_j$  can be estimated.

It has been suggested that long-term returns may be extrapolated by a weighted distribution of historical short-term returns and other relevant factors measuring economy and financial market conditions. Because the level of interest rates and stock returns are highly correlated (See Ibboston and Sinquefield 1976; Bodie and Rosansky 1980, Flannery and James

1984; and Campbell 1987), one can be used as a proxy for the other. This explains the presence of the term  $R_{i,t+1}$  in equation 1. Interest rates are more readily available and subject to more precise measurement than stock returns. Trying to pick out of a basket of, say, thirty stocks, the ten with the best promise for higher future returns using a model employing, for example, five lag periods will require the gathering of 150 return values; five returns per stock. By using the level of interest rates instead of stock returns in the model, one needs only to find five interest values, one per lag period; a much simpler task indeed.

Among those non-interest rate factors that need to be considered when estimating  $R_{j,t+1}$ , the most commonly cited determinant is the rate of inflation (for examples see Schwert 1981; Loo 1988; and Buono 1989). Other determinants that may be worth considering are unemployment rates, trade deficits, and exchange rates.

The expected interest rate factor index for the period ending at time  $t+1$ , i.e.,  $R_{i,t+1}$  in equation (1), may be projected in accordance with the following relationship (See Flannery and James 1984; Chambers, et. al., 1984; and Hilliard 1984):

$$R_{i,t+1} = a + \sum_{k=0}^m a_k \cdot r_{i,t-k} + B \cdot F_i \quad (2)$$

where

- $m$  = the number of lag periods to be used in estimating  $R_i$
- $k = 0, 1, 2, \dots, m$
- $r_{i,t-k}$  = the short term interest rate factor index at time  $t-k$
- $F_i$  = the factor including all other relevant variables for the estimation of  $R_i$  over the next decision period
- $B$  = the coefficient vector of the variables in  $F_i$
- $a_k$ 's = the weights of the lagged  $r_i$ 's

Following a similar line of reasoning, the projected equity market index over the same period ending at the time  $t+1$  may also be derived using the following relationship (see Kwan 1984):

$$R_{m,t+1} = C + \sum_{q=0}^n C_q \cdot I_{m,t-q} + DF_m \quad (3)$$

where

$n$  = the number of lagged periods to be used in estimating  $R_m$

$r_{m,t-q}$  = the short term equity market index at time  $t-q$

$F_m$  = the vector including all other relevant variables for the estimation of  $R_{m,t+1}$

$c_q$ 's = the weights of the lagged  $r_m$ 's

$D$  = the coefficients' vector of the variables in  $F_m$ .

From (2) and (3), the expected long term return on the  $j$ th stock for the  $t+1$  decision period, thus, may be related to current and historical short term interest rates, short term equity market indices, and other non-interest rate determinants by using the following expression:

$$\begin{aligned} R_{j,t+1} &= \alpha_0 + \alpha_1 \left( a + \sum_{k=0}^m a_k I_{i,t-k} + BF_i \right) \\ &\quad + \alpha_2 \left( C + \sum_{q=0}^n C_q I_{m,t-q} + DF_m \right) + V_{j,t+1}. \end{aligned} \quad (4)$$

$$\begin{aligned} &= A + \alpha_1 \sum_{k=0}^m a_k I_{i,t-k} + \alpha_2 \sum_{q=0}^n C_q I_{m,t-q} \\ &\quad + \alpha_1 BF_i + \alpha_2 DF_m + V_{j,t+1}. \end{aligned}$$

By incorporating relevant historical information, formula (4) accounts for the element of risk in the projection of  $R_j$ 's; something which is completely ignored under the practitioners' method.

III. CALCULATING THE  $R_j$ 's

Equation (4) will be used to calculate the  $R_j$ 's values which, as stated earlier, must be known before the IHSI model can be used to formulate the optimal inflation-hedge portfolio.

For the purpose of the current research, only the 30 companies making up the Dow Jones industrial average will be considered. The monthly returns on these companies' stocks from January 1979 through December 1989 were collected and converted to annual returns by applying the usual compounding technique. Additionally, monthly data on three-month Treasury bills rate, inflation rate, unemployment rate, trade deficit, and the exchange rate from January 1978 through December 1988 were also collected from Surveys of Current Business and Federal Reserve Bulletins. The inclusion of an additional year (1978) of information on the interest and non-interest rate variables was necessary to determine the lagged effects that such variables would have on stock returns for 1979.

Because of the sample size chosen, 30 expected long-term returns formulas, one for each of the stocks in the sample, would have to be calculated. To accomplish this task, the historical relationships between stock returns and the other relevant variables would have to be examined over the entire observation period using appropriate econometric techniques in order to determine the nature of such relationships, whether or not they are significant, and what coefficients to assign to the independent variables in the regression equations that will be used to calculate the  $R_j$ 's.

Some may suggest that long-term stock returns are negatively related to interest rates, employment rates, and trade deficits, but they are positively related to exchange rates. However, due to differences in the nature of operations, types of products, and sizes among companies, the effects of these four determinants could vary from one company to another and would, therefore, need to be examined empirically. A chronic trade deficit may adversely affect a domestic manufacturer while having a positive effect on a company with a sizable import/export business.

As to the impact of inflation rate on stock return expectations, Bodie and Rosansky (1980) found that the two are negatively related while a more recent study by Buono (1989) suggests that the relationship needs to be empirically determined and cannot be guessed beforehand.

The term-structure relationship for each stock was examined using ordinary least squares (OLS) techniques. The lagged effects of interest rates and inflation changes on long-term return expectation varied from one stock to another. Changes in the exchange rate was found to have insignificant effect on the determination of long-term returns for most of the stocks and, therefore, it was dropped from further consideration. The current rate of employment and trade deficit were found to have some effect on returns in about fifty percent of the cases.

As an example, the formula (equation 4) for determining return expectations for the IBM stock is listed below.

$$\begin{aligned}
 R_{\text{IBM}} = & 0.7023 & - & .0002R_{.1} & - & .0493R_{.2} & - & .0137R_{.3} \\
 & & & (.0277) & & (.0294) & & (.0311) \\
 & -.01R_{.4} & - & .0098R_{.5} & - & .0013R_{.6} & - & .045\Delta P_{.1} \\
 & (.0309) & & (.0291) & & (.0273) & & (.0322) \\
 & -.0113\Delta P_{.2} & - & .0573\Delta P_{.3} & - & .0214\Delta P_{.4} & - & .019\Delta P_{.5} \\
 & (.0370) & & (.0372) & & (.0375) & & (.0372) \\
 & -.104\Delta P_{.6} & - & .2698\Delta RUEMP & + & .000032TB. \\
 & (.0329) & & (.1077) & & (.000006)
 \end{aligned}$$

$$\text{Adjusted } R^2 = .4379 \quad \text{S.S.E.} = .2176$$

$R_{\text{IBM}}$  is the expected annual return on IBM's stock,  $R_{.m}$  is the treasury bill rate lagged by  $m$  periods,  $\Delta P_{.n}$  is the change in inflation rate lagged by  $n$  periods,  $\Delta RUEMP$  and  $TB$  are the current unemployment rate and trade deficit, respectively. The numbers in parantheses are the standard errors of the coefficient estimates. The adjusted  $R^2$  is not very high, but it is typical for this type of estimation. To conserve journal space the equations for the remaining 29 stocks are not listed, but can be obtained by writing the authors.

#### IV. THE IHSI MODEL

The traditional approach to portfolio formulation requires a priori information regarding the investor's risk preference [for example, see Tobin (1958) and Levy and Markowitz (1979)]. When the situation involves continuous decision-making activities, this approach may no longer be appropriate. For example, during a long decision-making period, the newly available information regarding changes in the economy and financial market conditions may lead the investor to change his/her risk preference. The original investment objective of, say, maximizing returns may become one of minimizing risk. With the traditional approach, the original long-term investment objective will have to be

abandoned if the investor has the freedom of restructuring the investment portfolio.

The IHSI Model, on the other hand, allows the decision maker to change the portfolio mixture frequently during a specified time span without having to abandon the original long-term investment objective. To ensure that such a change can be completed independently of the original objective, such an objective, as suggested in Tsai, et. al. (1990), is stated as part of the model in the form below:

$$\text{Max } p = \text{Prob } (R_e \leq R^* \leq R_u) \quad (6)$$

where  $R^*$  is the aggregate expected rate of return on the entire portfolio at the end of the specified time span.  $R_e$  and  $R_u$  are two arbitrarily selected rates enclosing  $R^*$  to ensure that the calculated probability is non-zero. In practice,  $R_e$  and  $R_u$  may represent the lower and upper boundary, respectively, of the rates of return available from all other investment alternatives at the time when the decision is to be made.  $R^*$  is determined by the following formula:

$$\mu = R_* = \sum_{j=1}^p C_{j,t+1} R_{j,t+1} \quad (7)$$

where  $R_{j,t+1}$  is estimated via equation (4),  $C_j$  is the share of  $j$ th stock in the formulated portfolio, and  $p$  is the number of stocks in the portfolio.

Because a stock's expected annual return could be correlated with those for other stocks under consideration, the results derived from the OLS estimation would have to be further tested using Seemingly Unrelated Regression (SUR) techniques. To illustrate how this is done, equation (4) is generalized to take the following expression:

$$R_j = X_j B_j + U_j, \text{ for all } j \quad (8)$$

where

$R_j$  = vector of all of the observed values of the dependent variables

$X_j$  = matrix of all observations of all of the independent variables

$B_j$  = coefficients vector.

The expected return for each individual stock under consideration is

$$E(R_j) = X_j \hat{B}_j \quad (9)$$

where  $\hat{B}_j$  is the estimated coefficient of the term structure equation for the  $j$ th stock via the following relationship:

$$\hat{B}_j = (X_j' X_j)^{-1} X_j' Y_j \quad (10)$$

The associated variance-covariance matrix of the expected returns is estimated by

$$\begin{aligned} \text{Var}(Y) &= X' \text{Var}(\hat{B}) X \\ &= X' (X' \sigma^{-1} X)^{-1} X = \Sigma \end{aligned} \quad (11)$$

where

$$Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_j \end{bmatrix} \quad X = \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X_j \end{bmatrix} \quad B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_j \end{bmatrix} \quad \sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1j} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{j1} & \sigma_{j2} & \dots & \sigma_{jj} \end{bmatrix}$$

The values of  $\sigma_{ii}$ 's and  $\sigma_{ij}$ 's are estimated by

$$S_{ii} = (Y_i - X_i \hat{B}_i)' (Y_i - X_i B_i) / (n - k_i) \quad (12.1)$$

$$S_{ij} = (Y_i - X_i \hat{B}_i)' (Y_j - X_j B_j) / (n - k_i)^{1/2} (n - k_j)^{1/2} \quad (12.2)$$

where  $n$  is the number of observations used to estimate the values of  $B_i$ , and  $k_i$  is the number of independent variables in  $i$ th equation.

The SUR test was applied to the ten stocks selected by the practitioners for 1988. The expected returns of these stocks along with the associated variance-covariance matrix are shown in Table 1. This information, when used with the portfolio selection model to be discussed next, will allow the user to select the best investment combination from among these ten stocks.

The solution to the objective function (6) must be subject to the following two constraints:

$$C'e = 1 \quad (13.1)$$

$$R = W_1R_e + W_2R_u \quad (13.2)$$

where  $C$  is a vector representing the selected portfolio, whose individual element,  $C_i$ , represents the percentage of all investment funds to be invested in the  $i$ th stock.  $W_1$  and  $W_2$  are the weights measuring the deviation between  $R_e$  and  $R^*$  and between  $R_u$  and  $R^*$ , respectively. It is required that  $W_1 > 0$ ,  $W_2 > 0$ , and  $W_1 + W_2 = 1$ . Thus, we have

$$W_1 = (R_u - R^*) / (R_u - R_e) \quad (14.1)$$

$$W_2 = (R^* - R_e) / (R_u - R_e). \quad (14.2)$$

$W_1$  and  $W_2$  will be used to derive the probability of generating the target rate of return by the formulated long-term portfolio.

Summarizing the objective function (6), its associated constraints (13.1) and (13.2), and the mean-variance of  $R^*$  as expressed in (11), we have the following Lagrangian:

$$G = C'\Sigma C - 2\lambda_1 (c'e - 1) - 2\lambda_2 [c'\mu - (W_1R_e + W_2R_u)]. \quad (15)$$

Table 1  
Expected Rates of Return

	R1 0.0984	R2 0.1304	R3 0.0950	R4 0.1005	R5 0.0900	R6 0.1794	R7 0.1692	R8 0.1203	R9 0.0662	R10 0.0537
<b>Variance-Covariance Matrix</b>										
R1	0.2035	-0.0004	-0.0129	0.0007	0.0223	0.0129	0.0507	0.0197	0.0486	-0.0037
R2	-0.0004	0.1870	-0.0108	0.1012	0.1097	-0.0526	-0.0131	-0.0008	0.0898	-0.0125
R3	-0.0129	-0.0108	0.0465	0.0269	0.0162	0.0350	0.0172	0.0003	0.0030	-0.0055
R4	0.0007	0.1012	0.0269	0.1785	0.1288	0.0633	0.0197	0.0429	0.0610	0.0088
R5	0.0223	0.1097	0.0162	0.1288	0.1422	-0.0038	-0.0150	0.0289	0.0626	-0.0079
R6	0.0129	-0.0526	0.0350	0.0633	-0.0038	0.2581	-0.0040	0.0635	-0.0403	0.0170
R7	0.0507	-0.0131	0.0172	0.0197	-0.0150	-0.0040	0.1754	0.0203	0.0404	0.0216
R8	0.0197	-0.0008	0.0003	0.0429	0.0289	0.0635	0.0203	0.0998	0.0173	0.0026
R9	0.0486	0.0898	0.0030	0.0610	0.0626	-0.0403	0.0404	0.0173	0.1051	-0.0211
R10	-0.0037	-0.0125	-0.0055	0.0088	-0.0079	0.0170	0.0216	0.0026	-0.0211	0.0190

From the first order condition of (15), we derive:

$$C^* = \lambda_1 \Sigma^{-1} e + \lambda_2 \Sigma^{-1} \mu \quad (16)$$

which is then used to derive the following simultaneous equations.

$$\lambda_1 e' \Sigma^{-1} e + \lambda_2 \mu' \Sigma^{-1} e = c'e = 1 \quad (17)$$

$$\lambda_1 e' \Sigma^{-1} \mu + \lambda_2 \mu' \Sigma^{-1} \mu = c'\mu = W_1 R_e + W_2 R_u \quad (18)$$

Equations (16), (17), and (18) will have to be solved simultaneously to derive  $C^*$ , the optimal portfolio in the sense that it would have the best chance of yielding the specified rate of return.

#### V. THE EXAMPLE

To illustrate what information can be generated by the proposed model, the expected rates of return and their associated variance-covariance matrix for the ten stocks reported in Table 1 were fed into the system of equations (16), (17), and (18). It was assumed that the investor set the target rate of return at 12 percent, with the upper and lower boundary rates at 13 percent and 11 percent, respectively. This hypothetical rate might reflect the real rate of return on some risk-free investment plus the expected rate of inflation over the investment time span. Solving for  $C^*$  using equations (16), (17), and (18) gives the optimal portfolio consisting of these ten stocks which is shown in Table 2.

The second column of Table 2 gives the percentage of total funds that need to be invested in each of the ten stocks. Clearly, the results suggest that the simple strategy as suggested by the practitioners may not be the optimal one, since

following the investment strategy as suggested by Table 2 would more than likely lead to buying an unequal number of shares of these ten stocks.

Table 2  
Optimal Portfolio - A

Portfolio Component	Portfolio Share (%)
C1	.0456
C2	.3654
C3	.3442
C4	-.3254
C5	.1054
C6	.1297
C7	.16119
C8	.1598
C9	-.1700
C10	.1843

The portfolio in Table 2 requires the investor to exercise the short-selling option, because the shares of two stocks, C4 and C9, turned out to be negative. Only then would the investor achieve the specified goal with the greatest probability. Exactly how much has to be short-sold depends on whether this is a revision of an existing portfolio or the establishment of a new one. Using portfolio component C4 as an example, if this is to be a new portfolio, C4 at time  $t$  would be zero. Therefore,  $.3254 * \omega_{t+1}$  represents the value of stock #4 that would have to be short-sold if  $\omega_{t+1}$  denotes the total dollar value of the new portfolio.

If this is to be a revision of an existing portfolio, then the following three cases represent the execution of short-selling options:

$$\begin{aligned}
 C_t \omega_t &> |C_{t+1} \cdot \omega_{t+1}|, \text{ if } C_t > \text{ and } C_{t+1} < 0 \\
 C_t \omega_t &< |C_{t+1} \cdot \omega_{t+1}|, \text{ if } C_t > \text{ and } C_{t+1} < 0 \\
 C_t \omega_t &< |C_{t+1} \cdot \omega_{t+1}|, \text{ if } C_t < \text{ and } C_{t+1} < 0
 \end{aligned} \tag{19}$$

In the first two cases,  $C_{t+1} \cdot \omega_{t+1}$  represents the value of the stock to be short-sold. In the last case, it is suggested that the investor continue short-selling the indicated stock, if possible, in order to have the best chance of achieving his long-term investment objective. The value of the stock to be short-sold would be in the neighborhood of  $|C_{t+1} \cdot \omega_{t+1}| - |C_t \cdot \omega_t|$ . It should be noted that in some instances the model may require the user to short-sell a stock at a volume in excess of current holdings.

If the short-selling option is neither possible nor desirable, the portfolio so derived following the procedures described above would be unattainable to the investor. Further restriction must be incorporated into the framework of the proposed model. Specifically,  $C_i$ 's values must be non-negative for any investment period. Thus, if the investor is to revise his existing portfolio, a negative value of  $C_i$  for the decision period  $t+1$  would mean that he should sell all his holdings of the  $i$ th stock. For the investor without an existing portfolio, a negative value of  $C_i$  would mean that he should take no action to buy any shares of the  $i$ th stock. In either case, the following restriction holds:

$$C_{i,t+1} = 0, \text{ for all negative } C_i \text{'s.}$$

A recursive enforcement procedure of replacing all negative  $C_i$ 's with the value zero will have to be incorporated into the model to derive the portfolio for the investor with no desire for short-selling options.

Using the same data as reported in Table 1, the best portfolio without involving short-selling is shown in Table 3.

Table 3  
Optimal Portfolio - B

Portfolio Component	Portfolio Share (%)
C1	.0362
C2	.2209
C3	.2406
C4	0
C5	0
C6	.1496
C7	.1919
C8	.1025
C9	0
C10	.0582

The second column of Table 3 indicates that in addition to the 4th and 9th stocks the investor is advised not to hold any shares of the 5th stock. Although the majority of the funds are still allocated to the 2nd and 3rd stocks, it is less concentrated in these two stocks as compared to the portfolio derived previously. If short-selling is not advisable, either because it is not a cost-effective strategy in long-term investments or because the investor does not want to use it, the

revised portfolio without involving short-selling would represent the next best alternative of meeting the investor's goal of hedging inflation by buying stocks.

#### VI. EMPIRICAL TESTING OF MODEL

The previous section provides an example showing how the IHSI Model works by selecting an investment strategy from among the ten stocks that were chosen for 1988 using the practitioners' method. What remains is to test the model empirically. For this purpose the selection pool has been widened to include the 30 stocks comprising the Dow-Jones industrial average. Rate of return information on these stocks was gathered from December 1987 through December 1991. It is worth mentioning again that the practitioners' method allocates the invested funds equally among the ten stocks that had the highest return during the previous year. On the other hand, the IHSI Model distribute the invested funds in varying proportion among different stocks whose number does not usually equal ten.

One question that needs to be answered before using the model is how to determine  $R_e$  and  $R_u$  representing the lower and upper boundaries for the entire portfolio's rate of return. One solution, which is adopted here, would be to calculate the mean ( $\mu$ ) of the returns on all of the 30 stocks along with the standard error ( $\sigma$ ) of the distribution of these returns and then set  $R_e = \mu - \sigma$  and  $R_u = \mu + \sigma$ . If  $\mu$  is negative, then one would use the smallest positive return as  $R_e$  and the highest positive return as  $R_u$ .

The testing results are reported in the Appendix which contains four tables for the years 1988 through 1991. The first column in every table lists the stocks by the number assigned to each in the sample. A list of the stocks by name, along with the associated number, can be obtained by writing the authors. The second column of the tables provides the actual annual rate of return on each of the stocks. The third column shows the ten stocks that were selected according to the practitioners' method. Such a selection, as stated before, is based on the ten best performers during the preceeding year. Thus, the ten stocks that were selected for 1989 had the highest returns during 1988 as shown in the second column of Table 1 of the Appendix. The fifth column in each table tells which stocks were selected for investment by using the IHSI Model. It also shows the percentage of total funds that was allocated to each of the selected stocks. The aggregated annual returns of each of the selected portfolio are shown at the end of the tables in the Appindex. Table 4 below provides a comparison among these returns:

**Table 4**  
**Comparison of Portfolio Returns**

Portfolio Formed For Year	Practitioners' Method	IHSI MODEL
1988	7.61%	39.14%
1989	5.68	16.51
1990	-29.46	-1.38
1991	19.26	19.89

It is clear from Table 4 that an investor who had selected the stocks suggested by the IHSI Model would have performed

appreciably better than another investor who followed the practitioners' recommendations. During the period of observation the practitioners' method yielded a negative rate of return close to 1.1% per year, where as the IHSI model generated a positive average annual rate of about 17.64%

#### VII. CONCLUDING REMARKS

In this paper, a term structure model is derived to estimate long-term returns on stock investments based on short-run market information. The projected rates of return and their associated risks are included in a portfolio formulation framework to derive an investment strategy that will have the greatest probability of achieving a desired objective. The model will enable the decision maker to set up the desired objective without involving the assessment of his risk preference. Furthermore, the decision maker is able to adjust the structure of the portfolio from time to time during the decision period without altering that long-term objective. Such flexibility is crucial for a portfolio consisting of stocks which are traded in a highly fluctuated market and may be purchased and sold at any time.

An additional restriction is later added to the model framework to accommodate the situation where short-selling option is not allowed. This is more realistic to those investors who are primarily interested in making long-term stock investment. Examples have been provided to illustrate the usefulness of the proposed model in providing valuable information for the formulation and restructuring of a portfolio in pursuing a desired long-term investment objective. An empirical testing of

the IHSI model showed that it is able to produce investment results far superior to those achieved by the practitioners' method.

Since a fraction of a share cannot be purchased, it may not be possible to formulate a portfolio in exactly the same way as recommended by the model. The real portfolio thus may fall within the neighborhood of the suggested one. Nevertheless, such a deviation is minor and should not prevent the formulated portfolio from being the one with the best chance of achieving the desired investment goal.

Unlike the strategy suggested by the practitioners, where the yields as reported at the time of making the decision is the sole criterion for choosing investment candidates, the methodology proposed here takes into account projections of future economy and financial market conditions. Thus investors are provided with much more meaningful information on which to base investment decisions.

Future research may expand the application of the IHSI model to stocks of other indices such as the SPOC 250 or the S&P 500. Also, the model may be applied to sectorial stock portfolio management.

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## APPENDIX

Table 1 (1988)  
Practitioners' Method vs. IHSI Model  
Stock Selections and Returns

Stock Number	Actual Return For 1988	Practitioners		IHSI Model	
		% of Funds Allocated	Weighted Returns	% of Funds Allocated	Weighted Returns
1	0.0125	0	0	0	0
2	0.2090	0	0	0	0
3	0.2299	0	0	0.2628	0.060418
4	0.1073	0	0	0	0
5	0.3741	0	0	0	0
6	0.6530	0	0	0	0
7	0.2232	0	0	0	0
8	0.1843	0	0	0	0
9	0.0427	0.1	0.00427	0	0
10	-0.0490	0	0	0.0616	-0.00302
11	0.2026	0.1	0.02026	0	0
12	0.0310	0	0	0	0
13	0.4675	0	0	0	0
14	-0.1150	0.1	-0.01150	0	0
15	0.0910	0	0	0	0
16	0.1103	0	0	0	0
17	0.1058	0.1	0.01058	0	0
18	-0.6190	0.1	-0.06190	0	0
19	0.0135	0	0	0	0
20	0.2353	0	0	0	0
21	0.2217	0.1	0.02217	0	0
22	-0.0030	0	0	0	0
23	0.0527	0	0	0	0
24	0.2302	0	0	0	0
25	0.3821	0	0	0.2759	0.105421
26	0.0275	0.1	0.00275	0	0
27	0.2303	0.1	0.02303	0	0
28	0.3111	0	0	0	0
29	0.0921	0.1	0.00921	0	0
30	0.5719	0.1	0.05719	0.3997	0.228588
$R_p = 0.12:$	$R_u = 0.36$	$\Sigma = 1.0$	$PAR = 0.07606$	$\Sigma = 1.0$	$PAR = 0.391407$

PAR = Portfolio Annual Rate of Return.

## APPENDIX

Table 2 (1989)  
Practitioners' Method vs. IHSI Model  
Stock Selections and Returns

Stock Number	Actual Return For 1989	Practitioners		IHSI Model	
		% of Funds Allocated	Weighted Returns	% of Funds Allocated	Weighted Returns
1	0.0136	0	0	0	0
2	0.3645	0	0	0	0
3	0.2727	0.1	0.02727	0	0
4	0.6009	0	0	0.1793	0.107741
5	-0.2570	0.1	-0.02570	0	0
6	-0.0230	0.1	-0.00230	0	0
7	0.5008	0	0	0	0
8	0.7468	0	0	0.0249	0.018595
9	0.4315	0	0	0	0
10	-0.0660	0	0	0	0
11	0.1693	0	0	0	0
12	0.4546	0	0	0.0240	0.01091
13	-0.4460	0.1	-0.04460	0.0521	-0.02324
14	-0.1200	0	0	0	0
15	-0.2010	0	0	0.0839	-0.01686
16	0.2543	0	0	0	0
17	-0.2720	0	0	0.1076	-0.02927
18	0.3678	0	0	0.0111	0.004083
19	0.2987	0	0	0	0
20	0.3333	0.1	0.03333	0	0
21	-0.0230	0	0	0.0354	-0.00081
22	0.2757	0	0	0	0
23	-0.1640	0	0	0	0
24	-0.0250	0.1	-0.00250	0	0
25	0.1951	0.1	0.01951	0.4817	0.09398
26	0.2025	0	0	0	0
27	-0.0705	0.1	-0.00705	0	0
28	0.3408	0.1	0.03408	0	0
29	0.4289	0	0	0	0
30	0.2473	0.1	0.02473	0	0
$R_p = -0.67$ $R_m = 0.38$		$\Sigma = 1.0$	$PAR = 0.05677$	$\Sigma = 1.0$	$PAR = 0.165127$

PAR = Portfolio Annual Rate of Return.

## APPENDIX

Table 3 (1990)  
Practitioners' Method vs. IHSI Model  
Stock Selections and Returns

Stock Number	Actual Return For 1990	Practitioners		IHSI Model	
		% of Funds Allocated	Weighted Returns	% of Funds Allocated	Weighted Returns
1	0.0141	0	0	0	0
2	-0.2120	0.1	-0.0212	0	0
3	-0.3810	0	0	0.1884	-0.07178
4	-0.3135	0.1	-0.03135	0	0
5	-0.1130	0	0	0.1060	-0.01198
6	-0.1880	0	0	0	0
7	0.1276	0.1	0.01276	0	0
8	-0.3880	0.1	-0.0388	0.0348	-0.0135
9	-0.6630	0.1	-0.0663	0	0
10	0.0675	0	0	0	0
11	0.0778	0	0	0	0
12	-0.0740	0.1	-0.0074	0	0
13	-0.1120	0	0	0	0
14	-0.5220	0	0	0	0
15	0.2559	0	0	0	0
16	-0.0190	0	0	0	0
17	-0.1475	0	0	0	0
18	0.1744	0.1	0.01744	0	0
19	0.1194	0	0	0	0
20	-0.9361	0.1	-0.09361	0	0
21	-0.1076	0	0	0	0
22	-0.1740	0	0	0	0
23	0.2530	0	0	0.2370	0.059961
24	-0.2781	0	0	0	0
25	0.0858	0	0	0.4104	0.035212
26	-0.1540	0	0	0	0
27	-0.2400	0	0	0	0
28	-0.0780	0.1	-0.0078	0	0
29	-0.5830	0.1	-0.0583	0	0
30	-0.4990	0	0	0.0234	-0.01168
$R_p = -0.11; R_u = 0.45$		$\Sigma = 1.0$	$PAR = -0.29456$	$\Sigma = 1.0$	$PAR = -0.01376$

PAR = Portfolio Annual Rate of Return.

## APPENDIX

Table 4 (1991)  
Practitioners' Method vs. IHSI Model  
Stock Selections and Returns

Stock Number	Actual Return For 1991	Practitioners		IHSI Model	
		% of Funds Allocated	Weighted Returns	% of Funds Allocated	Weighted Returns
1	0.0137	0	0	0	0
2	0.1724	0	0	0	0
3	0.0892	0	0	0	0
4	0.3440	0	0	0.1685	0.057964
5	-0.0661	0	0	0	0
6	0.0272	0	0	0	0
7	-0.0190	0.1	-0.0019	0	0
8	0.7764	0	0	0.0303	0.023525
9	0.2974	0	0	0	0
10	0.1801	0.1	0.01801	0	0
11	0.1802	0.1	0.01802	0	0
12	0.3253	0.1	0.03253	0.0312	0.010149
13	-0.1200	0	0	0.0435	-0.005220
14	1.7218	0	0	0	0
15	-0.1690	0.1	-0.0169	0.0808	-0.013660
16	0.3180	0.1	0.0318	0	0
17	0.3650	0	0	0.1072	0.039128
18	0.8913	0.1	0.08913	0.0132	0.011765
19	0.0991	0.1	0.00991	0	0
20	0.2941	0	0	0	0
21	1.5941	0	0	0.0360	0.057388
22	0.7678	0	0	0	0
23	0.0852	0.1	0.00852	0	0
24	0.4901	0	0	0	0
25	0.0344	0.1	0.00344	0.4885	0.016804
26	-0.0740	0	0	0	0
27	0.2667	0	0	0.0008	0.000213
28	0.1264	0	0	0	0
29	-0.3659	0	0	0	0
30	-0.1190	0	0	0	0
$R_p = 0.0675; R_u = 0.2559$		$\Sigma = 1.0$	$PAR = 0.19256$	$\Sigma = 1.0$	$PAR = 0.198062$

PAR = Portfolio Annual Rate of Return.

