A Note on Interest Rate Guarantees and Bonus: The Norwegian Case

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Abstract

Interest rate guarantees, or more precisely, annual minimum rate of return guarantees, seem to be included in life insurance products in most countries. In a companion paper Miltersen and Persson (2000) a model of interest guarantees which includes a surplus distribution mechanism between the insurance company and the customer, is presented. The goal of this paper is to extend this model to capture more characteristics of real-world insurance markets. More specifically, two different levels of the annual guaranteed rate of return are allowed, one level for the initial investment amount, another level (typically lower) for surplus credited during the contract period. Moreover, the effect of the requirement of a strictly non negative bonus account by the end of each year throughout the contract period is analyzed.

1 Introduction

Interest rate guarantees, or more precisely, annual minimum rate of return guarantees, seem to be included in life insurance products in most countries. Due to the recent rather low international interest rate level, such guarantees are of great practical concern. Descriptions for Denmark, Germany, Japan, and the Netherlands may be found in Hansen and Miltersen (2000), Mertens (1999), Matsuyama (1999), and Donselaar (1999), respectively.

The starting-point of this article is a model developed in the companion paper Miltersen and Persson (2000). A contract between *the insurer* and *the customer* specifies a benchmark return and annual

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minimum rate of return guarantees in addition to a surplus distribution mechanism, i.e. a rule for the distribution of the annual investment return above the minimum rate of return guarantee between the insurer and the customer.

Although interest rate guarantees have many common features across different countries, the exact implementation of such guarantees may be different, which may lead to fundamentally different market values. In this extended model we include two features which are present (at least) in the Norwegian market.

The first extension is the possibility of two different levels of the annual guaranteed rate of return, one level for the initial investment amount, another level (typically lower) for investment surplus credited during the contract period.

An important part of our set-up is a *bonus account*. This account is managed by the insurer throughout the contract period. In years when the realized annual rate of return on the benchmark portfolio is greater than the minimum rate of return guarantee, a positive amount will typically be credited to the bonus account. On the other hand, if the realized annual rate of return on the benchmark portfolio is less than the minimum rate of return guarantee, funds are transferred from the bonus account to cover the minimum rate of return guarantee. A positive balance of the bonus account at the expiration is credited to the customer[1]. Throughout the contract period the balance of this account represents undistributed investment surplus, and the account can be seen as, in some sense, a buffer between good and bad years. Although the exact terminology may vary from country to country it seems like accounts similar to our bonus account both are present and serve important purposes in modern insurance/pension systems.

The second extension is the requirement of a strictly non negative balance of the bonus account *by the end of each year* throughout the contract period. We refer to this requirement as the non negativity requirement.

Our model works as follows: At date zero the customer deposits an amount X to the insurer. This amount can be distributed between the following four accounts (in principle, almost any allocation is possible, although some allocations are more natural than others.).

The first account A^1 is the customer's account where the interest is accrued to the initial deposit $A_0^1 \leq X$ according to the guaranteed minimum rate of return g_1. Let A_i^1 denote the balance of this account by the end of year i. Thus, the balance of account A^1 evolves as

$$A_i^1 = A_{i-1}^1(1+g_1). \tag{1}$$

The second account A^2 is also the customer's account. Potential investment surplus, i.e., a fraction α of the return that exceeds the guaranteed return, is credited this account, therefore the natural initial condition of this account is $A_0^2 = 0$. The balance of A^2 has a minimum rate of return guarantee g_2. Let δ and A_i^2 denote the investment return of the insurance company in year i and the balance of A^2 by the end of year i, respectively. Then

$$A_i^2 = A_{i-1}^1(\alpha(\delta_i - g_1)^+) + A_{i-1}^2(1 + g_2 + \alpha(\delta_i - g_2)^+). \tag{2}$$

The value of the insurer's policy by the end of year i is thus given by $A_i^1 + A_i^2$.

The third account is the insurer's account. In (good) years when the investment return is above the guarantee levels, a fraction of the investment surplus is credited this account. Another purpose of this account is to cover potential negative (end of year) balances of the bonus account. The parameter β determines the fraction of the positive excess return that is credited the insurers account C.

$$C_{i-} = C_{i-1} + A_{i-1}^1\beta(\delta - g_1)^+ + A_{i-1}^2\beta(\delta - g_2)^+. \tag{3}$$

The minus sign in the notation C_{i-} indicates that a potential deficit on the bonus account has not been taken into account yet. Here C_0 is some arbitrary initial allocation.

The bonus account is analyzed both with and without the requirement that its balance must be positive by the end of each year. Let B denote the bonus account. Let X_i denote the market value by

[1] In Denmark, the final balance of this account is not returned to the customer. The bonus account thus only serves as a magnification of the capital base, which provides the customer (as well as the insurer) with higher returns on his insurance. In Denmark bonus accounts are pooled together for large groups of customers, creating a number of additional issues, see Hansen and Miltersen (2000). However, in Norway no such pooling takes place.

the end of year i of the reference portfolio. We first calculate the balance of the bonus account residually without taking into account the non negative requirement. Then

$$B_{i-} = B_{i-1} + X_i - X_{i-1} - A_i^1 + A_{i-1}^1 - A_i^2 + A_{i-1}^2 - C_{i-} + C_{i-1}. \tag{4}$$

The following condition incorporates the non-negative requirement:

$$B_i = B_{i-} + B_{i-}^- = B_{i-}^+. \tag{5}$$

For this account B_0 is some arbitrary initial allocation. Finally, a potential negative balance of the bonus account is subtracted from the insurer's account.

$$C_i = C_{i-} - B_{i-}^-. \tag{6}$$

In order to demonstrate how this contract works 3 numerical examples are given in the following 3 tables. Each table consist of (the same) 2 scenarios of the reference portfolio, but the contract parameters, such as the levels of the guarantees or the initial allocation to accounts, differ. In all examples the allocations to the different accounts follow from equations (1), (2), (3), (4), (5), and (6).

(a) Scenario 1 'Good'

Year	Return	X	A^1	A^2	B	C
0		100	100	0	0	0
1	30%	130	110	10	5	5
2	30%	169	121	23	14	11

(b) Scenario 2 'Bad'

Year	Return	X	A^1	A^2	B	C
0		100	100	0	0	0
1	30%	130	110	10	5	5
2	0%	130	121	11	0	-2

Table 1: The base case: Example of distributions between the accounts for two given scenarios, 'good' and 'bad', with the following parameter values: $g_1 = g_2 = 10\%$, $\alpha = 50\%$, $\beta = 25\%$, $A_0^2 = B_0 = C_0 = 0$, and $A_0^1 = X = 100$.

The example in table (1) is essentially the same as a similar example in Miltersen and Persson (2000). The only difference is the non negative requirement of the bonus account (thus $B_2 = 0$ and $C_2 = -2$ in scenario 2 instead of $B_2 = -7$ and $C_2 = 5$).

(a) Scenario 1 'Good'

Year	Return	X	A^1	A^2	B	C
2	30%	169	121	22.5	14.375	11.125

(b) Scenario 2 'Bad'

Year	Return	X	A^1	A^2	B	C
2	0%	130	121	10.5	0	-1.5

Table 2: Example of different levels of the two minimum guarantee levels. Here $g_1 = 10\%$ and $g_2 = 5\%$, whereas the other parameters are as in table (1). The values for year 1 and 2 are as in table (2).

The effect of $g_2 < g_1$ is demonstrated in table (2). The final balance of A^2 decreases, whereas the final balances of B and C increase. The market value of the annual guarantees is lower for this contract than for the base case in table (1).

In table (3) only 50% of X is credited A^1, whereas the remaining 50% of X is credited B_0. This contract differ from the base cas in two important areas. The initial amount A_0^1 which is the base for the guarantee g_1 is lower. The initial balance, and thereby also susequent balances of the bonus account are higher. Both these effects reduce the value of the annual guarantees compared to the base case.

(a) Scenario 1 'Good'

Year	Return	X	A^1	A^2	B	C
0		100	50	0	50	0
1	30%	130	55	5	67.5	2.5
2	30%	169	60.5	11.5	91.5	5.5

(b) Scenario 2 'Bad'

Year	Return	X	A^1	A^2	B	C
0		100	50	0	50	0
1	30%	130	55	5	67.5	2.5
2	30%	169	60.5	5.5	61.5	2.5

Table 3: Example of different initial allocation. Here $A_0^1 = 50$ and $B_0 = 50$, whereas the other parameters are as in table (1).

In these 3 examples the parameter values α, β, g_1, and g_2 are given. The main purpose of this paper is to determine, in a specific sense, consistent values of these four parameters.

The article is organized as follows: In the following section we decribe the model including logarithmic returns and the valuation principle. In section 3 a number of numerical examples is presented. Finally, section 4 concludes.

2 The model with logarithmic returns

From now on we work with logarithmic (or continuously compounded) returns in contrast to the arithmetic returns used in our initial examples. The equations (1), (2), and (3) must be modified[2] as follows:

$$A_i^1 = A_{i-1}^1 e^{g_1}. \tag{7}$$

$$\begin{aligned} A_i^2 &= A_{i-1}^1 \left(e^{g_1+\alpha(\delta_i - g_1)^+} - e^{g_1}\right) + A_{i-1}^2 e^{g_2+\alpha(\delta_i - g_2)^+} \\ &= A_i^1 \left(e^{\alpha(\delta_i - g_1)^+} - 1\right) + A_{i-1}^2 e^{g_2+\alpha(\delta_i - g_2)^-}. \end{aligned} \tag{8}$$

$$C_{i-} = C_{i-1} + A_{i-1}^1 \left(e^{\beta(\delta_i - g_1)^+} - 1\right) + A_{i-1}^2 \left(e^{\beta(\delta_i - g_2)^+} - 1\right). \tag{9}$$

The equations (4), (5), and (6) are on the same form as before and are not repeated here.

Assume that the interest rate r is constant and that the annual continuously compounded rate of return from the benchmark portfolio, δ_i, is normally distributed and independent over different years. Hence δ can be modeled (under an equivalent martingale measure Q) as

$$\delta_t = r - \frac{1}{2}\sigma^2 + \sigma(W_t - W_{t-1}). \tag{10}$$

[2]Keep in mind the relationship $1 + G = e^g$ between a simple arithmetic annual return G and the corresponding continuously compounded annual return rate g. Our notation does not emphasize this distinction.

To explain the development of equation (8) from equation (2), observe that

$$A_{i-1}^1(\alpha(\delta_i - g_1)^+) = A_{i-1}^1(1 + g + \alpha(\delta_i - g_1)^+ - (1 + g)).$$

The continuously compounded return version of the latter equation is

$$A_{i-1}^1(e^{g+\alpha(\delta_i - g_1)^+} - e^g),$$

again (mis-)using the same symbols for arithmetic and continuously compounded returns. Now equation (8) follows immediately.

where σ is the volatility of the rate of return on the benchmark portfolio and $W = \{W_t, t \geq 0\}$ is a standard Wiener process under the probability measure Q. Note that we have implicitly assumed that there are no dividend payments[3] on the assets included in the benchmark portfolio since the drift term of δ in equation (10)) is $r - \frac{1}{2}\sigma^2$. The return on the benchmark portfolio follows the process in equation (10) if e.g. we assume that the price process of the benchmark portfolio follows a standard geometric Brownian motion as e.g. in the Black-Scholes model, cf. Black and Scholes (1973) or Merton (1973).

Before the valuation principles are explained, the following standard T-account may be illuminating:

Assets	Liabilities
X_T	$A_T^1 + A_T^2 + B_T^+$
	C_T
X	X

The first post on the liability side represents the customer's claim at expiration, the two A accounts and the positive part of the B account.

In our model, cf. Miltersen and Persson (2000), the fundamental valuation principle dictates that initial the market value of the asset side of the balance must equal the initial market value of the first post on the liability side side, $A_T^1 + A_T^2 + B_T^+$. If we denote the market value at time t by $V_t(\cdot)$,

$$V_0(X_T) = V_0(A_T^1 + A_T^2 + B_T^+).$$

Obviously $V_0(X_T) = X_0 = X$ and we may write

$$X = V_0(A_T^1) + V_0(A_T^2) + V_0(B_T^+). \tag{11}$$

In actuarial terms the amount X represents the single premium of the contract. Equation (11) has an intutive interpretation: The market values at the inception of the contract of the future benefits from the contract equals the single premium, an interpretation in the spirit of the classical principle of equivalence.

The market values of A_T^1 and A_T^2 may be calculated in closed-forms, but no closed-form solution for $V_0(B_T^+)$ is available. In order to find consistent combinations of g_1, g_2, α, and β we need to resort to numerical methods.

First note that a simple consequence of our valuation principle is that

$$V_0(C_T) = 0.$$

We can therefore conclude that the initial market values of the capital inflows to account C is exactly identical to the initial market value of the capital outflows of account C. This can be written as

$$V_0\left(\sum_{i=0}^{T-1}\left(A_{i-1}^1\left(e^{\beta(\delta,-g_1)^+}-1\right)+A_{i-1}^2\left(e^{\beta(\delta,-g_2)^+}-1\right)\right)\right) = V_0\left(\sum_{i=1}^{T} B_{i-}^-\right).$$

3 Numerical results

We have implemented a numerical simulation algorithm in order to calculate the initial market values. On top of the simulation algorithm there is a numerical search algorithm searching for combinations of g_1, g_2, α, and β such that equation (11) is satisfied. For all examples we consider a 5-year contract, i.e. $T = 5$.

In figure (1) it is clear that when $g_2 < g_1$ the value of β is reduced for a given α compared to the case $g_2 = g_1$. The parameter β is connected to the financial option premium (cf. Miltersen and Persson (2000)) connected to the contract. A high β means a high option premium and vice versa. We may conclude from the figure that when $g_2 < g_1$ the financial option premium is lower than for the case $g_2 = g_1$ and the remaining contract parameters (α and β) may be improved, seen from the customers point of view, when $g_2 < g_1$ compared to the case when $g_2 = g_1$. From the figure it is also clear that the effect of reducing g_2 with 0.02 relative to g_1 is more pronounced for larger values of α. Finally, this effect is larger for $g_1 = 5\%$ than for $g_1 = 3\%$, and thus seems to be increasing in the level of g_1.

[3] An equivalent interpretation is that potential dividends on the assets included in the benchmark portfolio are immediately reinvested into the benchmark portfolio.

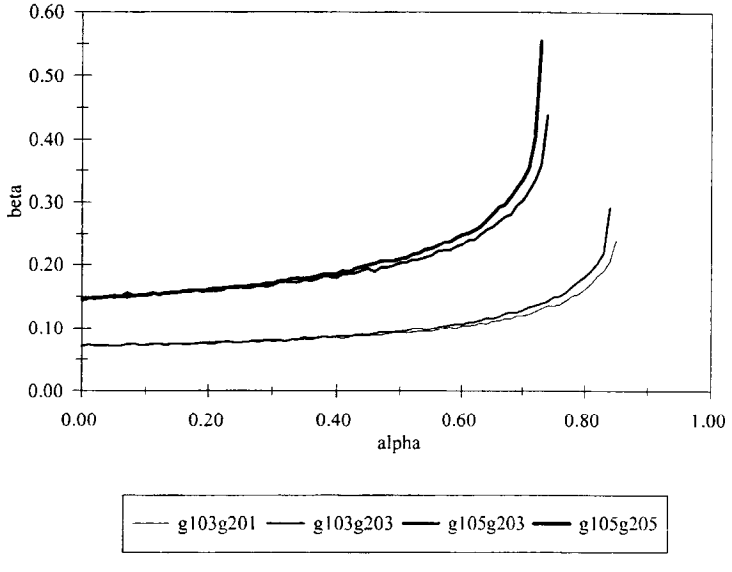

Figure 1: Corresponding values of α and β that imply fair contracts *including* the non negativity requirement for account B for four different combinations of guarantees ($g_1 = 3\%$, $g_2 = 1\%$; $g_1 = 3\%$, $g_2 = 3\%$; $g_1 = 5\%$, $g_2 = 3\%$; $g_1 = 5\%$, $g_2 = 5\%$), $r = 10\%$, $A_0^1 = 1$, $A_0^2 = 0$, and $B_0 = 0$.

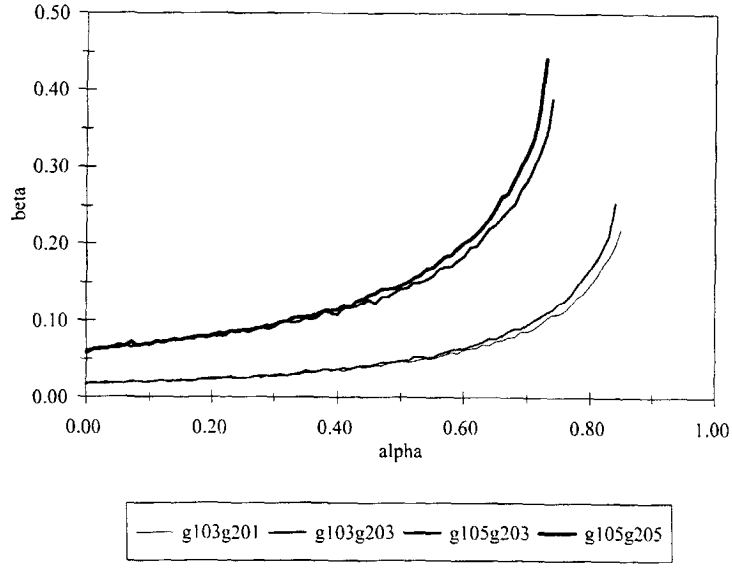

Figure 2: Corresponding values of α and β that imply fair contracts *without* the non negativity requirement for account B for four different combinations of guarantees ($g_1 = 3\%$, $g_2 = 1\%$; $g_1 = 3\%$, $g_2 = 3\%$; $g_1 = 5\%$, $g_2 = 3\%$; $g_1 = 5\%$, $g_2 = 5\%$), $r = 10\%$, $A_0^1 = 1$, $A_0^2 = 0$, and $B_0 = 0$.

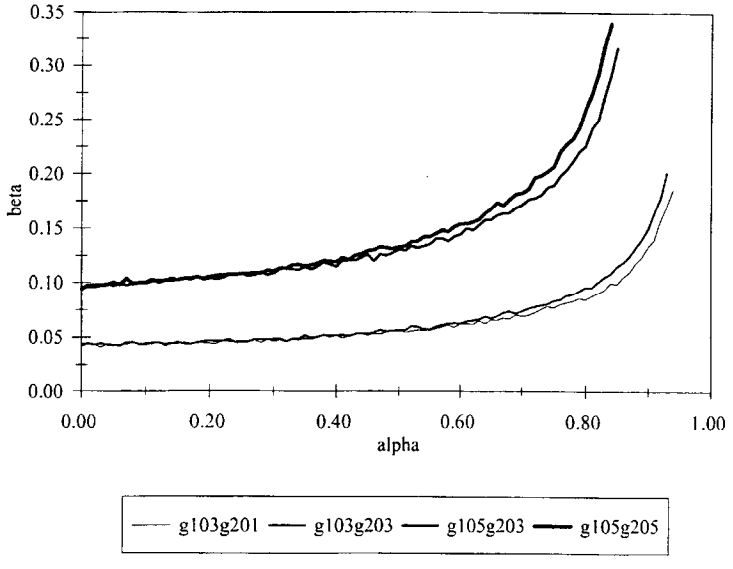

Figure 3: Corresponding values of α and β that imply fair contracts *including* the non negativity requirement for account B for four different combinations of guarantees ($g_1 = 3\%$, $g_2 = 1\%$; $g_1 = 3\%$, $g_2 = 3\%$; $g_1 = 5\%$, $g_2 = 3\%$; $g_1 = 5\%$, $g_2 = 5\%$), $r = 10\%$, $A_0^1 = 0.96$, $A_0^2 = 0$, and $B_0 = 0.04$.

By comparing figures (1) and (2) the effect of the positivity requirement may be analyzed. It is clear that the positivity requirement increases β for a given α and thus increase the financial option premium connected to the contract. For example for $\alpha = 70\%$ and $g_1 = 3\%$,the value of the parameter β is increased from, roughly, .10 to .12, i.e. around 20% because of the positivity requirement.

In figures (3) and (4) the effect of a different initial allocation is analyzed with and without the non negativity requirement. The initial balance of the account A^1 is 96% of X, whereas the remaining 4% of X is initially credited the bonus account. By comparing the figures (3) and (4) with the corresponding figures (1) and (2), we see that the effect on the parameter β for a given α is strong. For example for $\alpha = 70\%$ and $g_1 = 3\%$, the value of the parameter β is reduced by a factor of magnitude 50%.

4 Concluding remarks

This paper extends the Miltersen and Persson (2000) model to include the possibility of two different levels of guaranteed rate of returns and to include the requirement of a non negative balance of the bonus account at the end of each year.

Numerical examples indicates, as expected, that reducing g_2 compared to g_1, decrease the cost of the police compared to the case $g_2 = g_1$. For a fixed difference between g_1 and g_2, i.e. $g_1 - g_2 = k$, for some constant k, $g_2 < g_1$, the financial option premium seems to increase with g_1, as well as α, the fraction of the annual positive investment surplus which is credited the customer.

The non negativity requirement also increases the financial option premium, as expected. One of our examples indicates an increase in the parameter β, which is related to the cost of the policy, of roughly 20%.

We also present numerical examples of the effects of alternative initial distribution between the cus-

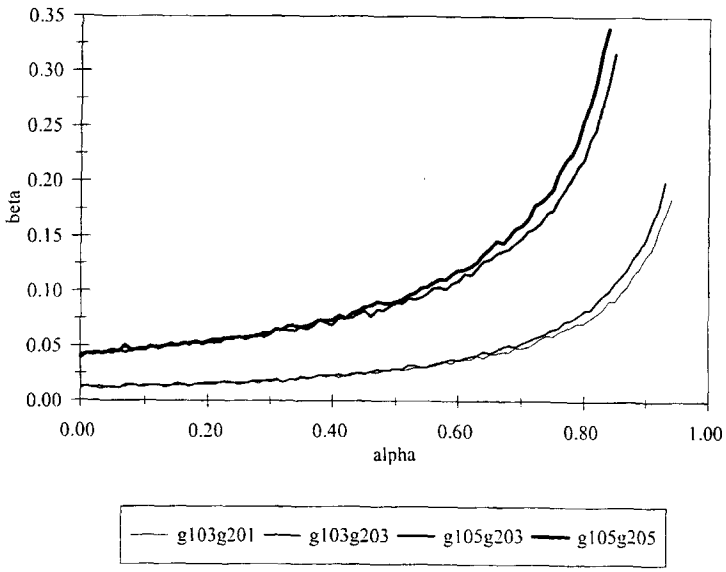

Figure 4: Corresponding values of α and β that implies fair contracts *without* the non negativity requirement for account B for four different combinations of guarantees ($g_1 = 3\%$, $g_2 = 1\%$; $g_1 = 3\%$, $g_2 = 3\%$; $g_1 = 5\%$, $g_2 = 3\%$; $g_1 = 5\%$, $g_2 = 5\%$), $r = 10\%$, $A_0^1 = 0.96$, $A_0^2 = 0$, and $B_0 = 0.04$.

tomer's accounts and the bonus account. The examples indicate a significant reduction of the parameter β (in the magnitude of 50%), by reducing the customer's account from 100% of the total investment amount X to 96% and depositing the remaining 4% into the bonus account.

So far we have not explicitly analyzed the connection between the parameter β and the financial option premium of the contract.

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