Investment Risks and the Solvency Margin

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Summary

Many statutory solvency requirements, e.g. the system prescribed by the EU, do not take into account the distribution of the investments. In practice, however, an adequate solvency margin of an insurance company depends strongly on the investment portfolio. This paper describes how the sufficient level of the solvency margin can be estimated. In this, only the margin depending on the company's investments is dealt with and the margin for the insurance risk is not included.

First, the paper describes the use of an asset liability simulation model for the calculation for the solvency margin. Then some formulas are developed for approximating the simulation results. These formulas depend on the means, standard deviations and correlation coefficients of the investment yields, as well as on the chosen risk level. In addition, the interest rate applied to the reserves is taken into account. Finally, the paper describes the new formula for the statutory solvency requirements of the Finnish employment pension institutions.

Keywords: solvency, investment risk, asset liability model, pension insurance

1 Introduction

The statutory minimum solvency margin used in the EU do not take into account the distribution of the investments of the insurance company. In practice, however, the solvency risk depends strongly on the investment portfolio. The risky assets, especially the shares, are likely to give the best returns in the long run, but the amount that can be invested in them is limited by solvency considerations.

When the investment portfolio and the solvency margin of a company are given, one can use various methods to estimate the probability of ruin, i.e. the probability that after some time the assets are no more sufficient to cover the liabilities of the company. The reverse of this problem is to estimate an adequate level of the solvency margin when the acceptable risk level is given. It is the latter problem that is dealt with in this paper.

The adequate solvency margin depends also on the length of time that is under consideration. In the following, the probability of ruin is calculated for one period in the future and, in practice, the period length is one year. Also, only the situation at the end of the period is considered. This means that the estimated margins are actually somewhat too low, since the ruin can also happen during the period even if the solvency margin is positive in the end. To estimate the effect of this, multiperiod of continuos methods could be used.

When estimating the adequate total solvency margin of an insurance company, the insurance risk should also be considered. However, this part of the margin is not treated here since only the effect of the investment portfolio is examined.

This paper is connected with the paper Tuomikoski (2000) in the AFIR 2000 Colloquium. The latter paper describes the system of the statutory solvency requirements of the Finnish employment pension scheme TEL. A basic concept in these requirements is the solvency border, which is calculated using a formula that depends on the investment distribution of the company. The purpose of this paper is to describe how the solvency border formula has been derived.

2 Simulation

A method to estimate the adequate solvency margin is to use an asset liability simulation model. This method is briefly described in this chapter.

The distribution of investments and the starting value of the solvency margin can be given as input to the model and the probability of ruin can be calculated from the simulation results. If there are N realisations and the number of ruins (non-positive solvency at the end of the period) is R, the estimated ruin probability ε is calculated as $\varepsilon = R/N$.

When the ruin probability ε is given, the objective is to determine the corresponding starting level of the solvency margin. It is not usually possible to find it directly, and so some kind of a search has to be conducted. First, a reasonable guess for the starting value U_1^0 of the solvency margin is selected for the simulations. The values of the solvency margin at the end of the period in N different realisations is denoted by $U_1^1, \dots, U_N^1$. The level of the solvency margin under which lie the proportion ε of these realisations is here denoted by $S_\varepsilon(U_1^0)$, where the dependence on the starting value U_1^0 is indicated. It can be calculated as

$$(1) \qquad S_\varepsilon(U_1^0) = U_{(k)}^1 \text{, where } k = \begin{cases} N\varepsilon & \text{when } N\varepsilon \text{ is an integer} \\ \lfloor N\varepsilon \rfloor + 1 & \text{otherwise} \end{cases}$$

where $U_{(1)}^1, \dots, U_{(N)}^1$ is the order statistics (the values in increasing order) corresponding to $U_1^1, \dots, U_N^1$.

Now the aim is to find the starting value U^0 so that $S_\varepsilon(U^0) = 0$, i.e. the ruin probability is ε when simulations are started from U^0. The initial value U_1^0 does not usually fulfil this condition directly. A better new starting value can be obtained by subtracting the amount $S_\varepsilon(U_1^0)$:

$$(2) \qquad U_2^0 = U_1^0 - S_\varepsilon(U_0^1).$$

Also this is the final solution only in simple cases. Complex models usually include decision rules that are dependent on the company's solvency margin. One may, for example, decide to give bigger bonuses for the customers or lower the premiums if the financial position of the company is strong. For this reason, changing the starting point also changes the shape of the distribution of the solvency margin at the end of the simulation period. Generally, therefore, $S_\varepsilon(U_2^0) \neq 0$. The value U_2^0 can, however, be used as the starting point of a new set of simulations, and the solution can be found iteratively in a few steps.

3 Some simulation results

As an example, a Finnish pension company was simulated using a stochastic asset liability model developed in Ilmarinen Mutual Pension Insurance Company. A description of the investment model used in the simulations is presented in Ranne (1998).

The investment portfolio in the example consists of 10% premium loans, 2.5% other loans, 2.5% market money (or cash), 50% bonds, 25% shares and 10% property. This is roughly similar to the present portfolio of Ilmarinen. When the ruin probability was set at 1% in the course of one year, the starting value of the solvency margin was found to be 9.47% of the reserves. The distribution of the solvency margin at the end of the year is shown in Figure 1. The mean value of the distribution is 11.58 and its standard deviation is 5.80 as per cent of the reserves.

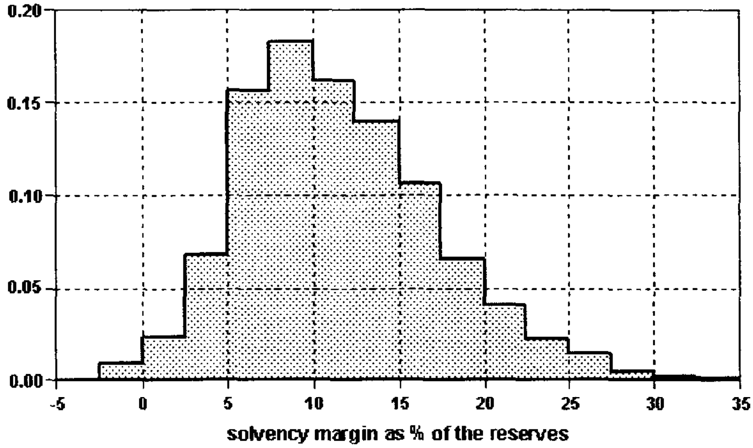

Figure 1. Distribution of the simulated solvency margin as per cent of the reserves after one year. The investment portfolio is 10% premium loans, 2.5% other loans, 2.5% market money, 50% bonds, 25% shares and 10% property. Starting value of the solvency margin is 9.47% corresponding to 1% probability of ruin.

It can be seen that the distribution has a fat tail to the right and, consequently, it is not normal. This can be seen also from the fact that 1% of the distribution lies left of the mean minus 2.00 times the standard deviation. For the normal distribution the corresponding point would be the mean minus 2.33 times the standard deviation. If the non-normality of the distributions is not taken into account, incorrect estimates of the adequate solvency margin may be made.

Next the proportions of shares and bonds in the investment portfolio were changed while the proportions of all the other asset classes were kept the same as before. The adequate solvency margins estimated by the simulation model are shown in Table 1. It can clearly be seen how the required margin strongly depends on the distribution of the investment portfolio.

Bonds %	60	55	50	45	40
Shares %	15	20	25	30	35
Solvency margin %	6.05	7.69	9.47	11.37	13.20

Table 1. The adequate solvency margin as % of the reserves when the ruin probability is 1%, calculated using the simulation model. The proportions of the asset classes other than bonds and shares are fixed: 10% premium loans, 2.5% other loans, 2.5% market money and 10% property.

4 Developing the basic formula

Because the calculation of a sufficient number of realisations by a simulation model takes some time, it is convenient to be able to get approximate results using a formula. Moreover, when the statutory solvency requirements for the Finnish employment pension scheme were being developed, it was decided they should be based on a mathematical formula common to all the pension institutions. The advantage of using a simulation model is that quite complex rules can be incorporated to describe the decision making of the company in different situations. Because it is not possible to include all these rules in a formula, the result is necessarily an approximation of the estimate obtained by simulation

The equation used for describing the development of the solvency margin of an insurance company in one year is

$$V_1 + U_1 = r(V_0 + U_0) + \sqrt{r}\,(P - X) - D. \quad (3)$$

Here V_0 and U_0 are the reserves and the solvency margin at the beginning of the year, V_1 and U_1 are the corresponding values at the end of the year, P is the premium income, X is the expenditure, D is the amount of dividends and $r = 1 + i$ is the interest rate factor corresponding to i, the total yield of the investments. For simplicity, equation (3) does not include the effect of the expenses. The term D can include both dividends to the shareholders and such bonuses as are immediately paid to the policyholders (e.g. as reductions of the premiums as in the Finnish TEL pension insurance).

That part of the yield that is transferred to the reserves is denoted by i_0 and $r_0 = 1 + i_0$ is the corresponding interest rate factor. It should be noted that i_0 includes not only the minimum guaranteed yield but also such additional bonuses to the policyholders, based on the investment profits, as are transferred to the reserves and used to increase future benefits or, alternatively, to reduce future premiums. In this paper the total yield i_0 is called the technical interest rate.

The equation connecting the reserves, the expenditure and the premiums is

(4) $$V_1 = r_0 V_0 + \sqrt{r_0}\,(P - X) \ .$$

In practice, the premiums are calculated based on a forecast of the expenditure and this is one cause of the variation of the total solvency margin of a company. However, as was mentioned earlier, this paper deals solely with the effect of the investment risks on the solvency margin and the fluctuations of the expenditure and the premiums is ignored. (The Finnish insurance companies have a separate part of the reserves, called the equalisation reserve, to act as a buffer against the insurance risks.)

Combining equations (3) and (4) results in

(5) $$V_1 + U_1 = r(V_0 + U_0) + \sqrt{r/r_0}\ (V_1 - r_0 V_0) - D.$$

For simplicity, the dividends D are from now on not included in the equations. The purpose of the formulas is to estimate a minimum required solvency margin, and it can be expected that a company in that position cannot probably pay a significant amount of dividends. However, in more precise calculations the known or estimated amount of the dividends paid during the year could be added to the required solvency margin.

Because it is approximately true that

(6) $$\sqrt{r/r_0} \approx 1 ,$$

equation (5) can be simplified to

(7) $$U_1 = r(V_0 + U_0) - r_0 V_0 = (1 + i)\, U_0 + (i - i_0) V_0.$$

Thus the solvency margin at the end of the year is the sum of the solvency margin at the beginning of the year, its total investment yield and the part of the yield on the reserves that exceeds the technical interest rate i_0.

The level of the solvency margin in proportion to the reserves at the beginning of the year is denoted by p and defined by the formula

(8) $$p = U_0 / V_0.$$

In the following p is called the solvency position of the company. Equation (7) is now transformed to

(9) $$U_1 = ((1 + i)p + i - i_0)\ V_0.$$

The requirement that the solvency margin is to remain positive with probability $1-\varepsilon$ can be expressed as

$$(10) \qquad \Pr\{(1+i)p + i - i_0 \le 0\} = \varepsilon$$

or equivalently

$$(11) \qquad \Pr\left\{ i - i_0 \le -\frac{p(1+i_0)}{1+p} \right\} = \varepsilon.$$

This is further simplified by the approximation

$$(12) \qquad \frac{1+i_0}{1+p} \approx 1.$$

It has been found that this approximation does not cause too great an error for the cases normally found when dealing with the Finnish pension institutions, if p is the level of the required solvency position calculated by the formulas below. In these cases p has been generally in the interval 5-12%, and the technical interest rate is currently 5.25%. However, if the proportion of shares in the investment portfolio grows very much, the error caused by approximation (12) may become significant and more complex formulas may have to be applied.

The equations (11) and (12) lead to the simple condition

$$(13) \qquad \Pr\{ i - i_0 \le -p \} = \varepsilon,$$

which is the equation on which the formulas presented in the sequel are based.

5 Constant technical interest rate

This chapter deals with the situation where the technical interest rate i_0 has a constant value. If $\mu[i]$ is the mean value and $\sigma[i]$ the standard deviation of the investment yield i, condition (13) leads to equation

$$(14) \qquad p = -(\mu[i] - i_0) + x_\varepsilon \sigma[i]$$

where x_ε is a coefficient depending on the chosen risk level ε.

Now, if the distribution of the investment yield were normal, the coefficient x_ε could be chosen as $\Phi^{-1}(1-\varepsilon)$, where Φ is the cumulative normal distribution function. For example, if the risk level was chosen as $\varepsilon = 2.5\%$, the coefficient would be $x_\varepsilon = 1.96$. However, as was pointed out in chapter 3, the yields are skewed to the right. If the value of x_ε from the normal distribution is used, the resulting estimate of the required solvency margin would be somewhat too high.

When the skewness of the distribution is taken into account, a problem is caused by the fact that it is not constant but depends on the composition of the investment portfolio. For example, the skewness of the yield is greater when the proportion of the shares grows. This means that the coefficient x_ε is not actually a constant but depends on the investment allocation. The solution adopted here is that x_ε is replaced by a constant, but one that is slightly lower than the value from the normal distribution.

The yields i_k of the different investment classes in the portfolio have the expected values μ_k, standard deviations σ_k and correlation coefficients ρ_{jk}. If the proportions of the classes are β_k (so that $\Sigma_k \beta_k = 1$), the mean value and the standard deviation of the whole portfolio are

$$(15) \qquad \mu[i] = \sum_k \beta_k \mu_k \qquad \sigma[i] = \sqrt{\sum_{j,k} \beta_j \beta_k \sigma_j \sigma_k \rho_{jk}}$$

Substituting these in equation (14) results in

$$(16) \qquad p = -(\Sigma_k \ \beta_k \mu_k - i_0) + a_\varepsilon \sqrt{\Sigma_{j,k} \ \beta_j \beta_k \sigma_j \sigma_k \rho_{jk}}\,,$$

where a_ε is the constant value replacing the variable x_ε. This is an approximate formula for the dependence of the required solvency position on the investment distribution.

6 Non-constant technical interest rate

When the technical interest i_0 rate is not constant, its dependence on the investment return i in equation (13) must be considered. This is the situation where the technical interest rate is determined each year in a way that takes into account the investment yields. For example, the chosen technical interest rate may be dependent on the general level of interest rates in the economy.

The previous chapter dealt with the mean value and the standard deviation of the investment yield i. When the technical interest rate i_0 is not constant, the corresponding values for the yield difference $i - i_0$ have to be estimated. These can be calculated in the same way as in equations (15):

$$(17)\qquad \begin{aligned} \mu[i-i_0] &= \sum_k \beta_k m_k \\ \sigma[i-i_0] &= \sqrt{\sum_{j,k} \beta_j \beta_k s_j s_k r_{jk}} \end{aligned}$$

where now m_k is the mean value of $i_k - i_0$, the difference between the return i_k and the technical interest rate i_0, s_k is its standard deviation and r_{ik} is the correlation coefficient between $i_j - i_0$ and $i_k - i_0$.

The values calculated in equations (17) lead to a formula for the solvency position analogous to (16):

$$(18)\qquad p = -\sum_k \beta_k m_k + a_\varepsilon \sqrt{\sum_{j,k} \beta_j \beta_k s_j s_k r_{jk}}\,.$$

If the mean value μ_0 and the standard deviation σ_0 of the technical interest rate are known, as well as its correlation coefficients $\rho_{0k}$with the returns of the various investment classes, the parameters in equation (18) can be calculated using the following formulas:

$$(19)\qquad \begin{aligned} m_k &= \mu_k - \mu_0 \\ s_k &= \sqrt{\sigma_k^2 + \sigma_0^2 - 2\sigma_k \sigma_0 \rho_{0k}} \\ r_{jk} &= \frac{(\sigma_j \sigma_k \rho_{jk} - \sigma_j \sigma_0 \rho_{0j} - \sigma_k \sigma_0 \rho_{0k} + \sigma_0^2)}{s_j s_k} \end{aligned}$$

The necessary values of the input parameters can be calculated form statistical data or, alternatively, estimated from simulation results if a stochastic asset liability model is available

7 Dependence of the interest rate on the solvency position

The previous calculations have been based on the assumption that the solvency position p and the technical interest rate i_0 in equation (13) are independent. This is not, however, necessarily true. If the solvency position p is good, the company may decide to pay to the policyholders higher bonuses (of the kind that are included in the term i_0) without risking its solvency.

In practice, the dependence of the technical interest rate on the solvency position p may be difficult to formulate. It is assumed here that the expected value of technical interest rate can be estimated to depend on the solvency position approximately by a linear formula

$$E[i_0 \mid p] = \mu_0 + \lambda p. \tag{20}$$

Using this equation, the expected value of the difference between the investment return and the technical interest rate can be expressed as

$$\mu[i - i_0] = \Sigma_k \; \beta_k m_k - \lambda p, \tag{21}$$

where the parameter m_k is defined as $m_k = \mu_k - \mu_0$. Based on this, equation (18) is transformed to

$$p = -\Sigma_k \; \beta_k m_k + \lambda p + a_\varepsilon \sqrt{\Sigma_{j,k} \; \beta_j \beta_k s_j s_k r_{jk}}\,. \tag{22}$$

Solving for p results in

$$p = -\frac{1}{1-\lambda} \Sigma_k \; \beta_k m_k + \frac{a_\varepsilon}{1-\lambda} \sqrt{\Sigma_{j,k} \; \beta_j \beta_k s_j s_k r_{jk}}\,. \tag{23}$$

8 The formula for the Finnish TEL pension scheme

The TEL pension scheme is part of the statutory pension system of Finland. The majority of the wage earners in the private sector, approximately one half of the total workforce, are insured in TEL. The scheme is run by six pension insurance companies and about 50 pension funds and foundations, which are called by the general name pension institutions.

The mechanism of the solvency requirements for the TEL institutions is described in detail in Tuomikoski (2000). The basis for the system of various limits is called the solvency border. When these solvency requirements were developed, it was decided that the solvency border should correspond approximately to the level where the probability of ruin in one year's time is 2.5%.

The formula for the solvency border in the Finnish statutory TEL pension scheme is based on equation (23). However, the formula was still modified a bit to the final form of

$$(24) \qquad p = c\,(-b \textstyle\sum_k \beta_k m_k + a \sqrt{\sum_{j,k} \beta_j \beta_k s_j s_k r_{jk}}\,).$$

The 2.5% ruin probability would correspond the value of 1.96 for the parameter a_ε in the previous formulas if the returns were normally distributed. As was pointed out in chapter 5, a somewhat lower value can be selected because of non-normality. When some examples of possible investment portfolios of TEL pension institutions were experimented with, the exact value for a_ε was found to vary between 1.65-1.96. The constant value selected was 1.83, which corresponds to the average investment portfolio of the TEL companies.

All the TEL pension institutions have a common technical interest rate whose value is decided for each year in advance. The method used in determining the technical interest rate is described in the paper of Ranne, Kivisaari and Mannonen (2000) in the AFIR 2000 Colloquium. The technical interest rate is determined so that it takes into account the average level of the solvency positions of the pension institutions. This is somewhat different from the situation described in the previous chapter where the interest rate depended on the company's own solvency position, not on the average of all the institutions. However, actually the solvency positions of all the pension institutions are strongly correlated because they operate in the same investment market. Therefore, the technical interest rate and the company's solvency margin are also correlated.

Based on the model used for determining the technical interest rate described in Ranne, Kivisaari and Mannonen (2000), the dependence of the technical interest rate on the solvency position can be estimated. The method is founded on the Markowitz efficient frontier model, and it results in a relationship that is not linear. However, the error in a linear approximation is not too great, and so equation (20) can be applied. Based on the values for the model's parameters selected by the Interest Rate Working Group, the value for parameter λ in equation (20) could be estimated to be 0.076.

The values of parameters a and b in equation (24) can now be calculated from

$$(25) \qquad a = \frac{a_\varepsilon}{1-\lambda} \text{ and } b = \frac{1}{1-\lambda},$$

with the results $a = 1.98$ and $b = 1.08$.

There is a further coefficient c in equation (24). The idea behind this parameter is the fact that the value of the technical interest rate can be lowered during the year if the solvency margins of the pension institutions are generally seen to be approaching a dangerously low level. This possibility decreases somewhat the required level of the solvency margin at the beginning of the year, and the value of the parameter c was set at 0.9. This value was chosen based on simulation experiments.

The other parameters m_k, s_k and r_{jk} in equation (24) were preliminary estimated using the simulation model. The values of some of the parameters were later revised based on the views of some investment experts. A further complication resulted from the decision to form the investment classes in the formula in a slightly different way from what was used in the model, and to add an extra class VII for all the investment instruments not included in the other classes. The final classification is outlined in Tuomikoski (2000), from which it is copied here:

Class I	Premium loans with TIR [technical interest rate] as interest rate; OECD money market instruments with high degree of security, etc.
Class II	Other premium loans; bonds issued by OECD states, communities or banks end denominated in EUR; housing with a certain type of state guarantee, etc.
Class III	Non-EUR bonds issued by institutions similar to those in Class II; EUR bonds issued by companies whose stock is traded on regulated OECD markets, etc.
Class IV	Housing, other than those included in Class II, other OECD bonds than those in Classes II and III, etc.
Class V	Other real estate than those in Classes II and IV
Class VI	Shares of OECD companies, traded on regulated OECD markets
Class VII	Other shares; investments not included in classes I-VI.

It was possible to estimate the parameters for these final investment classes based on the values used for the original categories. The final parameter values are presented in Table 2.

Class	m_i %	s_i %	r_{ij}						
I	0.1	1.0	1	-0.1	-0.2	0.0	0.0	-0.1	-0.1
II	0.6	3.5	-0.1	1	0.4	-0.1	-0.1	0.1	0.1
III	0.6	4.4	-0.2	0.4	1	-0.1	-0.1	0.1	0.1
IV	3.7	8.2	0.0	-0.1	-0.1	1	0.7	0.3	0.3
V	3.7	15.0	0.0	-0.1	-0.1	0.7	1	0.3	0.3
VI	6.2	21.4	-0.1	0.1	0.1	0.3	0.3	1	0.7
VII	6.2	29.9	-0.1	0.1	0.1	0.3	0.3	0.7	1

Table 2. Parameters of the solvency border formula (24)

Experiments made using various investment portfolios have shown that the solvency limits calculated according to equation (24) are generally not too far from the corresponding simulation results (when the parameters in the equation and the simulation model are consistent). Thus the solvency limits produced by the formula represent a constant risk level sufficiently well. However, when the proportion of the shares, or to a lesser extent the property, in the portfolio grows, the precision of the formula decreases. For such investment portfolios as are generally found in the Finnish pension institutions, the error has been considered not to be too great.

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