Modelling of Long-Term Risk

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A. Basel II

Amendment to the Capital Accord to Incorporate Market Risks (Basel Committee on Banking Supervision, 1996):

- “In calculating the value-at-risk, a 99th percentile, one-tailed confidence interval is to be used.”

- “In calculating value-at-risk, an instantaneous price shock equivalent to a 10-day movement in prices is to be used.”

- “Banks may use value-at-risk numbers calculated according to shorter holding periods scaled up to ten days by the square root of time.”
Basel II (cont.)

- **Market risk:** 10-day value-at-risk, 99%
  
  **Standard:** 1-day value-at-risk, 95%

- **Insurance:** 1-year value-at-risk, 99%
  
  1-year expected shortfall, 99%
VaR in Visual Terms

Loss Distribution

Mean loss = -2.4

95% VaR = 1.6

95% ES = 3.3

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B. Scaling

Question 1: how to get a 10-day VaR (or 1-year VaR)?

Solution in the praxis: scale the 1-day VaR by $\sqrt{10}$ (or $\sqrt{250}$).

Question 2: how good is scaling?

→ model dependent!
Scaling under Normality

Under the assumption

\[ X_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2), \]

\( n \)-day log-returns are normally distributed as well:

\[ \sum_{i=1}^{n} X_i \sim \mathcal{N}(0, n\sigma^2). \]

For a \( \mathcal{N}(0, \tilde{\sigma}^2) \)-distributed profit \( X \), \( \text{VaR}_p(X) = \tilde{\sigma} x_p \), where \( x_p \) denotes the \( p \)-quantile of a standard normal distribution. Hence

\[ \text{VaR}^{(n)} = \sqrt{n} \text{VaR}^{(1)}. \]
AR(1)-GARCH(1,1) Processes

A more complex process, often used in practice, is the GARCH(1,1) process \((\lambda = 0)\) and its generalization, the AR(1)-GARCH(1,1) process:

\[
X_t = \lambda X_{t-1} + \sigma_t \epsilon_t,
\]

\[
\sigma_t^2 = a_0 + a(X_{t-1} - \lambda X_{t-2})^2 + b \sigma_{t-1}^2,
\]

\(\epsilon_t\) i.i.d., \(E[\epsilon_t] = 0, E[\epsilon_t^2] = 1.\)

(typical parameters: \(\lambda = 0.04, a_0 = 3 \cdot 10^{-6}, a = 0.05, b = 0.92\))
Scaling for AR(1)-GARCH(1,1) Processes

Goodness of fit of the scaling rule, depending on different values of $\lambda$ (x axis) for different distributions of the innovations $\epsilon_t$.

For typical parameters ($\lambda = 0.04$, $\epsilon_t \sim t_8$), the fit is almost perfect.
C. One-Year Risks

Problems when modelling yearly data:

- Non-stationarity of data.
- Lack of yearly returns.
- Properties of yearly data are different from those of daily data.
How to Estimate Yearly Risks

- Fix a horizon $h < 1$ year, for which data can be modelled.

- Use a scaling rule for the gap between $h$ and 1 year.
Models for One-Year Risks

- Random Walks
- Autoregressive Processes
- GARCH(1,1) Processes
- Heavy-tailed Distributions
Backtesting

The suitability of these models for estimating one-year financial risks can be assessed by backtesting estimated value-at-risk and expected shortfall using observed return data for

- stock indices,
- foreign exchange rates,
- 10-year government bonds,
- single stocks.
Conclusions for One-Year Forecasts

- In general, the random walk model performs better than the other models under investigation. It provides satisfactory results across all classes of data and for both confidence levels investigated (95%, 99%). However, like all the other models under investigation, the risk estimates for single stocks are not as good as those for foreign exchange rates, stock indices, and 10-year bonds.

- The optimal calibration horizon is about one month. Based on these data, the square-root-of-time rule (accounting for trends) can be applied for estimating one-year risks.
The square-root-of-time scaling rule performs very well to scale risks from a short horizon (1 day) to a longer one (10 days, 1 year).

The reasons for this good performance are non-trivial. Each situation has to be investigated individually. The square-root-of-time rule should not be applied before checking its appropriateness.

In the limit, as $\alpha \rightarrow 1$, scaling a short-term $\text{VaR}_\alpha$ to a long-term $\text{VaR}_\alpha$ using the square-root-of-time rule is in most situations not appropriate any more.