A Dynamic Equilibrium Model for Capital Market Behaviour

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Abstract

This paper discusses the underlying assumptions of the Capital Asset Pricing Model and concludes that they are inconsistent with empirical observations of capital market behaviour. A dynamic equilibrium model is put forward as a more realistic theoretical framework.

Résumé

Cet article discute les suppositions du modèle de fixation des prix des actifs immobilisés et conclut qu'elles sont incompatibles avec le comportement réel dans le monde financier. On suggère un modèle d'équilibre dynamique comme alternative de plus grande puissance.

Keywords

Capital asset pricing model, equilibrium, rational behaviour.

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1.   INTRODUCTION

1.1   Although the Capital Asset Pricing Model (CAPM) is one of the cornerstones of modern finance theory, the literature contains very little discussion of the appropriateness or otherwise of the numerous underlying simplifying assumptions. Should these simplifying assumptions be inconsistent in any material respect with observed capital market behaviour, then interpretations of real world behaviour using the CAPM as a theoretical benchmark may lead not only to unsound theoretical conclusions but also to a higher level of inherent financial risk in attempted practical applications.

1.2   This paper first of all shows how the simplifying assumptions inherent in the CAPM are materially inconsistent with diverse bodies of empirical evidence. More plausible assumptions, particularly in the areas of levels of behaviour and equilibrium, are then used to develop a dynamic equilibrium model which is put forward for discussion as an alternative and much more powerful paradigm for capital market behaviour.

1.3   The most important difference between the CAPM and the dynamic equilibrium model developed in this paper relates to the dimension of time. The former is essentially static in nature, whereas the latter recognises the dynamic character of capital market behaviour as the resultant of complex adaptive systems.

1.4   This paper can be regarded as a more extensive exposition of certain ideas first propounded in Clarkson (1996).

2. THE CAPM, RATIONAL BEHAVIOUR, AND EQUILIBRIUM

2.1   The first account of the Capital Asset Pricing Model (CAPM) was in Sharpe (1964). Over the next few years various other financial economists re-examined the theoretical foundations and gave more explicit descriptions of the underlying assumptions. For the purposes of the present paper, I use the set of assumptions in Jensen (1968), a landmark paper which employed the CAPM to test for "strong level" efficiency in the context of US mutual funds.

2.2   The assumptions in Jensen (1968) can be summarised as follows:

A   All investors are averse to risk as measured by variability of return.

B   All investors maximise expected utility.

C   All investors have homogeneous expectations.

D   All investors use only expected return and variance of return when choosing amongst investments.

E   The capital market is in equilibrium.

F   Deviations from a least squares regression line of variance of return against expected return follow a normal distribution.

Assumptions A, B, C and D together define a particular version of what is generally called "rational behaviour", while assumptions E and F describe the capital market structure that is believed to result from this "rational behaviour". Brief comments on each of the assumptions are set out below.

2.3   It is implicit in assumption A that it is adequate to use the subjective probability distribution of the investor when assessing risk.   Also, considerations such as marketability (i.e. the ability to deal in appropriate size without moving the price unduly) are ignored.

2.4   The mathematical framework underlying assumption B is the axiomatic approach to utility theory pioneered by Von Neumann & Morgenstern (1944).   After many prominent economists expressed serious reservations about the validity of the axioms, various mathematicians proposed alternative axiomatic frameworks which, while similar in terms of their mathematical implications, could be described in relatively simple language.   The approach which found most favour with statisticians and mathematical economists is that described in Savage (1954), a modern commentary on which can be found in Anand (1993).

2.5   For all practical purposes, assumption C can be taken to imply that all investors have the same information base.

2.6   Assumption D implies that the investor (possibly after allowing for taxation) is indifferent as to whether the return is achieved through dividends or capital appreciation or a combination of the two.   Also, other considerations such as marketability are assumed to be irrelevant.

2.7   Assumption E is essentially equivalent to saying that exploitable anomalies would be identified and acted upon by astute investors so quickly that for all practical purposes such anomalies can be assumed not to exist, with the result that expected returns, possibly adjusted for risk in some way, will be the same for all securities.

2.8 Assumption F implies not only that a linear relationship exists between variance and expected return but also that the build-up of errors from the "true" underlying position is random over time.

3.   ANOMALOUS BEHAVIOUR

3.1 In many areas of scientific endeavour, an initial paradigm, or conceptual approach, while useful in setting up a framework which is of undoubted value in terms of enforcing for the first time a mathematically rigorous approach to the area, is often superceded by a quite different new paradigm once the old paradigm, despite valiant efforts by its supporters to refine its theories, is unable to explain various observed aspects of real world behaviour. In the physical sciences, this pattern of one temporarily successful paradigm being replaced by another is described in great detail in Kuhn (1970). In economic science, the need to question unsatisfactory paradigms despite the often bitter opposition that may arise is expounded very eloquently by Professor Maurice Allais under the heading of "New Ideas and the Tyranny of Dominant Doctrines" towards the end of the Nobel Lecture that he gave on 9th December 1988:

> "Finally, I have never hesitated to question commonly accepted theories when they appeared to me to be founded on hypotheses which implied consequences incompatible with observed data. Dominant ideas, however erroneous they may be, end up, simply through continual repetition, by acquiring the quality of established truths which cannot be questioned without confronting the active ostracism of the establishment."

3.2   The simplifying assumptions of the CAPM are discussed below to see whether or not a sufficiently strong case can be made out for a rival paradigm to be put forward in its place as a better conceptual framework for capital market behaviour.

3.3   Numerous economists and psychologists have shown that observed real world behaviour is often inconsistent with the approach to risk encapsulated in assumption A. In particular, Tversky (1990) questions the fundamental assumption that individuals are "risk averse" in the generally accepted sense and cites various counter-examples. Tversky also discusses the general bias towards overconfidence and suggests that this behaviour may cause individuals to fail to take into account information that is available to others.

3.4   The risk consequences of forecasts turning out to be wrong were highlighted more than forty years ago by Gwilt (1953), the then President of the Faculty of Actuaries, in a Sessional Meeting discussion on two investment papers:

> "If you will forgive a brief presidential platitude, I think myself it is wise in the investment of the assets of a life office to follow a policy of moderation and restraint, and to resist any temptation to strive for a spectacular profit if there is any possibility of a serious loss, if one's assumptions should prove to be wrong."

3.5   A recent editorial article by Bernstein (1996) entitled "Fearless forecasters, or fearless forecast consumers?" draws generalisations from crucial economic forecasts that proved wildly inaccurate:

> "The question is not whether wildly wrong forecasts will happen, but what we do about the high probability that wildly wrong forecasts will happen."

Bernstein's overall conclusion suggests that assumption A is totally inadequate as a basic guiding principle:

> "Pretending to believe that forecasts are going to be right may be the greatest risk of all."

3.6   Perhaps the most successful criticism of the axiomatic approach to utility theory underlying assumption B is Allais (1953), where it is asserted that:

> "Whatever their attraction might be, none of the fundamental postulates leading to the Bernoulli principle as formulated by the American school can withstand analysis. All are based on false evidence."

3.7   A quite different but still very detailed criticism of the axiomatic approach to utility theory is set out in Clarkson (1996).

3.8   Anyone with practical experience of investment management is aware that obtaining relevant information is a costly and time-consuming activity, so that in general the information set used to formulate investment judgments will increase with the resources (both financial and manpower) of the investor. In particular, institutional investors have access through stockbrokers to vast amounts of data not available to private investors. Accordingly, assumption C is highly unrealistic.

3.9   Even where investment analysts agree on basic information as regards company background and expected future profits, they often come to diametrically different investment conclusions. A detailed example of this nature for a well-researched UK brewery company is given in Clarkson (1981), where it is shown that analysts who look only at the current trading background believe that the shares should be bought whereas analysts who see the current high rating as unsustainable over other than the short term believe that the shares should be sold.

3.10   Assumption D relating to the irrelevance of attributes other than expected return and variability of return is again unrealistic in the extreme. Many investors restrict their universe of admissible securities, sometimes through legal requirements, to large capitalisation stocks where good marketability exists. Also, different investors have different requirements as regards immediate dividend income, so that, as shown in Clarkson (1981) the Miller & Modigliani (1961) "dividend irrelevance proposition" may be inconsistent with the observed market structure.

3.11   As regards equilibrium, Shiller (1989) shows that the volatility in equity markets is far higher than what would be expected on any equilibrium-based theory predicated on rational behaviour. Shiller suggests that this "excess volatility" may be due to the existence of two different types of investor, namely "noise traders", who follow fashions and fads, and, thereby, tend to overreact to changes in fundamentals, and "smart money traders", who invest according to fundamental value.

3.12   Peters (1991) uses the methodology of chaos theory, and in particular the Hurst exponent to show that there is a long-term memory effect in market prices that cannot be explained by any equilibrium-based theory. As described in Clarkson (1978, 1981), the Mean Absolute Deviation multiplier that is found to work best for individual gilt-edged securities and equities is 1.6 as opposed to the value of 2 which would be appropriate in an equilibrium-based market. Although the correspondence is not exact, there are strong parallels between this behaviour and the behaviour of the Hurst exponent as described by Peters.

3.13   Finally, as regards residuals being normally distributed in conformity with assumption F, there is a vast amount of empirical work from Mandelbrot (1963) onwards which shows a consistent pattern of "fatter tails" in capital market series. The Mean Absolute Deviation behaviour described above is very strong corroboration of this evidence that prices do not cluster around an equilibrium value in such a way as to give a normal distribution of residuals.

3.14   The overall conclusion must be that observed capital market behaviour bears virtually no resemblance to what would be expected if the simplifying assumptions of the CAPM were reasonable approximations to real world behaviour.

4.   LEVELS OF BEHAVIOUR

4.1   Suppose that we abandon the assumption of rational behaviour on the part of all economic agents and, guided by the anomalous empirical evidence cited above, postulate the existence of two types of investor - "intelligent" investors who assess the future prospects of a security against its current relative rating, and "unintelligent" investors who buy or sell on "good" or "bad" news respectively, irrespective of the current relative rating.

4.2   Consider now the exceptionally perceptive observation of Keynes (1936):

> "It might have been supposed that competition between expert professionals, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself. It happens, however, that the energies and skill of the professional investor and speculator are mainly occupied otherwise. For most of these persons are, in fact, largely concerned, not with making superior long-term forecasts of the probable yield of an investment over its whole life, but with foreseeing changes in the conventional basis of valuation a short time ahead of the general public. They are concerned, not with what an investment is really worth to a man who buys it 'for keeps', but with what the market will value it at, under the influence of mass psychology, three months or a year hence. Moreover, this behaviour is not the outcome of a wrongheaded propensity. It is an inevitable result of an investment market organised along the lines described. For it is not sensible to pay 25 for an investment of which you believe the prospective yield to justify a value of 30, if you also believe that the market will value it at 20 three months hence."

4.3   We now postulate the existence of a third kind of investor, namely "optimal" investors who not only understand the actions of both "intelligent" investors and "unintelligent" investors but also endeavour, by studying historic cyclical patterns as well as current sentiment and fundamentals, to exploit the "excess volatility" resulting from the actions of "unintelligent" investors.

5.   FORMAL DERIVATION OF THE DYNAMIC EQUILIBRIUM MODEL

5.1   As with the CAPM, some simplifying assumptions are required to translate the
      concept of different levels of investor behaviour into a mathematical model which
      can be tested against observed real world behaviour. These assumptions are set out
      below.

5.2   The first assumption is that investors are either "intelligent", "unintelligent", or
      "optimal"; gradations of skill and experience which would result in intermediate
      classifications are ignored.

5.3   The second assumption is that "unintelligent" investors are "trend-chasers" who, in
      the absence of intervention by other types of investor, would cause the rate of
      change of a share price over time to remain constant over the medium term.

5.4   The third assumption is that "intelligent" and "optimal" investors are "centralisers"
      whose aggregate scale of intervention (i.e. buying when the price is below the
      central price and selling when above) is proportional to the deviation of the price
      from its central value.

5.5   The fourth assumption, which will be relaxed later, is that the proportions of each
      type of investor remain constant over time.

5.6   Let the proportion of unintelligent investors be b, let the central price (assumed
      constant) be C, and let the share price at time t be P_t. Then, by analogy with the
      laws of mechanics (e.g. the behaviour of a pendulum) the equation of motion is:

$$k^2(1-b)(P_t-C) = -b\,\ddot{P_t},$$

where k is an arbitrary constant and $\ddot{P_t}$ is the second derivative of P_t with respect to time.

5.7   We note also that the maximum price is C + D, where D (also regarded for the moment as a constant) is the difference between each control limit and the central value. The general solution of the above second order partial differential equation subject to this boundary condition involving D can easily be shown, after introducing an error term E_t, to be:

$$P_t = C_t + D_t \sin\left(\sqrt{\frac{1-b}{b}}\;k + t_0\right) + E_t,$$

which gives simple harmonic motion with an (as yet unspecified) error term superimposed. In practice C and D will vary over time, albeit much more slowly than the share price, and the periodicity will also vary stochastically over time as the result of the essentially random nature of some of the contributory causal mechanisms.

6.   DAMPED HARMONIC MOTION AS THE NEW PARADIGM

6.1   While the first three assumptions used to derive the Dynamic Equilibrium Model are perhaps not unreasonable in the context of an initial attempt to break new ground in capital market theory, various arguments can be put forward to show that the fourth assumption may not be realistic over other than very short time periods.

6.2 The better performance of "intelligent" and "optimal" professional investors over the long term is likely to attract more funds for them to manage at the expense of "unintelligent" professional investors. This mechanism will also lead to some "unintelligent" investors ceasing to trade as a result of their becoming commercially unviable in the light of their poorer than average performance.

6.3 Another and possibly more powerful mechanism is the quest for self-improvement. Investors, whether private or professional, have very strong incentives to improve their levels of skill and experience by first of all observing what general philosophies appear to lead to superior performance and thereafter modifying their behaviour accordingly.

6.4 The overall effects are likely to be a decrease over time in the proportion of "unintelligent" investors and a consequent reduction in the band-width between the central value of a share price or market level and the upper or lower Mean Absolute Deviation control limits.

6.5 The paradigm of the CAPM is founded on capital market equilibrium as the result of the supposedly instantaneous and omniscient behaviour of investors in response to new information. My alternative paradigm of the Dynamic Equilibrium Model is based on an underlying process of damped harmonic motion as the result of investor behaviour being very far from omniscient but with an improving trend over time. I now investigate whether, in the light of observations of real world behaviour, this new paradigm of the Dynamic Equilibrium Model has greater explanatory power than the old paradigm of the CAPM.

7. GILTS

7.1   The marked rise in the mean square error of the Clarkson (1978) gilts model on
       sharp market movements and its subsequent fall as market levels stabilise is fully
       consistent with the new paradigm of the Dynamic Equilibrium Model.

7.2   A strong pointer in favour of the new paradigm is that the Mean Absolute
       Deviation multiplier of 1.6 which was found to work well in practice is very close
       to the value of $\Pi/2$, or about 1.57, which results from simple harmonic motion.

7.3   The band-width between the upper and lower control limits for long-dated stocks
       decreased from just under 1% in the late 'seventies before non-linear models had
       become a significant determinant of institutional investor behaviour to around
       0.2% in the late 'eighties. There is strong anecdotal evidence to suggest that an
       important factor in this marked reduction in the level of exploitable inefficiencies
       was the elimination, through unsustainable trading losses, of gilts market-making
       firms who did not have adequate price models in place. This is a classic example
       of the "survival of the fittest" mechanism described in Section 6.2 above.

8. EQUITIES

8.1   As with gilts, the usefulness in practical applications of a Mean Absolute Deviation
       multiplier of 1.6 as described in Clarkson (1981), and in particular in Appendix A
       thereof, is a strong pointer to the greater predictive power of the new paradigm.

8.2   The highly significant superior performance described in Clarkson (1981) and summarised in Section 8 of Clarkson (1996) is very strong evidence not only that exploitable inefficiencies exist but also that "traditional" methods of appraisal are likely to be far more profitable in practical applications than the methodologies of Modern Portfolio Theory, which are founded on the old paradigm of the CAPM.

9.   EQUITY MARKET LEVELS

The "confidence limits" approach of Clarkson (1978) for gilts and Clarkson (1981) for equities is extended for equity market levels in Mills (1991), where it is shown that a cointegration approach to equity and gilts market levels leads to very useful practical results. These results, while inconsistent with the old paradigm of the CAPM and the resultant implications of "stockmarket efficiency", are consistent with my alternative paradigm of the Dynamic Equilibrium Model.

10.   INVESTMENT MANAGEMENT

The superior explanatory power of the new paradigm in the areas discussed above suggests that it is indeed appropriate to investigate the implications for practical financial management. In the area of investment management, perhaps the most important immediate implication is that the expected returns on different securities within a particular capital market will vary markedly rather than being essentially constant, perhaps in some abstract risk-adjusted way, as implied by the market equilibrium assumption underlying the CAPM.

11. STOCKMARKET EFFICIENCY

11.1   Let Z be the "strong level" efficiency proposition that it is futile even for professional investors to attempt to achieve superior returns. Then, stripping out all the technical details, Jensen (1968) uses the following statement in terms of mathematical logic:

If A and B and C and D and E and F then Z,

where A, B, C, D, E and F are the simplifying assumptions of the Capital Asset Pricing Model as discussed in Sections 2 and 3 above.

11.2   Proposition Z is a compound statement to the effect that, if each of assumptions A, B, C, D, E and F is true, then there is no significant statistical evidence that actual mutual fund returns are inconsistent with what might have resulted from mere random chance. Accordingly, assumptions A, B, C, D, E and F are necessary conditions for proposition Z to be valid. Since the empirical evidence set out above suggests that it would be unsound in the extreme to regard all these assumptions as being true, we conclude that the analysis set out in Jensen (1968) is invalid as a scientific proof of "strong level" efficiency.

12. OPTION PRICING

As described in Clarkson (1995a), current theories of option pricing are based on numerous highly implausible simplifying assumptions, and in particular on the assumption that the capital market is in equilibrium. It is likely that a better theory of option pricing than that based on Black & Scholes (1973) would result if this apparently unjustified presumption of equilibrium were abandoned.

13.   ASSET/LIABILITY MODELLING

Conventional approaches to risk/return trade-offs in asset/liability modelling are based on the mean-variance framework of Markowitz (1952, 1959), which interprets the original utility theory approach of Bernoulli (1738) in a modern context. If, however, as argued above, the utility theory approach and its implicit assumptions of rational behaviour and equilibrium are unsound, an alternative downside approach to risk as set out in Clarkson & Plymen (1988) and Clarkson (1989, 1990) is likely to be more appropriate. The main characteristics of such an approach to asset/liability modelling are described in Clarkson (1995b).

14.   CONCLUSION

Although much further work remains to be done to develop the Dynamic Equilibrium Model outlined above into a robust new framework for capital market behaviour, it appears to offer a dynamic and non-linear approach that is potentially far more powerful than the static and linear methodologies derived from the Capital Asset Pricing Model.

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