POST-MODERN PORTFOLIO THEORY COMES OF AGE

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It has been a generation since Harry Markowitz laid the foundations and built much of the structure of what we now know as Modern Portfolio Theory ("MPT"). The greatest contribution of MPT is the establishment of a formal risk/return framework for investment decision-making. By defining investment risk in quantitative terms, Markowitz gave investors a mathematical approach to asset selection and portfolio management.

But as Markowitz himself and William Sharpe, the other giant of MPT, acknowledge, there are important limitations to the original MPT formulation.

> *Under certain conditions, the mean-variance approach can be shown to lead to unsatisfactory predictions of behavior. Markowitz suggests that a model based on the semi-variance would be preferable; in light of the formidable computational problems, however, he bases his analysis on the variance and standard deviation.*[1]

The causes of these "unsatisfactory" aspects of MPT are the assumptions that 1) variance of portfolio returns is the correct measure of investment risk, and 2) that the investment returns of all securities and assets can be adequately represented by the normal distribution. Stated another way, MPT is limited by measures of risk and return that do not always represent the realities of the investment markets.

Fortunately, recent advances in portfolio and financial theory, coupled with today's increased computing power, have overcome these limitations. The resulting expanded risk/return paradigm is known as Post-Modern Portfolio Theory ("PMPT"). MPT thus becomes nothing more than a special (symmetrical) case of the PMPT formulation.

This article discusses return and risk under MPT and PMPT and demonstrates an approach to asset allocation based on the more general rules permitted by PMPT.

La theorie du portefeuille post-moderne atteint l'âge de raison

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Une génération s'est écoulée depuis que Harry Markowitz a établi les fondations et construit une grande partie de la structure de ce que nous appelons à l'heure actuelle « MPT » (Théorie du portefeuille moderne). La plus grande contribution faite par la MPT est l'établissement d'un cadre officiel de risque/rendement pour la prise de décisions d'investissement. En définissant le risque de l'investissement en termes quantitatifs, Markowitz a mis à la disposition des investisseurs une méthode mathématique pour la sélection des avoirs et pour la gestion du portefeuille.

Mais, comme le reconnaissent Markowitz lui-même ainsi que William Sharpe, l'autre « grand » de la MPT, il existe d'importantes limitations à la formulation originale de la MPT.

> *Dans certaines conditions, l'analyse fondée sur la moyenne et la variance peut s'avérer mener à des prédictions de comportement insatisfaisantes. Markowitz suggère qu'un modèle fondé sur une semi-variance serait préférable ; à la lumière des énormes problèmes de calcul, il fonde cependant son analyse sur la variance et l'écart standard.*[1]

La raison de ces aspects « insatisfaisants » de la MPT est l'assomption que 1) la variance des rendements du portefeuille est la bonne mesure du risque de l'investissement et que 2) les rendements de l'investissement de toutes valeurs et avoirs peuvent être représentés de façon adéquate par la distribution normale. Autrement dit, la MPT est limitée par des mesures du risque et du rendement qui ne représentent pas toujours la réalité des marchés d'investissement.

Heureusement, de récents progrès effectués dans la théorie de la finance et du portefeuille, alliés à des capacités informatiques accrues, ont permis de surmonter ces limitations. Le paradigme du risque/rendement en résultant est connu sous le nom de PMPT (Théorie du portefeuille post-moderne). La MPT devient donc ainsi simplement un cas spécial (symétrique) de la formulation de la PMPT.

Cet article traite du rendement et du risque aux termes de la MPT et de la PMPT et démontre une méthode de répartition des avoirs fondée sur les règles plus générales permises par la PMPT.

MESURES DU RISQUE : DISTINCTION ENTRE BONNE ET MAUVAISE VARIABILITE

Dans la MPT, le risque est défini comme la variabilité totale des rendements autour du rendement moyen et est mesuré par la variance, ou alternativement, l'écart standard.[2] La MPT traite toutes les incertitudes de la même façon - les surprises (c'est-à-dire la variabilité) positives sont pénalisées de façon identique aux surprises négatives. En ce sens, la variance est une mesure *symétrique* du risque et va donc à l'encontre de l'intuition.

RISK MEASURES: DISTINGUISHING BETWEEN BAD AND GOOD VARIAIBLITY

In MPT, risk is defined as the total variability of returns around the mean return and is measured by the variance, or equivalently, standard deviation.[2] MPT treats all uncertainty the same--surprises (i.e., variability) on the upside are penalized identically to surprises on the downside. In this sense, variance is a *symmetric* risk measure, which is counter-intuitive for real-world investors. In fact, intuition argues just the *opposite*--in a bull market we should seek as much volatility as possible; only in a bear market should volatility be avoided! Furthermore, it is well known that individuals are more concerned with avoiding loss than with seeking gain.[3] In other words, from a practical standpoint, risk is not symmetrical--it is severely skewed, with the greatest concern going to the downside.

While variance captures only the risks associated with achieving the *average* return, PMPT recognizes that investment risk should be tied to each investor's specific goals and that any outcomes above this goal do not represent economic or financial risk. PMPT's downside risk[4] measure makes a clear distinction between downside and upside volatility. In PMPT only volatility below the investor's target return incurs risk; all returns above this target cause "uncertainty", which is nothing more than *riskless* opportunity for unexpectedly high returns (Figure 1).

In PMPT this target rate of return is referred to as the minimum acceptable return (MAR). It represents the rate of return that *must* be earned to avoid failing to achieve some important financial objective. Examples of MAR's for pension funds include the actuarial interest rate, the return required to eliminate an underfunded position, and the return required to reduce contributions to a specified level. For individuals, a typical MAR could be the return required to purchase a retirement annuity sufficient to replace a specified proportion of pre-retirement income. As this last example illustrates, the MAR can act as an explicit link between investors' financial requirements and their assets.

Because the MAR is explicitly included in the calculation of PMPT efficient frontiers, there is a unique efficient frontier for each MAR. In other words, for any given set of risk, return and covariance assumptions, there is an infinite number of efficient frontiers, each corresponding to a particular MAR.[5] This stands in contrast to MPT efficient frontiers, in which the investor's goals are never explicitly considered.

Another appealing attribute of PMPT is that the downside risk statistic[6] can be split into two independent components that can then be separately analyzed. In PMPT these two component statistics are known as downside probability[7] and average downside magnitude. Downside probability measures the likelihood of failure to meet the MAR. The average downside magnitude measures the average shortfall below the MAR, for only those instances when the MAR is not achieved. It is thus a measure of the *consequences* of failure. These two statistics provide useful additional perspectives on the nature of the investment risk for a portfolio or asset.

SYMMETRICAL vs. ASYMMETRICAL RETURN DISTRIBUTIONS: NOT ALL DISTRIBUTIONS ARE NORMAL

To represent the underlying uncertainty of asset forecasts, optimization procedures in both MPT and PMPT require a statistical return distribution to be specified for each asset. While MPT permits only the two-parameter normal or lognormal distributions, PMPT utilizes a broader class of asymmetrical distributions. For the analysis presented here, we use the four-parameter lognormal distribution.[8]

The question then arises as to how much asymmetry, or skewness, is observed in the real world. Table 1 shows skewness ratios for several major asset classes over different time periods. Ratios greater than 1.0 indicate distributions with more returns occurring above the median return (positive skewness); the converse is true for ratios less than 1.0 (negative skewness).

All the major asset classes in Table 1 have skewness ratios significantly different from 1.0 for all time periods analyzed. This is compelling evidence that the MPT requirement that all assets have symmetrical return distributions is inappropriate and likely to lead to incorrect results.

The PMPT formulation significantly reduces this problem. Because PMPT provides a more accurate representation of an asset's true shape, PMPT optimization studies will generally provide more accurate results. In addition, PMPT can accommodate severely skewed investment strategies such as portfolio insurance, option-writing, nuclear decommissioning trusts and other derivative-based programs.[9]

POST-MODERN PORTFOLIO THEORY AND PORTFOLIO OPTIMIZATION

Harlow succinctly states the appeal and benefits of using the downside risk framework for portfolio optimization:[10]

> *(The downside risk framework) ... view accords with most investment managers' perception of risk. Downside risk offers an attractive approach to asset allocation decisions. Theoretically more general than the traditional mean-variance technique, it also promises significant improvement in the risk-reward trade-offs to the investor.... That is, a downside risk approach can lower risk while maintaining or improving upon the expected return offered by mean-variance approaches.*

To illustrate the differences between the downside risk and mean-variance approaches, we compare the results using the most general assumptions permitted by each method.[11] The analysis is performed using historical data as proxies for expectations of future asset behavior. Although the results vary according to changes in the ex ante assumptions, the process of using historical data to assist in the formulation of future expectations is common. In any case, our focus is on exploring the usefulness of an alternative portfolio construction technique rather than on producing return forecasts.

Our example is fairly typical for a U.S.-based investor. There are five asset choices: large-capitalization stocks, small-capitalization stocks, non-U.S. stocks, bonds, and cash. The returns used in this example cover the 15-year period from January, 1978 to December, 1992 and are presented in Table 2.[12] Efficient portfolios of these assets are constructed using the PMPT and MPT techniques described below, a 10% MAR, and a five-year holding period.

The PMPT efficient frontier is calculated using an algorithm for downside risk developed by The Pension Research Institute[13] applied to the expected return, standard deviation and skewness values shown in Table 2. Figure 2 shows the downside risk efficient frontier and identifies the reference portfolios described below.

The MPT efficient frontier is calculated using standard Markowitz optimization techniques applied to the expected returns and standard deviations shown in Table 2. Figure 3 shows the MPT mean-variance efficient frontier and identifies the reference portfolios.

To assist in comparisons of portfolios generated from optimizations under the techniques, the following benchmark portfolios are used: (1) the minimum-risk efficient portfolio to represent the most risk-averse investor; (2) the maximum-efficiency portfolio to represent the purely rational investor; and (3) the Brinson Partners Global Securities Normal Portfolio[14] to represent the typical global investor.

DOWNSIDE RISK vs. MEAN-VARIANCE OPTIMIZATION

The Minimum-Risk Portfolios

Minimum-risk portfolios are shown in Table 3. Under mean-variance, the minimum-risk portfolio is practically an all-cash portfolio. As an indication of the limitations of mean-variance optimization, this portfolio is identified as the least risky; yet with certainty it will fail to deliver the 10% required return! This illustrates how mean-variance analysis can recommend illogical investment strategies and is a direct result of using standard deviation as the investment risk measure. The inefficiency of the mean-variance minimum risk portfolio is obvious in that it is located substantially below the downside risk efficient frontier in Figure 2.

Contrast this with the downside risk minimum-risk portfolio which has a substantial non-cash component. This diversification into higher-return, higher-volatility assets reflects the 10% MAR requirement which cannot be achieved by the low-return, low-volatility cash component on its own.

Table 4 shows the two components of downside risk--downside probability and average downside magnitude. These provide additional insights into how the riskiness of cash is viewed by the two optimization methods. For example, the low downside risk of cash is primarily a consequence of this asset's low average downside magnitude, despite the relatively high downside probability. Nonetheless, the downside risk minimum-risk portfolio holds substantially less cash than its mean-variance counterpart because the risk-reduction properties are significantly less under downside risk than under mean-variance. This is shown in Table 5, which displays the risk of each asset relative to cash. Notice, for example, that the mean-variance risk of large-cap stocks is 19 times greater than that of cash, but only 1.3 times greater using downside risk at a 10% MAR. This explains the disparity between the two optimization methods in the allocation to large-cap stocks.

A comparison of columns 2 and 3 in Table 5 shows how in the downside risk framework the MAR can change an asset's risk relative to cash. Notice the rapid climb in riskiness of the assets relative to cash when the MAR falls to 8% from 10%. This is a direct reflection of the sharp decline in the downside risk of cash as the lower MAR moves into its range of possible outcomes.

The Maximum-Efficiency Portfolios

Table 6 shows the downside risk and mean-variance maximum-efficiency[15] portfolios. The differences between these portfolios are explained by the efficiencies of the respective assets shown in Table 7.[16] Under mean-variance, cash is so much more efficient than the other assets that it dominates the maximum-efficiency portfolio with a 97% allocation. On the other hand, under downside risk, large-cap stocks are most efficient, which explains the 81% allocation to this asset. As noted previously, an asset's efficiency relative to other assets would be expected to change as the MAR changes.

The inefficiency of the mean-variance maximum-efficiency portfolio is obvious in that it is located substantially below the downside risk efficient frontier in Figure 2.

The Equivalent-Risk Portfolios

We define an equivalent-risk portfolio as an efficient portfolio with the same risk as a specified reference portfolio, for which we use the Brinson Partners Global Securities Normal Portfolio. Table 8 shows the mean-variance and downside risk equivalent-risk portfolios.

Notice that the downside risk portfolio has a higher allocation to large-cap stocks and lower weightings to foreign stocks and bonds than the mean-variance portfolio. These differences are attributable primarily to the assets' skewness. For example, the positive skewness of large-cap stocks makes this asset more appealing in the downside risk optimization (where the skewness is recognized) than in the mean-variance case (where the skewness is ignored). Similarly, the negative skewness of foreign stocks and bonds explains the relative underweighting in these assets under downside risk.

Importantly, this example illustrates the impact of ignoring asset skewness when performing mean-variance optimization: for large-cap stocks, mean-variance ignores some of the "good" returns on the upside, resulting in an *overestimation* of the actual risks inherent in this asset; for foreign stocks and bonds, mean-variance ignores some of the "bad" returns on the downside, resulting in an *underestimation* of the actual risks of these assets. This means that the mean-variance equivalent-risk portfolio is neither efficient nor optimal.

CONCLUSIONS

Recent advances in portfolio theory and computer technology today provide investors with capabilities unheard of even a few years ago. Among these is Post-Modern Portfolio Theory ("PMPT"), which uses downside risk and asymmetrical return distributions, providing analysts with flexibility and accuracy in constructing efficient portfolios unavailable under traditional Markowitz mean-variance methodology.

Examples of policy decisions using these two optimization techniques show how mean--variance analysis can produce illogical and counter-intuitive results and how PMPT can rectify these problems.

By providing a more accurate and robust framework for constructing optimal portfolio mixes, Post-Modern Portfolio Theory has made much needed improvements to the fundamental work done by Markowitz and Sharpe on portfolio theory.

ENDNOTES:

[1]Sharpe, W.F., (1964) *Capital Asset Prices: A Theory of Market Equilibrium under Considerations of Risk*, The Journal of Finance, XIX, 425. Markowitz recognized these limitations and proposed downside risk (which he called "semi-variance") as the preferred measure of investment risk. However, due to the complex calculations and the limited computational resources at his disposal, practical implementation of downside risk was impossible. He therefore compromised and stayed with variance.

[2]In Modern Portfolio Theory, the terms variance, variability, volatility, and standard deviation are often used interchangeably to represent investment risk.

[3]*Why Investors Make the Wrong Choices*, Fortune Magazine, January, 1987.

[4]There are many references to downside risk in the literature. The following is a partial list of those we feel are most relevant to our article. (See Bawa, V.S., *Stochastic Dominance: A Research Bibliography*, Management Science, June, 1982, for a complete bibliography.) Fishburn, P.C., *Mean-Risk Analysis with Risk Associated with Below-Target Variance*, The American Economic Review, March, 1977; Sortino, Frank and v.d. Meer, Robert, *Downside Risk: Capturing What's at Stake*, The Journal of Portfolio Management, Summer, 1991; Clarkson, R.S., *A Presentation to The Faculty of Actuaries (British)*, February 20, 1989.

[5]See Table 5 for an illustration. For example, notice in the downside risk framework the rapid climb in relative risk of each asset when the MAR falls to 8% from 10%. This is explained by the sharp decline in the downside risk of cash as the MAR moves into the cash range of possible outcomes.

[6]Downside risk is measured by target semi-deviation, the square root of target semi-variance. Downside risk is an asymmetric risk measure that calculates the probability-weighted squared deviations of those returns falling below a specified target rate of return (the MAR).

[7]Downside probability is also known as shortfall risk. See Leibowitz, M.L. and Langetieg, T.C., *Shortfall Risks and the Asset Allocation Decision*, Salomon Brothers, January, 1989.

[8]The additional parameters permit independent shifting of the left and right tails of the lognormal distribution. For many years financial theorists and practitioners have recognized that, for many common asset classes, traditional methods of representing return distributions do not adequately capture empirically observed results. The lognormal distribution used for the analysis here overcomes many of these objections.

[9]Lewis, A.L., *Semivariance and the Performance of Portfolios with Options*, Financial Analysts Journal, July-August, 1990; Study on CTA Managers, Sponsor-Software Systems, Inc., February, 1993; *Post-Modern Portfolio Theory Spawns Post-Modern Optimizer*, Money Management Letter, February 15, 1993.

[10]Harlow, W.V., *Asset Allocation in a Downside Risk Framework*, Financial Analysts Journal, September-October, 1991.

[11]Although our examples utilize PMPT's most general capabilities--asymmetric return distributions and downside risk--we emphasize that the benefits of downside risk described still apply if symmetrical return distributions are used. Space limitations preclude us from including further examples to illustrate this point. These are available from the authors on request.

[12]Throughout this article, the following market indexes are used as proxies for major asset classes: Standard & Poor's 500 Stock Index for large-cap stocks; Russell 2000 Stock Index for small-cap stocks; MSCI Europe, Australia, Far East Stock Index for foreign stocks; Lehman Brothers Aggregate Bond Index for bonds; 90-Day Treasury Bills for cash.

[13]The not-for-profit Pension Research Institute at San Francisco State University wrote the first commercial downside risk optimizer. This algorithm is used by *The Asset Allocation Expert*, a PC-based optimization software system developed and distributed by Sponsor--Software Systems, Inc. This system is used to generate the results described in this article.

[14]We use a summarized form of this portfolio: 39% large-cap stocks, 17% small-cap stocks, 19% foreign stocks, 20% bonds, and 5% cash. See *Investment Review*, Brinson Partners, Inc., 1992; page 7.

[15]In downside risk, the maximum-efficiency portfolio is the efficient portfolio with the highest Sortino ratio: (rate of return-MAR)/(downside risk). In mean-variance, the maximum-efficiency portfolio is the efficient portfolio with the highest modified Sharpe ratio: (rate of return)/(standard deviation).

[16]Covariances between assets also will affect the results. This information is available from the authors on request.

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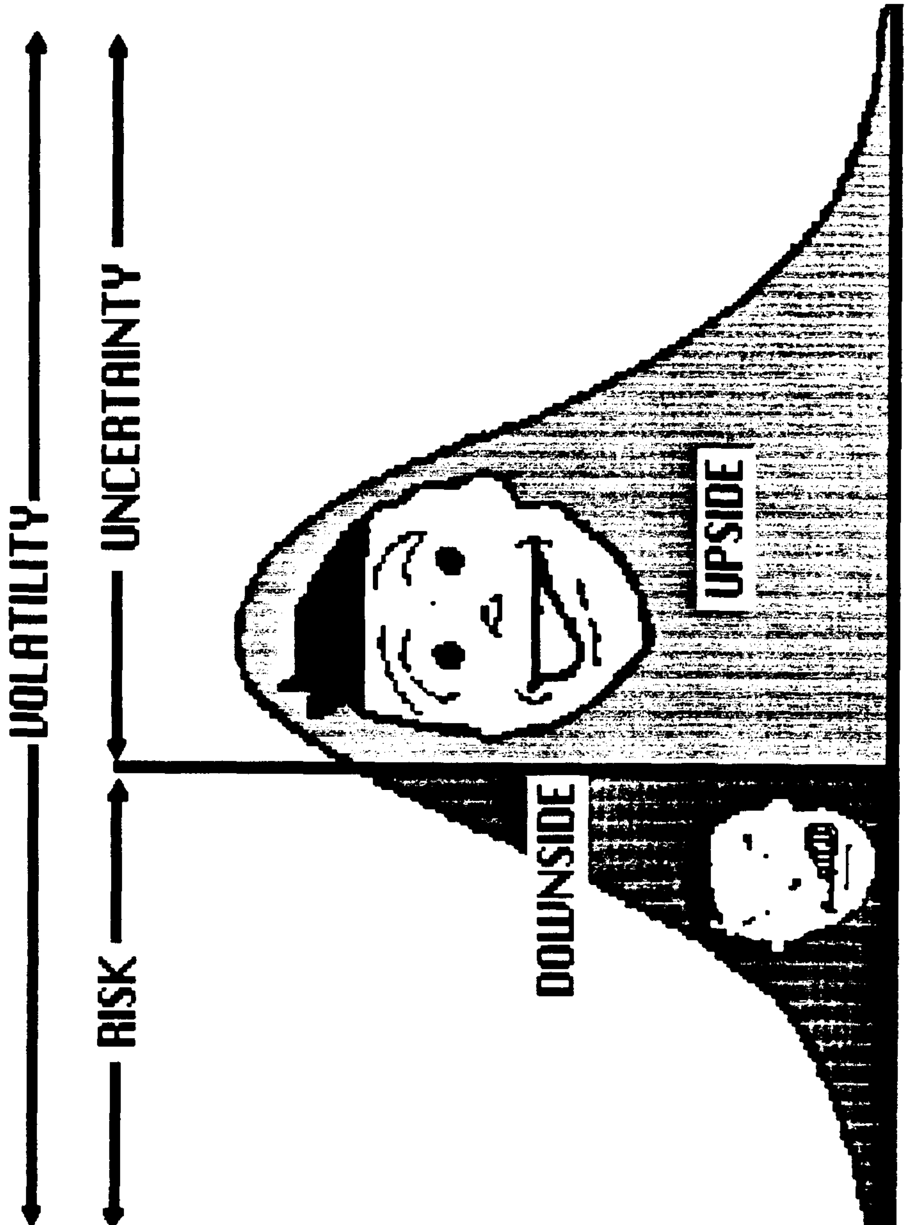
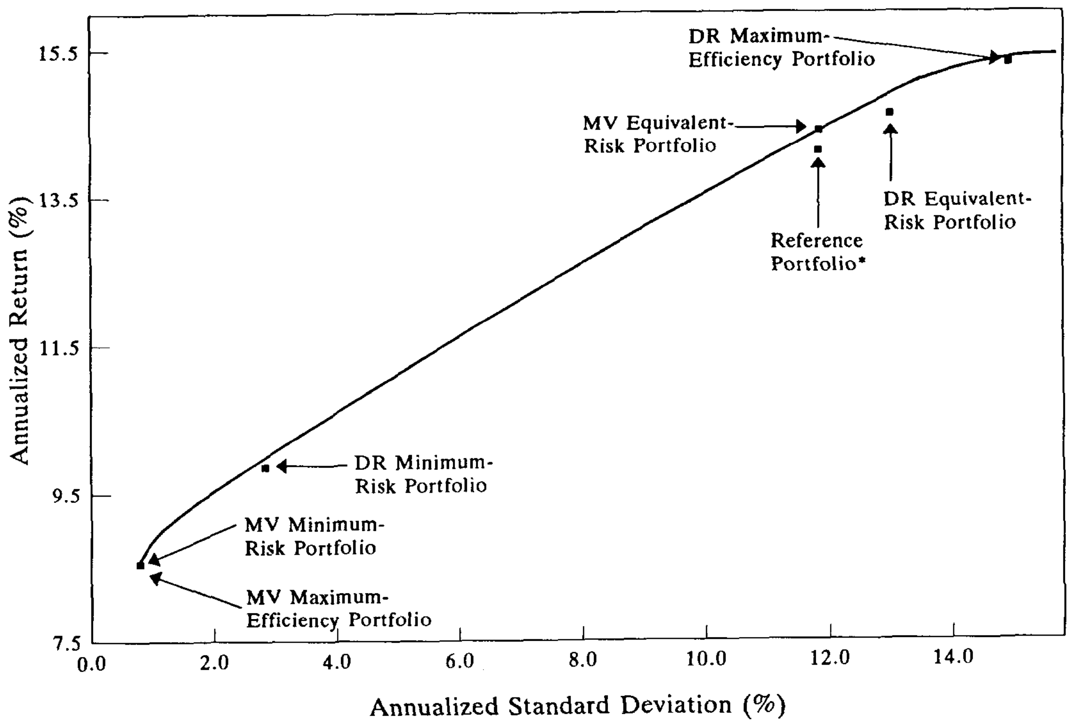
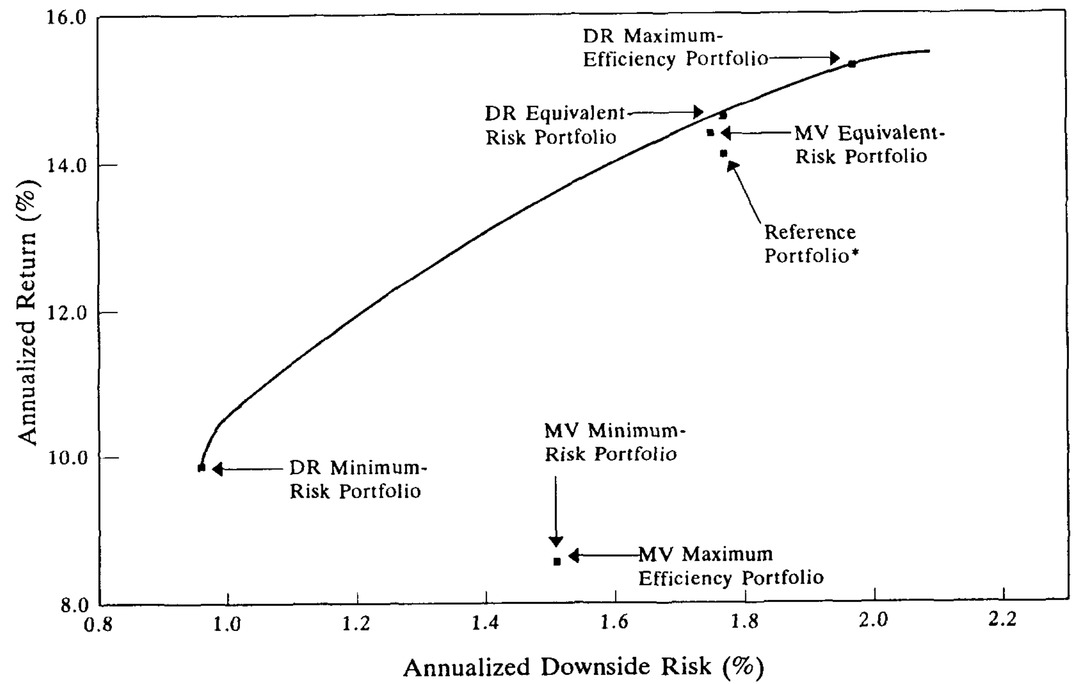

VOLATILITY
RISK
UNCERTAINTY
DOWNSIDE
UPSIDE

MPT EFFICIENT FRONTIER
5-Year Holding Period



DR=Downside Risk; MV=Mean-Variance
*Brinson Partners Global Securities Normal Portfolio.

PMPT EFFICIENT FRONTIER
5-Year Holding Period
10.0% Minimum Acceptable Return



DR=Downside Risk; MV=Mean-Variance
*Brinson Partners Global Securities Normal Portfolio.

Table 1
Skewness of Major Asset Classes and Inflation*

	Periods Ending 12/31/92		
Asset	10 Yrs	20 Yrs	30 Yrs
Large-Cap Stocks	1.80	1.23	0.89
Small-Cap Stocks	1.07	1.22	1.14
Foreign Stocks	0.92	1.10	NA
Bonds	0.83	0.94	0.97
Cash	0.64	1.25	1.11
Inflation	0.82	1.35	3.03

*Skewness equals (High 10th Percentile Return - Median Return)/(Median Return - Low 10th Percentile Return)

Table 2
Return and Risk Assumptions for Optimizations*

Asset	Expected Return	Standard Deviation	Skewness**
Large-Cap Stocks	15.45%	15.80%	1.22
Small-Cap Stocks	15.44	20.50	1.20
Foreign Stocks	15.30	18.26	0.85
Bonds	10.59	7.49	0.92
Cash	8.45	0.83	1.19

*Actual data from 1978 to 1992.
**Skewness equals (High 10th Percentile Return - Median Return)/(Median Return - Low 10th Percentile Return)

Table 3
Comparison of Minimum-Risk Portfolios
Five-Year Holding Period, 10.0% MAR

Portfolio Mix	Mean-Variance	Downside Risk	Difference
Large-Cap Stocks	0%	11%	+11%
Small-Cap Stocks	0	0	0
Foreign Stocks	1	4	+3
Bonds	2	18	+16
Cash	97	67	-30
Portfolio Characteristics			
Expected Return	8.55%	9.86%	+1.31%
Risk:			
Downside Risk	1.51%	0.96%	-0.55%
Standard Deviation	0.80%	2.84%	+2.04%
Downside Prob	100.0%	55.4%	-44.6%
Avg Downside Mag	1.51%	1.29%	-0.22%

Table 4
Components of Downside Risk
for Individual Assets
Five-Year Holding Period, 10.0% MAR

Asset	Downside Risk	Downside Probability	Average Downside Magnitude
Large-Cap Stocks	2.09%	23.2%	4.35%
Small-Cap Stocks	3.36	29.2	6.22
Foreign Stocks	4.49	27.1	8.64
Bonds	1.85	40.8	2.90
Cash	1.62	100.0	1.62

Table 5
Risk for Individual Assets
Relative to Cash
Five-Year Holding Period

Asset	Mean-Variance*	Downside Risk 10% MAR	Downside Risk 8% MAR
Large-Cap Stocks	19.0	1.3	14.2
Small-Cap Stocks	24.7	2.1	25.7
Foreign Stocks	22.0	2.8	37.6
Bonds	9.0	1.1	10.3
Cash	1.0	1.0	1.0

*Standard deviation

Table 6
Comparison of
Maximum-Efficiency Portfolios
Five-Year Holding Period, 10.0% MAR

Portfolio Mix	Mean-Variance	Downside Risk	Diff-erence
Large-Cap Stocks	0%	81%	+81%
Small-Cap Stocks	0	0	0
Foreign Stocks	1	17	+16
Bonds	2	1	-1
Cash	97	1	-96
Portfolio Characteristics			
Expected Return	8.55%	15.30%	+6.75%
Risk:			
Downside Risk	1.51%	1.97%	+0.46%
Standard Deviation	0.80%	14.93%	+14.13%
Efficiency Ratio:			
Downside Risk*	-0.96	2.70	+3.66
Mean-Variance**	10.71	2.48	-8.23

*The Sortino ratio = (Expected Return - MAR)/Downside Risk
**The Sharpe ratio = Expected Return/Standard Deviation

Table 7
Efficiency Measures and Rankings
for Individual Assets

	Mean-Variance		Downside Risk at 10% MAR	
Asset	Efficiency Ratio*	Rank	Efficiency Ratio**	Rank
Large-Cap Stocks	1.0	3	2.6	1
Small-Cap Stocks	0.8	5	1.6	2
Foreign Stocks	0.8	4	1.2	3
Bonds	1.4	2	0.3	4
Cash	10.0	1	-0.9	5

*The Sharpe ratio = Expected Return/Standard Deviation
**The Sortino ratio = (Expected Return - MAR)/Downside Risk

Table 8
Comparison of
Equivalent-Risk Portfolios
5-Year Holding Period, 10.0% MAR

Portfolio Mix	Mean-Variance	Downside Risk	Reference Portfolio*
Large-Cap Stocks	50%	65%	39%
Small-Cap Stocks	0	0	17
Foreign Stocks	29	18	19
Bonds	21	17	20
Cash	0	0	5
Portfolio Characteristics			
Expected Return	14.38%	14.60%	14.10%
Risk:			
Downside Risk	1.75%	1.77%	1.77%
Standard Deviation	11.86%	13.00%	12.23%
Efficiency Ratio:			
Downside Risk**	2.50	2.61	2.32
Mean-Variance***	1.21	1.12	1.15

*Brinson Partners Global Securities Normal Portfolio
**The Sortino ratio = (Expected Return - MAR)/Downside Risk
***The Sharpe ratio = Expected Return/Standard Deviation