

# INTERNATIONAL COMPARISON OF INTEREST RATE GUARANTEES IN LIFE INSURANCE

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PERSSON

ABSTRACT. Interest rate guarantees seem to be included in life insurance and pension products in most countries. The exact implementations of these guarantees vary from country to country and are often linked to different distribution of investment surplus mechanisms. In this paper we first attempt to model practice in Norway, Denmark and Germany by constructing a contract intended to capture practice in each country. All these contracts include rather sophisticated investment surplus distribution mechanisms, although they exhibit subtle differences. Common for these countries is the existence of a bonus account, an account where investment surplus is set aside in years with good investment returns to be used to cover the annual guarantee in years when the investment return is lower than the guarantee. These contracts are then compared with universal life insurance, a popular life product in the US market, which also include investment surplus distribution, but no bonus account. The contract parameters are calibrated for each contract so that all contracts have 'fair' prices, i.e., the theoretical market price of the contract equals the theoretical market price of all benefits at the inception of the contract. For simplicity mortality factors are ignored and the benefit is assumed to be paid out as a lump sum in 30 years. By comparing the probability distribution of the future benefit for the 4 contracts with the market index, our preliminary results indicate that for low levels of the financial risk premium the Danish and German systems behave just like the market index, i.e., the guaranteed rate and the investment surplus distribution have virtually no impact on the probability distribution of the benefit. The Norwegian benefit has a lower standard deviation than the German or Danish systems, whereas the benefit of the universal life contract offers the lowest standard deviation. Our preliminary numerical analysis therefore indicates that the relative simple US contract outperforms the more complex European counterparts if the objective is to provide the insurance customer a future benefit with low risk.

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## 1. INTRODUCTION

Interest rate guarantees, or more precisely, annual minimum rate of return guarantees, seem to be included in life insurance products in most countries. Due to the recent rather low international interest rate level, such guarantees are of great practical concern. The exact implementations of these guarantees vary from country to country and is often linked to different distribution of investment surplus mechanisms.

In a companion paper Miltersen and Persson (2003) interest rate guarantees are analyzed together with a distribution of investment surplus mechanism. The current paper is a natural extension to more realistically capture industry practice in Norway, Denmark and Germany which all include rather sophisticated investment surplus distribution mechanisms, although they exhibit subtle differences. Common for these three systems is the existence of a bonus account, which can be visualized as a buffer account on the liability side of the insurer's balance sheet, where investment surplus is set aside in years with good investment performance to be used to cover the annual guarantee in years when the investment return is lower than the guarantee. We construct a contract based on the systems in each of these countries.

These 3 contracts are compared with universal life insurance, a popular life product in the US market, which also include investment surplus distribution but no bonus account.

These four contracts are valued and the parameters calibrated using the standard no-arbitrage arguments from financial economics originating from Black and Scholes (1973) and Merton (1973).

The model does not take the life insurance specific factors *mortality* or *periodical premiums* into account. Specifying mortality and periodical premiums would limit the analysis to only a specific product but could easily be done. The questions we address in this article are common for a wide range of life insurance and pensions products, and thus of interest even if mortality is not specified.

Our model does not include stochastic interest rates.

Only one risky investment opportunity is available and we refer to this as the market index. By comparing the probability distribution of the future benefit for the 4 contracts with the market index, our preliminary results indicate that for low levels of the financial risk premium the Danish and German systems behave just like the market index, i.e., the guaranteed rate and the investment surplus distribution have virtually no impact on the cashflow profile of the benefit. The Norwegian benefit has a higher mean and a lower standard deviation than the German or Danish systems, whereas the benefit of the universal life contract offers the highest mean and the lowest standard deviation. Our numerical analysis therefore indicates that the relative simple US contract outperforms the more complex European counterparts if the

Insurer's balance sheet	
PORTFOLIO OF ASSETS $X$ -stocks -bonds	PREMIUM RESERVE -the customer's accounts $A^1$ with interest guarantee $g_1$ $A^2$ with interest guarantee $g_2$  UNDISTRIBUTED RESERVE -bonus account $B$  EQUITY -the insurer's account $C$

FIGURE 1. The balance sheet of the insurer.

objective is to provide the insurance customer a future benefit with low risk.

This paper is organized as follows: Section 2 describes the set-up. Section 3, 4, 5, 6 contain descriptions of the Norwegian, universal life (US), Danish, and German contracts, respectively. Section 7 explains the valuation principle used, section 8 presents the numerical results, and section 9 contains preliminary conclusions and suggestions to further research.

## 2. THE MODEL

Our model is based on an idealized picture of a life insurer's balance sheet depicted in Figure 1.

A fixed time horizon of  $T$  years is given and the initial point in time is denoted 0.

**2.1. The asset side.** We denote by  $X_t$  the market value of the market index. We assume that the investment portfolio of the insurance company is identical to the market index, even though in real life the investment portfolio may consist of various kinds of assets such as stocks, bonds, mortgages, or real estate. We denote the logarithmic return in year  $t$  by  $\delta_t$ . That is,

$$\delta_t = \ln \left( \frac{X_t}{X_{t-1}} \right),$$

where  $\ln(a)$  represents the natural logarithm of  $a$ . Observe that  $X_t = X_{t-1}e^{\delta_t}$ .

**2.2. The liability side.** The premium reserve at each time  $t$  represents the insurer's liability to the customer(s) at time  $t$ . We split the premium reserve into two components. Let  $A_t^1$  denote the balance of the first component of the premium reserve at time  $t$ . At this account interest accrues according to the guaranteed rate  $g_1$ , a constant. Let  $A_t^2$  denote the balance of the second component of the premium reserve at time  $t$ . Two premium reserve accounts are included to be able to model a different guarantee on additional investment returns distributed to the customer throughout the contract period. The guaranteed rate on the second component is denoted by the constant  $g_2$ . Typically  $g_2 < g_1$ . We sometimes refer to the two components just as  $A^1$  and  $A^2$ , respectively.

We denote by  $G_t$  the *sum guaranteed* at time  $t$ , i.e.,

$$G_t = A_{t-1}^1(e^{g_1} - 1) + A_{t-1}^2(e^{g_2} - 1)$$

Let  $I_t$  denote the time  $t$  investment return after guarantees, i.e.,

$$I_t = X_{t-1}(e^{\delta_t} - 1) - G_t.$$

Observe that  $I_t$  can be both positive or negative, depending on whether the investment return  $X_{t-1}(e^{\delta_t} - 1)$  is greater or less than, respectively, the sum guaranteed  $G_t$ . Therefore, we define

$$I_t^+ = (I_t)^+ = \max[I_t, 0]$$

as the *investment surplus* and

$$I_t^- = (I_t)^- = -\min[I_t, 0]$$

as the *investment deficit*.

We denote by  $B_t$  the balance of the bonus side at time  $t$ , and sometimes we refer to this account as account  $B$ . In years with a strictly positive investment surplus ( $I_t^+ > 0$ ), a part of the investment surplus is set aside to the bonus account for potential future use in years with a strictly positive investment deficit. In principle, the balance of the bonus account belongs to the customer, and a positive terminal balance of this account is credited the customer.

Similarly, we denote by  $C_t$  the equity of the company at time  $t$ , and refer to this account as account  $C$ . In years with a strictly positive investment surplus ( $I_t^+ > 0$ ), a part of the return is credit the insurer. These cashflows can be interpreted as a compensation for providing the annual guarantee and other embedded options of the contract. In our model these cashflows will be determined in a 'fair' way, i.e., the parameters of the model are calibrated so that the initial market value of the final balance of account  $C$  equals the initial market value of all future benefits of the contract.

We assume that initial distribution  $(A_0^1, A_0^2, B_0, C_0)$  between the four accounts on the liability side is given.

The distributions mechanisms between the different account at the liability side of the balance vary from country to country and they will therefore be explained for each case.

### 3. THE CASE OF NORWAY

The following description is meant to represent a stylized picture of practice in Norway for traditional life insurance products. This practice seems to be governed both by legislation, competition in the market, and established practice.

In 'good' years, when the investment surplus is positive ( $I_t^+ > 0$ ), fractions of the investment surplus  $I_t^+$  are distributed to the accounts  $A^2$ ,  $B$ , and  $C$ , determined by the parameters  $a$ ,  $b$ , and  $c$ , respectively.

In 'bad' years, when the investment deficit is positive ( $I_t^- > 0$ ), it is subtracted from  $B_t$ . However, the maximum deduction from account  $B$  at time  $t$  is limited to the amount guaranteed at time  $t$   $G_t$ . Any remaining deficit, i.e.,  $\max(I_t^- - G_t, 0)$ , is subtracted from the equity  $C_t$ . The balance of the two premium reserve accounts are given by

$$(1) \quad A_t^1 = A_{t-1}^1 e^{g_1},$$

and

$$(2) \quad A_t^2 = A_{t-1}^2 e^{g_2} + aI_t^+.$$

The guaranteed rate  $g_1$  determines the change of the balance of account  $A^1$  every year. If  $g_1$  is positive, the balance increases every year. Similarly, a positive  $g_2$  contributes to an increase in the balance of  $A^2$ . In the case of a positive investment surplus, the balance of  $A^2$  is further increased by a fraction  $a$  of the investment surplus. No deductions can be made from these accounts throughout the contract period.

The balance of the bonus account is given by

$$(3) \quad B_t = B_{t-1} + bI_t^+ - \min[I_t^-, G_t]$$

Also for account  $B$ , in the case of an investment surplus the balance is increased with a fraction  $b$  of the investment surplus. In the case of a investment deficit, the balance of  $B$  is reduced by the investment deficit limited to the sum guaranteed.

The balance of the equity at time  $t$  is

$$(4) \quad C_t = C_{t-1} + cI_t^+ - \max[I_t^- - G_t, 0].$$

Also for account  $C$ , in the case of an investment surplus the balance is increased with a fraction  $c$  of the investment surplus. To make sure that the complete (exhaustive!) investment surplus is distributed  $a + b + c = 1$ . In 'really bad' years, when the investment deficit is larger than the sum guaranteed ( $I_t^- > G_t$ ), the insurer must cover the remaining investment deficit.

#### 4. UNIVERSAL LIFE

In this section we consider a popular product in the US market called *universal life*. Universal life is more flexible than traditional life insurance in that the buyer is permitted to skip premium payments and vary the amount of premium payments. This property is however not incorporated in our model. The product has currently a market share of 22% in the US.

Apparently, the practice of reserving a part of the surplus in good years to prepare for future bad years is not common in the US industry. For an accurate description of this product we do not need to include the bonus account. Apart from this, the product is very similar to the Norwegian practice described in the previous section.

The balances of  $A^1$  and  $A^2$  are also for this product given by the equations (1) and (2).

There is no bonus account, so there does not exist any equation similar to equation (3) above. This fact implies that the insurer must cover the complete investment deficit in 'bad' years from the company's equity. The balance of account  $C$  is then

$$(5) \quad C_t = C_{t-1} + cI_t^+ - I_t^-$$

To make sure that the complete (exhaustive) investment surplus is distributed each 'good' year, we impose the condition that  $a + c = 1$ .

A more recent product in the US market is called *variable life* and includes the option to let the customer determine the asset allocation between bonds and typically, different mutual funds with different risk profiles. We do not model this option explicitly, but we will comment on this product in relation to our numerical results.

#### 5. THE CASE OF DENMARK

The following description is meant to capture practice for traditional life insurance products in Denmark. The Danish system is different from the two previous systems and is characterized by two special properties. See Grosen and Jørgensen (2000); Hansen and Miltersen (2002). First, insurance companies try to maintain the balance of the bonus account at a fixed predetermined ratio of the sum of the balances of the premium reserve and the equity. We denote this ratio by  $\gamma$ . The fraction  $\frac{B_t}{A_t + C_t}$  should therefore be close to the ratio  $\gamma$ . Second, the return on the insurance policy in year  $t + 1$ , is determined at time  $t$ , i.e., independent of investment performance in year  $t + 1$ . Finally, only one  $A$  account is used to describe Danish practice.

These features are incorporated as follows in our general model. First, the dynamics of the sum of the  $A$  and the  $C$  account is modeled as

$$(6) \quad (A + C)(t) = (A + C)(t - 1)e^{\max[g, \ln(1 + \alpha(\frac{B(t-1)}{(A+C)(t-1)} - \gamma))],}$$

where  $g$  denotes the annual guaranteed rate. Here  $\alpha$  can be interpreted as the fraction of the bonus account in excess of the desired level  $\gamma$  which is credited the accounts  $A$  and  $C$ . Second, the dynamics of only the  $A$  account is given by

$$(7) \quad A(t) = A(t-1)e^{\max[g, \ln(1 + \alpha(\frac{B(t-1)}{(A+C)(t-1)} - \gamma))] - \beta}.$$

The parameter  $\beta$  determines the deduction of the return credited account  $A$  and can be interpreted as a *cost* parameter.

Third, the balance of the  $C$  account only is calculated as the difference between the balance of the sum of the  $A$  and the  $C$  account and the balance of the  $A$  account as

$$(8) \quad C(t) = (A + C)(t) - A(t).$$

Finally, the balance of the bonus account is determined residually as

$$(9) \quad B(t) = B(t-1) + [X(t) - X(t-1)] + [(A + C)(t) - (A + C)(t-1)].$$

## 6. THE CASE OF GERMANY

--TO BE COMPLETED--

## 7. VALUATION

Let  $Z_s$  be the payoff payable at time  $s$ , dependent of the value of some underlying asset. Denote the market value at time  $t \leq s$  of  $Z_s$  by  $V_t(Z_s)$ .

At expiration of the contract the customer receives the final balances of the two premium reserve accounts  $A_T^1$  and  $A_T^2$  and a potential positive balance of the bonus account denoted by  $B_T^+$ . From the customer's point of view a fair valuation is expressed by

$$V_0(A_T^1) + V_0(A_T^2) + V_0(B_T^+) = V_0(X_0) = X_0,$$

i.e., the initial market value of the future benefits equals the (market value of) the initial premium, to be thought of as a single premium or initial deposit.

From the insurance company's point of view a fair valuation is expressed by

$$V_0(C_T) = V_0(B_T^-),$$

the initial market value of the company's future cashflow equals the initial market value of the potential negative balance of the bonus account, which may occur if the annual guarantee is consistently higher than the investment returns.

## 8. NUMERICAL RESULTS

8.1. **The model.** For the numerical analysis we use the original Black and Scholes (1973) set-up. We assume that the market value of portfolio of investments at time  $t$   $X_t$  is given by the following stochastic differential equation

$$(10) \quad dX_t = \mu X_t dt + \sigma X_t dW_t,$$

where the initial value of the process  $X_0$  is given and  $W_t$  is a Brownian motion. The parameter  $\mu$  can be interpreted as the annualized *instantaneous* expected logarithmic return, whereas  $\sigma$  is referred to as the volatility of the investment portfolio. The volatility is calculated as the square root of the *instantaneous* variance of the logarithmic return, and thus resembles an annualized *instantaneous* standard deviation of the logarithmic return, although the latter interpretation is not rigorously justified.

We assume that the continuously compounded rate of return is constant, denoted by  $r$ . According to standard financial terminology we define  $\pi = \mu - r$  as the (instantaneous) *risk premium*.

Equivalently, the logarithmic return in year  $t$  is

$$\delta_t = \mu - \frac{1}{2}\sigma^2 + \sigma(W_t - W_{t-1}).$$

For this specification of the model the market value operator takes the form

$$V_t(Z_s) = e^{-r(s-t)} E^Q[Z_s],$$

where  $E^Q[\cdot]$  denotes the expectation under the equivalent martingale measure.

8.2. **Choice of parameters.** We assume that the time horizon of the contract is  $T = 30$  and that the riskfree rate of return is constant and equal to  $r = 0.05$ .

The additional Danish parameter  $\gamma = 0.15$ , which is supposed to be a realistic value of this parameter. The initial deposit of the contract is normalized to  $X_0 = 1$ . The initial distribution between the different accounts at the liability side of the balance sheet is assumed to be  $A_0^2 = B_0 = C_0 = 0$  and  $A_0^1 = 1$ . We assume that the parameter  $a = 0.25$  ( $\alpha = 0.25$  for the Danish case),  $b = 0.25$  for the Norwegian case. The remaining distribution parameter  $c$  (Norway, universal life) and  $\beta$  (Denmark) is determined so that each contract is fair (determined numerically from 30000 simulations). We assume that the guarantees are  $g_1 = g_2 = 3\%$  (so there is no distinction between the two  $A$  accounts).

The following graphs show the probability distribution of the benefit. These graphs are based on 100 000 simulations.

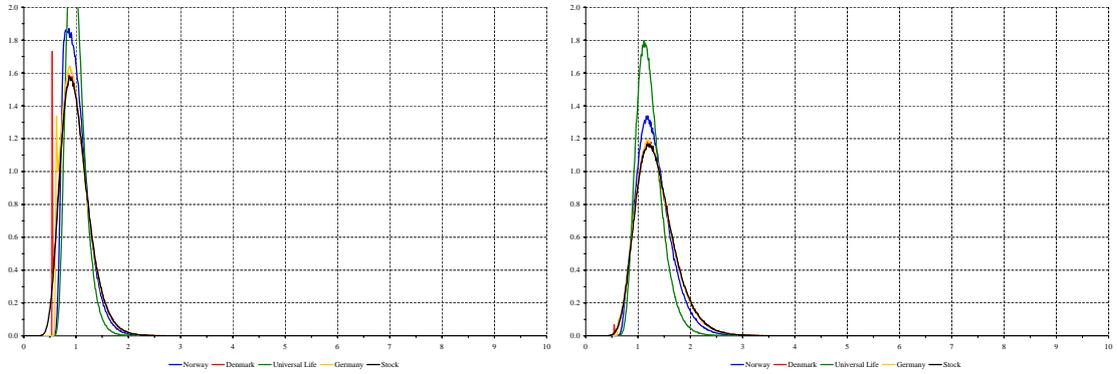
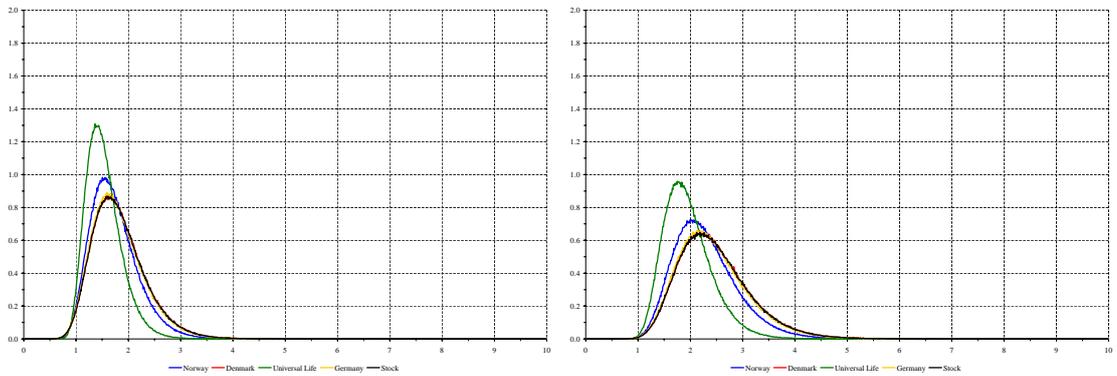
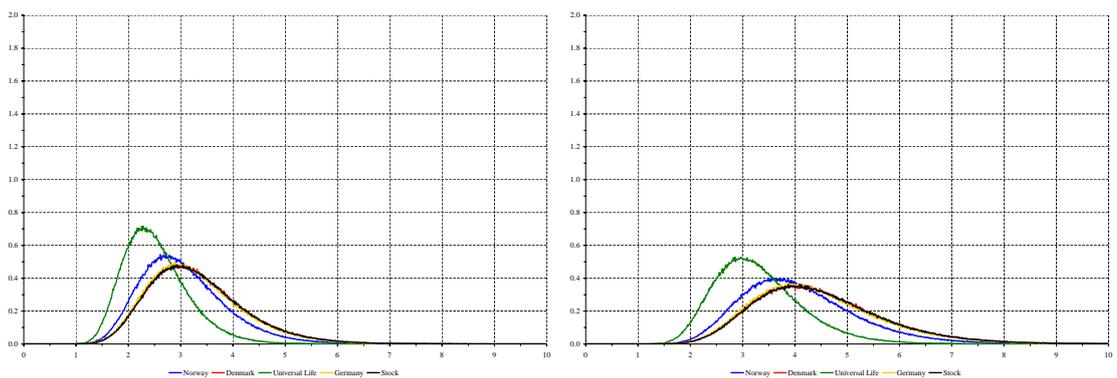
(a)  $\pi = 0$ .(b)  $\pi = 0.01$ .(c)  $\pi = 0.02$ .(d)  $\pi = 0.03$ .(e)  $\pi = 0.04$ .(f)  $\pi = 0.05$ .

FIGURE 2. Simulated probability distributions of the amount of benefit for the Norwegian, Danish, universal life, German contracts, and the market index for levels of the risk premium of 0 to 5% and low volatility,  $\sigma = 5\%$ .

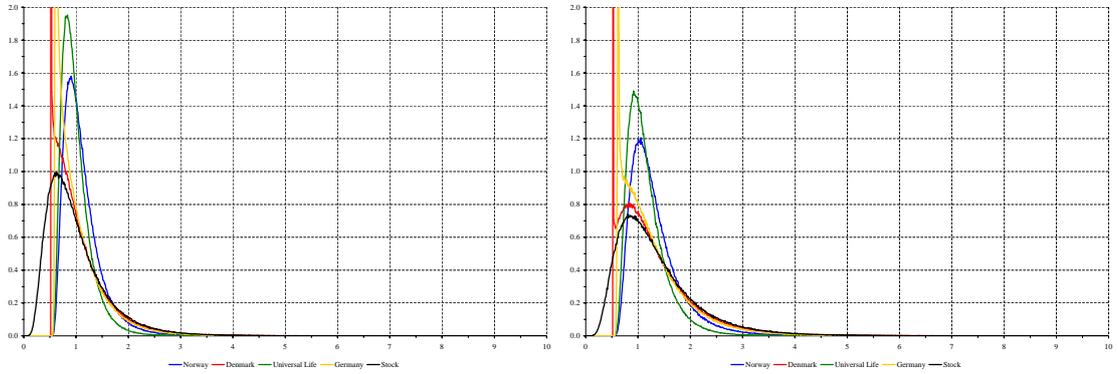
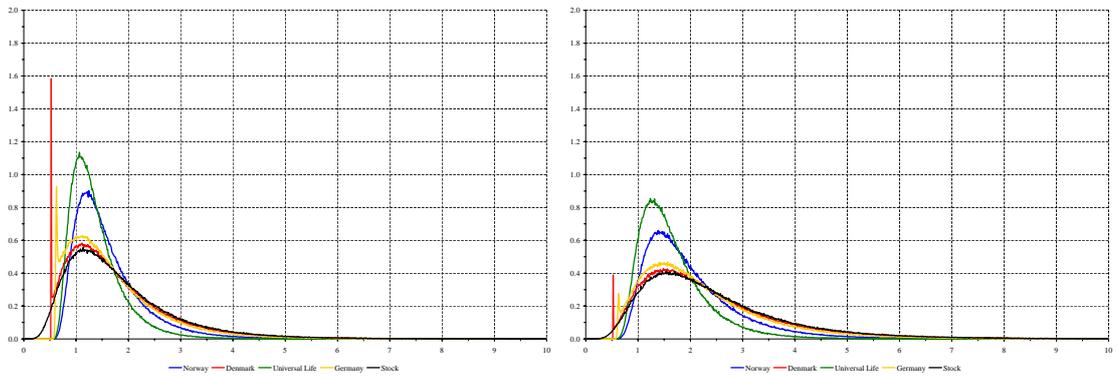
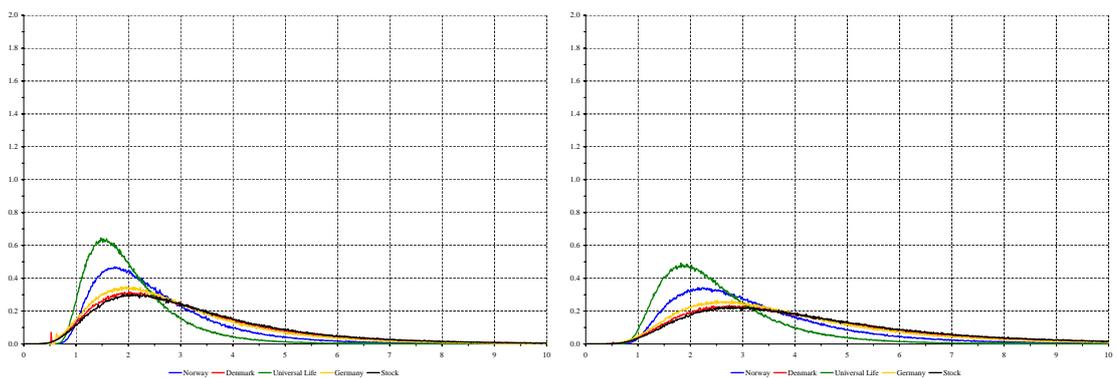
(a)  $\pi = 0$ .(b)  $\pi = 0.01$ .(c)  $\pi = 0.02$ .(d)  $\pi = 0.03$ .(e)  $\pi = 0.04$ .(f)  $\pi = 0.05$ .

FIGURE 3. Simulated probability distributions of the amount of benefit for the Norwegian, Danish, universal life, German contracts, and the market index for levels of the risk premium of 0 to 5% and medium volatility,  $\sigma = 10\%$ .

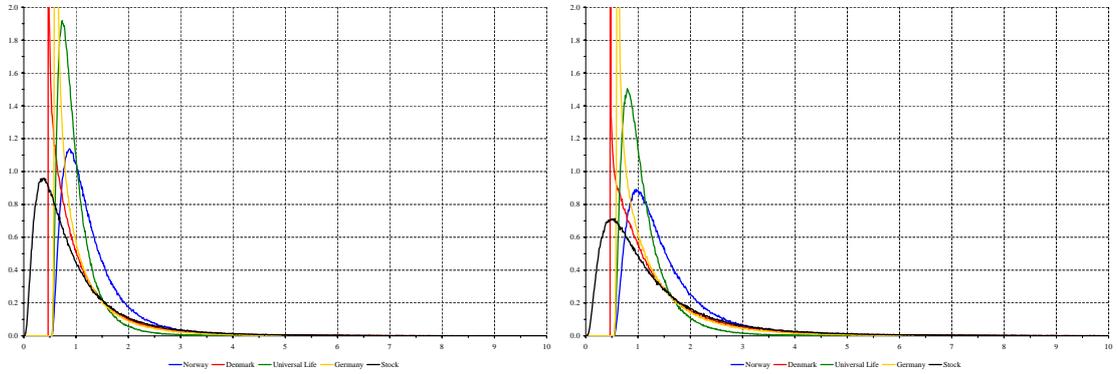
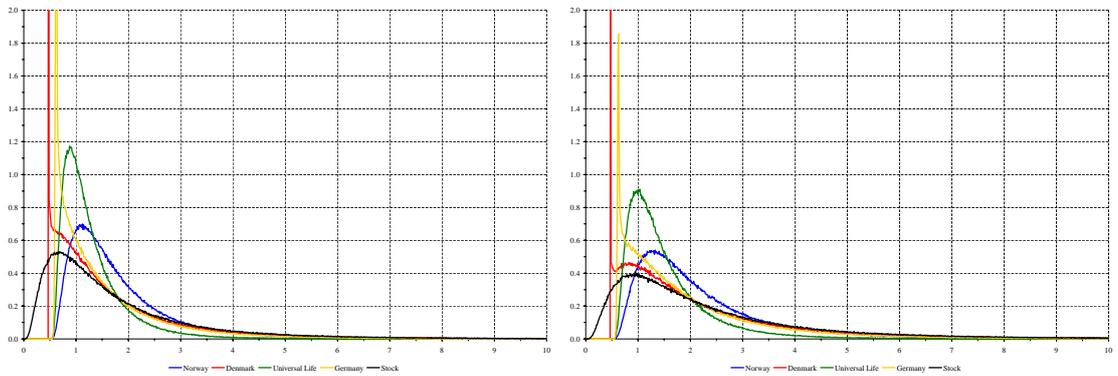
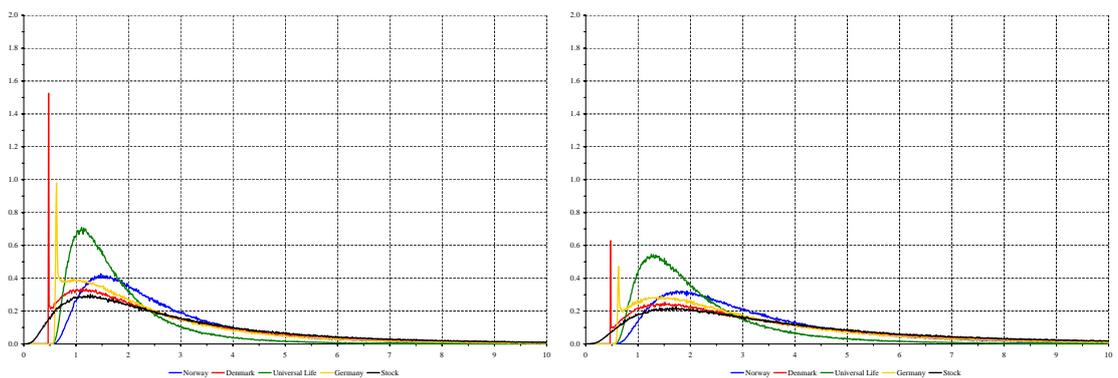
(a)  $\pi = 0$ .(b)  $\pi = 0.01$ .(c)  $\pi = 0.02$ .(d)  $\pi = 0.03$ .(e)  $\pi = 0.04$ .(f)  $\pi = 0.05$ .

FIGURE 4. Simulated probability distributions of the amount of benefit for the Norwegian, Danish, universal life, German contracts, and the market index for levels of the risk premium of 0 to 5% and high volatility,  $\sigma = 15\%$ .

**8.3. Preliminary numerical results.** In the absence of mortality the different plots represent what we get back in 30 years from investing one unit in the different contracts. The plots are probability distributions implying that the total area under the each plot is one. All plots have only one peak, the higher this peak is, the more the probability mass is concentrated, and thus the lesser uncertainty about the future payoff. Some of the plots contain spikes (typically the Danish and German contracts). These spikes are due to the embedded guarantees. The spikes are pronounced for high levels of volatility and risk premium, whereas not visible for low levels of volatility and risk premiums.

We have plotted risk premiums of 0% to 5% for 3 levels of volatility (5%, 10%, and 15%). For all plots universal life has the highest peak, the Norwegian contract has the second highest peak, whereas the Danish and German contracts are similar to the market index and they have all lower peaks than universal life and the Norwegian contract. In terms of variability or uncertainty this means that universal life gives the customer the least uncertain benefit, the Norwegian contract the second best contract, whereas the benefits of the German and Danish contracts are the most uncertain. The Danish and German contracts are surprisingly similar to the market index, but for low level of risk premiums and volatility spikes, representing the effect of the guarantees, are visible. The spikes illustrate the downside protection of the benefit due to the embedded guarantees. The spikes are visible for risk premiums of 0 and 1 % for 5% volatility, for risk premiums up to 4% for 10%volatility, and for all presented risk premiums (up to 5%) for 15 % volatility.

By increasing the risk premium the probability distributions are shifted to the right, indicating a higher benefit. This effect naturally reduces the peaks because the area under the curves must equal one for all plots.

Increasing volatility leads to more pronounced spikes, and lower peaks. The first effect is due to the embedded guarantees as mentioned, the second effect is due to increased uncertainty.

## 9. PRELIMINARY CONCLUSIONS AND FURTHER RESEARCH

Based on our numerical results we can conclude that the universal life contract has the least uncertain benefit. On second place, the Norwegian contract provides a less uncertain benefit than the German and Danish systems. The ranking seems to be independent of the levels of the financial risk premiums and volatility. We find it surprising that the simple universal life contract, which does not include any bonus mechanism, so clearly outperforms the more complex European counterparts, in terms of providing a future low risk benefit.

It is also surprising that the Danish and German contracts, which include sophisticated smoothing mechanisms, do not perform very different from the market index. However, the effect of the guarantees are pronounced for high volatilities and high risk premiums, indicating that these contracts provide a downside protection compared to just investing in the market index.

One question which one may raise (at least for low volatility and risk premium) is whether a rational investor should invest in an insurance contract like the Danish or German or just the market index. Clearly, the answer depends on whether the investor can change investment before the expiration. In this analysis we have implicitly assumed that the contracts are fixed for 30 years. We leave this question for future research.

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