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Non-Life Insurance Liability Measure: Mark to Statistical Model, Financial Market, and Clients

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Abstract

This paper suggests "Mark to Clients" base non-life insurance premium valuation. Market to Clients means that "clients' needs" decide the price (valuation). So far, statistical methods have been used for valuation, and recently, methods using financial engineering were introduced for valuation. Here, more clients' needs based, approaches have been tries to be developed.

The back-ground of clients' needs based valuation is that insurance products are made for solving the clients' risk management issues. To valuate through statistical data, there needs to be abundant data samples. Financial engineering gives us smart methods for insurance pricing and insurance risk management but it relies on market transaction. It often happens that there are few transactions or no transactions. Also financial engineering suppose arbitrage free, equilibrium, and, existence of risk adjusted measure. The clients' needs based method suggest solutions for those.

Keywords

Insurance Liability, Insurance Risk Management, Financial Engineering, Reduced Form Approach, Structural Approach, Clients, Needs

1. Introduction

Traditionally, a statistical approach has been used for pricing a non-life insurance premium and its risk management. Recently, financial engineering started to be used. Regarding "financial engineering," there are three aspects:

i) Pricing

Many pricing methods come from financial engineering technique. Not only by the law of large numbers and statistical technique, but also arbitrage pricing, equilibrium pricing, and, risk adjusted measure pricing are used for insurance premium calculation.

ii) Product development

Many insurance (derivative) ideas come from financial services such as earthquake-trigger-contingent loan accommodation, which show the characteristics of both insurance and financial market instruments.

Credit default swap is an example for alternatives, which substitute default risk insurance (guarantee insurance).

iii) Risk transfer

To hedge the risk, insurance companies started to use financial market investors. Examples are contingent surplus note, ART (alternative risk transfer) instruments and insurance risk transfer securitization type financial market instruments.

Regarding the first point, Moridaira (2003) summarized risk measure understanding for insurance premium calculation. In addition, Wang Transform by Wang (1995, 2001, 2002) is one of the popular methods for calculating insurance premium on the condition that they are neither arbitrage free, equilibrium pricing methods, nor, risk adjusted measure. Regarding second point, financial derivatives pricing theories can be made use of. Regarding third point, wider risk carriers allow us more risk hedge capacity.

From the next section, this paper summarize the concepts of traditional insurance premium valuation methods and then suggests clients' needs base pricing ideas.

2. Mark to Statistical Model

In the statistical model, three rules are governing.

i) The Law of Large Numbers

The risk premium ratio will be observed and will be gained stably by gathering large number of samples (insurance customers).

ii) Equilibrium / Premium vs. Expectation

The insurance premium ratio will be, or will be expected to be, equal to the expectation of the loss or damage probability of the risk.

iii) Sufficient-Enough

After a long time has passed, premiums earned by insurance companies and insurance payments by insurance companies will be match.

There is an exception to the above and an example is catastrophe damage. For such kind of losses, "extreme value theory" has developed.

3. Mark to Financial Engineering

(1) Mark to Financial Engineering

In financial engineering methods, following three rules are governing.

i) Diversification is the tool for risk hedge.

ii) Clients can be told by "certainty equivalent utility amount."

iii) Arbitrage free (No free lunch).

Financial engineering tools helps us to be more sophisticated. Arbitrage free conditions, equilibrium pricing, and, risk adjusted measure are examples. On the other hand, real world is often no-arbitrage free, no-equilibrium condition, and, no-risk adjusted measure conditions. As an example of the solution for that, Wang Transform gives us an insurance premium calculation method.

(2) Mark to Financial Market

When the utility of financial markets are explained, following three are pointed.

i) The role of the market is to gather participants, in order to make instruments pricing more unbiased.

ii) The role of the market is, risk burden sharing by participants to get new capacity.

iii) Risk distribution is the key for insurance companies.

Examples of the markets in which new type of insurances (derivatives) are priced are the electricity (derivatives), the weather (derivatives), and etc. Insurance companies started to use financial market in order to hedge their liability. Examples of those are ART (alternative risk transfer) instruments, securitization of catastrophe loss insurance, contingent surplus note, and etc.

Even the financial markets started to be used for insurance business, this paper would like to point the importance of the clients' needs of risk hedge and next section will treat it.

4. Mark to Clients

From the words "mark to clients," this paper expresses the most important point of view: clients needs, when risks are managed or prices of the risk are determined. Let us start from the model using financial technology, which goes into the reduced form model. For expressing clients' needs more in detail, here utility function will be discussed. Implementation of what is described is expected to be shown in the next paper. Here ideas are shown.

(1) Reduced Form Model

Here is a case of a valuation for a contingent loan by earthquake trigger. In the reduced form model, hazard ratio or probability are given from outside. Expression of the reduced form model is as follows. (In a financial market world, Duffie (1996) or Duffie and Singleton (1999) is applying for a bond pricing.)

$$C = E^{Q}{}_{t}[\theta(t) \bullet h(\mathrm{t}) \exp\{\int_{t0}^{t} - h(\mathrm{s})\mathrm{ds}\}]$$

C: insurance premium

E^Q: function of averaging default loss on the risk neutral measure by t

h( ): hazard ratio of default of borrower by the earthquake

θ ( ): loan spread increase at the time the earthquake happens

t0,t: t0 is time now, and t denotes at any time.

In case of the easiest calculation example, i.e., the probability of the happening of the earthquake is 0.01, the spread is 1.0%, and the loss averaging by factor is 0.5, then the insurance premium ratio is 0.01 * 1.0%* 0.5 = 0.50%.

Let us examine from a little more in detail. The insurance product (insurance derivative product) is that, the insurance seller will accommodate a loan to the insurance buyer, only in case the defined earthquake happens. The insurance buyer prepares for capital needs in case of happening the earthquake. Usually banks hesitate to accommodate a loan to the insurance buyer because of its uncertain circumstances. To explain the price of the insurance, two methods will be discussed. Suppose the insurance is the following conditions:

INSURANCE SELLER: x bank

INSURANCE BUYER: Y manufacturing company

INSURANCE PERIOD: 1 year from now.

CONDITION: only in case of happening the defined earthquake

LOAN ACCOMMODATION DATE: one week after the earthquake

LOAN ACCOMMODATION AMOUT: 100 million yen

LOAN PERIOD: 10 years

LOAN INTEREST: 3% (as is usual for company Y)

Form the financial market point of view, with the following additional information, we will get the price of the pure insurance premium C.

In the case an earthquake happens, the company Y's credit rating will deteriorate.

The risk premium will increase by 5%. (The interest rate should be not 3%, but 8%.)

The probability of happening the earthquake within one year is p=0.01

C = 5%*10 years*0.01*100 million yen = 0.50%*100 million yen = 0.5 million yen

(The pure insurance premium ratio is 0.50%.)

(2) Clients' Needs Approach

On the other hand, from the client point of view, there will be the company Y's conditions as shown in Table 1. In the case of light damage, company Y do not need to borrow money and there is no difference to borrow or not to borrow in order to continue its business. In the case of middle damage, only in case company Y can borrow money, company Y will survive and the other case company Y loses money ▲100 million yen. In the case of sever damage, even to borrow or not to borrow, company Y loses money ▲100 million yen.

Table 1

(million yen)	damage by the earthquake light	middle	sever	average loss
probability	50%	40%	10%	
loss with insurance	0	0	▲100	▲10
loss without insurance	0	▲100	▲100	▲50

Because the expected amount of loss for both cases are ▲10 million yen and ▲50 million yen with the insurance and without the insurance respectively. From the difference, the price of insurance is decided.

$C = (50\text{-}10)$ million yen * 0.01 = 400 thousand yen

(The insurance premium ratio is 0.40%.)

In this case, the loss is supposed to be equal to the loan amount. (For example, a 100 million value factory is destroyed and the loan is for its re-building.)

The difference between the premium ratios from financial engineering and from the client's own analysis comes from the sever earthquake case, when accommodated loan money is useless, i.e., 50 million yen * 0.01 vs. (50-10) million yen * 0.01. (Even though the average probability of the occurring the earthquake damage is 0.5 and it is equal to the sum of the probability of middle damage case plus sever damage case.) This point only can be analyzed by the client itself, and, it is very important part of insurance premium calculation.

Before starting the next section, one comment is set and it is that, as long as the sever case exist, the price of insurance by client needs approach is smaller than market data / financial engineering approach. I will be back to this issue later on.

(3) Structural Model by K. J. Arrow – J. Pratt Measure

Clients' needs based pricing is the following Structural Model and starts from K. J. Arrow – J. Pratt Measure. Insurance transactions (or insurance type derivative transactions) have their reasons for transactions. Structural modeling is an approach to the clients' characteristics (of utility functions). (Arrow et. al. 1983)

The insurance buyer decides the price of the premium of the insurance. Suppose he is risk averse. The price will be decided how the risk aversion is. Using K. J. Arrow – J. Pratt Measure, behavioral science and game theory, the price of the risk is explained. The basic concept is as follows.

The function u(x) is the utility function of the client. He has a chance to get x1 or x2, with the probability p1 and p2 respectively (p1 + p2 = 1). The average expected amount to get is xe, xe = x1*p1 + x2*p2 as shown bellow Fig. 1.

The client's utility of the case that the client may get x1 or x2, is p1*u(x1) + p2*u(x2) and it is smaller than u(xe) which is the utility of the case that the client surely can get xe.

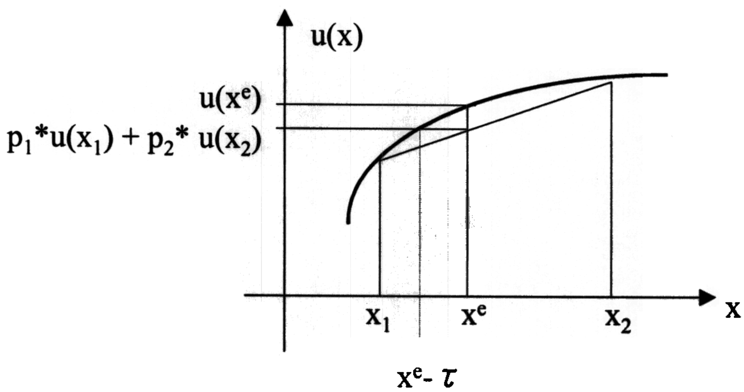


Fig.

This is because of the shape of u(x) and the case in Fig. 1 is the case of risk averse. There is a discount amount τ, which satisfies;

$$u(x^{e} - \tau) = p_1 \times u(x_1) + p_2 \times u(x_2)$$

τ means the price of the risk for the client. Even smaller x than xe by τ, since to get xe- τ surely is certainty equivalent with the bet x1 or x2. Generally,

$$u(E(X) - \tau) = E[u(X)]$$

In the case τ <<the variability of X << the size of X, the both side of the above equation will be as follows.

$$\begin{aligned} u(E(X)) - \tau \times u'(E(X)) &= E[u(E(X) + X - E(X))] \\ &= E[u(E(X)) + (X - E(X)) \times u'(E(X)) + \frac{1}{2}u''(E(X)) \times (X - E(X))^2] \\ &= u(E(X)) + \frac{1}{2}u''(E(X)) \times \sigma^2 \end{aligned}$$

From the equation, we can get as follows:

$$\tau = -\frac{u''(E(X))}{2 \times u'(E(X))} \times \sigma^2$$

We can say the price of the risk is proportional to both the absolute risk averseness and the variance of the risk. This much for one of the insurance premium principles "variance principle." It is interesting that the above is not consistent with other insurance premium calculation principles "standard variation principle," that the insurance premium is average loss amount plus standard deviation with a multiplier; which is very popular.

$$C = \mu + k\sigma$$

C : premium

μ : average of the loss

σ : standard deviation of the loss

k: multiplier.

In the above equation, the price of the risk is proportion to standard deviation, not to variance.

(4) Structural Model by Behavioral Science / Game Theory

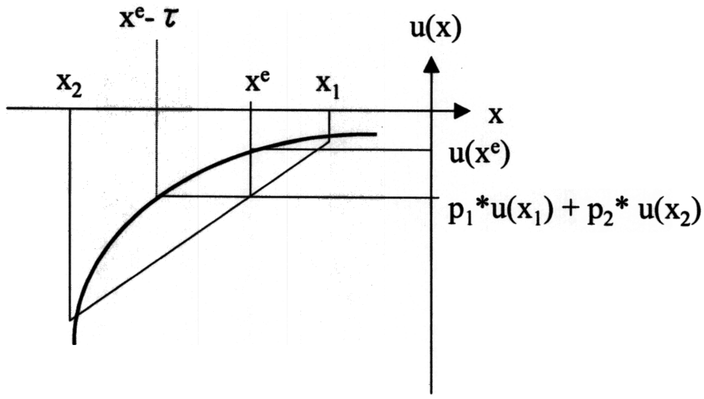


Fig. 2

The utility function in the case of loss is expressed in Fig. 2. Here x1, x2, and xe <0 and τ >0.

As is the same in the previous discussion, |u(xe) | is compared to |p1*u(x1)+p2*u(x2) |. |u(xe) | is smaller and there is τ >0 which satisfy u(xe- τ)= p1*u(x1)+p2*u(x2). We can say that this risk aversion person willingly pay insurance premium |xe- τ |, which is larger than |xe|. Here an insurance company could get extra money from him as a cost of insurance distribution or operation.

However, there will be a controversial point. From the point of behavioral finance theory, the shape of the famous utility function example is different. According to the Kahneman (2003, (See Poulton (1994))) and Smith (1991), the shape of the utility function in the loss aversion case is as follows.

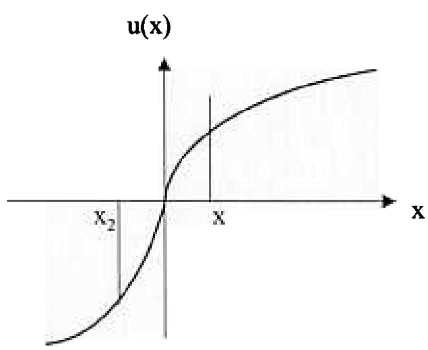


Fig. 3

In case one may get plus x1 or lose |x2| (x1=-x2), the shape of the utility function is as in Fig. 3.

The difference between the case Fig. 2 and Fig. 3 in a negative territory is that, in the case of Fig. 2, one surely loses but the magnitude is large or small, and , in the case of Fig. 3, one may gain or lose. That is why the shapes are different.

In the loss or loss condition, people willingly pay high premium larger than pure expectation loss amount for insurance premium in order to buy loss cover insurance policy.

Another example to support the shape of the utility function is the existence of the insurance of excess loss only. People willingly suffer the risk at the small magnitude for reducing premium amount.

The discussion below will be use the utility function shown in Fig. 4

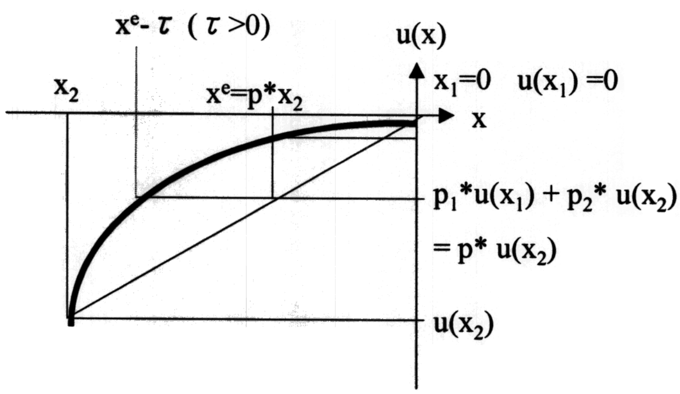


Fig. 4

Here we will treat catastrophe loss insurance like earthquakes. Suppose x1=0 and u(x1)=0. x2 (<0) is the loss by the catastrophe. Annual probability of the catastrophe is denoted p, so xe=p*x2. p*u(x2) is more worse than u(xe) so there is τ >0 which satisfy u(xe- τ)=p*u(x2). If insurance companies use xe as a premium, even expecting to realize the law of large numbers, insurance companies cannot afford to continue business. As shown in Fig.4, there will be additional payment form people by τ , which is used for insurance companies' costs, earnings, and etc.

i) Example - Government Support -

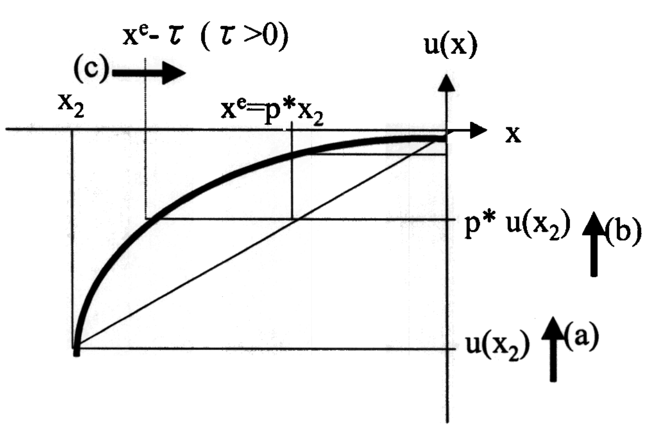


Fig. 5

In the case of catastrophe disaster, governments usually give money to disaster sufferers. Soon people begin to think they do not have to buy insurance policy to cover catastrophe losses because the government will help. As shown in Fig. 5, the thinking or behavior will change the market value of the insurance premium.

(a) The amount of u(x2) will become smaller because government will compensate at least part of the loss.

(b) The size of the expected average utility p*u(x2) also will become smaller.

(c) As a result, xe- τ becomes to near to xe, pure premium. People start to stop paying insurance premium as much as usual before. This make insurance companies difficult to continue insurance business.

ii) Example - Buy Insurance -

According to behavioral finance, people tend to think the size of the negative loss (utility) will be larger than as it is. It is said that that is why people buy insurance. Fig.6 shows this.

(a) People tend to think the size of u(x2) larger than as it is.

(b) The expected average utility also becomes worse.

(c) τ becomes larger and people become willingly pay insurance premiums.

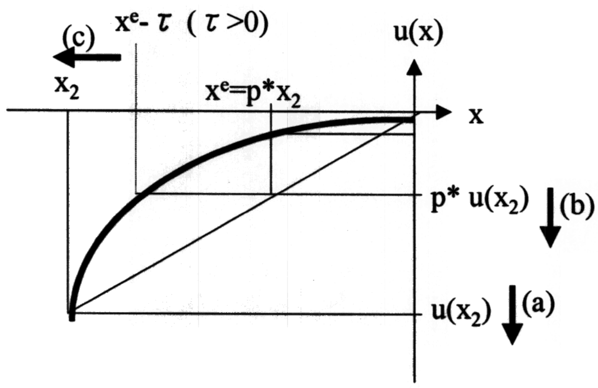
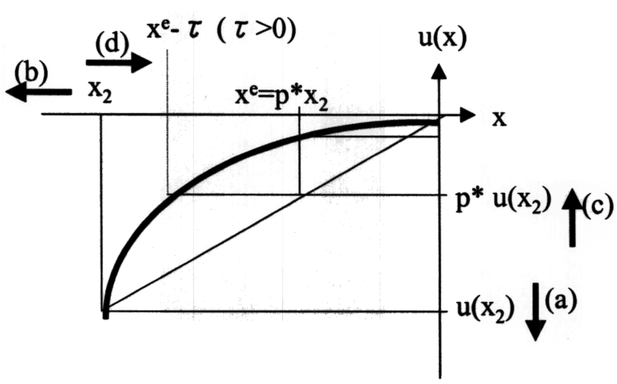


Fig. 6

iii) Example - Flame Theory -



People tend to think a small amount of money is not as valuable and as large amount of money is too sensitively valuable. Fig. 7 shows this.

(a) The size of u(x2) is large and people think the loss is too large.

(b) People feel the same of (a) for the loss x2.

(c) The size of p*u(x2) is small and people tend to think the loss is too small.

(d) Whether τ become large or smaller, it depends on the magnitude of x2 change and p*u(x2) change.

iv) Implementation - general case -

To make the mathematical calculation easy to understand, loss x<0 will be changed to x>0 and x2 becomes to positive.

$$u(x) = Const \times x^{d}$$

Using τ as described before, we can get the following equation.

$$(px_1 - \tau)^d = px^{d}$$

$$\tau = (p + p^{\frac{1}{d}})x_1 = (1 + p^{\frac{1}{d}-1})x^{e}$$

In case $\tau =$ (1+k * xe, which means τ is proportion to xe, and k <<1 and p <<1;

$$d = \frac{1}{1 + \dfrac{\ln(1+k)}{\ln p}} \approx 1 + \frac{\ln(1+k)}{\ln(1/p)} \approx 1 + \frac{k}{\ln\dfrac{1}{p}}$$

$\tau =$ (1+k) * xe means insurance companies decide costs of the insurance business in proportion to xe, pure insurance premium, i.e. average of expecting loss amount. The model shows the utility function is convex a little because d is larger than 1 by k/ln(1/p).

Example sets for (k, p, d) are shown in Table 2.

Table 2

k	0.1	0.1	0.2	0.2	0.3	0.3
p	0.001	0.01	0.001	0.01	[illegible]	[illegible]
d	1.014	1.021	1.027	1.041		

We can discuss the characteristic of real insurance business from this relationship as follows:

"d" expresses risk averseness, "1/p" expresses the length of the time period risk happens, and , k is the weight of insurance business costs vs. pure insurance premium.

(a) As u(x) is in proportion to "x to the d," the relative risk averseness (-du'(x)/u'(x) / dx/x) is fixed figure d-1 (constant).

(b) Suppose there is no difference in the relative risk averseness, if 1/p becomes larger, k will becomes (be able to make) larger. Larger, the length of time period, and larger, the cost weight k. (Insurance companies afford to cover costs.)

(c) So, insurance company can earn a relatively large amount of cost from rare occurring risks.

Insurance companies would change the ratio of insurance cost versus pure insurance premium according to the time period the risk happens (the longer, the larger).

v) Catastrophe Case

To make the mathematical calculation easy to understand, loss x<0 will be changed to x>0 and x2 becomes to positive. We introduce two types of utility functions, u1 and u2.

$$u1(x) = x^{1+D}$$

0<D <<1 and x is normalized by client utility unit. Fig. 8 shows the utilities supposing D=0.4.

The function u2 below is shown in Fig. 9.

$$u2(x) = 1 + (x-1)^3$$

The normalization has the following meanings. For a person, 100 thousand dollars is the unit of the impression for the money. (The salary per month is somehow such kind of.) So 100 unit (10 million dollars) is a large number and can be said catastrophe. (Example is the price of a house.)

Suppose the catastrophe probability is p, the catastrophe damage M, and , unit of utility M0. In case using u2 and p<M0/M (i.e., pM/M0>1), according to the shape of the function 0<x<1, people pay the premium unwillingly because the certainty equivalent utility is given by smaller than M0/M, pure premium ratio.

The function u2(x) can explain this and the function u1(x) cannot.

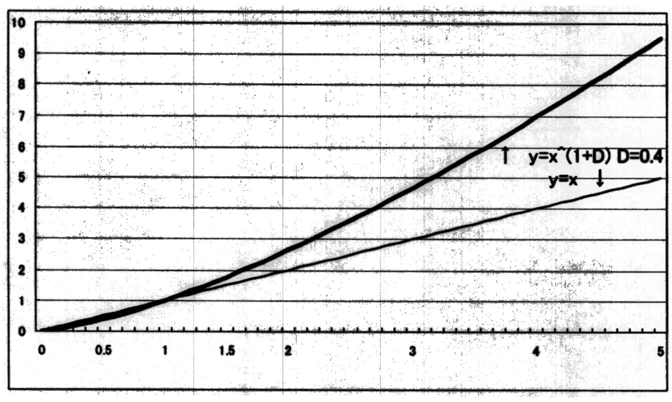
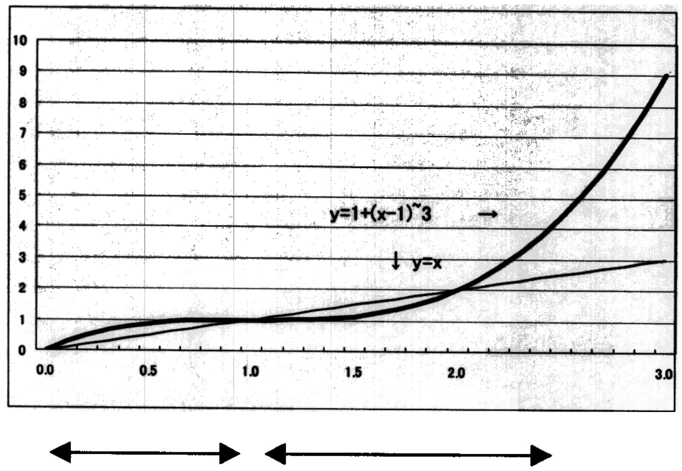


Fig. 8

This is the case the probability is too small. On the other hand, in case p>M0/M (i.e., pM/M0>1), people willingly pay more than the pure insurance premium insurance company has a chance to cover cost. In Japan, earthquake is the case p>M0/M, not the case p<M0/M. (More often the earthquake happens.)



For insurance companies,

Negative for Business

Positive for Business

Fig. 9

vi) Common Non-life Insurance

Under the utility function u2(x), in case M/M0～2, it will become that the outcome become significantly different, whether the figure of p is larger than 0.5 or not. In the real world of non-life insurance business, it is often the case. Non-life insurance company shows the necessity of the insurance showing the latest probability or trends of the probability in sales promotion.

vii) Clients' Needs Approach

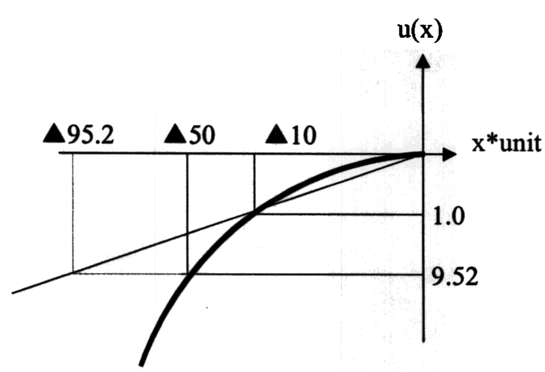


Fig. 10

Back to valuation for the premium of the contingent loan right with earthquake trigger.

According to the Table 2, using x unit for 10 million, and using utility function x^(1+D) type (D=0.4), even the average loss ▲50 million is felt severer, although ▲10 million is felt as ▲10 million. The calculation shows client feel ▲95.2 million for by the ▲50 million loss (|▲50/10|^(1+0.4)=9.52). The premium calculation goes as follows:

(95.2-10) * 0.01 / 100 = 0.852%

The pure insurance premium is 0.50% and there is room to add costs and others by 0.352% (70% of pure premium) as is often seen in the non-life insurance pricing. (See Fig. 10.)

4. Summary

This paper suggests "Mark to Clients" base non-life insurance liability (or premium) valuation. Solving the clients' risk management issues is the key to price the insurance. The financial engineering will be supported and strengthened by the clients' needs base method.

For the purpose, structural models using utility functions are suggested. The usage of clients' needs base calculation has led the following remarks:

In case financial market interweave the clients' needs, the solution will the same but usually there are differences.

Behavioral science helps to understand that people willingly pay for insurance premium to insurance companies more than pure premium ratio.

Structural Model using utility function shows several characteristics in insurance premium calculation.

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