EXPECTED PERFORMANCE AND RISKS OF VARIOUS PORTFOLIO INSURANCE STRATEGIES

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ABSTRACT

Guaranteed funds have had great success in France recently. These products are complex, and their behavior is not always well understood. What are their expected performance and risk profiles? While the technique of portfolio insurance (PI) has existed for some time, only the simplest case of the most popular method, the so-called "cushion method" of Black and Fisher, also known as Constant Proportion Portfolio Insurance (CPPI) has received any theoretical analysis.

In this paper, we consider this method in a realistic case, where there are constraints on the maximum exposure to the market. We also study PI techniques more sophisticated than this basic method, principally through stochastic simulations. We arrive at an understanding of the performances and risks of PI strategies little developed till now, including sensitivity to parameters such as the multiplier.

Our simulation results show that the imposition of a constraint on the maximum exposure significantly diminishes the performance of the method. Under the constraint, the leverage effect of the multiplier appears to be lost; but the introduction of a "ratchet" effect recuperates part of the loss.

Introduction[1]

This article concerns guaranteed funds that permit investment in the stock market while being assured of recuperating at least the amount invested. The idea behind these funds is the dynamic portfolio strategy called portfolio insurance.

Until recently portfolio insurance was generally done by replicating options - an idea introduced originally by Rubinstein and Leland [6]. In this technique, the portfolio is insured through a put with the appropriate maturity. In practice, since puts of sufficiently long maturity are not available, the strategy consists of rebalancing positions in the underlying portfolio (let us assume it is an index such as the CAC 250), and cash so as to replicate exactly a put option on the index. In this method, the exposure to the index at any moment is therefore given by the theory of options. An alternative approach, first popularised by Black and Jones [1] but having its origins in earlier work of Merton [5], also rebalances positions on the index and cash, but it is based on the following two ideas: first, the portfolio is always maintained above a certain minimum level called the floor, the difference or the "surplus" being called the "cushion"- the floor is assumed to grow at a fixed rate (for example, at the risk-less rate of interest) such that at the maturity of the fund, it is at least equal to the guaranteed amount; second, the exposure to the market at any moment is determined as a (non-decreasing) function of the cushion, usually a constant multiple of the cushion (Constant Proportion Portfolio Insurance, or CPPI). The first takes care of the insurance; the second determines the performance of the fund.

When it comes to the mutual funds market, the reluctance of many private investors, especially in continental Europe, to invest in stocks led CCF, like many other mutual fund managers, to design appropriate guaranteed funds. The products marketed in the early 90's at a time when interest rates were high enough to ensure an attractive pay-off, fell into two broad categories. In the first one entry and exit dates were fixed so that the guarantee is actually the same for every subscriber; for the sake of practicality, early withdrawal was generally possible, but with a big penalty on the performance. The second type of fund is open-entry open-ended, and tries to combine the well-

[1] Acknowledgement : We thank Emmanuel Campagne-Simon for extensive programming help

appreciated advantage of liquidity (without any penalty) and the enticing guarantee. Whereas options technology is suitable to manage the first category, it is no longer appropriate to the second. The CPPI technique on the other hand can accommodate for various entry dates and it is therefore useful to investigate the capacities and the limitations of this technique.

CPPI: Guarantee and performance

There are two aspects to the study of this technique: its ability to meet the guarantee, and its performance.

Under such a strategy, the exposure approaches zero as the cushion approaches zero; when the cushion is zero, the portfolio is completely invested in cash. Thus, in theory, the guarantee is perfect: the strategie of exposure ensures that the portfolio never descends below the floor; in the event that it touch the floor, the fund is "dead" - it can deliver no performance beyond the guarantee. In practice, the rebalancement of the portfolio being done at discrete intervals rather than continuously, there is a small risk of the portfolio crashing through the floor in between two rebalancements, as happened with some assured portfolios during the 1987 crash. In such a case, it is impossible even to meet the guarantee. Therefore, one objective of management might be to minimise this possibility.

With respect to performance, the objective is typically to attain some percentage of the performance that would have been attained if all the funds had been invested in the index. The performance of the method depends clearly on its two defining factors : the evolution of the floor, and the multiplier.

In theory, assuming that the portfolio is rebalanced continuously, that the floor grows at the riskless rate and that the price of the risky asset follows a log-normal model, it can be shown that the strategy has an option-like pay-off, which can be explicitly given as a function of the terminal value of the index - in particular, the value of the portfolio at maturity is path-independent. It is well known that portfolio insurance modifies the risk profile, in displacing it to the right but "paying" for the guarantee in terms of a lower likelihood of high gains. The attraction of the method lies in the leverage effect of the multiplier. The higher the multiplier, the higher the

exposure to the market, and the more the gains in strong market conditions; conversely, the more rapid the fall, accompanying a fall of the market. The portfolio therefore has a return which increases with the multiplier, but so does the risk.

Modifications of CPPI

This idealised portfolio has some obvious limitations, apart from the assumption of continuous evolution - to begin with, the existence of leverage constraints in practice. If we impose the realistic constraint, say, that the exposure to the risky asset cannot exceed a certain proportion of the value of the portfolio, the latter loses its attractive properties; in particular, it is no longer analytically tractable.

From the point of view of performance, another potential weakness of the method is the constant growth rate of the floor, since it determines the level of insurance: if the market climbs sufficiently, the floor threatens to become insignificant in relation to the value of the portfolio. A logical modification of the CPPI is therefore to have a variable floor: to incorporate a "ratchet" effect, or a jump in the floor, in order to take advantage of significant market rises.

The obverse of this case is when the market falls sufficiently for the cushion to approach zero, in which case the exposure approaches zero. Under the basic CPPI, when the exposure touches zero, the portfolio is completely invested in the non-risky asset - a situation to be avoided. A possible approach to tackle this eventuality is to adjust the floor *downward* to artificially augment the cushion - however, to avoid tampering with the guarantee, the floor could be, for example, initially augmented by some amount beyond what is prescribed by the CPPI method. The floor could in fact be dynamically adjusted in response to market changes, while keeping the basic CPPI rule for the exposure. We may refer to such strategies as CPPI with variable (growth) floor. A variable floor strategy, based on a somewhat different idea, is described in [3].

Such strategies may be too complex to be treated theoretically. For example, it can be shown that unless the floor grows exactly at the risk-less rate,the value of the portfolio at maturity is path-dependent. Thus, while the

performance promised is in terms of that of the index at maturity, the final value is not a simple function of the index in constrast to the case of put replication, but a function of its entire trajectory. The performance is thus by no means transparent. In addition, if we allow that re-balancing is not continuous, and the possibility of discontinuities (jumps) in the price of the risky asset, we have the problem of the guarantee not being perfect. On the other hand, under the idealised CPPI discussed above, the portfolio can be completely described analytically. Thus, this strategy, while unrealistic, can serve as a benchmark for the more complex modified strategies.

Theoretical framework

The CPPI was in fact shown by Merton [5] to be the optimal strategy corresponding to a certain utility maximization problem; Grossman and Vila [4] even give the optimal "multiplier" as a function of the risk aversion parameter. Further, under leverage constraints, they show that a modified strategy, under which the CPPI is followed until the leverage constraint becomes binding, is still optimal; the optimal multiplier is the same as in the unconstrained case. However, it is not clear that this theory can be extended to the case where both the multiplier and the floor strategy are parameters.

It is also not clear to what extent this expected utility formulation is useful in practice (the choice of the multiplier is not evident, as the risk-aversion parameter of the investor is not easily determinable; nor is even the choice of the appropriate performance criterion for an insured portfolio), however, the CPPI is a technique easy to understand and implement, and independent of time, unlike the synthetic put method, the other principal method of portfolio insurance. The CPPI in fact belongs to a class of time-invariant techniques referred to as "generalised portfolio insurance strategies" by Brennan and Schwartz [2]; each such strategy corresponds to a given final pay-off. This indicates a possible alternative frame-work to utility maximisation.

Many other interesting questions arise in the study of these dynamic strategies, not the least of which is the obvious analogy with option-based portfolio insurance. For example, given the final profile of the portfolio under a certain strategy, can the prime of the insurance be evaluated?

Objectives of study

In this article, we attempt to describe the profile of the insured portfolio under the modifications of CPPI discussed above, without addressing the question of optimality or other theoretical questions that naturally arise. Previous studies have looked at the guarantee and underperformance of the portfolio with respect to the index, under fixed rate floor CPPI, for different values of the multiplier. We do not consider the problem of the guarantee (see [7]); rather, we concentrate on the effect of a constraint on the leverage effect of the strategy, and the effect of variable floor strategies on the evolution of the portfolio. Our approach is to use the basic CPPI as a benchmark case (since it is well understood), and look at the effect of progressive modifications, through stochastic simulations. Our simulations are based on a log-normal model for the price of the index, so in spite of discrete rebalancement the guarantee is practically perfect.

We first present a mathematical description of the various strategies, and then present the results of our simulations. We conclude with some comments on further work.

Mathematical description of the strategies

Notation

V_t	value of the portfolio at time t, t>0
V_0	initial investment, guaranteed amount
P_t	value of the floor at time t
$C_t = V_t - P_t$	the "cushion" at time t
m	the "multiplier"
E_t	the "exposure", or the investment in the risky asset at time t
T	maturity of the fund
r	risk-less rate of interest
S_t	price of the risky asset at time t

The CPPI type methods of portfolio insurance ("floor based methods") are based on re-balancing the portfolio between a risky and a risk-less asset so that its value is always superior to a "floor", the "exposure" to the risky asset being at any time being a function of the "cushion" or the excess above the floor, until any constraints on the exposure become binding .

The evolution of the portfolio is described by the following stochastic differential equation.

$$dV_t = \frac{dS_t}{S_t}.E_t + r dt .(V_t - E_t)$$

(A) CPPI with fixed rate floor

Here we assume that the floor grows at a fixed rate $\bar{r}$, less than or equal to the risk-less rate of interest r. The guaranteed amount being V_0,

$$P_t = P_0 e^{\bar{r}t}$$

$$P_0 \geq V_0 e^{-\bar{r}T}$$

We suppose the price of the index follows the standard model

$$\frac{dS_t}{S_t} = \mu dt + \sigma d W_t$$

where W_t is standard Brownian motion, and μ and σ are positive constants.

(A.1) CPPI of Merton

In this case, $E_t = m C_t$, with no constraints on the maximum exposure (the minimum is naturally zero). In other words, no constraints on borrowing. Note that under auto-financing, any amount borrowed is instantly debited from the value of the portfolio at the risk-less rate. The multiplier can be chosen arbitrarily, or by chosing an initial exposure (assuming the initial floor is fixed) We then have, assuming the portfolio is auto-financing,

$$dV_t = C_t\, dZ_t + rP_t dt$$

where

$$d\,Z_t = (m(\mu - r) + r)dt + m\sigma d\,W_t$$

$$V_t = P_t + e^{(Z_t - \frac{1}{2}m^2\sigma^2 t)}\left[V_0 - P_0 + (r-\bar{r})\int_0^t e^{-(Z_s - \frac{1}{2}m^2\sigma^2 s)} P_s\, ds\right]$$

Let us assume $r = \bar{r}$, then the cushion grows according to a log-normal process. It is evident that if the cushion ever descends to zero then the portfolio is completely invested in the risk-less asset. In continuous time, the probability of this happening is zero, however the cushion can become arbitrarily small. The cushion can also be arbitrarily large, and thus the exposure also. Thus the method supposes an unlimited credit (at the risk-less rate).

The portfolio at maturity is completely described, and thus its properties can be thoroughly studied. When $r = \bar{r}$, in fact,

$$V_T = V_0 + C_0 e^{-b_m T}\left(\frac{S_T}{S_0}\right)^m$$

where $b_m = (m-1)(\frac{m\sigma^2}{2} + r)$. We have thus a "pay-off" explicited as a function of the index, and in particular, it is path-independent. We can also calculate explicitly

$$E(V_T) = V_0 + C_0 e^{(m(\mu-r)+r)T},\ V(V_T) = C_0^2 e^{2(m(\mu-r)+r)T}(e^{m^2\sigma^2 T} - 1)$$

We see that the expected value and the variance both increase with m.

(A.2) CPPI with constraint on exposure

The above method takes full advantage of market rises since there is no constraint on the exposure. Suppose we impose the realistic constraint

$$0 < E_t < p\,V_t$$

where p>0. Under the modified CPPI, then,

$$E_t = \min\{mC_t, pV_t\}$$

Then we have

$$dV_t = \begin{cases} V_t.rdt & V_t \le P_t \\ (V_t - P_t).d\,Z_t + P_t.rdt & P_t < V_t < \frac{m}{m-p}P_t \\ V_t.d\,X_t & V_t \ge \frac{m}{m-p}P_t \end{cases}$$

where $dX_t = (p(\mu - r) + r)dt + p\sigma dW_t$.

The probability of the first eventuality is once more zero in continuous time. It does not seem possible to resolve this equation analytically; note the value of the portfolio at any time must be path-dependent.

(B) Variable floor strategies

It is clear that the (constrained) CPPI with a fixed growth floor "fails" in the case of a strong climb of the index, when the floor becomes insignificant in relation to the value of the portfolio:

$$C_t \approx V_t$$

The method then fails to capture the rise; also, the portfolio is then highly exposed throughout the rising period, a risky situation in case of a downturn. Conversely, if the cushion becomes too small, the portfolio is too little exposed, and may have trouble taking off in case of a future rise. An intelligent modification of the strategy is to adjust the floor, to capture definitively the performance of the index and to diminish the cushion in the case of a rise, and to increase it in case of a fall.

(B.1) "Ratchet" effect

In the case of a rise, the "ratchet" effect puts the "excess" cushion in the floor. The part of the cushion that is not used due to the constraint on the exposure is

$$\frac{mC_t - pV_t}{m} = \frac{m-p}{m}V_t - P_t$$

We increase the floor precisely by this amount:

$$\text{if } mC_t > pV_t:$$

$$P_t^{new} := \frac{m-p}{m}V_t$$

$$E_t^{new} := m(V_t - P_t^{new}) = pV_t$$

This has the effect of increasing the level of insurance. Note that this strategy only makes sense in discrete time.

(B.2) The initial floor and the strategy of the margin

Recall that the minimum initial floor to assure the guaranteed amount at the end is

$$P_o = V_o e^{-rT}$$

and that the initial exposure is

$$E_0 = \min\{m(V_0 - P_0), V_0\}$$

It may happen that the initial exposure is too high; the idea then is to set the initial floor at a value higher than the minimum, and to use the "margin" later on to augment the exposure when it falls too low. Suppose we denote this extra floor as M_0:

$$P_o = V_o e^{-rT} + M_o$$

so that the initial exposure is diminished.

We can then fix a lower limit for the exposure as the portfolio evolves, in either absolute or relative terms and "play" a part of the margin each time this limit is reached, in effect by diminishing the floor by a certain amount, consequently increasing the cushion. Note that the margin evolves at the risk-less rate also. Some possibilities:

Choose an initial margin. Play half the margin remaining each time the exposure falls below half of the current lower limit E_t^* :

Whenever $E_t < .5E_t^*$,

$$P_t^{new} := P_t - .5M_t$$
$$M_t^{new} := .5M_t$$
$$E_t^{new} := mC_t = m(V_t - P_t^{new})$$

The fractions can of course be chosen differently.There are any number of possibilities for setting the lower limit of the exposure.

(a) Reset it each time the margin is played:

$$E_t^{*(new)} := E_t^{(old)}, \text{ with } E_0^* = .5E_0$$

Example:
$V_0 = 100$, $r = 5\%$, $m = 4 \Rightarrow P_0^{min} = 77.88$; set $M_0 = 8.48$, then $E_0 = 80$

Suppose t is such that :
$E_t = 39$ $(<.5*80 = 40)$, then $E_t^{*(new)} = 39$, $M_t^{new} = 4.24$
Half of this margin will be used next when the exposure falls below 19.5.

(b) It could be a fixed value, for example: $E_t^* = E_0 \quad \forall t$

(c) It could be relative to the value of the portfolio: $E_t^* = V_t$

The margin strategy has two connected purposes: to prevent the exposure from falling too low, and to improve performance. If the initial margin is too small, it will not have much impact on either. A more efficient margin strategy might be to make use of any "excess" floor due to the ratchet effect, rather than depend solely on the initial margin.

Note that the strategies we have described above are only applicable in the practical context of discrete re-balancing, being based on jumps in the price of the index. They cannot be integrated in the previous continuous framework. It may be possible, however, to devise continuous strategies of modification of the floor, as a function of the variation of S_t, and to develop evolution equations for the two-dimensional process (V_t, P_t). We may call all such strategies as CPPI strategies with variable floor, since the exposure at all times is a multiple of the cushion.

Comparison of strategies

The strategies we have developed above are of progressively increasing complexity; no analytical resolution being possible except in the unconstrained case, we study their performance through simulations. We consider the CPPI as the benchmark strategy, and consider the effect of the constraint on exposure and the variable floor strategies on the performance of the portfolio, and its risk profile.

We employ the following terminology in discussing the various strategies discussed above:

CPPI
Constrained CPPI
Ratchet strategy (constrained CPPI with ratchet)
Margin strategy (ratchet strategy with initial margin)

Details of the simulations

We generate the prices of the risky asset through a log-normal process. The unit of time is taken to be one day; we assume 250 trading days in the year. To be realistic, we re-balance not every day, but following market

movements, the criterion being a 2% variation in the price of the index; however, transaction costs are ignored.

The results we present are for a single set of model parameter values; our objective is to identify the effect of the parameters of the method, the multiplier and the floor strategy, qualitatively; we find that these effects are robust to change in the model parameters. We chose a reasonable set of these values:

$$\mu=9.5\%,\ \sigma=21\%,\ r=5\%,\ T=5 \text{ years},\ V_0 = 100$$

The guarantee is assumed to be equal to V_0. Under the fixed floor strategies, the floor is assumed to grow at the risk-less rate; the initial floor is set at

$$P_0 = V_0 e^{-rT}$$

We assume that the constraint on exposure takes the form

$$E = \min\{mC, V\}$$

i.e. we take p = 1. We considered a number of different margin strategies ; the results being not significantly different, we present only one in this article:

$$\underline{\text{Margin strategy}}:\ \text{Strategy (B.2)(a)},\ E_0 = 80\ (M_0 = 8.48)$$

Finally, we considered the following values of m, sufficiently high to have a significant leverage effect:

$$m \in \{4,5,6\}$$

All the simulations discussed in this paper are based on the above choice of parameters.

As we have seen above, the portfolio under CPPI can be completely described analytically, assuming a continuous variation of the price of the index and continuous rebalancement of the portfolio. However, for purposes of comparison with the other strategies, we simulated the CPPI strategy also following the same re-balancement discipline.

We validated our simulations by comparing results in the case of the CPPI strategy with the analytical results, and found good agreement. (see Appendix)

Performance measures

For each strategy, we study statistical properties of the following terminal quantities:

$$V_T,\ \Omega_T = \frac{V_T}{S_T},\ \text{and}\ P_T$$

(the last being interesting in the case of variable floor strategies). The second measure is pertinent, since as we mentioned above the performance objectives of guaranteed funds are typically in terms of the performance of the index. We also consider the evolution of the portfolio and its composition over time :

$$\{V_t\},\ \{E_t\},\ \{P_t\}$$

Simulation results

(A) Return and risk

Figures 1 and Table I show the expected values and standard deviations of the performance measures V_T and Ω_T, under the four strategies. We include the statistics for the uninsured portfolio (S_T) as well, for comparison.

We first discuss the results for V_T. The most striking effect is seen in the case of CPPI: the expected return and risk increase significantly with m; for high m, the return is very attractive, exceeding significantly the performance of the index, but the risk is enormous. On the contrary, we do not see a dramatic change due to m under the constrained strategies; however, there does seem to be a slight decreasing trend, contrary to the case of CPPI. (see Table I). The high risk in the case of the CPPI is understandable, since the exposure is unlimited: the multiplier amplifies the risk of the index multifold.

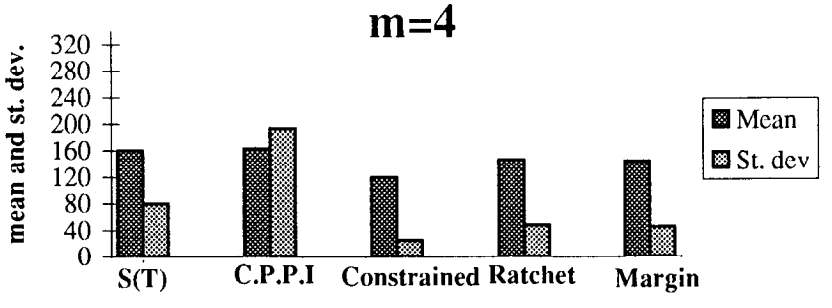
Equally interesting is the effect of constraining the exposure. Not surprisingly, there is a significant drop in the expected return, but also in the risk. The introduction of the ratchet effect recuperates some of the performance, by incorporating it in the floor (ie the level of insurance); the increase in risk can also be explained by the risk of the final floor value:

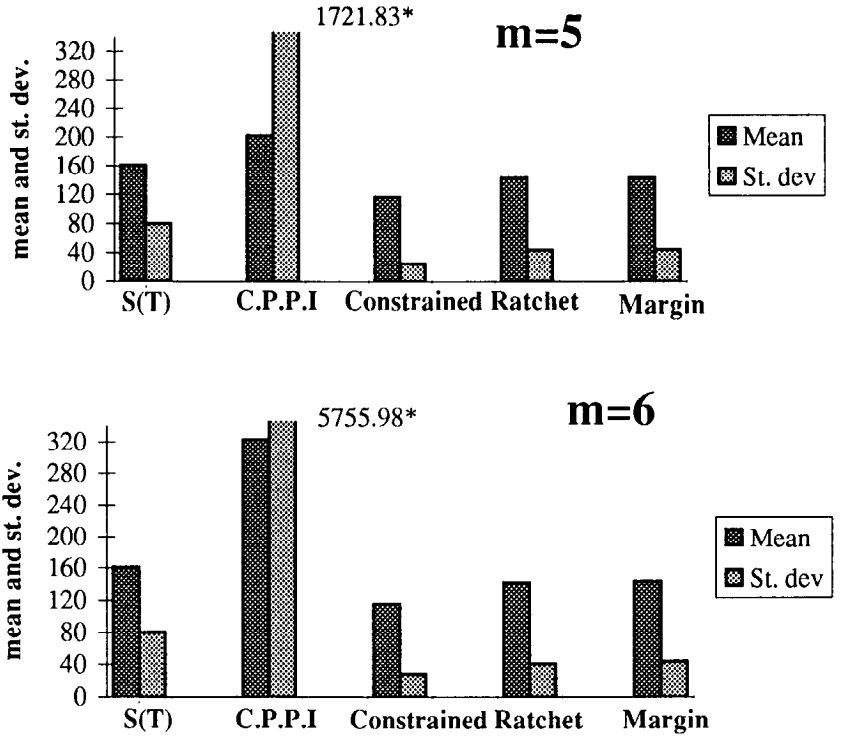
$$\mathrm{Var}(V_T^{constr}) + \mathrm{Var}(P_T^{ratchet} - V_0) \approx \mathrm{Var}(V_T^{ratchet})$$

Note also that the final floor value represents a high percentage of the portfolio.These effects are qualitatively the same for all the m we consider.

The margin strategy appears to have no appreciable effect on the performance results; however, we note that it reduces the value of the final (expected) floor value, which is coherent with the strategy (since it consists of lowering the floor to increase the exposure). While the objectives of this strategy

FIGURE 1: MEAN AND ST. DEV. OF V_T.





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are valid: to prevent the exposure from falling too low, and to increase the performance, its utility might be seen more clearly in specific market conditions, and even better, if anticipation about future movements could be incorporated, rather than in averaging the results of simulations based on a random walk hypothesis for the price movements of the index. Also, what is an intelligent choice of the initial margin, and the strategy of utilisation of this margin? We attempted a number of strategies and found no appreciable difference in the results. Note also that in our strategies, we do not use any of the "excess" floor accrued due to the ratchet effect; this is assumed to be definitively incorportated into the insured part of the portfolio.

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* For accuracy of estimate, see Appendix

It is interesting to consider how the leverage effect of m is modified in the presence of the constraint on exposure. This constraint becomes important whenever there are strong market conditions. A higher multiplier causes the constraint to be attained quicker; in the case of a ratchet effect, the size of the ratchet is higher also ($\frac{m-p}{m}V-P$) i.e. the jump in the floor is higher, consequently lowering the cushion, and thus the exposure - in this case, in fact, the leverage effect might be expected to be transferred to the floor. We do observe a slight (but significant) increasing trend in the average final value of the floor, with increasing m (Table I). One possible conclusion from these results is that a lower multiplier might be more effective in strong market conditions. It would be interesting to study this effect when constraint is less stringent - for example, if the portfolio can be exposed at 200% its value.

In the case of Ω_T, the effect of the constraint on the performance is similar to that noted above for V_T. If the performance objective of the fund is defined in terms of Ω_T, then these results are quite re-assuring on the average, though the dispersion about the average is high. This dispersion is highly amplified by m, in the case of CPPI, as to be expected; otherwise m has no appreciable effect. It might be more instructive to consider Ω_T under various scenarios for S_T, rather than averaged over all possibilities.

We next consider how the probability distribution of the returns - this helps us to understand more precisely the effect of the constraint. The mean and the variance are in fact not sufficient to describe the distribution of final returns, since they are path dependent under all strategies except the CPPI with fixed floor.

TABLE I: PERFORMANCE UNDER VARIOUS STRATEGIES (V_0=100)

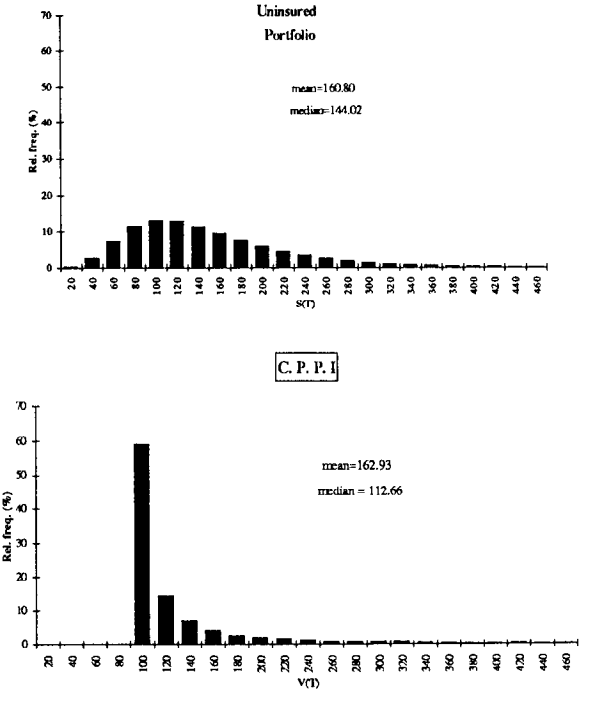
	m=4		m=5		m=6	
	$E(V_T)$	$Stdev(V_T)$	$E(V_T)$	$Stdev(V_T)$	$E(V_T)$	$Stdev(V_T)$
C.P.P.I	162.93	193.30	202.15	1721.84	323.05	5755.98
C.P.P.I constrained	119.89	25.23	116.17	25.32	115.70	27.95
Ratchet strategy	145.96	48.60	142.94	44.26	141.89	41.17
Margin strategy	144.24	45.20	143.35	44.93	143.14	44.15
	$E(\Omega_T)$	$Stdev(\Omega_T)$	$E(\Omega_T)$	$Stdev(\Omega_T)$	$E(\Omega_T)$	$Stdev(\Omega_T)$
C.P.P.I	1.067	0.421	1.013	1.895	1.241	6.732
C.P.P.I constrained	0.932	0.354	0.911	0.368	0.891	0.346
Ratchet strategy	1.085	0.342	1.087	0.389	0.969	0.213
Margin strategy	1.098	0.333	1.101	0.353	1.090	0.356
	$E(P_T)$	$Stdev(P_T)$	$E(P_T)$	$Stdev(P_T)$	$E(P_T)$	$Stdev(P_T)$
Ratchet strategy	131.62	37.50	134.93	37.04	137.19	37.05
Margin strategy	128.30	34.28	132.93	37.31	136.62	39.41

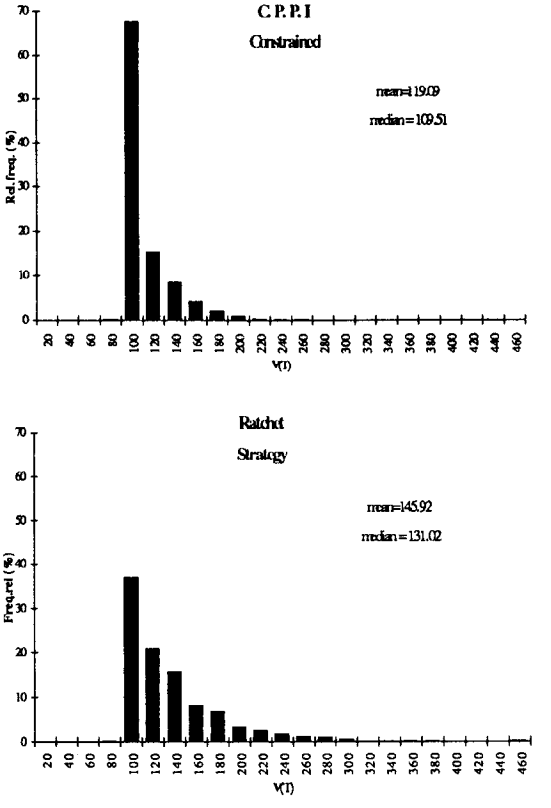
(B) Risk profile

Figures 2 and 3 show the changes in the probability distribution under the various strategies, for m=4. The asymmetry of the distribution of the insured portfolio is immediately clear. Under CPPI, the distribution has a very long tail, reflecting the possibility of very high performance; however, most of the mass is concentrated at values close to the floor, as reflected by the value of the median. This is the case also in the case of constrained CPPI, but the striking difference is the much shorter tail, reflecting the loss of performance. Under the variable floor ratchet strategy, the mass is shifted to the right, reflecting both a higher performance and a lower risk. The distribution has

still a shorter tail than that under CPPI: the constraint still has the effect of diminishing the likelihood of large gains.

FIGURE 2: HISTOGRAMS OF V_T FOR m=4 (V_0=100)

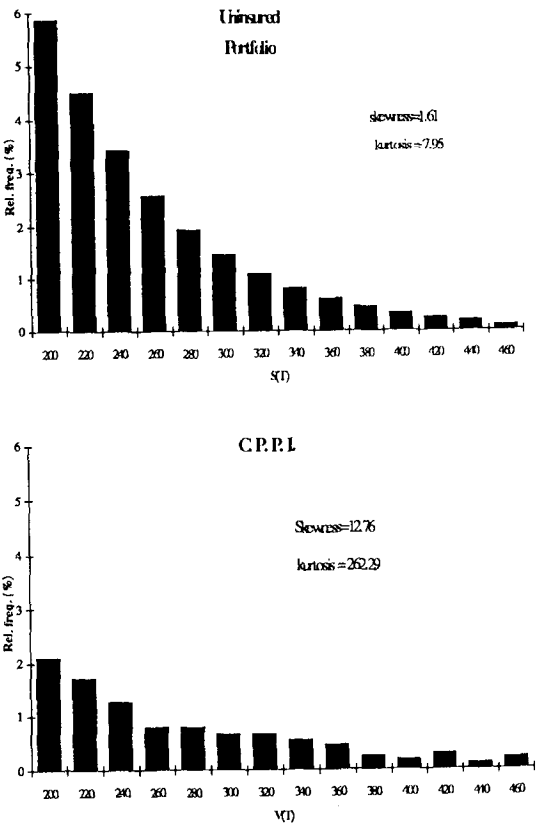


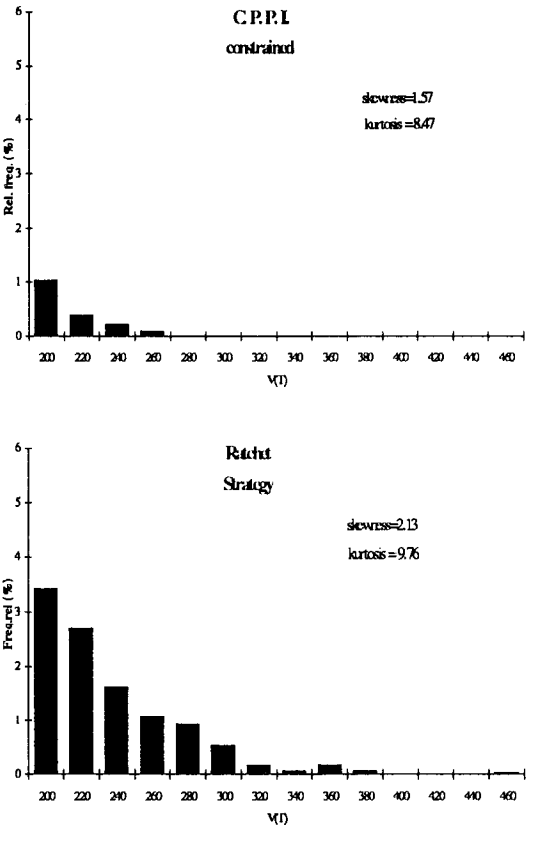


In summary, we see clearly that fixed floor CPPI with constrained exposure has poor performance, and is clearly dominated by the unconstrained CPPI; between the two, the insured portfolio under the ratchet strategy has a desirable risk profile: it captures most of the performance of the index, and has a smaller risk, while never descending below a minimum value. If the ceiling on exposure is raised (p higher), then the risk profile under the ratchet might be clearly superior to that of the uninsured portfolio.

We next look at the performance of the portfolio under various strategies as a function of the performance of the index.

FIGURE 3: TAILS OF HISTOGRAMS FOR m=4.





(C) Terminal value as function of the index

Figure 4(A) is a plot of simulated values of the portfolio at maturity against the index. In the case of the CPPI it is a curvilinear function of the index, as noted above - remark the resemblance to the pay-off of a "power" option (the dispersion observed here being due to discretisation); for the other strategies the significant dispersion reflects the path-dependence of the terminal value, which is thus not solely a function of the index.

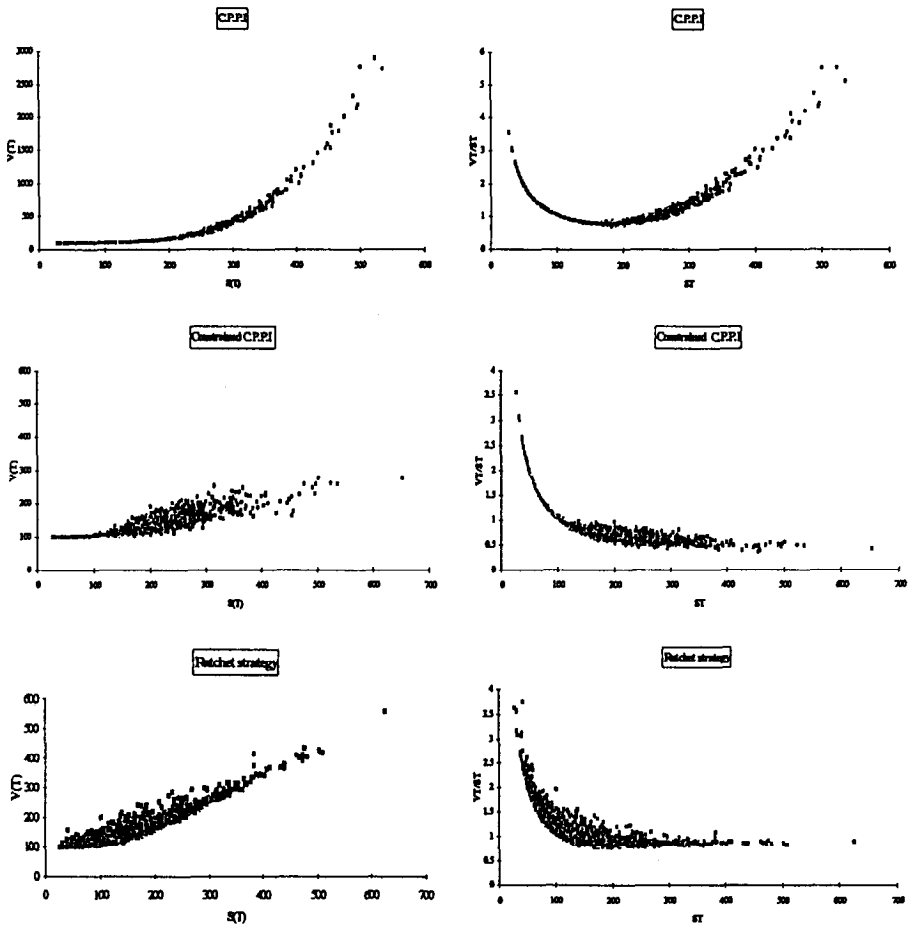
This plot confirms our observations above but has additional information: the CPPI amplifies the performance of the index multifold, however, at the higher end of the scale, while the ratchet strategy has a reasonable performance even when the market does not perform well; its performance increases much more

rapidly at high levels of the index than the constrained strategy with fixed floor. The resemblance to the pay-off profile of a put option is as expected.

In Figure 4(B) we have a plot of Ω_T against the index. At both lower and higher ends of the scale, the performance of the ratchet strategy is clearly superior to the constrained strategy, while both miss the leverage effect of the multiplier at the higher end.

FIGURE 4

(A) V_T vs S_T (B) Ω_T vs S_T



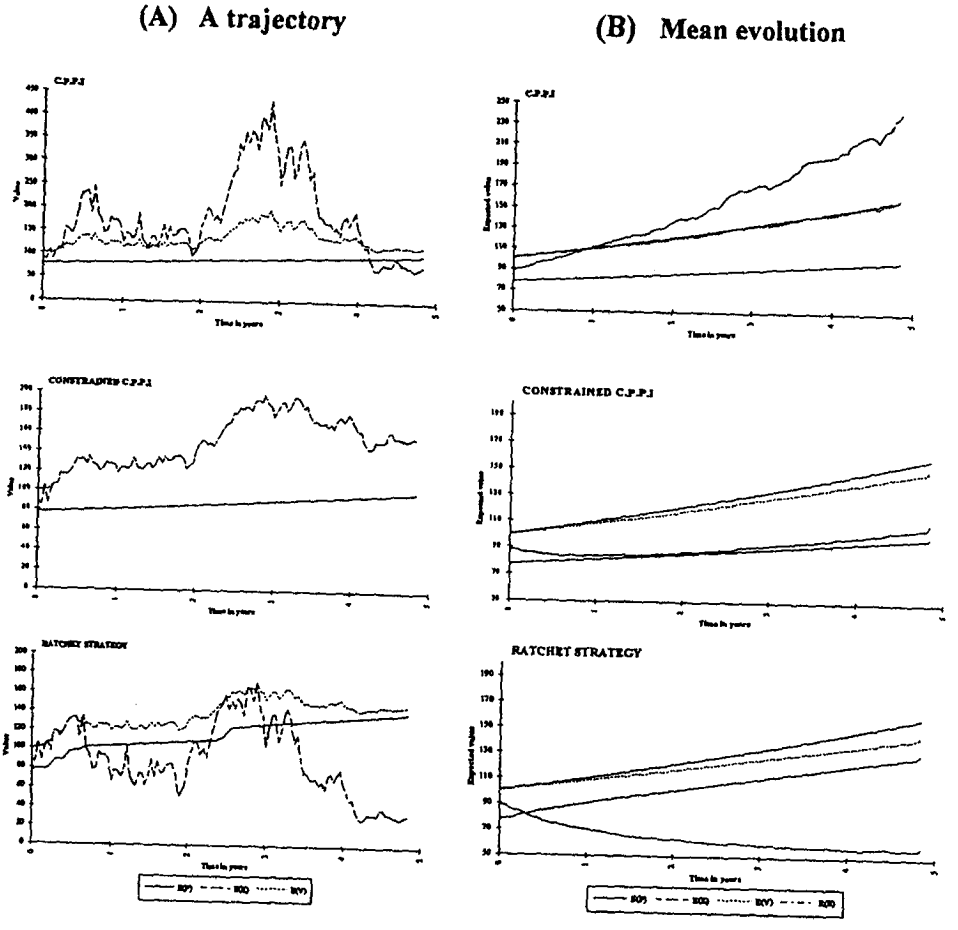
(D) Evolution of the portfolio

Finally, we consider the evolution of the portfolio under the different strategies. Figure 5(A) shows a particular realisation of the portfolio under the strategies, while Figure 5(B) shows mean trajectories of the portfolio, the exposure and the floor. The purpose of these graphs is to understand the performance of the strategies better by looking at the intermediate composition of the portfolio.

In Figure 5(A) we see that the exposure exceeds the value of the portfolio most of the time; since the portfolio is autofinancing, the amount borrowed is debited at the risk-less rate. In the second case, the exposure reaches the upper limit (100%) very soon after the beginning, and the portfolio is exposed at 100% the rest of the time. Under the ratchet strategy, we have the evolving floor, a much diminished cushion and consequently a much lower exposition.

In Figure 5(B) the differences are clearly shown. The portfolio grows exponentially in all cases, and its performance is clearly between that of the riskless and the risky asset. Under CPPI its evolution on the average follows closely that of the index; the exposure rapidly overtakes the portfolio. Under the constraint, it remains stable at quite a high proportion of the portfolio, while under the ratchet strategy it decreases exponentially. One explanation for this can be found in the fact that the floor being at a high proportion of the portfolio due to successive applications of the ratchet, the cushion gets smaller and smaller. This surely has an effect on the performance of the portfolio. Thus we have a trade-off between two opposing factors: the ratchet effect recuperates some of the loss due to the constraint at times of strong market performance, but at the same time it diminishes the cushion significantly thus lowering exposure. A potential disadvantage of this strategy then is that it could lead to the portfolio being relatively little invested in the risky asset.

FIGURE 5: Evolution of the portfolio for m=4



IV CONCLUSION

We have presented results of simulations of various strategies based on the CPPI method of portfolio insurance. We find that the under a realistic constraint on exposure, the method loses the advantage of the leverage effect of the multiplier. The ratchet effect recaptures some of the lost performance, but could end in the portfolio being little invested in the risky asset. The risk profile of the portfolio, already reflecting the price of the guarantee in terms of lower probabilities of high gains, is further modified by the constraint in the same direction.

Our purpose was to compare modified CPPI strategies with an analytically tractable simple case; however, the assumption of a log-normal model for the price movements of the index does not take into account discontinuities, in particular the possibility of a steep fall between two re-balancements. In our simulations, the fund never crashes through the floor. While this is quite reasonable under the ratchet strategy - since the floor is successively adjusted upwards, the probability of a crash through the actual floor becomes smaller and smaller - to get more realistic results, we could use alternative models, such as diffusions with jumps, or even stable processes.

It would be interesting to see the effect of increasing the maximum allowable exposure, to say 150 or 200% of the value of the portfolio (which is allowable under certain regulatory conditions), under the ratchet strategy. The question of the choice of an initial exposure (or margin), and a strategy of raising or lowering the floor dynamically to optimise performance (and possibly to control the level of exposure) while following the "CPPI" rule for exposure seems to be a promising direction for further work, in view of our empirical results. However, the definition of an appropriate performance criterion, and "optimality" remains to be clarified. As we have seen, the "pay-off" of the CPPI portfolio resembles that of a call option - can the profile of the portfolio under a given CPPI type strategy be reproduced or approximated by some combination of options?

APPENDIX

Validation of simulation results

Table II compares the results of simulations of the CPPI strategy with theoretical values, for various values of m. There is excellent agreement at a sample size of 5000 for values of m less than 4. For higher m, the estimate of the mean of V_T converges at a higher sample size (10000 appears to suffice), but that of the variance does not converge even at a sample size of 20000, notably in the case of m = 6. But for our purposes (comparison of strategies), it suffices to remark only the order of magnitude of the variance for higher values of m, in the case of the CPPI, since it differs significantly from that for the other strategies we consider.

On the other hand, this slow convergence for higher values of m creates the problem of the choice of an appropriate simulation run length for the simulation of the other strategies. Table III shows the estimates of the mean and variance of V_T under a margin strategy (with constraint on exposure and ratchet effect), for increasing sample size. The results are similar for the other strategies we consider. In this case, we have no theoretical values to compare to, but the estimates clearly appear to stabilise for sample sizes higher than 1500. We chose a sample size of 3000, at which the estimate of the mean has a standard error of about .9%.

TABLE II: Convergence of simulation estimates of $E(V_T)$ and Std. devn(V_T)

CPPI strategy, r=6.5%, σ=21%, V_0=100

	μ = 9%		μ = 13%		μ = 9%		μ = 13%	
	mean	*std. devn*		*std. devn*	*mean*	*std. devn*	*mean*	*std. devn*
Sample size								
		m=1				**m=4**		
5000	144.14	21.86	153.65	26.56	164.83	242.55	227.73	421.41
10000	144.03	21.93	153.73	26.73	164.31	278.74	246.66	880.14
20000					167.55	429.41	246.93	966.22
Theor. Value	***143.51***	***21.61***	***153.15***	***26.4***	***163.32***	***364.03***	***240.91***	***810.16***
		m=2				**m=5**		
5000	150.71	57.45	176.7	87.58	164.01	434.6	340.96	3798.28
10000	149.85	58.11	175.05	88.67	170.19	1022.1	289.83	1266.35
20000					173.3	811.57	297.97	2213.54
Theor. Value	***149.31***	***58.67***	***173.56***	***87.53***	***171.75***	***1127.1***	***295.03***	***3063.66***
		m=3				**m=6**		
5000	158.32	157.28	204.23	299.13	160.87	756.48	344.25	2332.07
10000	157.85	131.49	207.18	239.23	178.01	2482.6	314.5	1758.54
20000					184.59	2155.7	330.24	2480.11
Theor. Value	***155.88***	***139.97***	***201.81***	***255.05***	***181.3***	***4302.6***	***369.92***	***14284.9***

TABLE III: Convergence of simulation estimates of $E(V_T)$ and Std. devn(V_T)

Margin strategy (B.2)(b), μ=9.5%, r=5%, σ=21%, E_0=50, V_0=100

	m=4		m=6	
sample size	*mean*	*std. devn*	*mean*	*std. devn*
100	154.23	49.44	150.42	74.77
500	144.53	44.66	140.3	40.89
1000	142.65	48.73	141.09	42.77
1500	149.35	58.49	141.36	42.7
3000	145.7	49.62	142.31	44.41
5000	146.18	51.65	141.15	42.48
7000	147.13	52.96	143.45	46.88
10000	146.33	51.89	142.41	44.7

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