

RANDOM BENEFITS AND STOCHASTIC INTEREST RATES IN LIFE INSURANCE

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Abstract

Interest rate risk is a major source of uncertainty for the insurance industry. Unit-linked or equity-linked contracts are examples of life insurance contracts where the amount of benefit is contingent on the market value of some financial asset. Using contingent claims theory and traditional actuarial theory, we derive an economic valuation theory for such contracts in a model with random interest rates and present some new formulas for certain unit-linked contracts.

Keywords : Life Insurance, Contingent Claims Analysis, Arbitrage Pricing Theory, Principle of Equivalence, Heath, Jarrow and Morton Term Structure Model.

1. INTRODUCTION

1.1 Motivation

Recently many new kinds of life insurance products have been introduced. This paper deals with types of equity-linked or unit-linked insurance contracts which link the amount of benefit to a financial asset, usually a mutual fund. These products seem to offer the insurance companies as well as the insurance customers advantages compared to traditional products. The insurance industry may benefit from higher yields in the financial markets and therefore offer products more competitive compared to alternative ways of saving. These products also offer the customers some flexibility in that they may choose more or less freely to which mutual fund to link the benefit, thereby influencing the amount of financial risk of their policies.

One important issue for the insurance industry is the interest rate risk. The last two decades a number of papers in the actuarial literature introduce stochastic models of interest rates. In this paper we use a stochastic model of the interest rate, but rather than simply specifying a stochastic process, the interest rate follows from the specified financial model using economic valuation theory. This rate may be interpreted as the interest rate prevailing in the economy.

The focus of this paper is pricing of life insurance contracts, taking both random benefits and stochastic interest into account. The principle of equivalence, which traditionally has been the basis of the pricing of life insurance policies, neither deals with stochastic interest rates nor stochastic amounts of benefits.

Our set-up includes a simple model of a financial market. This approach is standard in the theories of financial economics, but is not common in the actuarial literature. In this market the mutual fund and default free bonds are traded and from the market prices of the bonds, the interest rates can be deduced. In order to keep the model simple, we restrict ourselves to two sources of uncertainty. The first reflects risk connected to the interest rate, the second risk connected to the mutual fund to which the policy is linked. For the stochastic interest rate we apply a theory developed by Heath, Jarrow and Morton (1992) (henceforth referred to as HJM). From observed market prices of bonds one may deduce relevant information about interest

rates prevailing over different time intervals. The HJM-model assumes that the development of the forward rates and the initial term structure are given. By assuming that no arbitrage opportunities exist, the drift term of the forward rate is restricted in a way so that pricing can be done from knowledge of only the volatility processes of the forward rate and the initial term structure.

The philosophy behind the traditional principle of equivalence is that, abstracting from administrative expenses, a company's income (premiums), and expenses (paid benefits) should balance in the long run. Traditionally the discount factor used for the valuation purpose is interpreted as the company's return on its investments. In the described financial environment this return will depend on the chosen investment strategy, which again depends on the company's attitude towards financial risk. Here we adopt a conservative point of view, i.e., we assume that the company does not want to accept more financial risk than it is forced to. This view corresponds to the common opinion that the insurance companies should not "play with other people's money", in most countries manifested by legislation restricting the insurance industry's investment possibilities. We do not address the important question whether an insurance company should accept more financial risk and, if yes, how much more. However, we should expect that companies which choose more risky strategies on average will get a higher yield on their investments and hence could offer cheaper insurance than the conservative companies. At the same time risky investment strategies increase the probability of bankruptcy of the insurance company.

We see few reasons to treat mortality risk connected to unit-linked contracts different from mortality risk connected to traditional contracts. An important assumption we maintain throughout the paper is that the insurance company is risk neutral with respect to mortality risk. That is, the insurer does not receive any economic compensation for accepting mortality risk. This assumption is also implicit in the traditional principle of equivalence and does not imply that mortality risk is unimportant in potential applications of the theory. The assumption is justified by the traditional argument saying that the insurer can, at least in principle, eliminate mortality risk by increasing the number of identical contracts in his portfolio.

We consider two types of contracts: Pure endowment contracts, which expire if the customer is in a certain state (alive) at a certain point in time

(term of the contract), and term insurance, which expire upon a certain event (the policyholders dead) within the term of the contract. These simple contracts include the characteristics of most interesting life insurance policies on single lives. Furthermore, the treatment of financial risk is independent of how complex the insurance contract is, and the results of this paper can easily be generalized to more complex policies.

Another important, though reasonable assumption is that the financial factors are independent of mortality factors.

The first treatments of unit-linked contracts with guarantees of which we are aware, are Brennan and Schwartz (1976), (1979a), (1979b) and Boyle and Schwartz (1977). They recognised that the payoff of a unit-linked contract with guarantee is related to the payoffs of certain financial options, and applied the option pricing theory initiated by the results of Black and Scholes (1973). Delbaen (1980), Bacinello and Ortu (1993), and Nielsen (1993) extended this analysis by using the martingale-pricing theory. Contrary to the mentioned papers Aase and Persson (1994), Persson (1994) and Persson and Ekern (1995) use time-continuous death probabilities which is common in the actuarial literature. This leads to results that can be directly compared to the corresponding classical actuarial, as well as the pure financial, counterparts. The current paper differs from all the above by considering both time-continuous death probabilities and random interest rates.

Stochastic interest rates in actuarial models are treated, e.g., in the following papers: Boyle (1976), Panjer and Bellhouse (1980), (1981), Waters (1978), Wilkie (1976), Devolder (1986), Beekman and Shiu (1988), Beekman and Fuelling (1990, 1991) and Parker (1992), (1994). In all these works the interest rate is directly modelled by stochastic processes. An approach different from our, where we deduce the interest rate in the economy from a financial model.

The paper is structured as follows: In section 2 we describe the financial market, section 3 explains the pricing principles. In section 4 some examples of contracts are given, and section 5 contains some concluding remarks.

2 THE FINANCIAL MARKET

2.1 The financial assets

A time horizon T is fixed and the financial uncertainty is generated by a 2-dimensional standard Brownian motion (V, W) defined on a probability space (Ω, \mathcal{F}, P) together with the usual filtration $(\mathcal{F}_t, 0 \leq t \leq T)$, representing the revelation of information. All trade is assumed to take place in a frictionless market (no transaction cost or taxes and short-sale allowed with continuous trading opportunities).

Definition

A unit discount bond is a financial asset that entitles its owner to one unit currency at maturity without any intermediate coupon payments. We denote by $B_t(s)$ the market price at time t for a bond maturing at a fixed date $s \geq t$. From the definition $B_s(s) = 1$ (assuming no default risk).

We assume there is a continuum of such bonds maturing at all times $s, 0 \leq s \leq T$.

Furthermore, we assume the price of the mutual fund is given by the following stochastic differential equation,

$$dS_t = \eta S_t dt + \sigma_1 S_t dW + \sigma_2 S_t dV, \quad (1)$$

where η , σ_1 and σ_2 are constants and the initial value S_0 is given. Here η may be interpreted as the instantaneous expected rate of return of S_t and $\sqrt{\sigma_1^2 + \sigma_2^2}$ as the instantaneous standard deviation of return. As will soon be apparent, the Brownian motion $V(t)$ is used to model mutual fund specific risk.

2.2 The HJM model

The starting point for the term structure model of Heath, Jarrow and Morton (1992) is the relationship between the instantaneous forward rates and the bond price,

$$B_t(s) = \exp \left(- \int_t^s f_t(u) du \right)$$

Here $f_t(u)$ is the instantaneous forward rate for time u prevailing at time t . The forward rates are modelled by Ito -processes of the form,

$$f_t(u) = \alpha_t(u)dt + \sigma_t(u)dW,$$

for all u , $0 \leq u \leq T$, where the initial values $f_0(u)$, $0 \leq u \leq T$, are given.

For fixed u , the function $\alpha_t(u)$ is called the drift process and the function $\sigma_t(u)$ is called the volatility process of the instantaneous forward rate, respectively. Both these processes are assumed to satisfy technical conditions so that the above expressions are well-defined Ito -processes. Later we impose the more restrictive assumption that these processes are deterministic functions. In potential applications these functions may be estimated from observable data. The initial forward rates $\{f_0(u) : 0 \leq u \leq T\}$ are assumed given, hence also the bond prices at time zero are known. We will refer to the collection $\{\sigma_t(u) : 0 \leq t \leq T, t \leq u \leq T\}$ as the volatility structure. In particular, the short interest rate r_t is given by

$$r_t = f_t(t).$$

Subject to some technical conditions on the drift and the volatility processes it follows from the two above expressions and Ito's lemma that

$$dB_t(s) = [r_t + b(t,s)]B_t(s)dt + a(t,s)B_t(s)dW,$$

where

$$b(t,s) = \frac{1}{2} a(t,s)^2 - \int_t^s \alpha_t(u)du$$

and

$$a(t,s) = - \int_t^s \sigma_t(u)du.$$

We notice that the market price process of the bond with maturity s is an Ito-process with parameters expressed in terms of the primitives, i.e., the drift and the volatility processes of the forward rates. This model of the bond price also has the theoretical advantage that the volatility process tends to zero as time approaches maturity.

No-arbitrage opportunities guarantees the existence of a function $\lambda^1(t)$, see e.g., Heath, Jarrow and Morton (1992), such that

$$\alpha_t(s) = \sigma_t(s)[\lambda^1(t) - a(t,s)], \text{ for all } s, t \leq s \leq T.$$

This condition is called the forward rate restriction. The function $\lambda^1(t)$ is called the market price of risk and is related to the first source of uncertainty generated by the Brownian motion $W(t)$.

Let

$$\lambda^2(t) = \frac{1}{\sigma_2}(\eta - r_t - \sigma_1 \lambda^1(t)).$$

This quantity can be interpreted as the market price of risk related to $V(t)$, the second source of risk in this model .

We define

$$\xi_t = \exp \left(- \int_0^t \lambda^1(u) dW - \int_0^t \lambda^2(u) dV - \frac{1}{2} \int_0^t [\lambda^1(u)^2 + \lambda^2(u)^2] du \right)$$

and a probability measure Q by

$$Q(A) = \int_{\Omega} 1_A(\omega) \xi_T dP(\omega).$$

The quantity

$$v(t) = \exp\left(-\int_0^t r_u du\right)$$

is sometimes called the discount function and represents the present value at time zero of one unit currency at time t .

It follows that

$$B_t(s) = E^Q\left(\frac{v(s)}{v(t)} \mid F_t\right) \tag{2}$$

where $E^Q(\cdot \mid F_t)$ denotes the conditional expectation under the probability measure Q .

Under the probability measure Q we obtain the following processes

$$f_t(u) = f_0(u) + \int_0^t \sigma_v(u) \int_v^u \sigma_t(s) ds dv + \int_0^t \sigma_v(u) dW^*,$$

$$r_t = r_0 + \int_0^t \sigma_v(t) \int_v^t \sigma_t(s) ds dv + \int_0^t \sigma_v(t) dW^*,$$

$$dB_t(s) = r_t B_t(s) dt + a(t,s) B_t(s) dW^*,$$

and

$$dS_t = r_t S_t dt + \sigma_1 S_t dW^* + \sigma_2 S_t dV^*,$$

where $V^*(t)$ and $W^*(t)$ are two independent Brownian motions under the probability measure Q . Here we notice that the instantaneous expected returns of the mutual fund as well as the bonds are identical to the short interest rate r_t . These expressions do not include the drift processes of the forward rates $\{\alpha_t(s), 0 \leq t \leq T, t \leq s \leq T\}$.

The HJM-model takes the initial term structure, volatility structure and the market price of risk as primitives. The market prices of the bonds depend on the initial term structure and the volatility structure. The HJM-model has the advantage that it does not require direct knowledge of the market price of risk. Still $\lambda^1(t)$ and $\lambda^2(t)$ must satisfy some technical conditions, an issue we have not addressed here, to ensure that the measure Q is well-defined (uniformly boundedness is a sufficient condition).

3. PRICING PRINCIPLES

As pointed out, the traditional principle of equivalence is not suitable to calculate single premiums when financial risk is present. The following methodology is a slight modification of the results of Aase and Persson (1994) and Persson (1994), where the following pricing principle is denoted the principle of equivalence under Q .

Let $C(t)$ denote an arbitrary insurance benefit payable at time t possibly dependent on the market value at time t of the mutual fund.

Let the random variable T_X denote the remaining life time of an x -year old person. We assume that the probability density function for T_X exists and denote it f_X . Let ${}_t p_X = P(T_X > t)$ denote the survival probability of an x -year old policy buyer. Here T_X is assumed independent of $W(t)$ and $V(t)$, hence it is independent of all processes reflecting financial quantities.

From the assumed risk neutrality with respect to mortality and the independence between mortality and the financial risk it follows that the market price at time t of an endowment insurance contract payable at time s if the insured is alive is

$$\pi(t) = {}_{s-t}p_{X+t} E^Q \left[\frac{v(s)}{v(t)} C(s) | F_t \right].$$

Similarly, the market price at time t of a term insurance payable upon death before time T is

$$\pi^1(t) = \int_t^T \mathbb{E}^Q \left[\frac{v(s)}{v(t)} C(s) | \mathcal{F}_t \right] f_x(s) ds.$$

These formulas really represent a new pricing principle for life insurance contracts. The transformation from the original probability measure P to the probability measure Q represents the correct adjustment for financial risk, in a certain sense, in this model.

4. EXAMPLES OF CONTRACTS

In this section we consider three different kinds of benefits:

$$1) \quad C^1(t) = 1,$$

$$2) \quad C^2(t) = \text{Max}[S_t, G_t],$$

and

$$3) \quad C^3(t) = \text{Max}[\text{Min}[S_t, K_t], G_t].$$

In the first example the benefit is deterministic, as in the traditional contracts. This example is included to isolate the effect of the stochastic interest rate. In the next example the benefit is the maximum of the value of one unit of the fund and a guaranteed amount G_t . In principle there is no maximum amount the insurance company is obliged to pay under this contract so in example 3 we have included a maximum amount, a cap, in this contract.

The quantity $\beta(t,s) = -\ln\left(\frac{v(s)}{v(t)}\right) = \int_t^s r_u du$ is included in both pricing formulas of the last section. It follows that

$$\beta(t,s) = \ln[B_t(s)] + \frac{1}{2} \int_t^s a(v,s)^2 dv - \int_t^s a(v,s) dW^*,$$

under the risk adjusted probability measure Q .

We now assume that the volatility structure is deterministic, then $\beta(t,s)$ is Gaussian.

The solution to the stochastic differential equation (1) is

$$S_t = S_0 \exp \left(\beta(0,t) - \frac{1}{2}(\sigma_1^2 + \sigma_2^2)t + \sigma_1 W^*(t) + \sigma_2 V^*(t) \right).$$

We will calculate the single premiums of the policies at time zero. First we turn to example 1 and calculate the market premiums of the pure endowment insurance and the term insurance. From the formulas of the last section and by using the expression (2), we obtain

$$\pi(0) = {}_T p_x B_0(T)$$

and

$$\pi^1(0) = \int_0^T B_0(s) f_x(s) ds.$$

For the endowment insurance the single premium is the market price at time zero of one unit currency payable at time T multiplied with the probability for the benefit is actually being paid.

For the term insurance $B_0(s) f_x(s) ds$ can, similarly, be interpreted as the market value at time zero for the expected payoff in the time interval $(s, s+ds)$. The single premium of the contract is then the sum of these expected payoffs over the whole term of the contract.

These formulas resemble the corresponding classical formulas. However, note carefully the important difference, the usual present values of future payoffs are replaced with market values of future payoffs.

For the second example we let

$$P^1(t) = E^Q [v(t) \text{Max}[S_t, G_t]].$$

Here $P^1(t)$ can be interpreted as the market value at time zero of a benefit which expires at time t with probability 1.

It follows that

$$P^1(t) = EQ \left[\text{Max} \left[G_t \exp \left(-\frac{1}{2} \Gamma_t + X \right), S_0 \exp \left(-\frac{1}{2} \Delta_t + Y \right) \right] \right],$$

where the covariance matrix between X and Y is

$$\Sigma_{x,y} = \begin{pmatrix} \Gamma_t & \Psi_t \\ \Psi_t & \Delta_t \end{pmatrix}$$

where $\Gamma_t = \int_0^t a(s,t)^2 ds$, $\Psi_t = \sigma_1 \int_0^t a(s,t) ds$ and $\Delta_t = (\sigma_1^2 + \sigma_2^2)t$.

The calculation of the above expectation is presented in the following lemma.

Lemma 1

The market value at time zero of the benefit $\text{Max}[S_t, G_t]$ payable at time t is

$$P^1(t) = S_0 \Phi(d^1) + G_t B_0(t) \Phi(-d^2),$$

where

$$d^1 = \frac{1}{\Theta_t} \left(\frac{1}{2} \Theta_t^2 + \ln \left(\frac{S_0}{B_0(t) G_t} \right) \right)$$

$$d^2 = d^1 - \Theta_t.$$

and

$$\Theta_t^2 = \Gamma_t + \Delta_t - 2\Psi_t.$$

Proof.

This formula follows from straight forward calculations from the previous expression, involving properties of the bivariate normal random variables X and Y.

The resulting formula depends on the volatility structure of the forward rates (represented by $a(t,s)$) and the initial forward rates

($B_0(t)$) and 5 parameters. These are: 3 parameters of the mutual fund price process (S_0, σ_1, σ_2), the guarantee (G_t), and time to expiration (t). We also notice that it does not depend on η the instantaneous expected return of the mutual fund.

Rabinovitch (1989) values the claim $\text{Max}[(S_t - G_t), 0]$ in a similar setting. His formula reproduced in our notation is $S_0\Phi(d^1) - G_t B_0(t)\Phi(d^2)$. Observe that $\text{Max}[S_t, G_t] = \text{Max}[(S_t - G_t), 0] + G_t$. The market price at time zero of the last term is $B_0(t)G_t$. The formula of lemma 1 is then the sum of the two time zero market values.

Incorporating the insurance aspects, the single premiums of the two contracts can now be expressed by

$$\pi(0) = \text{TP}_X[S_0\Phi(d^1) + G_T B_0(T)\Phi(-d^2)]$$

and

$$\pi^1(0) = \int_0^T [\Phi(d^1) + G_s B_0(s)\Phi(-d^2)] f_X(s) ds.$$

Simple calculations show that these formulas reduce to the results in Theorem 1 and 2 in Aase and Persson (1994) when the interest rate is constant.

We now turn to the third example. Define

$$P^2(t) = \text{EQ} [v(t)\text{Max}[\text{Min}[S_t, K_t], G_t]].$$

This expression can similarly be interpreted as the market value at time zero of the benefit $\text{Max}[\text{Min}[S_t, K_t], G_t]$ payable at time t .

It follows that

$$P^2(t) = \text{EQ} \left[\text{Max} \left[G_t \exp \left(-\frac{1}{2} \Gamma_t + X \right), \text{Min} \left[K_t, S_0 \exp \left(-\frac{1}{2} \Delta_t + Y \right) \right] \right] \right]$$

where the covariance matrix between X and Y is $\Sigma_{X,Y}$.

Lemma 2

The market value at time zero of the benefit $\text{Max}[\text{Min}[S_t, K_t], G_t]$ payable at time t is

$$P^2(t) = S_0[\Phi(e_2) - \Phi(e_1)] + G_t B_0(t) \Phi(e_1) + K_t B_0(t) \Phi(-e_2),$$

where

$$e_1 = \frac{1}{\Theta_t} \left(\frac{1}{2} \Theta_t^2 + \ln \left(\frac{G_t}{S_0} \right) \right)$$

and

$$e_2 = \frac{1}{\Theta_t} \left(\frac{1}{2} \Theta_t^2 + \ln \left(\frac{K_t}{S_0} \right) \right)$$

Proof.

Also this formula follows from straight forward calculations from the previous expression, involving properties of the bivariate normal random variables X and Y .

By comparing this formula to the formula for $P^2(t)$, we observe that this formula depends on one more parameter, namely K_t , the upper cap. Simple calculations show that $P^3(t) < P^2(t)$. The explanation for this is that the high values ($> K_t$) of the mutual fund do not lead to higher benefit for the insured because of the cap.

Similarly, by incorporating the insurance aspects, the single premiums of the two contracts can now be expressed by

$$\pi(0) = \tau p_X [S_0[\Phi(e_2) - \Phi(e_1)] + G_T B_0(T) \Phi(e_1) + K_T B_0(T) \Phi(-e_2),]$$

and

$$\pi^1(0) = \int_0^T [S_0[\Phi(e_2) - \Phi(e_1)] + G_s B_0(s) \Phi(e_1) + K_s B_0(s) \Phi(e_2)] f_X(s) ds.$$

By the arguments above, it follows that these contracts will be cheaper than the corresponding contracts under example 2.

5 CONCLUDING REMARKS

This paper presents some new formulas for unit-linked life insurance contracts in a model with stochastic interest rate generated by an HJM-model. While idealized models of financial markets are commonly used in the theory of financial economics, this approach seems to be quite new in the theory of the actuarial sciences, though quite natural, at least when dealing with unit-linked products, which are linked to financial assets.

Maybe the approach used here in itself is more interesting than the specific formulas obtained. By this approach a simple model of a financial market is included as a part of an actuarial pricing model. The single premiums are found as expectations with respect to mortality as well as financial uncertainty, but where the expectations is taken under a risk adjusted probability measure.

The so-called hedging strategies the insurance company may employ in the financial market to reduce the associated financial risk are not explicitly computed in this paper.

The volatility structure of the HJM-model are given in terms of $\sigma_t(u)$, the volatility processes of the forward rates. By constructing more specific examples of the volatility processes, one could obtain a set of the formulas presented here for each example. This framework is also suitable for other benefits and insurance policies on more than one life.

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