

Risk Neutral Valuation of With-Profits Life Insurance Contracts

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Abstract

The valuation of insurance contracts using concepts from financial mathematics, typically referred to as *Fair* or *Risk Neutral Valuation*, has recently attracted considerable interest in academia as well as among practitioners. We will investigate the valuation of so-called participating or with-profits contracts, which are characterized by embedded interest rate guarantees and some bonus distribution rules. We will focus on the specific regulatory framework in Germany, in particular cliquet-style guarantees; however, our analysis can also be applied to similar insurance markets.

We will present a framework, in which the different kinds of guarantees or options can be considered and priced separately. We will also discuss the practical implementation of such models. Our numerical analysis, in which we use a discretization approach and a Monte Carlo method to price the considered options, reveals information on fair parameter settings of the contract.

Our analysis reveals that life insurers offer embedded interest rate guarantees below their risk-neutral value. Furthermore, we find that the financial strength of an insurance company affects the value of the contract considerably. In addition, we study the sensitivity of the contract value with respect to the participation rules applied, the regulatory framework and the relevant asset value movements.

Keywords: with-profits life insurance contracts, risk-neutral valuation, interest rate guarantees, embedded options

1 Introduction

Many with-profits life insurance policies contain an interest rate guarantee. In many products, this guarantee is given on a point-to-point basis, i.e. the guarantee is only relevant at maturity of the contract. In other products (which are predominant e.g. in the German market), however, there is a so-called cliquet-style (or year-by-year) guarantee. This means that the policy holders have an account to which each year a certain rate of return has to be credited. Usually, the life insurance companies provide the guaranteed rate of interest plus some surplus on the policy holders' account every year. Considering the big market share of such products in many countries, the analysis of traditional life insurance contracts with a cliquet-style guarantee is a very important topic.

However, it has hardly been analyzed in the academic literature so far. Most of the existing work focuses on life insurance contracts where the development of the liabilities is linked directly to the performance of some reference portfolio, so called unit-linked, equity-linked or variable products (e.g. [1] or [11]).

A central feature in the analysis of with-profits policies with a cliquet style guarantee is a realistic model of bonus payments. For instance the approach presented in [6] explicitly models a bonus account, which permits smoothing the reserve-dependent bonus payments. Smoothing the returns is often referred to as the *“average interest principle”*. Aside from the guarantee and a distribution mechanism for excessive returns, Grosen and Jørgensen's model includes the option for the policy holder to surrender and *“walk away”* – in this case, the policy holder obtains his account value whereas the reserves remain with the company. Since the account value is path dependent, they are not able to present closed form formulas for the risk neutral value of the liabilities. Monte Carlo methods are used for the valuation and the analysis.

Similarly, in [10] a cliquet-style guarantee, a bonus account and a distribution mechanism are considered. Here, the bonus is modelled as a fixed fraction of the excessive return, i.e. the return exceeding the guaranteed level is distributed between the policy holders' account, the company's account and an account for terminal bonus. If the return drops below the guaranteed rate, the bonus account can be used in order to fulfill the guarantee. In particular, the bonus account can be negative, but the insurer has to consolidate a negative balance at the end of the insurance period. A positive balance is

however completely credited to the policy holders.

In [7], Miltersen and Hansen present a hybrid of the models by Grosen and Jørgensen ([6]) and Miltersen and Persson ([10]). They use the same model for the distribution mechanism as in [6], but the account structure from [10]. Besides a variety of numerical results, they focus on the analysis of the “*pooling effect*”, i.e. they analyze the consequence of pooling the undistributed surplus over two inhomogeneous customers.

In [2] and [3], Bacinello also considers cliquet-style guarantees. In [2], she prices participating insurance contracts with a guaranteed rate in a Black-Scholes market model. Here the bonus payments are also modelled as fixed fractions of the excessive return. She is able to find closed form solutions for the prices of various policies. In [3], she additionally permits for the surrender of the policy and presents numerical results in a Cox-Ross-Rubinstein framework.

Grosen, Jørgensen and Jensen introduce a different numeric approach to their valuation problem from [6] using the Black-Scholes Partial Differential Equation and arbitrage arguments in [5]. They show that the value function follows a known differential equation which can be solved by finite differences. Tankanen and Lukkarinen extend and generalize this approach in [9]. They use a finite mesh method in order to solve the differential equation. Their model permits for multiple distribution mechanisms, including those considered in [10] and [6].

However, these models can not be applied to e.g. German contracts, since the surplus distribution mechanisms considered in these papers are not consistent with German regulation and/or market practice, where hidden reserves are used to smooth annual surplus participation. In this case, the reserve quota is of great influence on the decision of how much surplus is distributed and thus on the value of an insurance contract.

The present paper fills this gap. In particular, our model for the development of the liabilities is very general. Surplus at time t can be determined and credited depending on the development of the assets (book or market value) and some management decision rule based on information available at time t . Furthermore, minimum surplus distribution laws that exist in many countries are considered. In particular, our model is general enough to represent all relevant features of the German market, including legal and supervisory issues as well as predominant management decision rules. Since most of the above mentioned models are included as special cases, the situation of other markets can also be modelled by a suitable choice of parameters.

We use a distribution mechanism that is typical for the German market which has been introduced in [8]. As opposed to [8] where the authors investigate, how the different parameters, such as the initial reserve quota, legal requirements, etc., affect the default probability of a contract and how these factors interact, we are interested in the fair value of the corresponding contracts.

The rest of this paper is organized as follows. In section 2 we introduce our model and the distribution mechanisms, i.e. the rules according to which earnings are distributed among policy holders, shareholders, and the insurance company. The goal of this paper is to find a fair price for an insurance policy using methods from financial mathematics. However, certain conditions must be fulfilled in order to obtain a “meaningfull” price. We will determine, under which circumstances a risk neutral valuation makes sense. Section 3 deals with this issue. Since the considered insurance contracts are complex and path-dependent derivatives, it is not possible to find closed form formulas for their price. Numerical methods have to be used. In section 4, we present a Monte Carlo Algorithm, which allows for the seperate valuation of the embedded options and an extension of Tanskanen and Lukkarinen’s finite mesh approach (see [9]), that allows us to consider a *surrender* or *walk-away option*. Our results are presented in section 5. Besides the values of the contracts and the embedded options, we examine the influence of several parameters and give economic interpretations. Section 6 closes with a summary of the main results and an outlook for future research.

2 Model

This section introduces our model. First, we model the reserve situation of the insurance company’s balance sheet. Our analysis of the interaction of assets and liabilities takes into account the ability of insurance companies to build up and dissolve hidden reserves over time. Afterwards, the insurance contract is considered and the corresponding liabilities are defined. Here, we refer to some specific aspects of German regulation and present two different kinds of distribution schemes.

2.1 Insurance Company

We use a simplified illustration of the insurer’s financial situation given in Table 1.

Assets	Liabilities
A_t	L_t
	R_t
A_t	A_t

Table 1: Model of insurer's financial situation

By A_t we denote the market value of the insurer's assets at time t . The liability side comprises two entries: L_t is the time t book value of the policy holders' account; the second account is the reserve account R_t , which is given by $R_t = A_t - L_t$, and consists of asset valuation or hidden reserves as well as other components, e.g. equity.

To simplify notation, we assume that payments to equity holders D_t occur once a year, at times $t = 1, 2, \dots, T$. Thus, $A_t^+ = A_t^- - D_t$ denotes the value of the assets after these payments.

2.2 Insurance Contract

For the sake of simplicity, we look at a very simple life insurance contract, a single-premium term-fix insurance and ignore any charges. The premium P is paid at $t = 0$, and a benefit is paid at time T , regardless if the policy holder is still alive or not. Thus, no mortality effects need to be considered. The benefit paid at time T depends on the development of the insurer's liabilities and is given by $P \frac{L_T}{L_0}$.

Even though this contract is not very common in most markets, it permits a nice interpretation: Given that the cost of insurance and the death benefit payments neutralize each other in every period, this term-fix insurance is an image of the company's general financial situation and thus the evolution of the liabilities can be thought of as the development of the insurer's liabilities as a whole.

2.3 Development of the Liabilities

The decision which surplus (i.e. interest exceeding the guaranteed rate) is given to the policy holders has to be made by the insurance company's management each year. Our general model allows for any management decision rule at time t that is \mathcal{F}_t -measurable. In the numerical analysis, however, we will focus on two different bonus schemes. The first one only considers obligatory payments to the policy holders (MUST-case). Addi-

tionally, we present a mechanism, which closely models the behavior of typical insurance companies in the German market over recent years (IS-case). Distinguishing these two methodologies is motivated by the different points of view of people interested in values of an insurance contract: Of course obligatory payments should be considered in any meaningful valuation. In addition, corporate political issues might be of interest for companies' actuaries, who are interested in the value of their product.

2.3.1 The MUST-case

In what follows, we include important features of the current German regulatory and legal framework. Nevertheless, specific aspects of other countries may be considered analogously.

Under German legislation, there must be a year-by-year cliquet-style guarantee on the liabilities. Currently, German life insurance companies guarantee the policy holders a minimum rate of interest of $g = 2.75\%^1$, i.e.

$$L_t \stackrel{!}{\geq} L_{t-1} (1 + g), \quad t = 1, 2, 3, \dots, T.$$

This guarantee has to be given for the whole term of the policy, even if the guaranteed rate will be changed by the regulators for new business. Thus, all policies that were sold when guaranteed rates were higher are still entitled to the guarantee rate that prevailed when the contracts were sold (e.g. 3.25% or even 4% p.a.). Therefore, the average guaranteed interest rate over the policy-portfolio of a typical German insurer is currently about 3.5%.

Furthermore, the law requires that at least $\delta = 90\%$ of the earning on book values have to be credited to the policy holders' accounts. Hence, δ is called the minimal participation rate. Since earnings on book values are subject to accounting rules, they are not necessarily equal to earnings on the market value $A_t^- - A_{t-1}^+$. Following [8], we assume that the insurer can always dissolve hidden reserves without restrictions by selling the corresponding assets. Building up reserves is, however, subject to restrictions. We assume that at least a portion y of the increase in market value has to be identified

¹More precisely, there is a maximum rate of return, policy reserves may be calculated with. Since this rate is used for almost all products, this implies that policy holders have a year-by-year guarantee of this interest on their account value

as earnings on book values in the balance sheet. Thus, we get

$$L_t = L_{t-1} \cdot \left(1 + \max \{g, \delta y (A_t^- - A_{t-1}^+)\}\right), \quad t = 1, 2, 3, \dots, T. \quad (1)$$

We assume that in the MUST-case the remaining portion of the earnings on book values is being paid out as dividends, i.e.

$$D_t = \max \{(1 - \delta) y (A_t^- - A_{t-1}^+), 0\}, \quad t = 1, 2, 3, \dots, T.$$

Finally, the policy holder has the possibility to cancel the contract at any policy anniversary date t_0 , receiving his account value L_{t_0} .

2.3.2 The IS-case

In the IS-case, we model actual behavior of typical German insurers. Obviously, bonus payments may not be lower than in the MUST-case. Thus, we get

$$L_t \geq L_{t-1} \cdot \left(1 + \max \{g, \delta y (A_t^- - A_{t-1}^+)\}\right). \quad (2)$$

We will now focus on a specific bonus distribution scheme that appears to prevail in Germany. In the past, German insurance companies have credited a stable rate of interest to the policy reserves each year. In adverse market conditions, they used hidden reserves that had been accumulated in earlier years to keep the surplus stable. Only when the reserves reached a rather low level, they started reducing the surplus. Therefore, we apply the following decision rule that has been introduced in [8]:

A target rate of interest $z > g$ is credited to the policy reserves, as long as the so called reserve quota $x_t = \frac{R_t}{L_t}$ stays within a given range $[a, b]$. Only if the reserve quota becomes too low (too high), will the surplus be reduced (increased). We assume, that the dividends amount to a portion α of any surplus credited to the policy reserves. Thus,

$$L_t = (1 + z) L_{t-1} \text{ and } D_t = \alpha (z - g) L_{t-1}, \quad t = 1, 2, 3, \dots, T$$

as long as this leads to $a \leq x_t \leq b$.

If crediting the target rate z implies a reserve quota below a and crediting the guaranteed rate g ends up with a reserve quota above a , then the company credits exactly the rate of interest that leads to $x_t = a$. Hence, we have

$$\begin{aligned} L_t &= (1 + g) L_{t-1} + \frac{1}{1 + a + \alpha} [A_t^- - (1 + g)(1 + a) L_{t-1}], \text{ and} \\ D_t &= \frac{\alpha}{1 + a + \alpha} [A_t^- - (1 + g)(1 + a) L_{t-1}]. \end{aligned}$$

If even crediting only the guaranteed rate of interest leads to a reserve quota level below a , then the guaranteed rate of interest is granted and no dividends are paid, i.e.

$$L_t = (1 + g) L_{t-1} \text{ and } D_t = 0.$$

If crediting the target rate of interest leads to a reserve quota above the upper limit b , the company credits exactly the rate of interest to the policy holders that meets the upper reserve quota boundary $x_t = b$, i.e.,

$$\begin{aligned} L_t &= (1 + g) L_{t-1} + \frac{1}{1 + b + \alpha} [A_t^- - (1 + g)(1 + b) L_{t-1}] \text{ and} \\ D_t &= \frac{\alpha}{1 + b + \alpha} [A_t^- - (1 + g)(1 + b) L_{t-1}]. \end{aligned}$$

Finally, we still need to check whether these rules comply with the minimum participation rate, i.e. if condition (2) is fulfilled. Whenever necessary, we increase the surplus defining L_t as in (1) and by letting

$$D_t = \alpha [\delta y (A_t^- - A_{t-1}^+) - g L_{t-1}].$$

3 Risk Neutral Valuation

As usual in this context, we assume that there exists a risk neutral measure Q under which payment streams can be valued as discounted expected values. Existence of this measure also implies that the financial market is arbitrage free. Using $(B_t)_{t \in [0, T]}$ as the numéraire process, the general pricing formula for a contract in our model is

$$P^* = E_Q \left[B_T^{-1} P \frac{L_T}{L_0} \right] \stackrel{L_0 = P}{=} E_Q [B_T^{-1} L_T]. \quad (3)$$

In contrast to unit linked products, the underlying security in our case is not traded on the financial markets. It is a portfolio of assets which is subject to management decisions and the company's investment. However, it is possible for a financial intermediary, e.g. an investment bank, to approximate the insurers reference portfolio by a traded benchmark portfolio, which permits the risk neutral valuation approach. In what follows, we call the relevant portfolio the reference portfolio.

The above formula (3) is only useful to price the contract in total, but not appropriate to analyze the embedded features (such as the guarantees). Hence, we choose a different approach. We assume that the insurer invests his capital in the reference portfolio A and leaves it there. Using this reference portfolio we can model the following two relevant cash-flows:

- Dividends are paid to the shareholders. These payments leave the company and thereby reduce the value of the reference portfolio but not its composition.
- If the return of the reference portfolio is so poor, that granting the minimum interest guarantee at time t would result in negative reserves, capital is needed in order to fulfill the obligations. This capital shot C_t increases the value of the reference portfolio. The composition stays the same.

Within the risk neutral valuation approach, we can now price these cash-flows. In case of the capital shot the time $t = 0$ risk neutral value is

$$F_0 = E_Q \left[\sum_{t=1}^T B_t^{-1} C_t \right].$$

Thus, F_0 can be interpreted as the value of the interest guarantee.

Similarly, one can determine the fair value of the dividend payments Z_0 and of the change of the reserve situation ΔR_0 , i.e.

$$Z_0 = E_Q \left[\sum_{t=1}^T B_t^{-1} D_t \right] \text{ and } \Delta R_0 = E_Q [B_T^{-1} R_T] - R_0.$$

For a “fair” contract, the value of the guarantee should coincide with the values of the dividend payments and the change of the reserve account, i.e.

$$F_0 \stackrel{!}{=} Z_0 + \Delta R_0, \tag{4}$$

since then the policy holder’s benefit from the guarantee is levelled out by the financial disadvantage due to dividend payments and undistributed final reserves. Thus, (4) represents an equilibrium condition for a fair contract. We also obtain an equivalent representation by taking into account, that for the value of the contract we have

$$P^* = E_Q \left[B_T^{-1} P \frac{L_T}{L_0} \right] = P + F_0 - Z_0 - \Delta R_0. \tag{5}$$

Therefore, the equilibrium condition (4) is equivalent to

$$P^* = E_Q \left[B_T^{-1} P \frac{L_T}{L_0} \right] \stackrel{!}{=} P. \tag{6}$$

As mentioned above, the policy holder has the possibility to surrender his contract at time t_0 and obtain his account value L_{t_0} . The reserves and the unneeded capital for granting the minimum interest guarantee in $[t_0, T]$ remain with the insurer. For

the sake of simplicity, we only allow for surrenders at the policy anniversary dates, i.e. $t_0 \in \{0, 1, 2, 3, \dots, T\}$. If $Z_{[t_0, T]}$ denotes the value at t_0 of dividend payments in $[t_0, T]$, and analogously, $F_{[t_0, T]}$ denotes the value at t_0 of future capital shots, the policy holders gain from surrendering is

$$\text{WA}_{t_0} = \max \left\{ Z_{[t_0, T]} + B_t E_Q \left[B_T^{-1} R_T \mid \mathcal{F}_t \right] - R_t - F_{[t_0, T]}, 0 \right\}.$$

Thus, the value of the walk-away option (surrender option) is given by

$$\text{WAO}_0 = \sup_{\tau \in \Upsilon_{[0, T]}} E_Q \left[B_\tau^{-1} \text{WA}_\tau \right],$$

where $\Upsilon_{[0, T]}$ denotes all stopping times with values in $\{0, 1, 2, 3, \dots, T\}$ ².

Considering the surrender option, the equilibrium condition (4) changes to

$$F_0 + \text{WAO}_0 \stackrel{!}{=} Z_0 + \Delta R_0.$$

However, (6) stays valid, since

$$P^* = E_Q \left[B_T^{-1} P \frac{L_T}{L_0} \right] = P + F_0 - Z_0 - \Delta R_0 + \text{WAO}_0. \quad (7)$$

Implicitly, by (5) and (7) we also answered the initial question for the risk neutral value of an insurance contract in our model. Even though this representation is equivalent to the one initially given by (3), it is more meaningful in our approach, since it takes into account the special circumstances when pricing insurance contracts. In particular, it allows for a separate valuation and thus hedging of the components of the contract.

4 Numerical Analysis

We assume a frictionless, arbitrage-free and continuous market. Ignoring payments to equity holders for a moment, we let A_t evolve according to a geometric Brownian motion with constant coefficients

$$dA_t = A_t (r dt + \sigma dW_t),$$

where W_t denotes a Wiener process on the stochastic basis $(\Omega, \mathcal{F}, Q, \mathbf{F} = (\mathcal{F}_t)_{t \in [0, T]})$, r denotes the risk free rate of interest, and σ denotes the volatility of the reference portfolio.

²Note that surrendering at time $t = 0$ can be interpreted as not concluding the contract.

Taking into account dividend payments and capital shots as defined in sections 2 and 3, we obtain

$$\begin{aligned} A_t^- &= A_{t-1}^+ \exp \left\{ \left(r - \frac{\sigma}{2} \right) + \sigma (W_t - W_{t-1}) \right\} \text{ and} \\ A_t^+ &= \max \{ A_t^- - D_t, L_t \}, \quad t = 1, 2, 3, \dots, T. \end{aligned}$$

This can be conveniently used in Monte Carlo algorithms. Furthermore, we let $B_0 = 1$ and obtain $B_t = e^{rt}$.

We implemented a standard Monte Carlo algorithm, which provides the possibility to separately determine the risk neutral price of a contract and the price of the embedded options. We also implemented a discretization approach extending the ideas of Tankskanen and Lukkarinen. In [9], they present a method based on discretization via a finite mesh. In what follows, we provide the basic ideas of this approach and our extensions. For a more detailed description of their model and our extensions, respectively, we refer to [9] and [4].

Between two policy anniversaries, differences in the value process $V(t)$ of the contract are only caused by movements in the asset process. Thus, $V(t)$ fulfills the Black-Scholes PDE. By a change of variables, this equation can be transformed into a one dimensional heat equation, from which an integral representation of the solution can be derived given the final value. The value process at $t = T$ is known ($V(T) = L_T$), and the value process is a function of the prior account values as state variables. Thus, we can obtain the value process at $t = T - 1$ given the state variables at $T - 1$.

Arbitrage arguments show that the value process has to be continuous at any policy anniversary, since the distribution of the surplus is carried out according to deterministic, known functions. This enables us to obtain $V(t^-)$ for given state variables by a transformation of the state variables according to the distribution mechanism. Again by solving the Black-Scholes PDE with given state variables at $t - 1$, one can obtain $V(t - 1)$ given these state variables. This leads to an iterative algorithm to obtain $V(0)$ for given state variables.

Since there is an infinite number of possible state variables, discretization methods are used. Account values are obtained by interpolation, and the integral representation of the solution for the Black-Scholes PDE is also approximated by interpolation methods.

As opposed to [9], in our model x_t describing the insurer's current reserve situation is needed as an additional state variable besides L_t and A_t . Since including this state

variable would increase the dimensionality of the problem and thus slow down the computations considerably, we examined the arbitrage arguments for a discontinuous asset process, and found that the argumentation remains valid. Thus, we can lower the dimensionality by letting A be discontinuous. However, additional and different interpolation schemes are needed, since now not only the transformation of the policy holders' account value L is necessary, but also A_t^- has to be transformed to A_t^+ .

This method enables us to consider non-European style contracts, e.g. contracts with a surrender option. We can price the value of the walk-away option as the difference of a European and a non-European contract with identical parameters. We used C++ for the practical implementation. Different distribution mechanisms can easily be incorporated in our program.

5 Discussion of Results

In what follows, we will provide risk neutral values of the contract and the considered embedded options. Furthermore, we will perform sensitivity analyses with respect to the most important parameters. Some dependencies are obvious and rather trivial; therefore, we will focus on the more complex and interesting relationships, in particular the interaction of several parameters. Unless stated otherwise, for all our calculations, we let the guaranteed rate of interest $g = 3.5\%$, the minimum participation rate $\delta = 90\%$ as motivated earlier, the insurer's initial reserve quota $x_0 = 10\%$, the target distribution $z = 5\%$, the reserve corridor $[a, b] = [5\%, 30\%]$, the portion of earnings that is provided to equity holders $\alpha = 5\%$, the asset volatility $\sigma = 7.5\%$, and the risk free rate of interest $r = 4\%$, since these values represent the situation and behavior of a typical German insurance company. Furthermore, we let the time horizon $T = 10$ and the initial investment $P = 10,000$. Besides, we assume the portion of market value earnings that has to be identified as book value earnings to be $y = 50\%$. An estimation of y seems to be rather hard to perform, since the accounting rules are rather complex. However, our results indicated only minor changes for $0\% \leq y \leq 60\%$ and since values of y above 60% seem unrealistic, we fix $y = 50\%$.

Base Case results

Table 2 shows the risk neutral value of the contract and its components.

	MUST-Case	IS-Case
initial investment	10,000.00	10,000.00
+ Value of Interest Guarantee	+ 868.42	+ 998.99
- Value of Dividends	- 238.16	- 74.36
- Change of Reserve	- 275.76	- 20.30
Fair Value European Contract	10,354.50	10,904.33
+ Value of Surrender Option	0	0
Fair Value American Contract	10,354.50	10,904.33

Table 2: Contract Values

Note that we used Monte Carlo methods for the calculation of all values except the value of the surrender option, that was calculated as the difference of the risk neutral value of the contract with and without the surrender option, using the Finite Mesh method. Due to discretization and interpolation errors within the Finite Mesh approach, there are slight differences to the results of the Monte Carlo simulation. However, the differences are negligibly small. In what follows, we use the Finite Mesh method for the calculation of the surrender option only. All other results are calculated using Monte Carlo techniques.

One can see from table 2, that the value of the contract (10,354.50) exceeds the initial investment (10,000) even in the MUST-case. This shows that the contracts are not fair according to our equilibrium condition from section 3. Insurance companies currently seem to offer their products with embedded interest rate guarantees under their fair value. Furthermore table 2 shows that the interest rate guarantee is an essential part of a life insurance contract. Its value equals 868.42 in the MUST-case and 998.99 in the IS-case.

In the MUST-case, any earnings that are not required to be credited to the policy holders' accounts are paid out as dividends. This is the reason why the value of the dividend payments is rather high in this case. The value of the final reserve and therefore the change of reserve tends to be higher in the MUST-case. This is obvious since higher ongoing surplus in the IS-case leads to a lower reserve quota.

For our base case assumptions, the surrender option is worthless in the MUST-case as well as in the IS-case. In what follows, we therefore only will consider European type

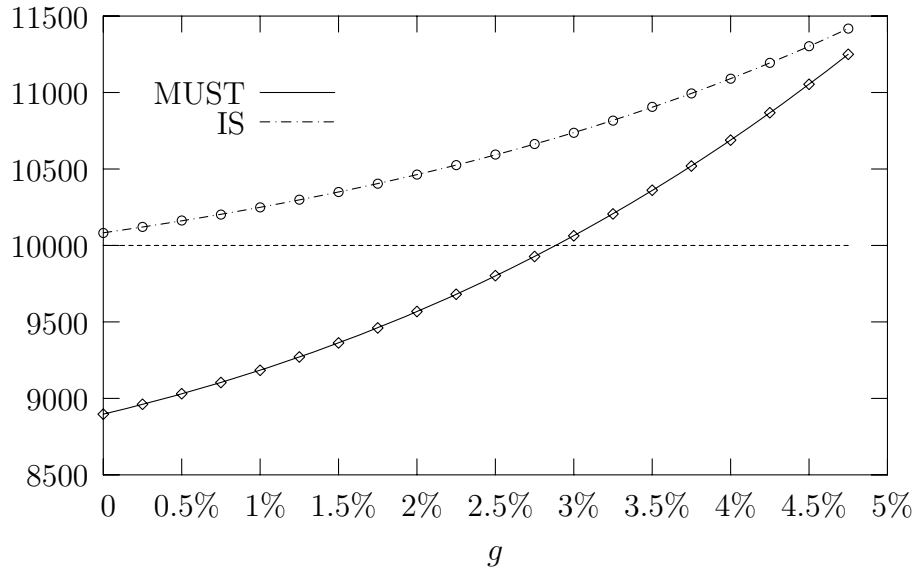


Figure 1: Influence of the guaranteed rate on the contract value

contracts unless stated otherwise.

The influence of the guaranteed rate of interest

Figure 1 shows the risk neutral value of the liabilities as a function of the guaranteed rate of interest g .

The value of a contract in the IS-case, i.e. under a realistic distribution scheme, exceeds the initial investment even for a guaranteed rate of 0%. This is because a rather high target rate of interest is credited as long as the reserve quota doesn't fall below 5%. This shows that in times of rather low market interest rates, life insurance companies give away valuable interest rate guarantees below value.

In the MUST-case, for $g = 2.75\%$ the contract value coincides with the initial premium. Thus, at current guaranteed rates a contract would be fairly priced if insurers only credited to the policy holders' accounts what they are required to.

Furthermore, the influence of g is more pronounced in the MUST-case. In the IS-case, the guaranteed rate is of less importance, since often the target rate is credited, which is assumed to be the same for all values of g . However, for growing g , the influence becomes more dominant in the IS-case, too. In particular, the two curves become close for large values of g , since then the guarantee is the dominant factor under both schemes.

The influence of the risk free rate of interest

In Table 3 contract values and the value of the surrender option are shown for different levels of the risk free rate r .

	$r = 3.5\%$	$r = 4\%$	$r = 5\%$
Fair Value European MUST-Case	10,775.80	10,360.60	9,612.42
Surrender Option	0	0	369.44
Fair Value European Is-Case	11,282.10	10,904.80	10,240.50
Surrender Option	0	0	102.80

Table 3: Contract Values with different levels of risk free rate

Obviously the contract value decreases when market interest rates increase since the guarantees become less valuable. In particular, the values decrease by about 6% when r is increased from 4% to 5%. This shows that the fair value of life insurance liabilities is rather sensitive to changes in r . Thus, extending our model to include stochastic interest rates in a next step seems worthwhile.

However, in the IS-case even for $r = 5\%$ the risk neutral value of the contract exceeds the initial investment. Also, it is worth noting that the surrender option becomes more valuable with increasing r since alternative assets become more and more attractive, in particular when the insurer's reserves are low.

The influence of the insurer's initial financial situation

Figure 2 shows the risk neutral value of the European contract as a function of the initial reserve quota x_0 .

The insurer's initial financial situation has a big impact on the contract value. In the IS-case, the contract value increases in x_0 since companies with higher reserves are more likely to be able to credit the target rate over a long period. This implies that customers should consider the financial situation of a company when buying participating life insurance, not only for credit risk reasons. Clearly, higher policy interest rates can be expected from insurers with higher reserves.

Also in the MUST-case, the insurer's financial situation is of importance. Since return on the reserves is part of the total return of a period, on average higher surplus has to be credited for higher values of x_0 . This influence is, however, rather small.

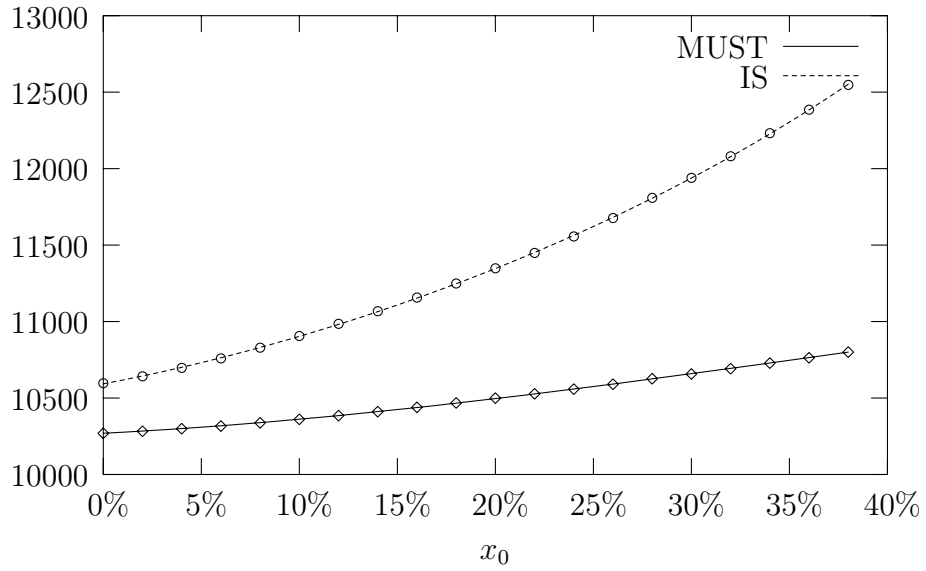


Figure 2: Influence of x_0 on the contract value in the MUST-case

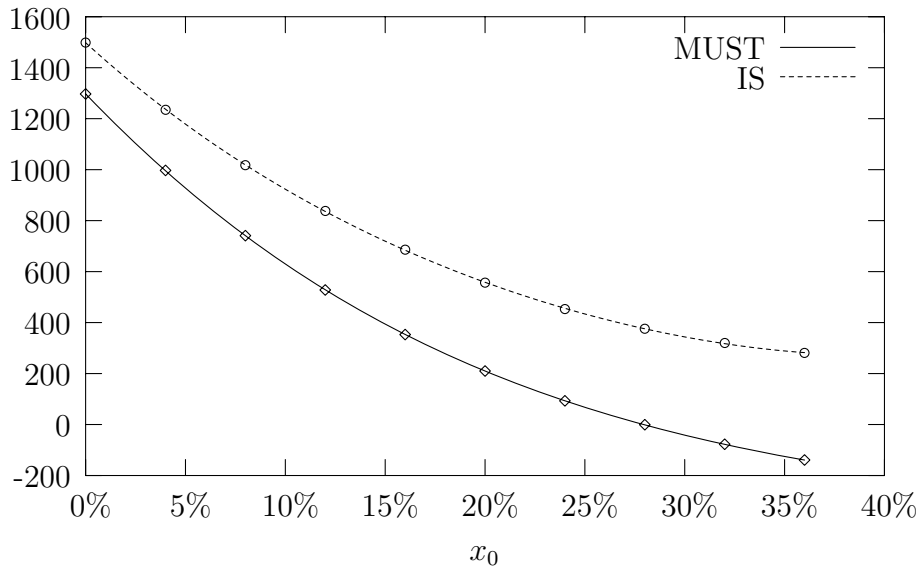


Figure 3: Influence of x_0 on the subsidy in the MUST-case

Figure 3 shows the risk neutral value at $t = 0$ of the subsidy that has to be provided to fulfill the interest rate guarantee as a function of the initial reserve quota x_0 . By subsidy, we mean the value of the capital shots less the value of the dividend payments.

As opposed to the contract value, the value of the subsidy is decreasing in the initial reserve quota x_0 . Companies with higher reserves need significantly lower capital shots since money only has to be provided from outside if reserves fall below 0. Thus, a strong financial situation is an advantage for the company as well as for the policy holder. Since this advantage is financed by the reserves, future policy generations will possibly not have a similar advantage.

The insurer's flexibility in the distribution of asset returns

The parameters y and δ describe the flexibility insurers have in distributing the returns. While y determines which portion of the market returns has to be shown as book return, δ determines which portion of the book return has to be given to the policy holders. Current German accounting rules give a lot of freedom (i.e. low values of y), but minimum participation rates imposed by regulators are rather high ($\delta = 90\%$). Since it is obvious that the contract value is increasing in δ and y , the interaction of these parameters is worth analyzing. If asset returns can no longer be used to increase hidden reserves (via y), insurers need the possibility to keep part of the book returns, i.e. to build reserves via δ .

Figure 4 shows combinations of δ and y , which lead to identical contract values of 10,000 units (fair contracts according to the equilibrium condition) and combinations of δ and y that lead to identical contract values of 10,360 units (contracts that have the same value as the base contract in the MUST-case).

It is obvious that more restrictive asset valuation rules (high y) have to be compensated by reducing the minimal participation rate δ , to keep the value of the liabilities on the same level. Regulatory authorities have to pay attention to the interdependence of the various parameters, in particular when changing accounting rules towards more market value oriented accounting, e.g. IFRS 4³.

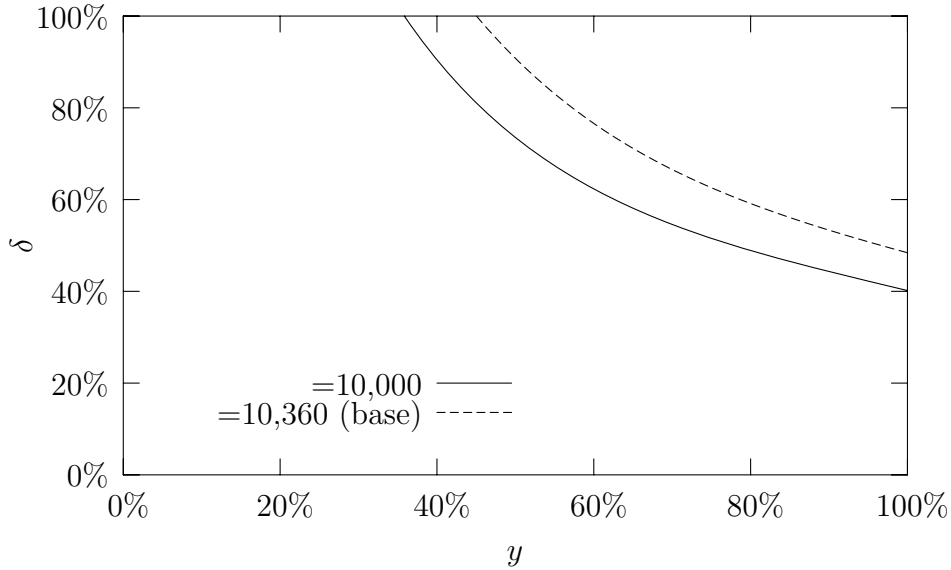


Figure 4: Parametercombinations of δ and y in the MUST-case

The interaction of guarantees and surplus distribution

Historically, German life insurers used to credit the same total rate of interest (i.e. guaranteed rate plus surplus) to different “generations” of contracts. Recently, some insurers credited a higher total rate to contracts with a lower guaranteed rate, since these contracts obviously bear a higher downside risk. Regulators intervened and claimed that this is an unfair discrimination amongst policy holders.

However, our model indicates the contrary. As can be seen in Figure 5, a decrease in g should be compensated by an increase in the target rate, in order to keep the liabilities on the same level. Thus, splitting the target rate does not constitute a discrimination of customers with higher guarantees but rather not splitting seems to be to the disadvantage of customers with lower guarantees.

However, this relationship is more complex, since other parameters also have an impact on the interaction of g and z . In Figure 6, the same relationship is shown, but this time for a company with higher reserves. We can see that the difference in target rates for different guarantees should be the lower, the higher reserves are. Thus, the insurer’s individual situation has to be considered when making such a decision.

³International Financial Reporting Standard 4 deals with the accounting of insurance liabilities.

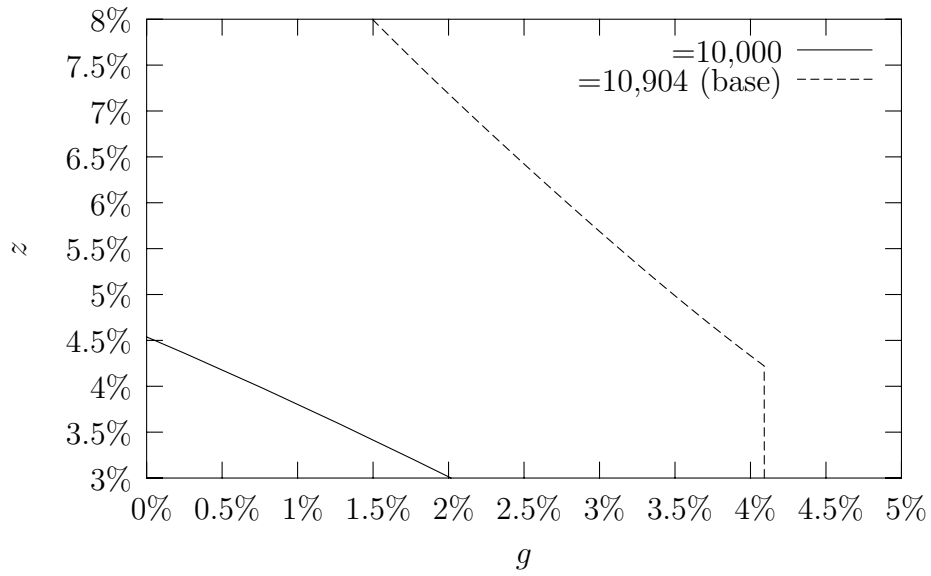


Figure 5: Parametercombinations g and z in the IS-case - $x_0=10\%$

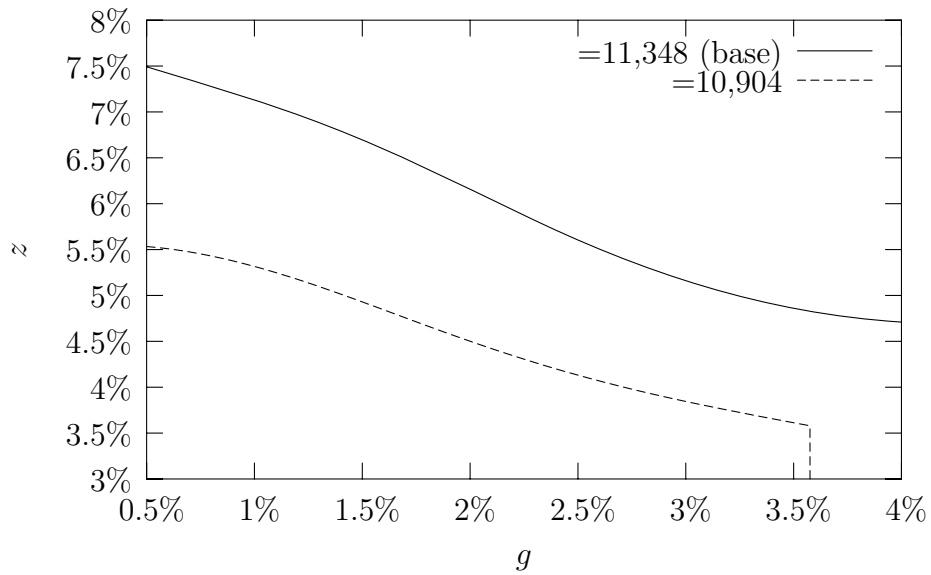


Figure 6: Parametercombinations g and z in the IS-Case - $x_0=20\%$

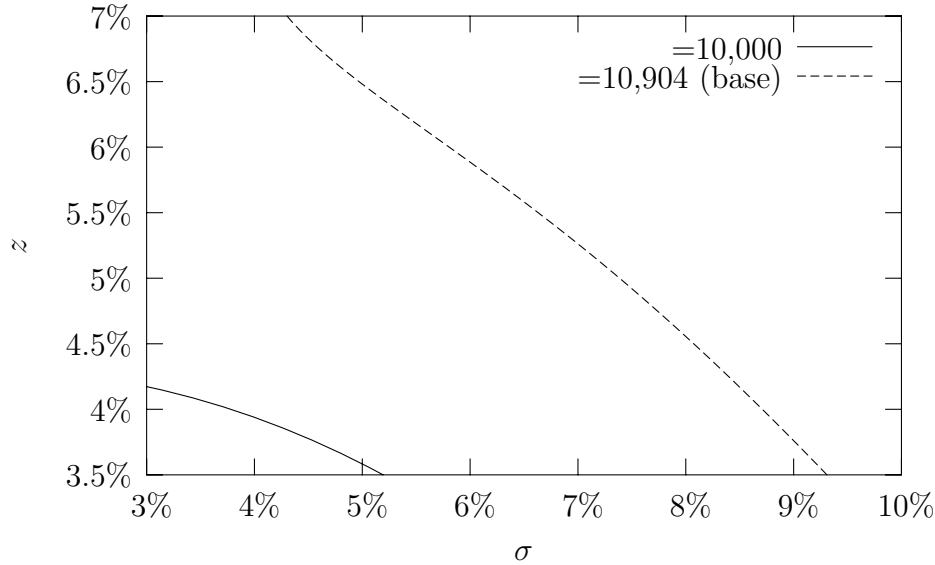


Figure 7: Parametercombinations σ and z in the IS-case

The interaction of asset allocation and surplus distribution

There are many other parameters that influence the contract value, for example, the asset allocation determining the volatility of the reference portfolio.

Figure 7 shows combinations of the asset volatility σ and the target rate of interest z , which lead to identical contract values of 10,000 units, and 10,904 units (base contract, IS-case), respectively.

With higher volatility, the policy holders' possibility to participate in high earnings is increased, while the downside risk is eliminated by the minimum interest rate guarantee. Thus, a conservative asset allocation strategy is advisable for the insurer, whereas the policy holder would benefit from high volatilities.⁴ For the base contract, changing the volatility by 1% can be compensated by changing the target rate by about 1.5% in the opposite direction.

If the insurance company aims to offer fair contracts, only rather conservative combinations of σ and z are acceptable, e.g. an asset volatility of $\sigma = 5\%$ combined with a target rate of interest $z = 3.5\%$ or an asset volatility of $\sigma = 3\%$ combined with a target rate of interest $z = 4.2\%$.

⁴If default risk is ignored.

6 Conclusion

We presented a model for evaluating and analyzing participating insurance contracts and adapted it to the German regulatory framework. Besides considering the obligatory payments (MUST-Case), we also included a distribution mechanism which is typical for German insurers (IS-Case). We applied the model to value and analyze contracts. We also discussed under which conditions and prerequisites a risk neutral approach is meaningful. We presented a cash flow model, which takes into account the special circumstances of the valuation of German insurance contracts and also provides the possibility to separately value and analyze embedded options and other components of the contract.

Since these types of contracts are complex and path-dependent contingent claims, we relied on numerical methods for the evaluations. Besides an efficient Monte Carlo algorithm, which enables us to consider the contract components separately, we presented a discretization algorithm based on the Black Scholes PDE, which allows us to include a surrender option.

We examined the impact of various parameters on the value of a contract. We found that this value is significantly influenced by the insurer's financial situation and the provided minimum interest and participation guarantees, whereas the surrender option is of negligible value. In particular, a contract with a financially strong company in general is more valuable. The fact, that the value of the contract exceeds the initial investment – and therefore the price – of a contract is alarming and partially explains current problems of the German life insurance industry. Furthermore, the ability to build up hidden reserves is crucial for offering these types of contracts. Thus, changing the financial reporting standards towards a more market based valuation of assets has to be compensated by weakening the interest or participation guarantees.

Our model supports the venture of several insurance companies in the German market to provide different target rates of interest for different contract generations with different guarantee levels. However, other factors than just the interaction of target and guarantee rates have to be taken into account in order to not disadvantage either “old” or “new” customers.

We particularly found that the interactions of the parameters describing the regulatory framework, the financial market, the insurance company's situation, and the

insurance contract are rather complex. An isolated analysis of the impact of one (set of) parameters does not seem appropriate. Since our results are rather sensitive to changes in the risk free rate of interest r and since the considered time horizon is rather long, including stochastic interest rates in the model should be the next step.

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