Performance Concentration

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The essence of performance analysis is to measure the value added by the service provided by the portfolio management. For investigating whether a fund manager helps to add value, in the context of the debate between pro and con indexation, we propose a new concept named "performance concentration", and a new type of performance measure which is related to this concept, the characteristic of which is twofold : it can be identified from market data; it is independant of any asset pricing model. By using the term "performance concentration", we means that the performance of a given portfolio is highly concentrated on very few stocks or very few days. The purpose of this paper is to exhibit and emphasize this new stylized fact and to introduce this new performance measure. A real managed portolio data set is also used to demonstrate how the measurement method developed here can be applied.

Introduction

Performance measurement and performance attribution are key issues for asset managers. If we consider the value chain of a given investment process, the performance function is like a feedback loop, designed to monitor the investment process and to scrutinize the value-added created by the asset manager for its clients. In the other hand, on the theoretical side, performance measurement is also one of the key issues in the old debate dividing fund managers and investors about the usefulness of indexation compared to active portfolio management and more recently alternative asset management. In the context of this debate, the purpose of this paper is to exhibit and emphasize a new *stylized fact* and a new performance measure, the characteristic of which is twofold : it can be identified from market data; it is independant of any asset pricing model[1].

Sometimes, asset managers say that the most profitable days (or the worst days) are responsible for more than 10% (or 20% or other...) of the total performance for the period under analysis; others think the same thing for stock picking, and say that some specific securities are responsible for more than 10% (or 20% or other...) of the total performance. The common idea characterizing this thought can be translated in statistical terms and stated as "the total level of the performance is mainly determined by few upper order statistics". But the classical templates of performance reportings, including the models of attribution analysis, fail to exhibit this phenomenon. In this paper, we would like to address this specific issue and we propose to introduce a new concept in the broad field and huge litterature of performance measurement[2] named "performance concentration", and a new type of performance reporting which is related to this concept. By using the term "performance concentration", we means that the performance of a given portfolio is highly concentrated on very few stocks or very few days, in the sense where the extreme values of the market fluctuations capture the main part of the gains so as the losses.

This article is organized as follows. In Section 1, we outline the basic framework, and introduce the performance concentration phenomenon by defining two indices of concentration, for market timing and stock picking. Section 2 provides performance concentration numbers, by conducting an empirical analysis of on the diversified portfolio of the french insurance company SMA BTP, between 31/12/2001 and 23/09/2004. Section 3 concludes the article.

The reason of choosing the particular portfolio of SMA BTP is twofold :

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[1] The flaws of asset pricing-based performance measures are well known : for example, in the CAPM context, see Roll [1978], and the further analysis by Dybvig and Ross [1985] and Green [1987].

[2] In this paper, I do not intend to conduct a literature review. To list just a few : since the early formal measures of Treynor [1965], Sharpe [1966] and Jensen [1968], numerous new performance measures have been proposed : the APT-based measure of Connor and Korajczyk [1986] and Lehmann and Modest [1987], the period-weighting measures of Grinblatt and Titman [1989], and the intertemporal marginal rates of substitution-based measures of Glosten and Jagannathan [1994]. On the empirical side, there have been studies to examine how different measures may rank : see for example Chen *et al.* [1987].

1. The style of the investment process of SMA BTP : the asset management departement of SMA BTP has choosen an actively managed style with high level of stock picking operations, deliberately not indexed on any type of given benchmark.

2. The high overperformance of SMA BTP against the two main benchmarks for French diversified portfolios : the performance of the SMA BTP diversified portfolio is $+17,24\%$ over the CAC 40 index and $+19,42\%$ over the Eurostoxx 50. The issue is therefore to look for the sources of this overperformance.

1 The performance concentration and its measure

1.1 The performance concentration on few days

Let $\{V_t, \ t \geq 0\}$ the process of the market value of the portfolio. We defines the continuously compounded return process $\{R_t, \ t \geq 0\}$ such that

$$R_t = \ln V_t - \ln V_0 \tag{1}$$

We introduce the notation $\Delta R(t,\tau) = R_t - R_{t-\tau}$ for defining the periodic continuous return for an interval of length τ, and $\Delta R(t,1) = \Delta R_k = R_k - R_{k-1}$. The analysis of the portfolio between the dates 0 and T (with $T = N\tau$) leads to a sample of size N : $\{\Delta R_1, \cdots, \Delta R_N\}$. Next, we split the whole sample into two subsamples, and we consider the n positive returns (the same reasoning can be conducted for the n' negative returns). We rank the subsample by decreasing order, and define the ordered sample

$$\Delta R_{(1)} > \Delta R_{(2)} > \Delta R_{(3)} > \cdots > \Delta R_{(n)}$$

where $\Delta R_{(1)} = \max(\Delta R_1, \cdots, \Delta R_n)$, and $\Delta R_{(n)} = \min(\Delta R_1, \cdots, \Delta R_n)$. Hence $\Delta R_{(k)}$ is the k-th upper positive return. The cumulative positive performance of the k highest positive returns, noted $R_{(k)}$, is

$$R_{(k)} = \Delta R_{(1)} + \Delta R_{(2)} + \cdots + \Delta R_{(k)} = \sum_{i=1}^{k} \Delta R_{(i)}$$

Let us now consider the contribution of the $100\,p\%$ upper positive returns, compared to 100% of the total positive performance. By using Embrecht *et al.* [1997], we introduce now the new concept :

Definition 1 (Market timing performance concentration index)
The contribution of the $100\,p\%$ upper positive returns to the total performance is the ratio

$$J_+(p) = \frac{\Delta R_{(1)} + \cdots + \Delta R_{([np])}}{R_{(n)}} \qquad \frac{1}{n} < p < 1 \tag{2}$$

where $[np]$ is the integer part of np.

The quantity $J_+(p)$ is a concentration measure in the exact sense where the value of $J_+(p)$ quantifies the way of which the $100\,p\%$ upper positive returns contribute to 100% of the aggregate positive performance. For example, if $p = 0,2$, and if $J_+(0,2) = 0,8$, it means that 20% of the individual positive returns are responsible for more than 80% of the total positive return. It is the so-called "law of 80/20". Sometimes, fund managers say that the most profitable day (or the worst day) contributes about 10% (or 20% or other...) of the total performance for the period under analysis, but the classical performance reportings,

SET cumulative frequency (X-axis)	TOTAL POSITIVE PERFORMANCE cumulative positive ordered contributions	RELATIVE MASS cumulative frequency (Y-axis)
$1/n$	$R_{(1)} = \Delta R_{(1)}$	$R_{(1)}/R_{(n)}$
$2/n$	$R_{(2)} = \Delta R_{(1)} + \Delta R_{(2)}$	$R_{(2)}/R_{(n)}$
$\vdots$		$\vdots$
k/n	$R_{(k)} = \Delta R_{(1)} + \cdots + \Delta R_{(k)}$	$R_{(k)}/R_{(n)}$
$\vdots$		$\vdots$
1	$R_{(n)} = \Delta R_{(1)} + \cdots + \Delta R_{(n)}$	1

Table 1: Gini-Lorenz curve of positive market timing concentration

including the attribution analysis, fail to exhibit this phenomenon. The methods proposed here allows to characterize portfolios where this $80/20$ rule applies.

Practically, we are now able to build the so-called Gini-Lorenz curve, by using the empirical index

$$J_n(k) = \frac{\Delta R_{(1)} + \cdots + \Delta R_{(k)}}{R_{(n)}} = \frac{R_{(k)}}{R_{(n)}}$$

The table 1 details the method for tracing out the curve. The same methodology is used for tracing out the Gini-Lorenz curve of negative concentration.

It is now possible to address a new issue interesting the fund managers : the cumulative effect of *both* positive and negative concentration. We start from the total performance (negative and positive)

$$R_N = \underbrace{\Delta R_{(1)} + \Delta R_{(2)} + \cdots + \Delta R_{(n)}}_{\text{ordered positive returns}} + \underbrace{\Delta R_{(n+1)} + \Delta R_{(n+2)} + \cdots + \Delta R_{(N)}}_{\text{ordered negative returns}}$$

next we drop out the upper return, and recaculate the new total performance as

$$R_N^{(1)} = R_N - \Delta R_{(1)} = \Delta R_{(2)} + \Delta R_{(3)} + \cdots + \Delta R_{(N)} = R_N - R_{(1)}$$

and for the two upper returns

$$R_N^{(2)} = R_N - \left(\Delta R_{(1)} + \Delta R_{(2)} \right) = \Delta R_{(3)} + \cdots + \Delta R_{(N)} = R_N - R_{(2)}$$

etc. The result of this is a new ordered sample of adjusted performance :

$$R_N^{(0)} > R_N^{(1)} > R_N^{(2)} > \cdots > R_N^{(n)}$$

with $R_N^{(0)} = R_N$. We use the same methodology for recalculating the adusted performance without the lower returns.

1.2 The performance concentration on few stocks

Now we turn our attention to the contribution of stocks for a given (sub)period. In so far as the analysis is conducted over only one period, we now drop the time index t. Let R_P be the total performance of a given portfolio over one periode (one day, one week, one month...). We introduce the stock j contribution, denoted C_j, such as

$$R_P = \sum_{j=1}^{N} C_j$$

Next, we split the whole portfolio into two subportfolios, and we consider the n positive contributions (the same reasoning can be conducted for the n' negative contributions). We rank the subsample by decreasing order, and define the ordered sample

$$C_{(1)} > C_{(2)} > C_{(3)} > \cdots > C_{(n)}$$

where $C_{(1)} = \max(C_1, \cdots, C_n)$, and $C_{(n)} = \min(C_1, \cdots, C_n)$. Hence $C_{(k)}$ is the k-th upper positive contribution. Let us now consider the contribution of the $100\,p\%$ upper positive contributions, compared to 100% of the total positive performance. We introduce now the second new concept :

Definition 2 (Stock picking performance concentration index)
The contribution of the $100\,p\%$ upper positive contributions to the total performance is the ratio

$$T_+(p) = \frac{C_{(1)} + \cdots + C_{([np])}}{C_{(1)} + \cdots + C_{(n)}} \qquad \frac{1}{n} < p < 1 \tag{3}$$

where $[np]$ is the integer part of np.

The quantity $T_+(p)$ is a concentration measure in the exact sense where the value of $T_+(p)$ quantifies the way of which the $100\,p\%$ upper positive contributions contribute to 100% of the aggregate positive performance. For example, if $p = 0, 2$, and if $T_+(0, 2) = 0, 8$, it means that 20% of the individual positive specific securities returns are responsible for more than 80% of the total positive return. Again we find the "law of $80/20$". When specific securities contribute about 10% (or 20% or other...) of the total performance for the period under analysis, the classical performance methods including the attribution analysis fail to exhibit this contribution. The methods proposed here allows to characterize portfolios where this concentrated contribution applies.

To build the Gini-Lorenz curve, we use the empirical index

$$T_n(k) = \frac{C_{(1)} + \cdots + C_{(k)}}{C_{(1)} + \cdots + C_{(n)}}$$

The table 2 details the method for tracing out the curve. The same methodology is used for tracing out the Gini-Lorenz curve of negative stocks contributions.

SET cumulative frequency (X-axis)	TOTAL POSITIVE PERFORMANCE cumulative positive ordered contributions	RELATIVE MASS cumulative frequency (Y-axis)
$1/n$	$C_{(1)}$	$T_n(1)$
$2/n$	$C_{(1)} + C_{(2)}$	$T_n(2)$
$\vdots$		$\vdots$
k/n	$C_{(1)} + C_{(2)} + \cdots + C_{(k)}$	$T_n(k)$
$\vdots$		$\vdots$
1	$C_{(1)} + C_{(2)} + \cdots + C_{(k)} + \cdots + C_{(n)}$	1

Table 2: Gini-Lorenz curve of positive stock picking concentration

To address the cumulative impact of *both* positive and negative concentration of specific securities returns, we use the same methodology as previous. We start from the total performance

$$R_P = \underbrace{C_{(1)} + C_{(2)} + \cdots + C_{(n)}}_{\text{ordered positive contributions}} + \underbrace{C_{(n+1)} + C_{(n+2)} + \cdots + C_{(N)}}_{\text{ordered negative contributions}}$$

next we drop out the upper contribution, and recaculate the new total performance as

$$R_P - C_{(1)} = C_{(2)} + C_{(3)} + \cdots + C_{(N)}$$

and for the two upper contributions

$$R_P - \left(C_{(1)} + C_{(2)} \right) = C_{(3)} + \cdots + C_{(N)}$$

etc.

1.3   A first look on the tails

The relationship between the order statistics and the empirical distribution function of a sample is well known. The empirical df $F_n(x)$ of a random variable X is defined by

$$F_n(x) = \text{Fr}(X \leq x) = \frac{1}{n} \text{card}\{i : 1 \leq i \leq n,\ X_i \leq x\} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{\{X_i \leq x\}}, \quad x \in \mathbb{R}$$

where $\mathbf{1}_A$ denotes the indicator function of the set A. Hence, for the market timing concentration, we have

$$F_n(\Delta R_{(k)}) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{\{\Delta R_i \leq \Delta R_{(k)}\}} = \frac{n-k}{n}$$

therefore

$$\text{Fr}(\Delta R_i \geq \Delta R_{(k)}) = 1 - F_n(\Delta R_{(k)}) = 1 - \frac{n-k}{n} = \frac{k}{n} \tag{4}$$

7

where Fr is the empirical cumulative distribution function of ΔR_k. A rank/ordering technique can now be used to trace out the charts of the two tails. The figures 1 and 2 present the results of this analysis. The systematic exploratory data analysis for the extremes of the portfolio is led for a further research.

insert figures 1 and 2 here

2 Results obtained with the portfolio of SMA BTP

The overall period of analysis for the portfolio of SMA BTP is : $31/12/2001 - 23/09/2004$, i.e. 719 days. The three subperiods are $31/12/2001 - 31/12/2002$ (266 days); $31/12/2002 - 31/12/2003$ (262 days); $31/12/2003 - 23/09/2004$ (191 days).

The global performance of the portfolio over 719 days is $-3,76\%$, to be compared to CAC 40 index $(-21,00\%)$ and Eurostoxx index $(-28,13\%)$. The number of securities over the period is 114, hence

$$R_P = \sum_{j=1}^{114} C_j = -3,76\%$$

The ordered sample of stock contributions $C_{(1)} > C_{(2)} > C_{(3)} > \cdots > C_{(n)}$ is

$$C_{\text{Areva}} > C_{\text{PernodRicard}} > C_{\text{Essilor}} > C_{\text{CréditLyonnais}} > \cdots > C_{\text{Sanofi}}$$

The results are summarized in the tables : 3 (market timing concentration), 4 (stock picking concentration). The table 5 details the most important stock performances, and their impact on the total performance of the portfolio.

insert tables 3, 4, 5 here

The Gini-Lorenz curves of market timing and stock picking concentration are traced out and given in figures 3 to 6. The figure 7 is a graphical illustration of table 5.

insert figures 3 to 7 here

	overall period		subperiod 1		subperiod 2		subperiod 3	
	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$
$J(0,10)$	0,30	0,35	0,30	0,35	0,30	0,30	0,30	0,25
$J(0,30)$	0,65	0,68	0,63	0,70	0,62	0,65	0,60	0,58
$J(0,50)$	0,85	0,85	0,83	0,87	0,82	0,85	0,80	0,80
$R(5)$	+8%	$-$8%						
$R(10)$	+15%	$-$14%						
$R(15)$	+20%	$-$20%						

Table 3: Market timing performance concentration

This table presents the results of market timing performance concentration for the four periods analyzed : overall (31/12/2001 – 23/09/2004), subperiod 1 (year 2002), subperiod 2 (year 2003), subperiod 3 (31/12/2003 – 23/09/2004). For each period, we give the results of $J_-(p)$ and $J_+(p)$ for $p = 10\%, 30\%, 50\%$. We note a relative stability of results : it appears that 10% of positive (resp. negative) returns are responsible for more than 30% of the aggregate positive (resp. negative) performance, 30% of positive (resp. negative) returns are responsible for more than 65% of the aggregate positive (resp. negative) performance, and 50% of positive (resp. negative) returns are responsible for more than 85% of the aggregate positive (resp. negative) performance. The last three lines $R(.)$ give the value of annualized performance recalculated after the withdrawn of the 5, 10, and 15 upper (resp. lower) returns of the overall period.

	$-$	$+$
$T(0,10)$	0,40	0,45
$T(0,30)$	0,75	0,75
$T(0,50)$	0,88	0,90
$R(5)$	+3%	$-$9%
$R(10)$	+7%	$-$11%
$R(15)$	+9%	$-$13%

Table 4: Stock picking performance concentration

This table presents the results of stock picking performance concentration for the overall period (31/12/2001 – 23/09/2004). We give the results of $T_-(p)$ and $T_+(p)$ for $p = 10\%, 30\%, 50\%$. It appears that 10% of positive (resp. negative) contributions are responsible for more than 40% of the aggregate positive (resp. negative) performance, 30% of positive (resp. negative) contributions are responsible for more than 75% of the aggregate positive (resp. negative) performance, and 50% of positive (resp. negative) contributions are responsible for more than 90% of the aggregate positive (resp. negative) performance. The last three lines $R(.)$ give the value of annualized performance recalculated after the withdrawn of the 5, 10, and 15 upper (resp. lower) contributions of the overall period.

Rank	STOCK	Ordered contributions	IMPACT OF CONCENTRATION : New total performance after withdrawn of cumulative contribution
1	Areva	+1, 94%	−5, 70%
2	Pernod Ricard	+1, 08%	−6, 78%
3	Essilor	+0, 92%	−7, 70%
4	Crédit Lyonnais	+0, 70%	−8, 40%
5	Technip	+0, 68%	−9, 08%
6	Spir comm.	+0, 59%	−9, 67%
7	Vinci	+0, 57%	−10, 24%
⋮	⋮	⋮	
108	Lafarge	−0, 79%	+5, 13%
109	France Telecom	−0, 92%	+4, 34%
110	Suez	−1, 11%	+3, 42%
111	Aventis	−1, 32%	+2, 31%
112	Scor	−1, 37%	+0, 99%
113	Vivendi	−1, 48%	−0, 38%
114	Sanofi	−1, 90%	−1, 86%
	TOTAL	−3, 76%	

Table 5: Stock picking concentration : impact of the best and worst bets

This table presents the seven best and worst bets of the management of the portfolio SMA BTP over the period 31/12/2001 – 23/09/2004, and their impact on the global performance. The global portfolio performance is −3, 76%, resulting from 114 specific contributions. The seven best choices are responsible for 6,48%, and the seven worst choices are responsible for −8, 89%. If management can avoid the seven worst stocks, the global performance grows up from −3, 76% to +5, 13%. But if the management misses the seven best stocks, the global performance falls from −3, 76% to −10, 24%. Now 7 stocks over 114 represent 6% of the global portfolio: that means 6% of the portfolio can gain +8, 89% (2,36 times the result) of loss −6, 48% (1,72 times the result) over two years of active management. The main conclusion is the extreme concentration of the performance on very few stocks which capture the main part of the global performance. See figure 7 for a graphical illustration of this phenomenon.

3   Concluding remarks

In this article, we have developed a general framework for evaluating the performance concentration of a managed portfolio. This framework provides a useful mean of conducting performance evaluations independent of asset pricing models. In this sense, it appears particularly useful to avoid model misspecifications of performance measures that build on equilibrium asset pricing models. Using this framework to assess existing performance concentration of the SMA BTP portfolio exhibits a clear phenomenon of performance concentration.

Further developments could adress :

1. The comparability of different portfolios by using the performance concentration indices, and the classification of the portfolios for which the 80/20 rule holds. The stability of the concentration index over different subperiods.

2. The characterization of the tail behaviour of the portfolio by using the extreme value theory, and the choice of a tail parameter to define the risk of the portfolio, instead of the volatility parameter.

3. The value added by an investment process not designed to follow a given benchmark.

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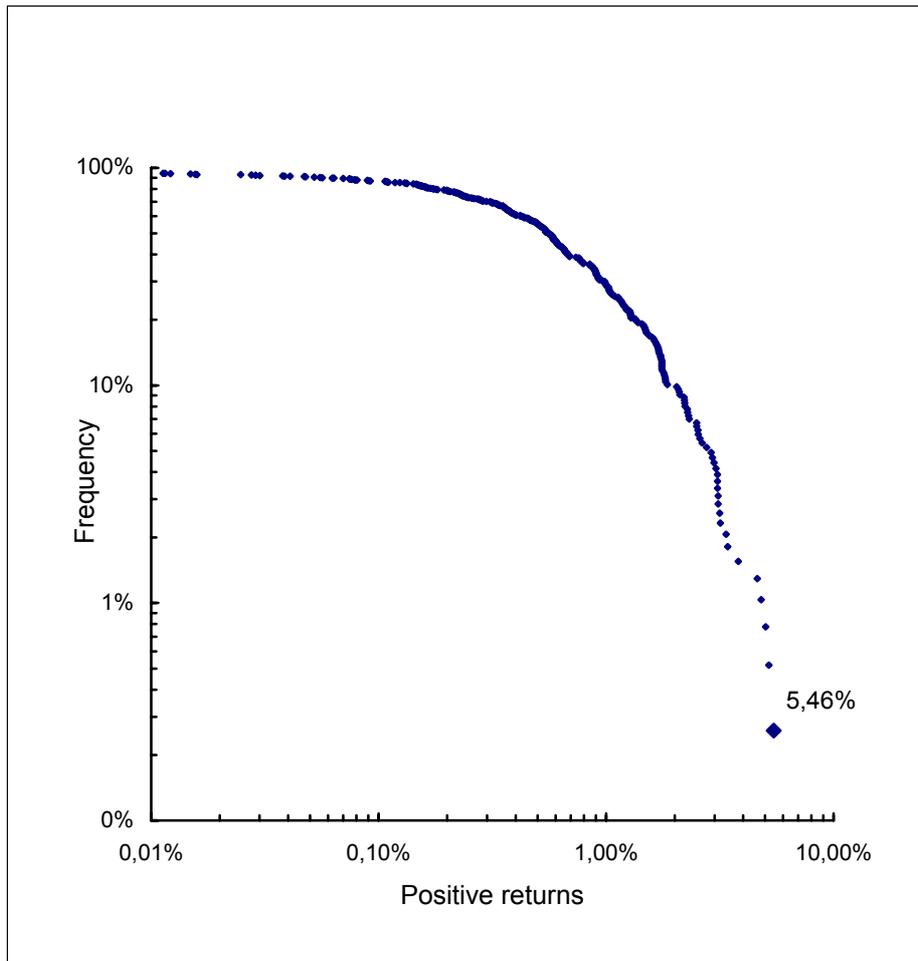
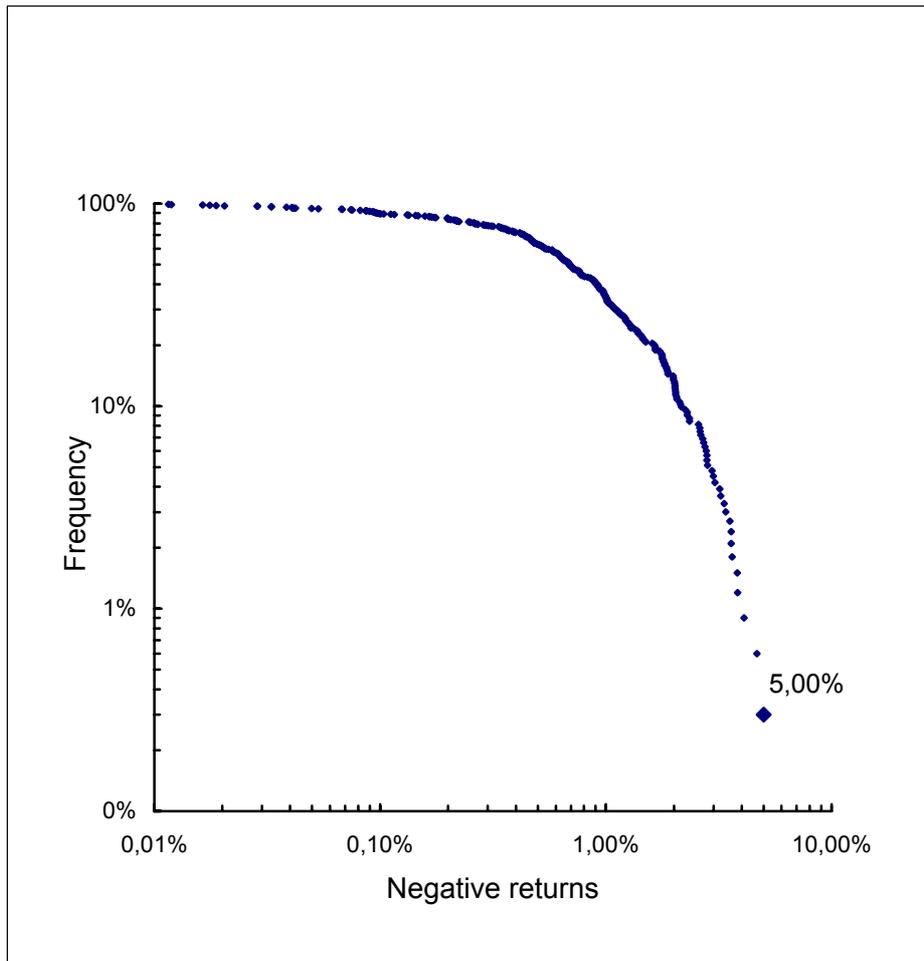Figure 1 : Positive tail of the managed portfolio of SMA BTP

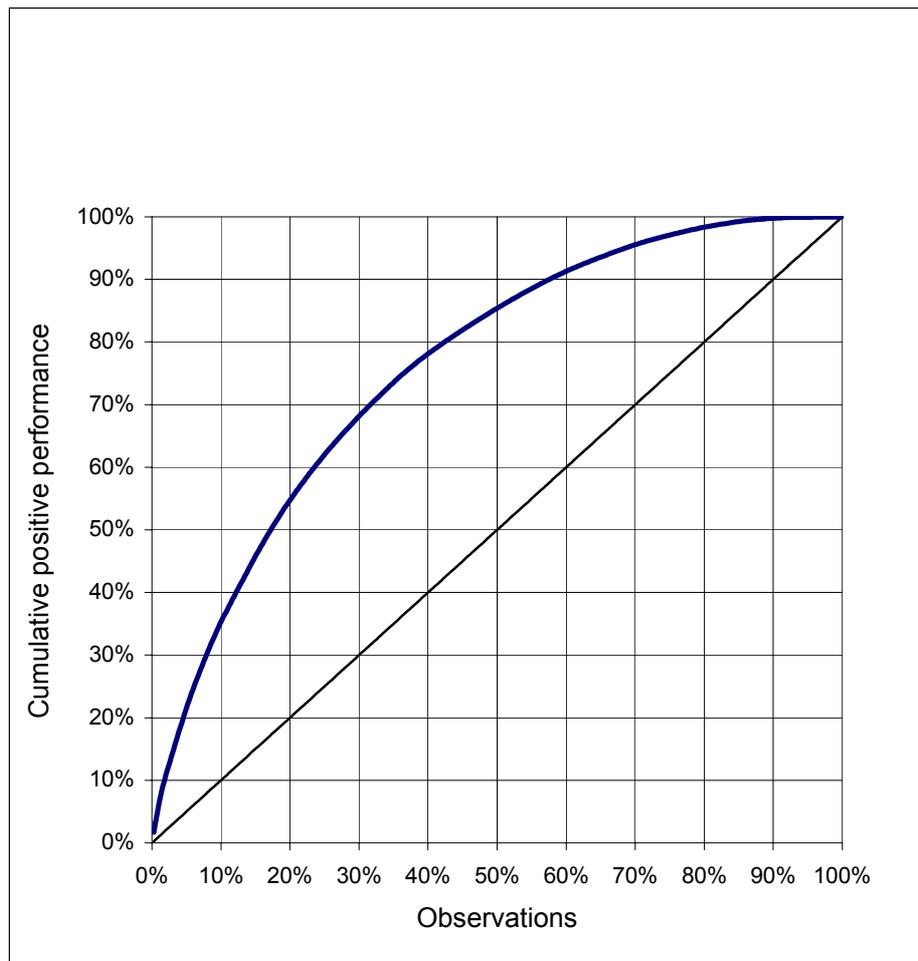Figure 2 : Negative tail of the managed portfolio of SMA BTP

Figure 3 :
Market timing concentration of positive performance of managed portfolio of SMA BTP

The above Gini-Lorenz curve exhibits the market timing concentration of the total positive performance of managed portfolio of SMA BTP over the period 31/12/2001—23/09/2004, i.e. 719 working days. Over 719 working days, there are 386 (100%) positive returns, ranked from 1 to 386. The X-axis represents these 386 positive returns (in % of the total) ranked by decreasing order, from the upper (on the left) to the lower (on the right). The Y-axis represents the cumulative positive performance (the total mass of positive performance) in % of the total. The curve exhibits the repartition of the cumulative contributions of the positive returns (days) as a function of the proportion of these days in the overall period. If each day contributed equally to the total mass of positive performance, then 50% of positive returns would be responsible for 50% of the total positive mass. We note that 50% of days are responsible for 85% of the total positive mass : $J_+(0,50) = 0,85$. Reciprocally, the contribution of half of the days is almost zero. More precisely, 10% of positive returns are responsible for 35% of the total positive mass : $J_+(0,10) = 0,35$. 30% of positive returns are responsible for 68% of the total positive mass : $J_+(0,30) = 0,68$. The structure of repartition is clearly concentrated.
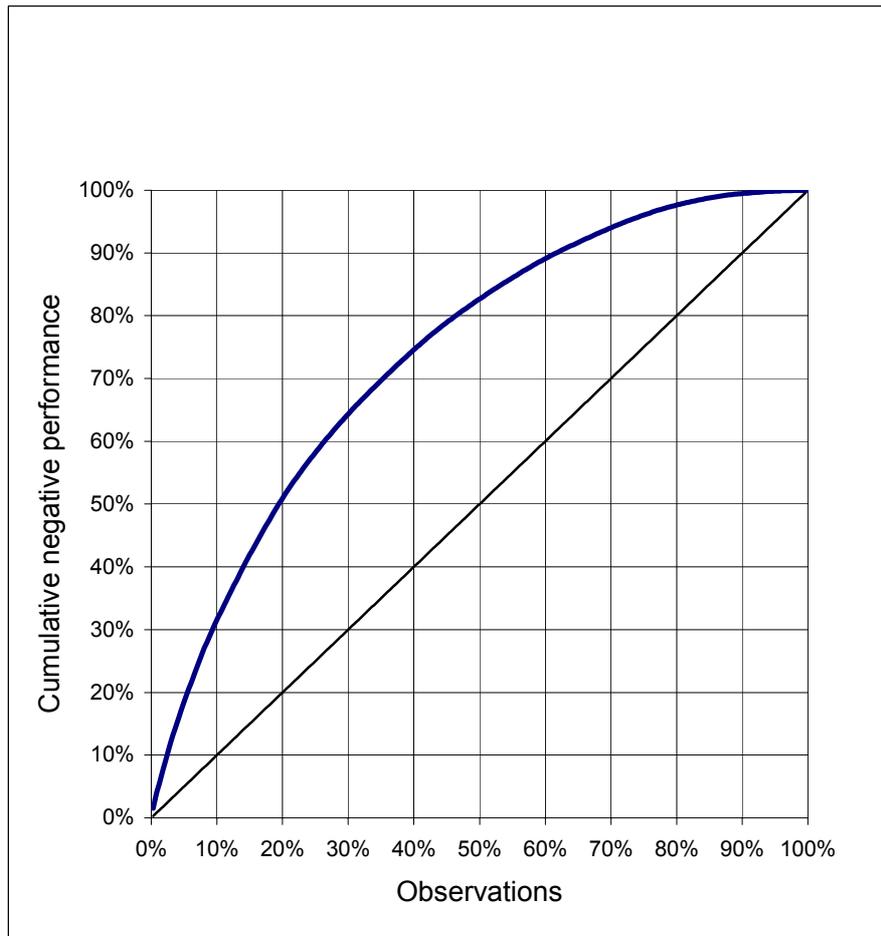
Figure 4 :
Market timing concentration of negative performance of managed portfolio of SMA BTP

The above Gini-Lorenz curve exhibits the market timing concentration of the total negative performance of managed portfolio of SMA BTP over the period 31/12/2001—23/09/2004, i.e. 719 working days. Over 719 working days, there are 333 (100%) negative returns, ranked from 1 to 333. The X-axis represents these 333 negative returns (in % of the total) ranked by decreasing order, from the upper (on the left) to the lower (on the right). The Y-axis represents the cumulative negative performance (the total mass of negative performance) in % of the total. The curve exhibits the repartition of the cumulative contributions of the negative returns (days) as a function of the proportion of these days in the overall period. If each day contributed equally to the total mass of negative performance, then 50% of negative returns would be responsible for 50% of the total negative mass. We note that 50% of days are responsible for 85% of the total negative mass : J (0,50) = 0,85. Reciprocally, the contribution of half of the days is almost zero. More precisely, 10% of negative returns are responsible for 30% of the total negative mass : J(0,10) = 0,30. 30% of negative returns are responsible for 68% of the total negative mass : J(0,30) = 0,65. The structure of repartition is clearly concentrated.
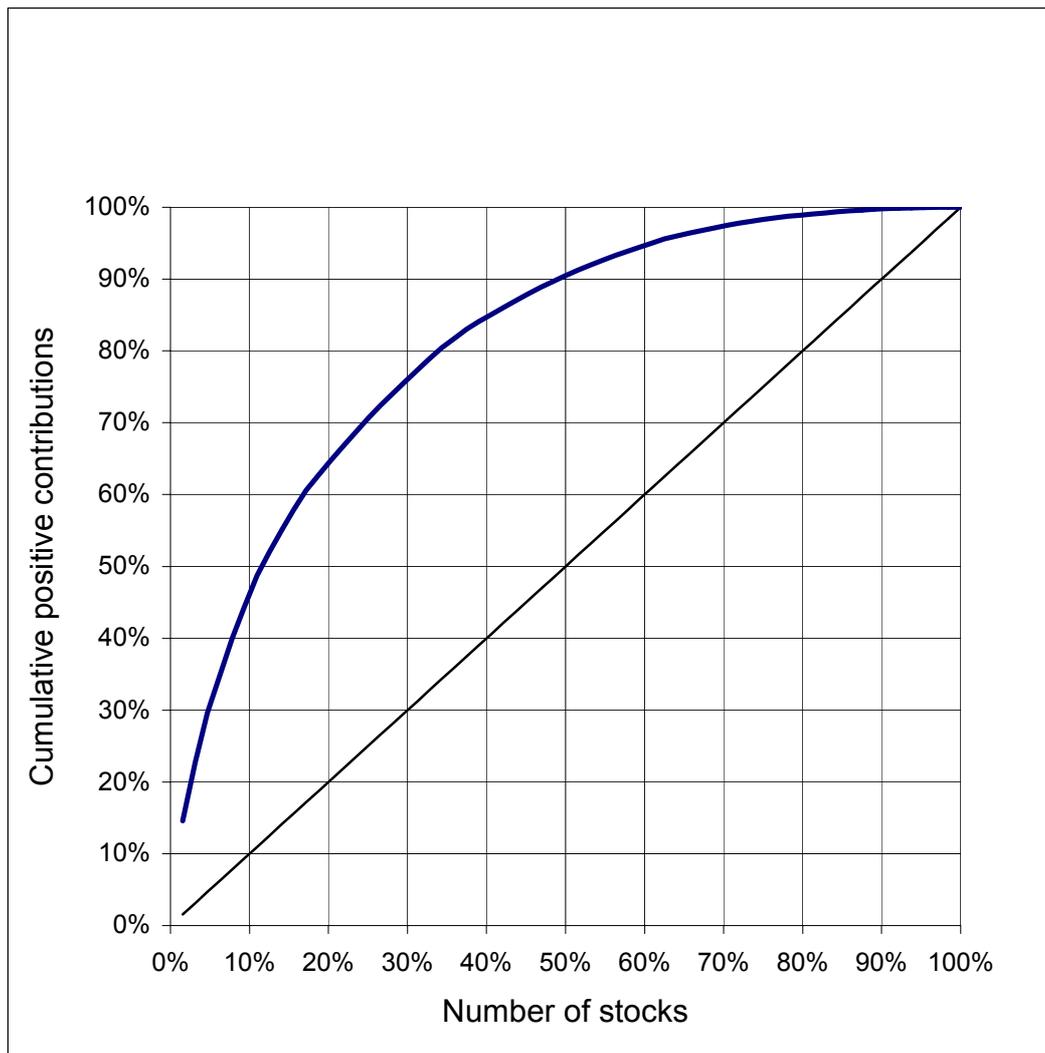
Figure 5 :
Stock picking concentration of positive performance of managed portfolio of SMA BTP

The above Gini-Lorenz curve exhibits the stock picking concentration of the total positive performance of managed portfolio of SMA BTP over the period 31/12/2001—23/09/2004. The number of stocks is 114. Over 114 stocks, there are 64 (100%) the contribution of which is positive, ranked from 1 to 64. The X-axis represents the number of "positive" stocks (in % of the total) ranked by decreasing order of positive contribution, from the upper (on the left) to the lower (on the right). The Y-axis represents the cumulative positive performance (the total mass of positive performance) in % of the total. The curve exhibits the repartition of the cumulative contributions of the positive stocks as a function of the proportion of these stocks in the overall period. If each stock contributed equally to the total mass of positive performance, then 50% of positive contributions would be responsible for 50% of the total positive mass. We note that 50% of positive stocks are responsible for 90% of the total positive mass : $T_+(0,50) = 0,90$. Reciprocally, the contribution of half of the stocks is almost zero. More precisely, 10% of positive stocks are responsible for 45% of the total positive mass : $T_+(0,10) = 0,45$. 30% of positive stocks are responsible for 75% of the total positive mass : $T_+(0,30) = 0,75$. The structure of repartition is clearly concentrated.
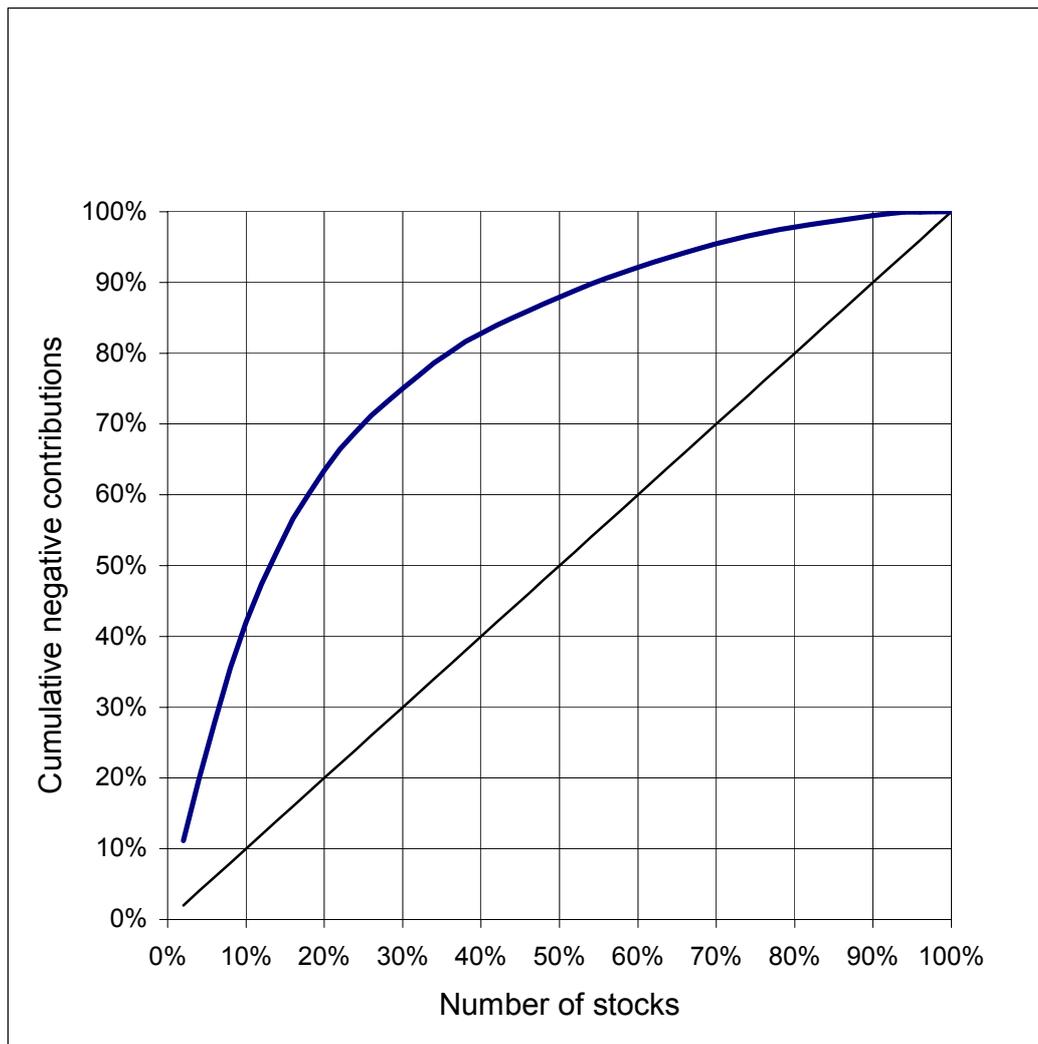
Figure 6 :
Stock picking concentration of negative performance of managed portfolio of SMA BTP

The above Gini-Lorenz curve exhibits the stock picking concentration of the total negative performance of managed portfolio of SMA BTP over the period 31/12/2001—23/09/2004. The number of stocks is 114. Over 114 stocks, there are 50 (100%) the contribution of which is negative, ranked from 1 to 50. The X-axis represents the number of "negative" stocks (in % of the total) ranked by decreasing order of negative contribution, from the upper (on the left) to the lower (on the right). The Y-axis represents the cumulative negative performance (the total mass of negative performance) in % of the total. The curve exhibits the repartition of the cumulative contributions of the negative stocks as a function of the proportion of these stocks in the overall period. If each stock contributed equally to the total mass of negative performance, then 50% of negative contributions would be responsible for 50% of the total negative mass. We note that 50% of negative stocks are responsible for 88% of the total negative mass : $T_-(0,50) = 0,88$. Reciprocally, the contribution of half of the stocks is almost zero. More precisely, 10% of negative stocks are responsible for 40% of the total negative mass : $T_-(0,10) = 0,40$. 30% of negative stocks are responsible for 75% of the total negative mass : $T_-(0,30) = 0,75$. The structure of repartition is clearly concentrated.
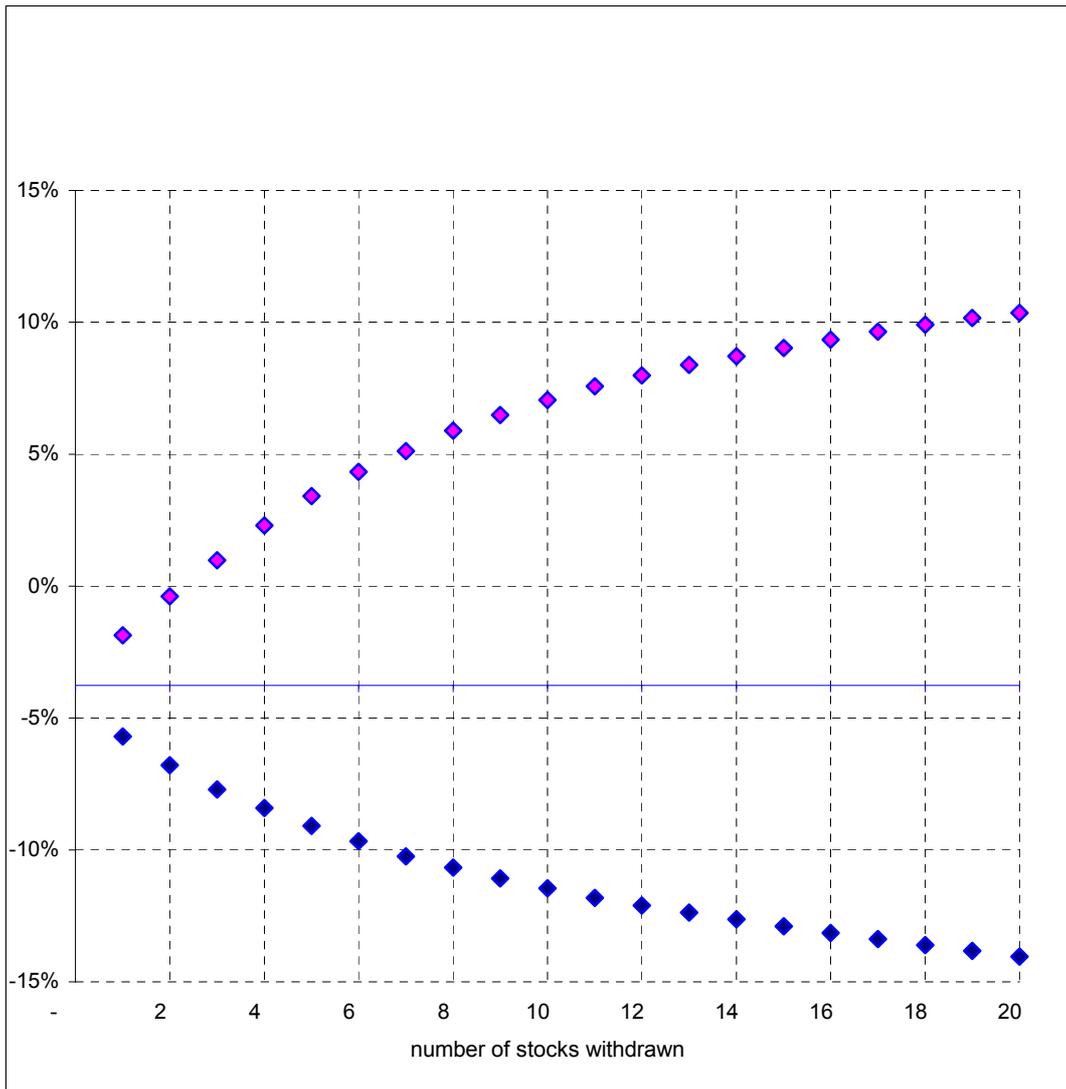
Figure 7 :
Stock picking concentration of total performance of managed portfolio of SMA BTP

The above curve exhibits the change of the total performance of managed portfolio of SMA BTP over the period 31/12/2001—23/09/2004 with the withdrawal of the best and worst stock picking bets. It is the graphical representation of table 5. We note the extreme concentration of the performance on very few stocks which capture the main part of the global performance or the global loss.