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Measuring Risk in Internationally Diversified Bond Portfolios

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Summary

In this paper we argue that the traditional measures of risk in bond portfolio management (duration and convexity) are irrelevant in an international context. Starting from a discussion of the drawbacks of duration within a domestic setting, we develop a multifactor model as a prime alternative. This multifactor model allows for non-parallel shifts in the term structure and accommodates the risk inherent in changing yield spreads.

This multifactor model can be easily generalized to apply in a multicurrency environment. In an international context, currency risk, which tends to be totally ignored by the traditional duration measure, plays a crucial role. The multiple factor framework can be extended to specifically incorporate this currency risk in addition to the cross market term structure effects. In this respect the multi-factor risk model could become the natural framework for the measurement of international fixed income risk.

Résumé

Mesurer le Risque dans des Portefeuilles d'Obligations Diversifiées Internationalement

Dans cet article, nous démontrons que les mesures traditionnelles de risque dans la gestion de portefeuilles d'obligations (durée et convexité) sont inappropriées dans un contexte international. En partant d'une discussion sur les inconvénients de la durée dans un cadre national, nous développons un modèle multifacteur en tant que principale alternative. Ce modèle multifacteur autorise les déplacements non-parallèles dans la structure de durée et adapte le risque inhérent en changeant les distributions de rendement.

Ce modèle multifacteur peut être facilement généralisé pour être applicable dans un environnement multi-devise. Dans un contexte international, le risque de devise, qui a tendance à être entièrement ignoré par la mesure de durée traditionnelle, joue un rôle crucial. Le cadre de facteur multiple peut être étendu pour inclure spécifiquement ce risque de devise en plus des effets de structure de durée inter-marchés. De ce point de vue, le modèle de risque multi-facteurs pourrait devenir le cadre naturel du calcul du risque de revenu fixe international.

1. INTRODUCTION

The benefits of international diversification are well documented : most pension funds and insurance companies have typically increased their international exposure during the last decade. It is interesting to note though that this growing internationalization of investment management has mostly been attributable to the equity portion of these funds.

Indeed, with a few exceptions, there has been little theoretical analysis of the investment implications of international fixed income investment (see Cholerton, Pieraerts and Solnik [1986], Jorion [1987 and 1989]). Without doubt this is partly due to the comparative lack of data covering the major international bond markets. There is however also a general feeling that the arguments for international bond investment are weaker because the large impact of currency on the risk - return profile of multicurrency bond portfolios.

Our aim in this article is to discuss and quantify the various dimensions of risk in international fixed income markets.

2. DATA

For this study we have restricted our attention to the government bond markets of the major international countries. It should obviously be recognized that the government bond sector is in a number of cases only a small part of a country's fixed income market. This is especially the case in the U.S. and Japan. Moreover the Euromarkets are omitted completely due to the extremely poor quality of the available data for these markets. It should also be noted that the government bond sector will have different characteristics from other fixed income sectors in the same market (such as the junk bonds in the U.S. , the index-linked sector in the U.K. or the mortgage backed bonds in Denmark). The government sector - although representative of the basic default free term structure of interest rates - may therefore only partially capture the universe of all bonds and will therefore not completely reflect the 'true' global bond market.

A final caveat concerns the liquidity of the individual issues. Many of the government bonds - perhaps up to 60% of the issues in some countries - are not sufficiently liquid for their pricing to be reliable. Further, in some markets only a few issues may trade in sufficient size to accommodate large institutional funds. The following Table summarizes the liquidity of the different Government Bond Markets as assessed by JP Morgan Securities

Table 1 : Relative liquidity of different government bond markets

Market	Number Traded	Actively traded	Benchmark
U.S.	145	20	7
Japan	45	10	1
Germany	65	25	1
U.K.	85	25	10
France	35	17	4

3. THE PROBLEM WITH DURATION (AND CONVEXITY)

When assessing the risk characteristics of internationally diversified bond portfolios, the traditional measures of risk in the context of a single market - (option adjusted) duration and convexity - break down. Before suggesting an alternative approach we will first briefly discuss the relevance of these concepts within a domestic setting.

The duration of a bond approximately captures the price change associated with a given yield change (see figure 1). The duration measure implicitly assumes that the price-yield relation is linear (as measured by the tangent to the price- yield curve). The error implied by the duration measure will therefore be a function of its curvature (or convexity).

A portfolio manager who would only quantify the interest rate risk of his portfolio through its duration may be using an inadequate measure of price sensitivity except for very small changes in yield. In addition duration implicitly assumes identical shifts in yield for instruments with different maturities i.e. duration will only be correct for very small and parallel shifts in the yield curve. It will therefore be obvious that duration will not completely describe the inherent interest rate risk in a fixed income portfolio and that the degree of approximation is a function of the nature of the term structure shift. As a rule of thumb one could say that for the major bond markets across the world duration typically explains about 70% of the cross sectional differences in returns between different bonds, although the measure is much less accurate for Japan (see Table 2).

Table 2 : Average adjusted R - Squared
Weekly data January - December 1989

Model	U.S.	Japan	U.K.	Germany	France
Duration	71.8	47.1	71.7	63.5	71.0
Duration + Convexity	83.8	52.9	78.9	73.4	72.9

Convexity partly remedies some of the shortcomings of duration. Technically convexity is related to the second derivative of the price - yield curve and reflects the speed with which the price changes when the yields change. Although the combined use of duration and convexity gives us a more complete picture of the price sensitivity of a fixed income portfolio, they will only be fully descriptive of the price yield dynamics if the relationship between the two is exactly quadratic. The combined use of duration and convexity on average (for the major international bond markets) accounts for about 75 % of the cross sectional differences in bond returns.

In short, the usage of duration and convexity crucially depends on the assumption of parallel shifts in the yield curve. In this respect they rely upon a fairly simplistic description of the term structure dynamics which can be reduced to one factor : a uniform up - or downward shift. To the extent that other interest rate factors are at play in the market, they will be ignored by this approach.

In other words when interest rates only move up or down uniformly, we would be able to completely explain the differences in return between various bonds through their differences in duration or convexity (i.e their different sensitivities to this market factor). The objections against duration and convexity therefore break down in two parts :

- there is probably more than one factor needed to describe the dynamics of the term structure : it may change in shape as well as shift up or down. In addition there may be other factors of value such as a price for liquidity, quality, tax advantages (some UK gilts being free of tax to residents abroad for instance) etc...
- there are probably other characteristics beyond the duration and convexity of the bond needed in order to capture the exposure of the instrument to these additional sources of value

4. MULTIPLE SOURCES OF RISK

As outlined in the previous paragraph, the framework within which duration and convexity are used is fairly limiting. Even within a domestic setting it would be more satisfying if additional factors of value and price sensitive bond characteristics could be taken into account.

As far as the term structure dynamics are concerned, one could learn from the historical yield curve behaviour which particular changes have taken place in the past. One way of assessing these dymnamics is to describe the returns to pure zero coupon bonds of differing maturities (the yield curve dynamics will then be expressed in terms of the changes to zero coupon bonds or discount factors applying at a certain point along the maturity spectrum). Using this approach, the yield curve can be described in terms of the price of pure discount bonds of different maturities, each of which can move independently.

Other sources of value can be captured in terms of yield spreads associated with liquidity, quality, current yield, tax treatment etc... The changes in these spreads through time will again give an indication of the inherent riskiness of each of these characteristics.

Formally let us describe the following valuation specification :

$$P = \sum_{i=1}^{N} \frac{CF(t_i)\ d(t^i)}{(1 + \sum_j a_j x_j)^{ti}} + e$$

where P = bond market price

$CF(t_i)$ = expected cash flow at time t_i
$d(t_i)$ = discount factor applicable at date t_i (or equivalently the current price of a zero coupon bond maturing at time t_i)
x_j = bond exposure to factor j

a_j = market price (or equivalent yield spread) associated with factor j

e = bond pricing error

Note that the above equation assumes that cash flows only occur on dates t (typically these grid dates are judiciously chosen along the maturity spectrum such as 1, 2, 3, 4, 5, 7, 10, 20 and 30 years into the future), Cash flows occurring between these dates are proportionately allocated to the previous and following grid date under preservation of present value and duration.

The price of each bond is therefore given by the present value of its future (re- allocated) cash flow stream. This present value is adjusted in function of any bond specific characteristics which may affect this particular instrument. Hence the value of any instrument will be influenced by the default- free term structure as it applies to all issues in a given market as well as by the distinguishing bond characteristics which will have a price in the market. Note that these distinguishing bond characteristics may be market specific (the UK has the Free of Tax to residents Abroad feature, the US has flower bonds, Japan has very strong liquidity effects etc...).

The relevant parameters of the valuation function (dt_i and a_j), can then be estimated for a cross-section of representative bonds on a given date. This estimation can be done subject to all sorts of constraints which for instance impose smoothness on the functional form of the term structure. (for a detailed discussion see for instance Sevilla and Beckers 1990).

Our research has shown that the above specification is very robust across markets and through time. In most of the major markets the term structure can be defined using a limited number of zero coupon rates (typically 1, 2, 3, 4, 5, 7, 10, 20 and 30 years to maturity). On average there are another 2 to 3 factors (possibly different from market to market) which contribute to the explanation of differential asset prices. The following Table summarizes the relevant yield spreads for the major markets

Table 3 : Relevant yield spreads for the major markets

Spread	Markets where applicable
Benchmark status	ALL
Liquidity	ALL
Current Yield	ALL
Free of tax to Residents abroad	U.K.
Callable	U.S., U.K.
Perpetuals	U.K.

Figures 2 through 6 summarize the relevance of the different factors of value in each market (using the standard T-statistic as the criterion for significance). It can be inferred that current yield is a very important source of value in Japan (over and above the term structure factors). Also, Liquidity spreads have a significant impact on bond prices in Japan and Germany.

Another insight into the accuracy of the full-factor model can be gained by evaluating the increase in explanatory power obtained by the inclusion of the different factors of value : As can be inferred from Table 4 , on average the model achieves an increase in R-Squared of about 8%.

Table 4 : Average Adjusted R-Squared
Weekly Data January 1989 - December 1989

Model	U.S.	Japan	U.K.	Germany	France
Duration + Convexity	83.8	52.9	78.9	73.4	72.9
Full Factor Model	91.3	71.9	86.7	80.3	78.1

The improvement is smallest for the French government bond market, possibly because for the period under consideration (1989) parallel shifts in the term structure were most prevalent.

The above relationship can be estimated at regular intervals (daily, weekly , monthly), herewith capturing the markets sources of value [a_j and $d(t_i)$]. From the time series behaviour of these parameters we can also estimate their risk (volatility) and correlations. In particular we can estimate the volatility of the 1 year zero coupon rate, its correlation with changes in the 7 year zero coupon rate, the volatility of the liquidity yield spread and its correlation with the 5 year zero coupon rate. The core of a bond risk model is therefore an estimate of the variances and covariances of the term structure and yield spread factors.

Beyond the market-wide sources of risk, there remains a - typically small- component of bond specific risk. This risk may be due to unique , idiosyncratic characteristics associated with an individual issue. In the above model specification this specific risk is captured by the standard deviation of the pricing error e. Especially for government bonds the specific risk is mostly negligibly small (or equivalently the above model will capture the majority of the pricing relationship). Specific risk is by definition non-hedgeable but will on the other hand diversify very quickly since it is unrelated across different instruments.

It will now be obvious that we have extended the single factor duration/convexity concept into a multidimensional framework : The price risk of a 5 year bond for instance will be a function of its sensitivity to the 1, 2, 3,4 and 5 year zero coupon rates since the price risk of its coupon and principal payments will be a function of the volatility of these rates (as well as of the size of the cash flows which will be paid out). If the 1 year rate is more volatile than the 3 year rate (i.e. if the term structure typically moves more at the short end than at the longer end), this will be reflected in a relatively higher risk (volatility) being assigned to the 1 year coupon than to the 3 year coupon.

5.RISK MEASUREMENT FOR MULTICURRENCY PORTFOLIOS

The multifactor approach outlined above can be generalized quite easily to a multicurrency context. In fact this is the only viable approach since the notions of duration and convexity - although useful within a given base currency - loose all their meaning in internationally diversified portfolios.

Indeed , although duration and convexity have the portfolio property (i.e. the duration and convexity of a portfolio are the capitalization weighted averages of the individual bonds) and it would therefore be technically feasible to compute the duration and convexity of a multicurrency portfolio, these measures would be meaningless. The computation of duration and convexity within a multicurrency portfolio would only be relevant to the extent that there would only be a single interest rate process (i.e. that interest rate movements across the different markets are perfectly correlated and move in a parallel fashion) and that there is no currency risk.

For example a portfolio which contains a US Treasury with a duration of 5 years and a German government bond with a duration of 3 years (with an equal weight in each) could be said to have a duration of 4 years. Note that this measure would be identical irrespective of the base currency of the investor (i.e. the duration based measure would imply that the risk of this portfolio is the same irrespective of whether one is US dollar, DM or Yen based). In other words the duration calculation totally ignores the currency dimension. In addition the duration would refer to a sensitivity to changes in a 'global' term structure which would somehow be a combination of the US and German term structures (which in turn would be assumed to be perfectly correlated).

While duration and convexity have lost all meaning, the extension of the multiple factor approach to international portfolios is very straightforward : A risk model can be estimated for each important international market (in excess local currency terms). We can then extend the model by not only considering the within country variances and correlations but also taking into account the cross-currency correlations : we could calculate the correlation between changes in the 3 year zero coupon rate in US dollars and the 3 year rate in Sterling, similarly for the correlation between the 5 year zero coupon rate in DM and the 3 year rate in Yen or the correlation between changes in the French Franc current yield spread and the Japanese liquidity spread.

6. CURRENCY RISK IN INTERNATIONAL BOND PORTFOLIOS

Note that so far we have completely ignored the currency dimension. In this paragraph we will argue that currency risk - properly defined - can be separated from the local market risk.

Currency is a complicating issue in international asset pricing. It is well known that deviations from Purchasing Power Parity at any given point in time are significant and persist for long periods , fluctuating in a random way. Investors with different base currencies would therefore face different expected real returns when looking at the same asset, resulting in a base-currency specific portfolio construction.

To be more specific, let us introduce the following notation. We will distinguish between the base or numeraire currency of the investor and between local returns that apply to a 'local' investor.

Let

rx	be the random rate of return due to changes in exchange rates e.g. the rate of depreciation of Sterling versus DM
rc	be the random rate of return due to changes in exchange rates plus any interest received as a result of an investment in the foreign risk-free rate
rfl	be the short term risk free interest rate a local investor would receive

Formally :

$$1 + rc = (1 + rx) (1 + rfl)$$

Further let

rl be the total rate of return on a bond as experienced by a local investor

rn be the total rate of return on a foreign bond expressed in the investor's numeraire currency

Hence

$$1 + rn = (1 + rx) (1 + rl)$$

or

$$rn = (rx + rfl) + (rl - rfl) + (rx) (rl)$$

$$= rc + (rl - rfl) + (rx) (rl - rfl)$$

The last term captures the cross product between exchange rate movements and each bond's excess return over the local short term risk free rate. This term will typically be small, especially over periods such as a month or shorter. In appendix 1 we illustrate the magnitude of this cross-product term for aggregate bond market returns from a US dollar base currency. The effect doesn't exceed more than a few basis points per month on average for aggregate market returns.

Therefore :

$$rn \cong rc + (rl - rfl)$$

In other words, the foreign bond rate of return in the numeraire currency is approximately equal to the currency rate of return plus the excess rate of return of the foreign bond in the local market. Further, the excess rate of return of the bond in the numeraire currency is

$$rn - rfn \cong (rc - rfn) + (rl - rfl)$$

where rfn is the risk free rate of return in the domestic (or numeraire) currency. In other words the excess rate of return in the numeraire currency is (approximately) equal to the excess rate of return on the currency plus the excess rate of return of the foreign bond in its local market.

[As an aside , an investor who would hedge his exchange rate risk through a (fairly priced) forward sale of his initial investment would incur a cost of rfn - rc. The currency hedged foreign bond rate of return in the numeraire currency is then

$$rn_{hedged} \doteq rfn + rl - rfl$$

i.e. the currency hedged rate of return on the foreign asset expressed in the numeraire currency is approximately equal to the numeraire risk free rate plus the excess rate of return of the foreign asset (over its risk free rate). [For the arguments around currency hedging see Kaplanis and Schaefer (1990) and Thomas (1988) and (1989)] .

It will therefore be natural for most international investors to look at the currency policy separately from the local market decision. In particular the risk an investor is exposed to can now be subdivided into a market risk component (which is independent of his base currency), a currency risk component and the cross product term between the two (i.e. the correlation between currency and local market movement).

The currency risk can be measured straightforwardly as the volatility of currency movements (measured from a given numeraire currency) and their correlations through time.

7. AN INTERNATIONAL FIXED INCOME MULTIFACTOR RISK MODEL

From a risk measurement point of view,the currency risk block completes the international risk model. In particular the model will consist of the following building blocks :

- the within country block capturing the volatility of the term structure and yield spreads within a given market (expressed in local currency terms)

- the cross - country block reflecting the correlations between term structure and yield spread movements across different markets (each expressed in local currency terms)

- the currency block which measures the volatilities and correlations between exchange rate movements (measured from the investor's base currency perspective)

- the currency - local market block which contains the correlations between exchange rate movements (measured from the investor's base currency) and interest rate or yield spread movements in different markets (expressed in local market terms).

The risk model captures the essential (fundamental) sources of risk in the international fixed income market. Given this model the risk of any individual asset or portfolio can be determined in function of its

senstitivity to these risk factors. These exposures can be Identified straightforwardly since they are a function of
-the exposure of the bond (portfolio) to different parts of the term structure in each market (i.e. which cash flow will be received at each grid date)
- the exposure to yield spread risk in each market (which will be a function of the bond's coupon, liquidity, benchmark status etc...)
- the exposure to currency risk as a function of the fraction of the present value of each bond (portfolio) denominated in different base currencies than the numeraire currency

The combination of exposures and their riskiness allows one then to quantify the expected return volatility (i.e. the uncertainty around the expected return). This will be expressed in standard deviation terms, reflecting by how much the realized return could deviate from the expected return .

The accuracy of the risk measure crucially depends on the stability of the historic risk measures. Imbedded in the risk model are the historic term structure (and currency) movements. To the extent that structural changes take place within each of these market (Sterling joining the EMS, Belgium tying its term structure to that of Germany ...), the historically observed patterns may no longer be valid into the future.

Experience has shown though that the use of historic risk measures is a valuable and accurate predictor of future return volatility in most cases.

8.SUMMARY

In this paper we argue that the traditional measures of risk in bond portfolio management (duration and convexity) are irrelevant in an international context. Starting from a discussion of the drawbacks of duration within a domestic setting, we develop a multifactor model as a prime alternative. This multifactor model allows for non-parallel shifts in the term structure and accommodates the risk inherent in changing yield spreads.

This multifactor model can be easily generalized to apply in a multicurrency environment. In an international context, currency risk plays a crucial role which tends to be totally ignored by the traditional duration measure. The multiple factor framework can be extended to specifically incorporate this currency risk in addition to the cross market term structure effects.

THE PRICE -YIELD RELATIONSHIP.

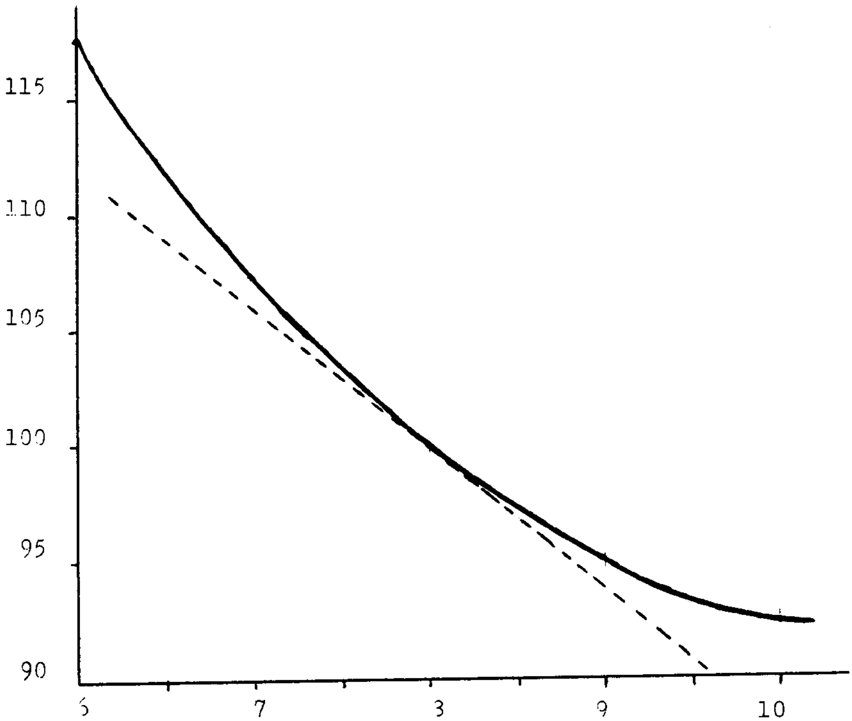


FIGURE 1

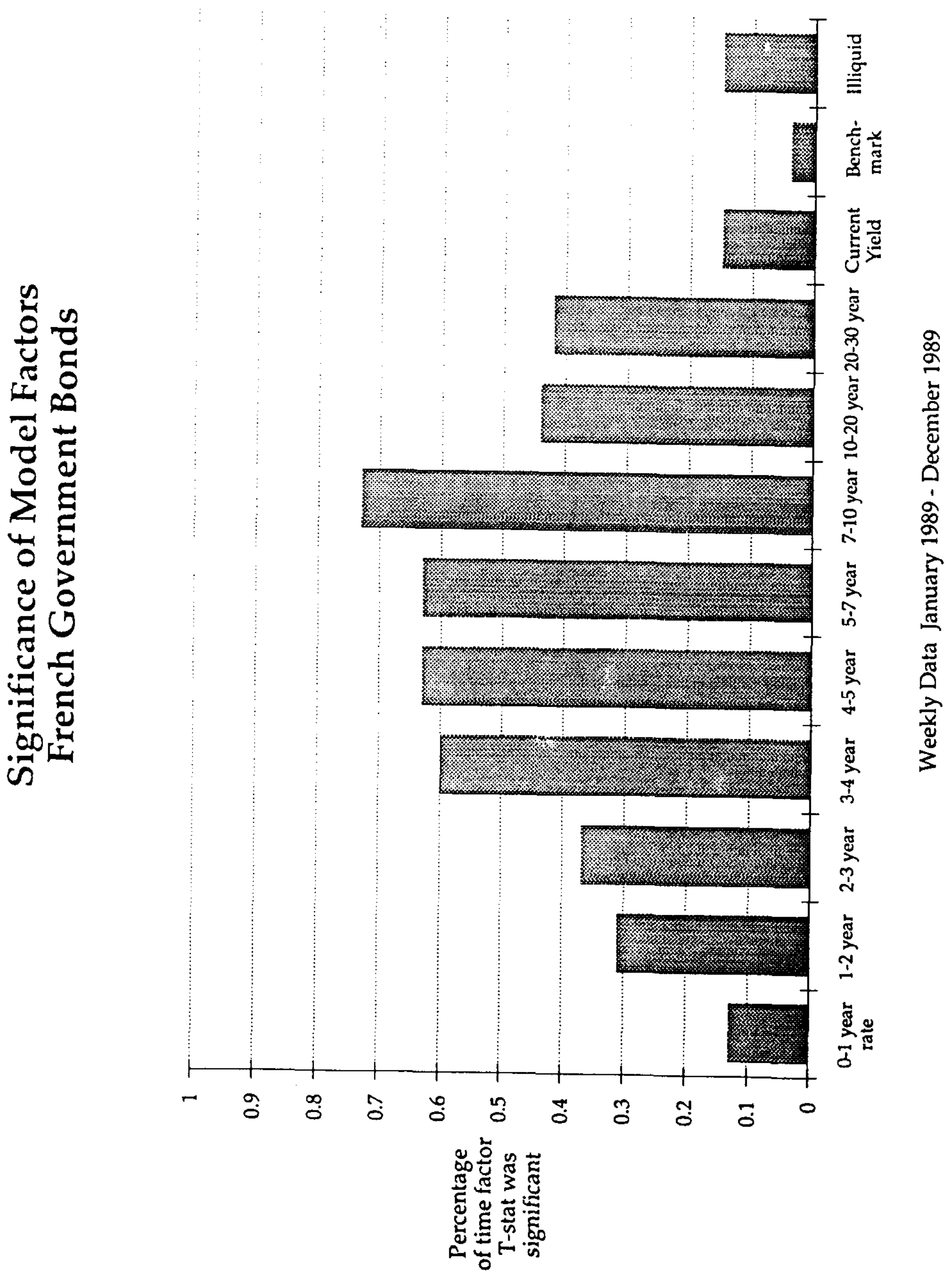

Significance of Model Factors
French Government Bonds
Percentage of time factor T-stat was significant
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0-1 year rate
1-2 year
2-3 year
3-4 year
4-5 year
5-7 year
7-10 year
10-20 year
20-30 year
Current Yield
Bench-mark
Illiquid
Weekly Data January 1989 - December 1989

FIGURE 2

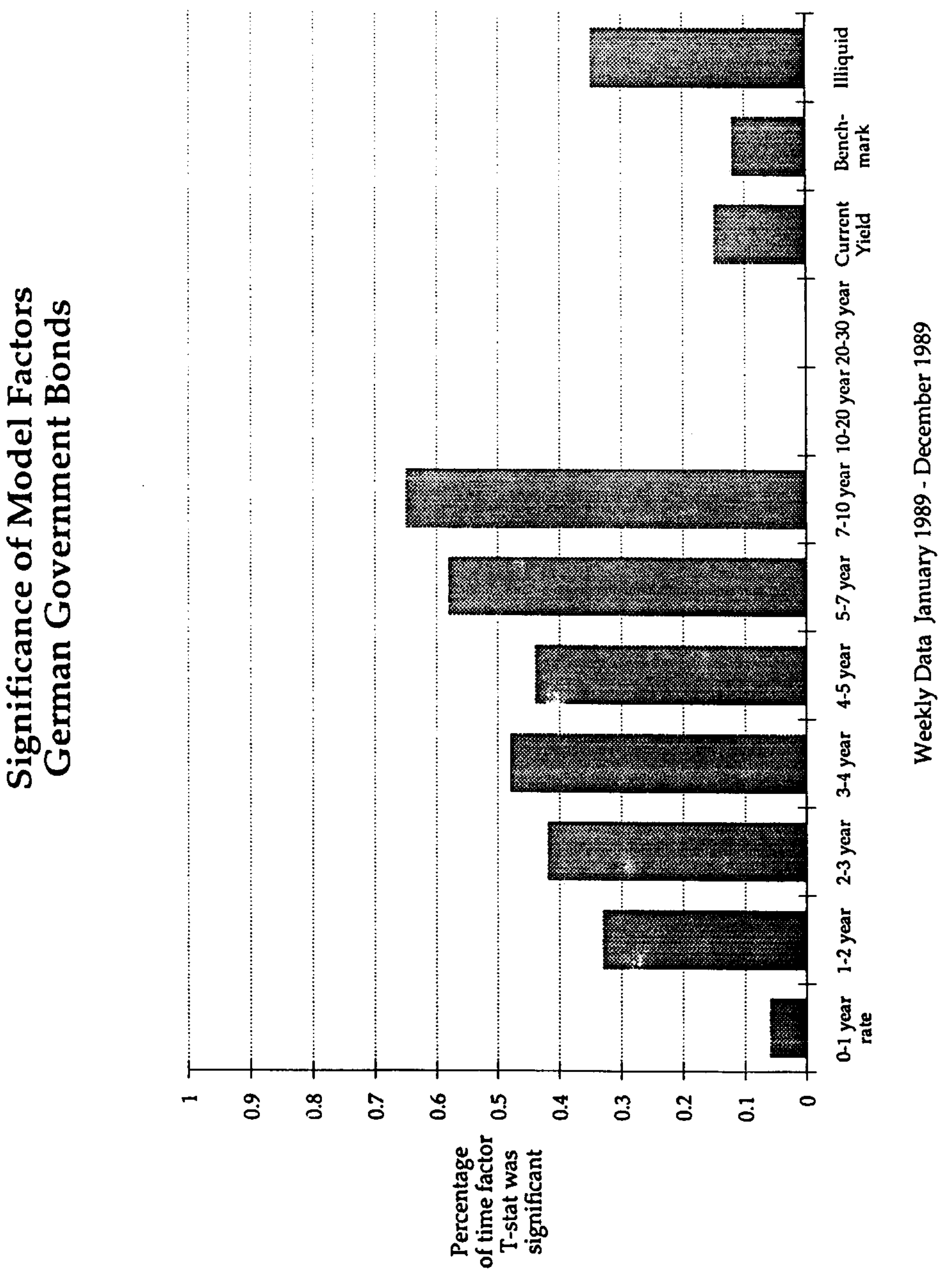
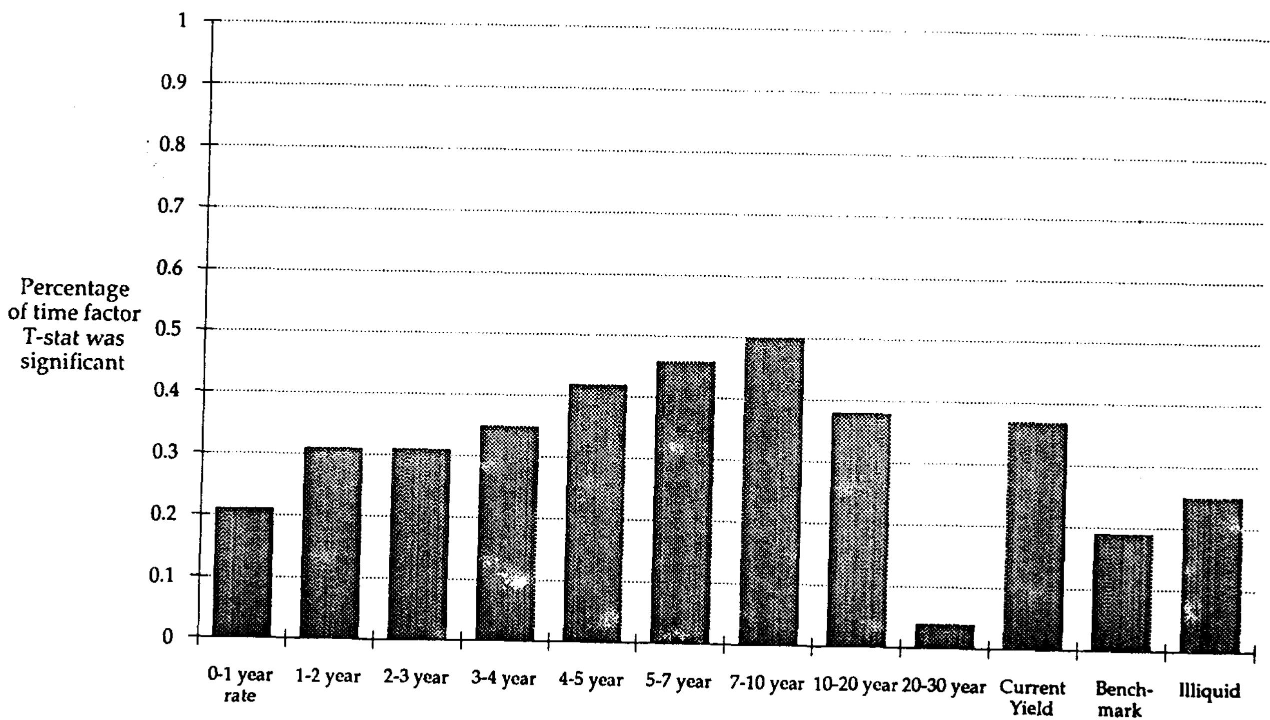

Significance of Model Factors
German Government Bonds
Percentage of time factor T-stat was significant
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0-1 year rate
1-2 year
2-3 year
3-4 year
4-5 year
5-7 year
7-10 year
10-20 year
20-30 year
Current Yield
Bench-mark
Illiquid
Weekly Data January 1989 - December 1989

FIGURE 3

Significance of Model Factors
Japanese Government Bonds



Weekly Data January 1989 - December 1989

FIGURE 4

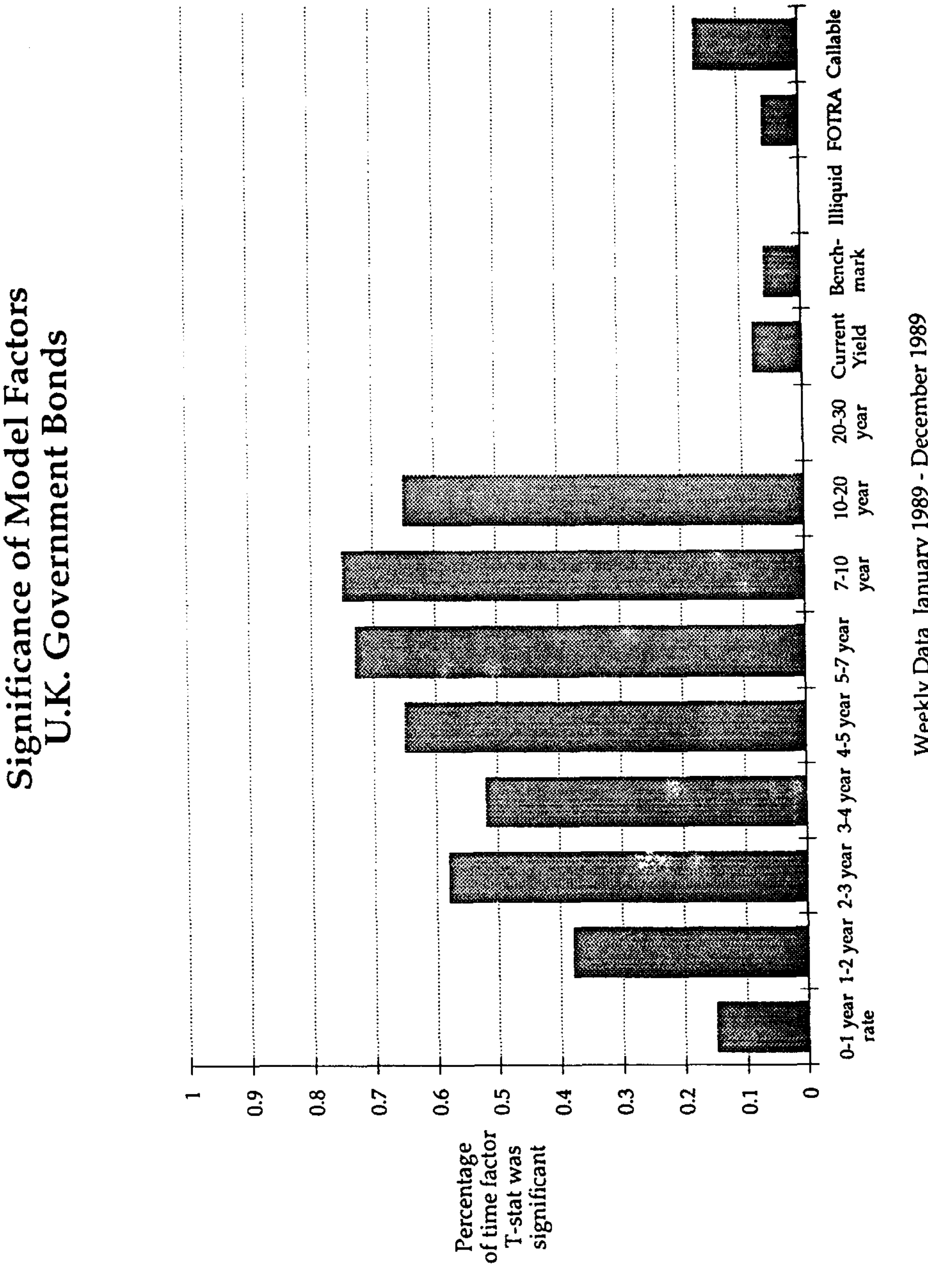
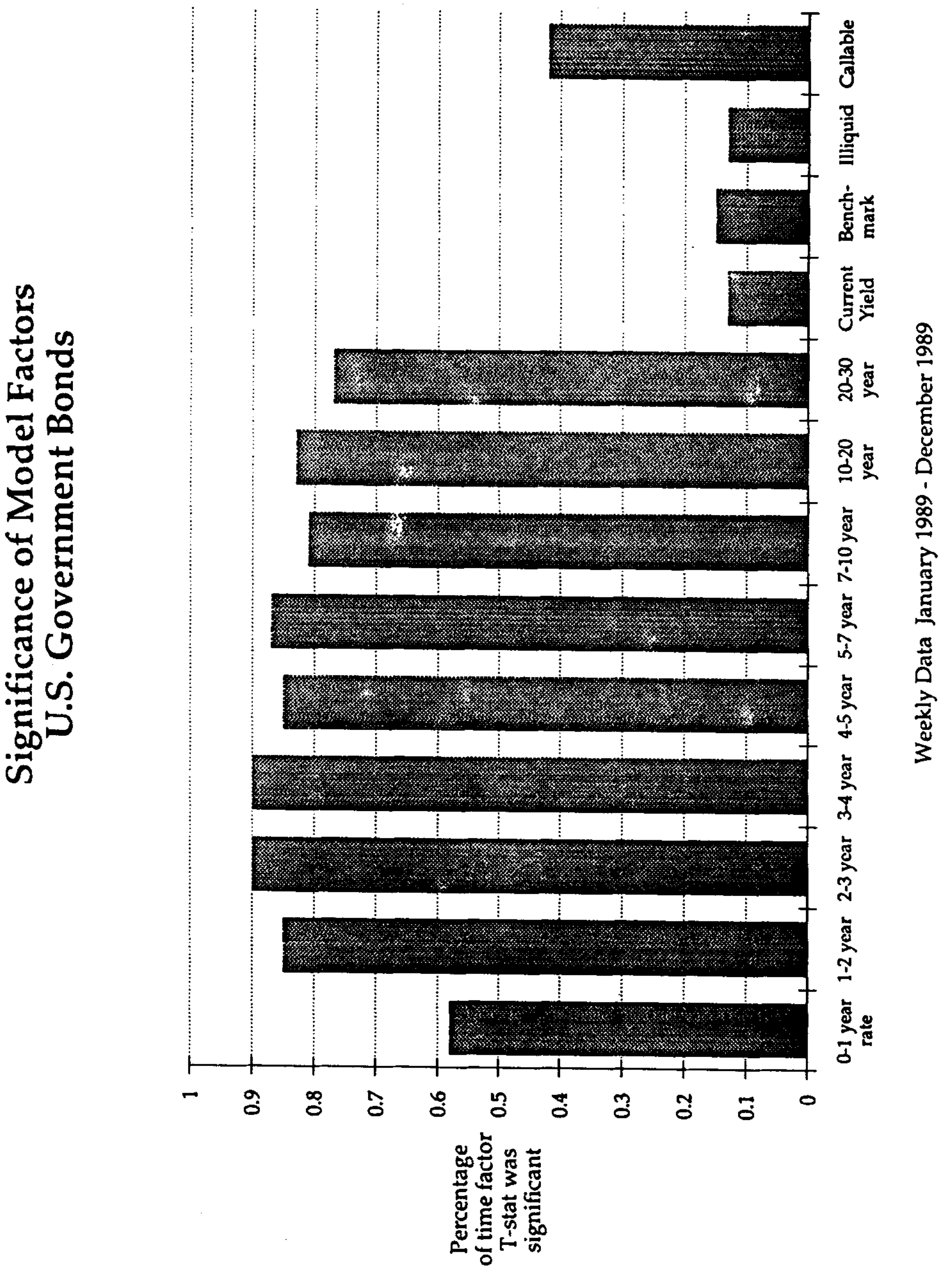

Significance of Model Factors
U.K. Government Bonds
Percentage of time factor T-stat was significant
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0-1 year rate
1-2 year
2-3 year
3-4 year
4-5 year
5-7 year
7-10 year
10-20 year
20-30 year
Current Yield
Bench-mark
Illiquid
FOTRA
Callable
Weekly Data January 1989 - December 1989

FIGURE 5



Significance of Model Factors
U.S. Government Bonds
Percentage of time factor T-stat was significant
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0-1 year rate
1-2 year
2-3 year
3-4 year
4-5 year
5-7 year
7-10 year
10-20 year
20-30 year
Current Yield
Bench-mark
Illiquid
Callable
Weekly Data January 1989 - December 1989

FIGURE 6

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APPENDIX 1: MAGNITUDE OF CROSS PRODUCT OF EXCHANGE RATE MOVEMENTS AND LOCAL MARKET EXCESS RETURNS (monthly data)

Base Currency : U.S. Dollar
Observation period : January 1980 - February 1989
Data Series : Salomon Government Bond Series

Country	Cross Product		Annual Standard Deviation	
	Mean	Std	with cross prod. terms	without cross prod. terms
Australia	.00040	.00107	.18155	.18226
Canada	.00015	.00051	.14706	.14681
France	.00004	.00083	.14516	.14399
Germany	.00030	.00079	.16495	.16399
Japan	.00035	.00087	.16919	.16721
Netherlands	.00020	.00073	.15587	.15444
Switzerland	.00013	.00045	.15756	.15724
UK	.00031	.00099	.18467	.18353