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A Stochastic Approach to Asset Allocation within a General Insurance Company

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Summary

This paper starts from the cashflow model of a General Insurance Company as set out in "Assessing the Solvency and Financial Strength of a General Insurance Company" by Daykin et al. (1987)

The authors use the model office proposed in this paper, together with an asset model, to consider the effects on a General Insurance company of investing in a variety of portfolios consisting of cash, fixed interest securities and equities.

As a result of correlations between the returns on the various asset classes and the inflation linked liabilities it is possible to ascertain those portfolios which, for a given expected value of remaining assets after a given time period, minimise the expected variability in this expected value.

Although the practical example given in this paper is a simple one, the authors believe that the application of such techniques provides management with an important analytical tool which could be used to investigate the likely effects on solvency of many different investment strategies.

Résumé

Une Approche Stochastique à la Répartition des Actifs dans une Compagnie d'Assurance IARD

Cet article part du modèle de "cash-flow" d'une compagnie d'assurance toute branche, exposé dans "Evaluation de solvabilité et de force financière d'une compagnie d'assurance toute branche" de Daykin et al. (1987)

Les auteurs utilisent le bureau modèle proposé dans cet article, ainsi qu'un modèle des actifs, pour étudier les effets sur une compagnie d'assurance toute branche du placement dans divers portefeuilles consistant de cash, valeurs à intérêt fixe et actions ordinaires.

A la suite des corrélations entre les rendements des différentes classes d'actifs et les engagements indexés à l'inflation, il est possible d'établir les portefeuilles qui, pour une valeur attendue donnée d'actifs restants après une période de temps donnée, minimisent la variabilité attendue dans cette valeur attendue.

Bien que l'exemple pratique donné dans cet article soit simple, les auteurs pensent que l'application de telles techniques offre aux gestionnaires un outil analytique important qui pourra être utilisé pour étudier les effets probables sur la solvabilité de nombreuses stratégies d'investissement différentes.

1.0 Introduction

1.1 This paper has its beginnings in a paper entitled "Assessing the Solvency and Financial Strength of a General Insurance Company" by Daykin et al. (1987). We refer to this paper as SFGIC.

1.2 One of the authors of this paper was a co-author of SFGIC. The other author has used asset and liability modelling techniques in order to provide practical advice on the asset allocation decision to the trustees and sponsors of UK pension funds.

1.3 We believe that many General Insurance companies are following an investment policy which does not realise the full potential of the assets held by the company because:-

i) Their investments are chosen with little regard to the nature of their liabilities, which are to a large extent (especially in the case of long tailed business) linked to inflation.

ii) To the extent that claim reserves are matched by fixed interest assets and cash, leaving free reserves to be matched by equities, there is scope for a more efficient investment strategy.

1.4 We believe that part of the reason for this is that in the past very little guidance has been available as to how one might go about devising a medium to long term investment policy. In this paper we put forward some ideas as to how to design just such a strategic asset allocation policy.

1.5 In section 2 we provide a simple illustration of the point outlined in 1.3 (ii).

In section 3 we provide brief details of the model office we have used. Further details are given in Appendix 1. For a fuller explanation the reader is referred to SFGIC.

In section 4 we outline the investment model which we have used. Further details are given in Appendix 2.

In section 5 we consider the problem of asset allocation from a theoretical point. In section 6 we provide a practical example of the techniques.

Section 7 considers how one would develop the ideas contained in the previous two sections in order to provide practical solutions to some of the problems which might be encountered in practice. The limitations of any such method are also considered.

2.0 Part of the problem

2.1 In 1.3 (ii) we inferred that for a General Insurance company to restrict its holding of equities to the level of its free reserves could be inefficient. A simple example may help to demonstrate the problem (this example is based on one given in a paper by P R Lockyer (1989).

2.2 Suppose that we have a liability of £110 to meet in one year's time. Suppose further that we can purchase a one year zero coupon bond with a redemption yield of 10%.

If we have current assets of only £100 then we can invest them all in the bond and ensure that we can meet the required payment in one year's time.

Suppose now that we have assets of £101 and that we are also permitted to invest in equities. The expected total return (dividend income combined with capital growth) on equities for the coming year is 15%; however returns of +25% and +5% are also possible.

The conventional approach might be to invest £100 in the bond and £1 in the equity, consider though an investment of £22 in equities and £79 in the bond. As the table below shows, in both cases we still have sufficient assets to meet our liability at the end of the year.

Amount in gilts	£100.00	£79.00
Amount in equities	£1.00	£22.00
Return on equities	Proceeds at the end of the year	
+25%	£111.25	£114.40
+15%	£111.15	£112.20
+5%	£111.05	£110.00

If we fear that the worst we can expect is a return on equities of -10% over the year then we can still invest £5.50 in equities and the remaining £95.50 in the bond. If our fears are realised then at the end of the year we can still meet our liability:-

(£95.50 x 1.1) + (£5.50 x 0.9) = £110.00

2.3 Investing only the free reserves in an asset class which we expect to provide a higher return implicitly assumes that the investor believes this asset could be valueless at the time the liability is to be met.

3.0 The model office

3.1 The equation below provides a brief outline of the main features of the model office as set out in SFGIC. Further details of the parameters we have used are given in Appendix 1. For further details of the model we refer readers to the original paper.

3.2 The model is based on a consideration of the cashflows in successive years.

In simple terms the SFGIC model defines for the development year from time t to time t+1:

Fund (t+1) = Fund (t) increased by asset growth +
Net cash inflow (t)

Where: Net cash inflow (t) = Net premium income (t)
- Claims paid (t)
+ Investment income (t)
- Taxes and dividends paid (t)
+ Half a year's interest on the excess of premium income over claims paid (t)

In this paper we assume that the office is closed to new business at time zero.

4.0 **The choice of an asset model**

4.1 The investment and inflation model used in SFGIC is one proposed by Wilkie.

4.2 Wilkie's investment model as used in SFGIC is based on an analysis of annual statistics over the period 1919-1982. The derived model is a cascade model: first the inflation series is derived, the results of this are then used to generate series for the yield on 2.5% Consols, the dividend yield on shares together with an index of the level of share dividends.

4.3 For the purposes of our own research we have substituted Wilkie's model with our own investment model. Our own model is again a cascade model, but with quarterly steps. The parameters were determined from an analysis of quarterly returns from 1963-1986. Further details are given in Appendix 2.

4.4 Our model contains series for the following variables:-

a) Economic growth
b) Retail price inflation
c) Cash
d) Dividend growth
e) Equity yields
f) UK Government fixed interest securities

In the interests of simplicity this paper considers only those asset classes which were considered in SFGIC (ie cash, gilts and equities). An extension to include asset classes such as index-linked Government securities and overseas securities would not affect the principles involved.

4.5 In order to use the results of any analysis in an intelligent and critical manner it is vital that the user is aware of the strengths and weaknesses of a particular model. The parameters chosen for a particular asset model will depend on the purposes for which the model is being used. The parameters of our own model are chosen to provide reasonable estimates of the variabilities of, and the correlations between, the various asset classes over a 5-15 year time horizon. We believe that such a time horizon is appropriate in setting strategic asset allocation policy.

4.6 We would encourage any users of the techniques set out in this paper to investigate the effects on the conclusions drawn of altering the chosen parameters. These comments apply not only to the asset model but to other features of the model such as the claim run offs, reserving basis and variability of claim amounts.

5.0 The application of the model office to the problem of asset allocation

5.1 The simulations carried out for the purposes of SFGIC assumed (for the purposes of the standard basis) that initially technical provisions were covered by a 50% holding in cash and a 50% holding in gilts, the asset margin comprising a 100% holding in equities. After each year of the projection assets were bought or sold as necessary in proportion to the existing holdings.

5.2 The starting point for our own work is to consider the effect of holding a portfolio of 100% in each of the three asset classes. 5,000 simulations of the model are carried out with these fixed asset portfolios and the market values of the total assets held at the end of each year tabulated. The graph in Appendix 3 gives an indication of the spread of the remaining assets after 12 years.

5.3 As one would expect, a 100% holding in equities produces a wide range of returns whereas a 100% holding in cash produces a much lower spread of returns although with a lower mean market value of the assets after any given time period.

5.4 If we focus our attention on the results after 12 years; there will be a non-zero correlation between the market values after 12 years for holdings in each asset class. This arises both from the fact that for example a high return on equities is likely to be linked to a high return on gilts over a 12 year time period and also from the fact that a high return on equities over such a time period is more likely to be linked to a period of high inflation leading to larger claim settlements being made.

5.5 As a result of these non-zero correlations it is possible to use quadratic programming to answer questions such as the following.

For a given expected return on a portfolio comprising α_1 in equities, α_2 in gilts and α_3 in cash which portfolio has the lowest variance?

5.6 Mathematically this is equivalent to solving the following problem:-

Minimise

$$f(\alpha_1,\alpha_2,\alpha_3) = \alpha_1{}^2\mathrm{Var}(X_1) + \alpha_2{}^2\mathrm{Var}(X_2) + \alpha_3{}^2\mathrm{Var}(X_3) + 2\alpha_1\alpha_2\mathrm{Cov}(X_1,X_2) + 2\alpha_3\alpha_1\mathrm{Cov}(X_3,X_1) + 2\alpha_2\alpha_3\mathrm{Cov}(X_2,X_3)$$

Subject to

$$g(\alpha_1,\alpha_2,\alpha_3) = \alpha_1X_1 + \alpha_2X_2 + \alpha_3X_3 - k = 0$$

$$h(\alpha_1,\alpha_2,\alpha_3) = \alpha_1 + \alpha_2 + \alpha_3 - 1 = 0$$

and $\alpha_i > 0$ for $i = 1 \ldots 3$

5.7 The set of all such portfolios forms what is known as an efficient frontier. Given a fixed mean return (which must of course lie between the mean returns on the lowest and highest returning asset classes) the portfolio represented by a point on the efficient frontier is the one with the lowest standard deviation.

5.8 Clearly the above analysis can be extended to incorporate as many asset classes as required. There are many books which address this subject. For further details we refer the reader, initially, to Portfolio Selection: Efficient Diversification of Investments by H M Markowitz (1959). The use of similar techniques for assessing optimal product mixes for General Insurance companies was considered by Biger and Kahane (1977).

6.0 A practical example

6.1 In order to help the reader to understand the processes involved we consider the results of one run of 5,000 simulations as described above.

Considering the position after 12 years we have:-

Investment	Mean Values	Standard Deviation	Correlation Coefficients Cash	Equities	Gilts
Cash	20.9	7.56	1.000	-	-
Equities	40.9	19.99	-.03	1.000	-
Gilts	23.4	10.17	-.07	.53	1.000

Using the model set out in section 5.6, the following points lie on the efficient frontier:-

Mean Value k	**Cash** α_1	**Equities** α_2	**Gilts** α_3	**St Dev** $\sqrt{f(\alpha_1, \alpha_2, \alpha_3)}$
21	96.7%	-	3.3%	7.3
24	60.7%	12.0%	27.2%	6.3
27	57.1%	28.7%	14.1%	7.8
30	53.6%	45.4%	1.1%	9.9
33	39.5%	60.5%	-	12.4
36	24.4%	75.6%	-	15.2
39	9.4%	90.6%	-	18.1
40	4.4%	95.6%	-	19.1

6.2 A graph of the efficient frontier is included in Appendix 4 together with a brief commentary. From the above we can make the following observations. Any portfolio represented by a point lying on the lower section of the efficient frontier graph can be replaced by one representing a point on the upper section having the same standard deviation but a higher expected final asset value. For example instead of a portfolio composed as follows:-

Point	**Cash**	**Equities**	**Gilts**	**Mean**	**St Dev**
A	96.7%	-	3.3%	21	7.3

One could hold the following portfolio:-

B	58.3%	23.2%	18.5%	26	7.2

This has a slightly lower standard deviation together with a higher expected final asset value. The two points, A and B, are indicated on the efficient frontier graph in Appendix 4.

6.3 It is important also to consider the shorter term implications of a chosen strategic investment strategy. A relatively high level of equity investment may provide prospects of long term growth but may, in the shorter term, prejudice solvency. Although we do not do so in this paper, the next stage in the above analysis would be to consider the position after 3 or 6 years.

7.0 A more detailed approach

7.1 We appreciate that the solution to the problem outlined in section 6 is perhaps of limited practical use. An obvious extension to the above is to effectively carry out a full valuation of the model office and to prepare profit & loss accounts and balance sheets at regular intervals. This allows more complex objectives for strategic asset allocation to be set. Daykin and Hey (1990) considered such an extension to FSGIC.

7.2 Clearly the number of asset classes under consideration can be expanded to give a more realistic approach.

7.3 A more realistic problem might be formulated as follows:-

"What asset distribution is implied by assuming that we wish to minimise (at the 10th percentile) the probability of the solvency margin falling below 40% whilst maximising (at the 75th percentile) the expected solvency margin?"

"How should I modify the answer bearing in mind that I also need to have regard to the solvency margin in the shorter term (eg say after 3 years or 6 years and also with reference to the statutory returns to be produced in one year's time)?"

7.4 It is to address questions similar to this one, albeit in the area of pension funds that a model similar to the above is currently being used to provide practical advice to plan sponsors. We feel that management within the general insurance environment could profitably use such models as a management tool.

7.5 Having made this comment it is of course clear that the problem as specified in 7.3 does not have a unique solution. There are many problems which anyone trying to use such a model to address this problem needs to consider, for example:-

i) The distributions of the variables will not be normal. How can one best characterise or approximate them?

ii) How sensitive are the answers to the many assumptions which will have been made? (This can of course be tested using the model itself).

7.6 In using the model it must be appreciated that it is not a black box. On the contrary it is primarily a management tool, a relatively straight forward set of mathematical equations and techniques which encapsulate a reasonable amount of the information underlying the workings of a General Insurance company. Before relying on the results a user should fully understand the model's sensitivities and assumptions. We would advocate that anyone contemplating the use of such models develop their own as a first step to a thorough understanding of the principles.

8.0 Conclusions

8.1 The model office as put forward in SFGIC provides a powerful tool to assist in the management of a General Insurance company.

8.2 The model can be used, together with an economic model, as a tool to make sensible statements about strategic asset allocation and the associated inherent degree of risk involved. This is because different asset classes have, historically, shown long term correlations to a greater or lessor degree both with inflation and between themselves. The claim payments are also linked to inflation.

8.3 Risk in this context may not be the same as risk as currently perceived by the management of General Insurance companies. It is perhaps only a slight exaggeration to say that "risk" is currently avoided by investing in assets which have historically shown short-term price stability. Our definition of "risk" would take into account the opportunity cost of investing in, what have, over the longer term, historically been, poorer performing asset classes.

8.4 We firmly believe that the emerging cost, stochastic approach outlined above provides a useful insight into the workings of a General Insurance company. We would welcome the opportunity to discuss the ideas contained within this paper with those involved in the management of General Insurance funds.

9.0 Acknowledgements

We would like to express our thanks to all those who have helped both to develop the models used and to those who have contributed to the ideas behind this paper. In particular we would like to thank Miss P Herz who has been involved in much of the development work underlying this paper.

We are particularly indebted to the authors of the following papers to which we have made reference in the text:-

Biger, N & Kahane, Y (1977) Balance Sheet Optimization in Inflationary Circumstances, The Journal of Insurance June 1977.

Daykin, C D, Bernstein, G D, Coutts, S M, Devitt, E R F, Hey, G B, Reynolds, D I W & Smith, P D (1987a) Assessing the Solvency and Financial Strength of a General Insurance Company JIA **114**, 227-310.

Daykin, C D & Hey, G B (1990) Managing Uncertainty in a General Insurance Company, JIA **117**, 173-259.

Lockyer, P R (1990) Further Applications of Stochastic Investment Models, presented to SIAS March 1990.

Markowitz H M (1959) Portfolio Selection: Efficient Diversification of Investments, Cowles Monograph No 16 John Wiley, New York.

Wilkie, A D (1986) A Stochastic Investment Model for Actuarial Use, TFA **39**, 341-73.

Finally we must emphasise that the opinions given in this paper are entirely our own personal views on the subject.

Appendix 1 **The model office (standard basis)**

Assumption	**Value used**
Net Written Premiums	£10m per annum
Proportion of long-tailed business	40% of net written premiums
Past Growth	In line with inflation
Future Growth	In line with inflation
Mean claim ratio (short-tailed)	100% of net written premiums
Standard deviation of claim ratio (short-tailed)	10% of net written premiums
Mean claim ratio (long-tailed)	100% of net written premiums
Standard deviation of claim ratio (long-tailed)	15% of net written premiums
Variability of outgo (a)	0.15
Variability of outgo (b)	Initial value: 75
Initial asset margin	40% of net written premiums

Appendix 2 **The investment model**

The long-term assumptions based on price inflation of 6% p.a. and real growth in GDP of 2.25% p.a. are as follows:-

	Actual return/increase	Real return/increase relative to price inflation
Equities	11.6% p.a.	5.3% p.a.
Fixed interest	10.0% p.a.	3.8% p.a.
Cash	8.7% p.a.	2.5% p.a.

The standard deviations and correlations for the returns used are as follows:-

	Standard Deviation of returns (%)	Correlation			
		1	2	3	4
1. Price Inflation	2	1.0			
2. UK Equities	18	0	1.0		
3. Fixed Interest	15	0	-.3	1.0	
4. Cash	5	.1	-.2	-.4	1.0

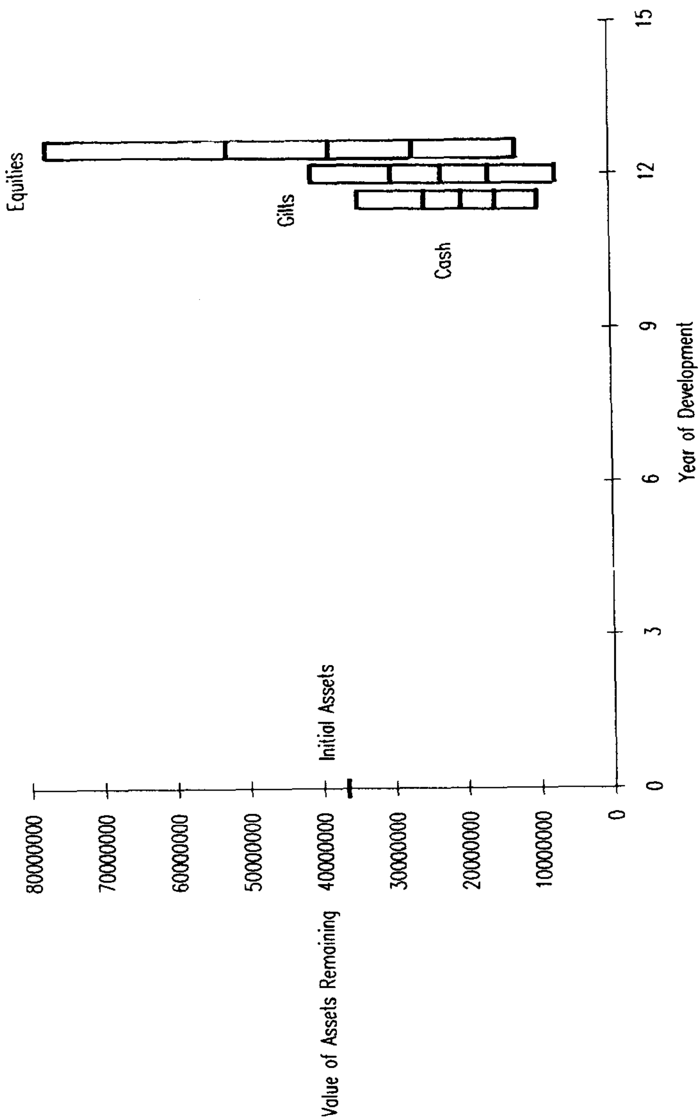
The correlation between different asset classes is a measure of the extent to which the returns on different asset classes are linked. Positive correlation between two asset classes indicates that if, for example, a particular asset class produces a return in excess of its mean value then the other asset class is also more likely to produce a return in excess of its mean value. A negative correlation between two asset classes indicates that if a particular asset class produces a return in excess of its mean value then the other asset class is more likely to produce a return below its mean value.

The correlations shown above relate to returns measured over a one year period. Different correlation coefficients will be obtained when considering the mean returns over different time periods.

Appendix 3 **5,000 simulations**

The graph shows, for each of the three asset classes, the spread of assets remaining after 12 years. The horizontal line at the top of each of the three bars represents the 95th percentile. In other words in 249 simulations out of the 5,000 the value of assets remaining after 12 years exceeded this level.

The remaining horizontal lines indicate the 75th, 50th (or median), 25th and 5th percentiles respectively.

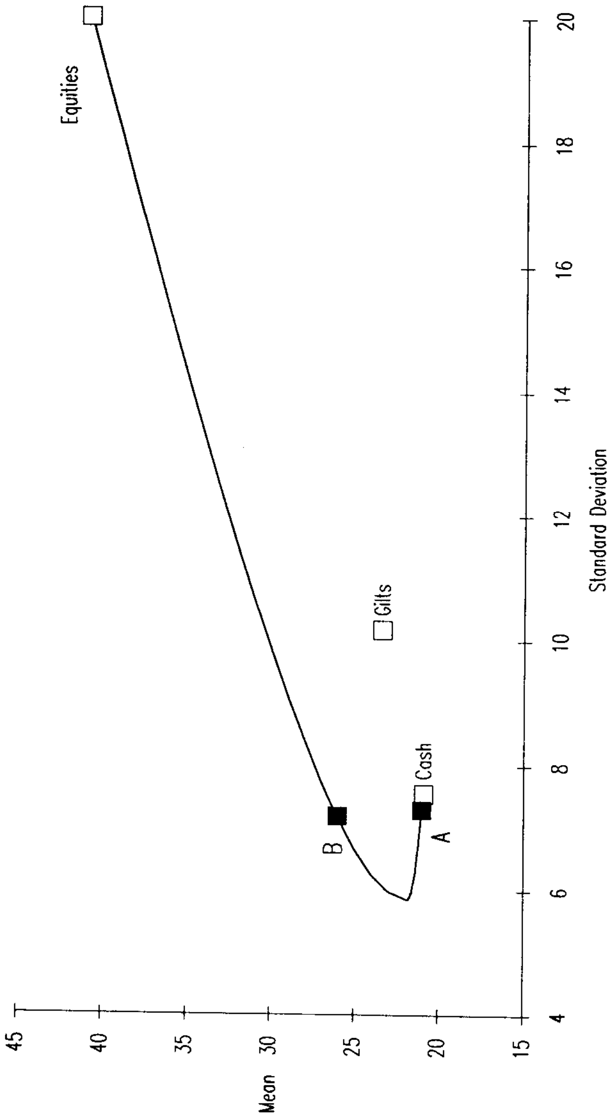


Value of Assets Remaining
80000000
70000000
60000000
50000000
40000000
30000000
20000000
10000000
0
Initial Assets
Equities
Gilts
Cash
0
3
6
9
12
15
Year of Development

Appendix 4 **The efficient frontier**

The graph first shows the expected value of the assets remaining after 12 years, together with the standard deviation from this mean if the assets were invested solely in each of the three asset classes. These three points are marked and labelled on the graph.

For each expected value of the remaining assets it is possible to combine the three asset classes in an optimum manner so as to produce the lowest standard deviation in the level of remaining assets. The totality of all such portfolios is known as the efficient frontier and is represented by the continuous line on the graph.


Efficient Frontier (12 years)
Equities
Gilts
Cash
B
A
Mean
45
40
35
30
25
20
15
4
6
8
10
12
14
16
18
20
Standard Deviation