

A Study of Methods for Coping with Typhoons Cash-Flow Simulation Using CAT Bonds

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Abstract

The non-life insurance industry has experienced record catastrophe claims over the past decades all over the world. Traditionally, insurers purchased reinsurance to manage their catastrophe exposure.

As catastrophes occur with greater severity and frequency, insurers have found it difficult to obtain reinsurance coverage because of the lack of reinsurance capacity.

Recently, some insurers have developed a new risk-transfer alternative to help insurers manage their underwriting exposure.

The purpose of this paper is to compare CAT bonds to reinsurance for typhoon-loss coverage, using our simulation model of cash flow effects with various scenarios.

Keyword

catastrophe risk, securitization of insurance risks , cat bonds , cash-flow simulation

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V. Closing

I. Introduction

As demonstrated by 1991's Typhoon Mireille and the Great Hanshin Earthquake of 1995, Japanese non-life insurance companies remain vulnerable to the effects of natural disasters. As a hedge against Catastrophe risks and to help ensure stable profits, Japanese non-life insurers have generally relied on contingency reserves and reinsurance.

Relatively recently, a mechanism has been developed to transfer insurance risks to capital markets using the technique of insurance-risk securitization. Using standard financing vehicles such as bonds and options, the mechanism allows insurers to transfer the insurance risks posed by natural calamities to investors. This development provides insurers with a secure hedge against catastrophe risks, and directly links the insurance industry to capital markets. A number of insurers have begun applying this hedging technique – primarily overseas, but in Japan as well.

In this paper, we compare bonds, a vehicle of insurance risk securitization, with commonly used reinsurance products. Using a storm model and simulations, we attempt to clarify the differences and advantages of the two methods as hedges against ordinary catastrophe risks for non-life insurers from a cash-flow perspective.

In a previous Japan-specific paper, entitled “How to Cope with Storms” by Tetsuji Mayuzumi, presented at the 25th ICA in 1995, the author created a storm model, and used it to analyze cash-flow effects on non-life insurer operations through cash-flow simulation of reinsurance and contingency reserves. As a follow-up to this previous paper, we use a storm model to examine the effects of insurance risk securitization on non-life insurer operations.

II. State of Claim Payments for Natural Disasters in Japan

Net direct insurance premiums for Japan in fiscal 1996 totaled approximately ¥10 trillion, including nearly ¥2 trillion (17.9%) for fire insurance. Over the past ten years, claims payments for major natural disasters (excluding earthquakes) have exceeded ¥200 billion annually. In 1991, Typhoon Mireille had a profound impact on non-life insurance insurers, leading to ¥497.5 billion in claims payments. The world's ten worst natural disasters by claims paid (1970 - 1997) are listed in Table II-1.

Table II-1: The World's 10 Worst Natural Disasters by Insurance Claims Paid (1970 - 1997)

No	Claims Paid (in millions of dollars)	DATE	Natural Disaster	Country
1	\$18,286	92. 8.24	Hurricane "Andrew"	USA
2	\$13,529	94. 1.17	Northridge earthquake in southern California	USA
3	\$6,542	91. 9.27	Typhoon "Mireille"	Japan
4	\$5,636	90. 1.25	Winter storm "Daria"	Europe
5	\$5,427	89. 9.15	Hurricane "Hugo"	USA
6	\$4,230	87.10.15	Autumn storm	Europe
7	\$3,917	90. 2.26	Winter storm "Vivian"	Europe
8	\$2,712	88. 7. 6	Explosion on offshore platform "Piper Alpha"	Britain
9	\$2,603	95. 1.17	Great Hanshin/Awaji Earthquake	Japan
10	\$2,211	95.10. 4	Hurricane "Opal"	USA

III. Securitization of Insurance Risks

1. Definition of Securitization of Insurance Risks

In this document, "securitization of insurance risks" refers to methods for raising funds from capital markets to pay insurance claims. Of the various means of securitizing insurance risk, this paper primarily discusses catastrophe bonds (CAT bonds).

2. Development of Securitization of Insurance Risks

Reinsurance, whereby one insurance company insures another, is used to transfer some of the risk of the ceding insurance company in order to stabilize its operations. Recent market developments have resulted in alternative risk transfer mechanism, in addition to the traditional reinsurance approach.

As shown in Table II-1, natural disasters wreaked enormous damage globally in 1990s. In Japan, the staggering sum of nearly ¥500 billion in fire insurance claims was paid in the wake of Typhoon Mireille of 1991. U.S. insurers also paid enormous claims for

damage produced by a succession of natural disasters, including Hurricane Andrew and the Northridge earthquake.

Since direct insurers made substantial recoveries from their reinsurers, the profitability of the reinsurance market was significantly affected by the high frequency of major calamities. Reinsurers responded by reducing underwriting limits and raising reinsurance premiums, making reinsurance capacity more difficult to obtain for many insurers. This demonstrated the lack of reinsurance capacity and the limits of obtaining reinsurance coverage in the traditional manner.

Faced with a lack of reinsurance capacity, American and European direct insurers began using alternative measures to hedge their risks in 1994. The methods explored included risk diversification through the swap of catastrophic risks among direct insurers, and the funding of insurance claims through capital markets by securitizing insurance risks.

The absence of recent major natural disasters and an increase in underwriting capacity due to the emergence of new reinsurers in Bermuda and other areas led to a subsequent softening of the reinsurance market. The trend toward securitization of insurance risk has also abated, with some insurers placing securitization plans on hold. Nonetheless, a number of insurers, including several in Japan, have announced new plans in preparation for the future hardening of the reinsurance market.

3. Main Characteristics of Reinsurance Coverage for Catastrophe Risk

The main characteristics of reinsurance coverage for catastrophe risk are as follows.

- The reinsurance market lacks stability, mainly because the estimated size of the global market is only US\$15 billion. Major natural disasters lead to a sharp decrease in supply or significant increases in reinsurance premiums.
- Reinsurance recoveries are typically made soon after a disaster occurs, making reinsurers' credit risk (solvency rating, etc.) a significant factor.
- Reinsurance covers are usually one-year contracts and long-term (e.g. 10-year) contracts are normally not available.
- There are only about 30 major reinsurers. In recent years the market has become increasingly oligopolic, as a result of mergers and acquisitions. Price competition is likely to diminish.

4. Securitization of Insurance Risks and Its Potential

Recently, securitization of a variety of risks has occurred outside the insurance sector. In light of the problems with reinsurance previously mentioned, the securitization of insurance risks was studied by a number of insurance companies as a new vehicle for transferring insurance risk.

Securitization of risks can be accommodated by financial markets, which are vastly larger than the reinsurance market (see note).

Note: Financial markets are enormous and highly stable. The size of the financial market (volume of funds) in the U.S. alone is estimated at some US\$19 trillion.

The key point in the securitization of insurance risks is defining the “loss occurrence” (i.e. setting the trigger point). The “loss occurrence” could be defined as follows.

- (1) When the total amount of losses incurred by the insurer exceeds a certain value
- (2) When the total amounts of losses incurred by the industry exceeds a certain value
- (3) When an objective index published by another entity (e.g. a meteorological agency) exceeds a certain value

For investors, the first definition lacks transparency; and for long-term contracts, it is difficult for investors to ascertain changes in an insurer’s portfolio of insurance policies. While more transparent, the second one remains insufficient in objectivity. The third definition is the best of the three, since it makes it easier for a third-party entity to evaluate the risk assured.

From the insurer’s perspective, the first definition is a perfect substitute for reinsurance. The second and third definition, in particular the third, pose the risk of a relatively low correlation between the trigger point and an insurer’s own losses, producing an insufficient hedge. If investors accept the first or second definition, any risk can be securitized. Not only does the third definition involve difficulties with index definition, but also it is inherently limited, being impractical for risks other than storms and earthquakes.

Since securitization of insurance risks can not be a perfect substitute for reinsurance, it is unlikely to supplant reinsurance. However, it seems likely to develop as a supplement to reinsurance, particularly as a source of additional capacity. Some reinsurers predict that securitization of insurance risk will eventually become a US\$40 billion business.

5. CAT Bonds

(1) Overview

CAT bonds are insurer-issued bonds that pay normal interest plus a sum equivalent to a conventional reinsurance premium. If no catastrophe occurs during the bond's term, the bonds are redeemed at face value. If a catastrophe occurs, the bonds' principal is totally or partially forfeited to pay insurance claims. Thus, CAT bonds function similarly to reinsurance.

A number of insurers have used CAT bonds to hedge against natural disasters, using various risk indices (see note 1).

In practice, insurers are unable to issue securities directly, for accounting and other regulatory reasons, therefore, insurers normally structure the deal in a form of the outgoing reinsurance contract, with a special-purpose company (see note 2), which then issues the bonds.

Note 1: CAT bonds were issued by AIG and St.Paul Re in 1996; Winterthur, USAA, Swiss Re, and Tokio Marine and Fire in 1997; and Yasuda Fire and Marine, among others, in 1998.

Note 2: Unlike reinsurance, the accounting procedures for the issuance of bonds are yet to be established in Japan. One objective of establishing a special-purpose company is to obtain a higher rating for the bonds by removing the insolvency risk of the insurance company.

In Japan, regulations restrict insurers from issuing bonds except for such purposes as capital investment. Direct cessions to a special-purpose company are considered a violation of the restriction, therefore, Japanese insurers circumvent this restriction by ceding to an ordinary reinsurer first, which then cedes to the special-purpose company.

The following matters also need to be considered.

- ① Assessment of estimated risk value and fixing the terms of issuance (setting the interest rate in addition to the risk free rate, etc.)
- ② Accounting procedures when bonds are issued directly by the insurer
 - Recognition as a reinsurance transaction
 - Classification as asset or liability
 - Impact on solvency ratio
 - Clerical procedures and costs which accompany to issuance

(2) Characteristics

From the insurer's perspective, CAT bonds have the following characteristics.

- Investors pay the notional amount to the issuing company in advance, avoiding credit risk problems.
- Different investors have varying time frames, making long-term contracts of around 10 years possible.
- Major market players are institutional investors (pension funds, investment advisors, investment trusts, etc.).

From the investor's perspective, a CAT bond has the following characteristics.

- The correlation with the risk that conventional investors normally incur (e.g. interest-rate risk, currency risk, stocks or bond (conventional) market risk) is low, therefore, investors can achieve risk diversification by including CAT bonds in their portfolios.
- Investors earn high yields if no catastrophe occurs.

IV. Simulation

In this section, we compare CAT bonds to reinsurance for typhoon-loss coverage. To see the differences between CAT bonds and ELC reinsurance in terms of cash flow effects, we create a simulation model, conduct an actual simulation, and compare the resulting profit /loss.

1. Setting Company Size and Profit-and-Loss Calculation Variables

This simulation assumes the following company size (initial value) and Profit-and-Loss (P/L) variables (Table IV-1).

The contingency-reserve system implemented by Japanese non-life insurance companies is excluded from P/L variables to make the simulation model as clear and easily understood as possible. Investment income and income taxes are also excluded from the calculations.

Table IV-1: P/L Variable Settings

(in billions of yen)

	Variables	Value	
(a)	Ordinary income		=(b)
(b)	Net premiums		=(c)-(d)-(e)
(c)	Direct premiums	40	constant (initial setting)
(d)	Reinsurance premiums paid		E (see IV.3 below)
(e)	Interest payable on bonds		$\Delta r \cdot B$ (see IV.4.6 below)
(f)	Ordinary expenses		=(g) - (k) - (l) + (m)
(g)	Net claims paid		=(h) - (k)
(h)	Direct claims paid		=(i) + (j)
(i)	Claims paid for recurring losses	15	Set at (c) \times 37.5% (initial setting)
(j)	Claims paid for typhoon losses		$\sum_j L_{ij}$ (see IV.2.5 below)
(k)	Claims recovered from reinsurance		M_i (see IV.3 below)
(l)	Amount of bond principal forfeited		N_i (see IV.4.3 below)
(m)	Net operating expenses	16.0	set at (c) \times 40.0% (initial setting)
(n)	Ordinary profit (loss)		=(a) - (f)
(o)	Average premium rate	1.60%	initial setting
(p)	Amount of claims retained	25,000	=(c)/(o)

2. Modeling

In this section, we set up the typhoon-loss model to be used in the simulation, using the techniques described in "How to Cope with Storms" (presented at the 25th ICA in 1995).

(1) Definition of Distribution

The typhoon-loss model was formulated using the following data.

- Number of damaged/destroyed houses (classified as totally destroyed, half-destroyed, and partially damaged) (1967 - 1996): Meteorological Handbook (Meteorological Agency)
- Number of households (1967 - 1996): Japanese Census (Management and Coordination Agency)

The annual typhoon incidence distribution conforms to a Poisson distribution, with an average λ of 2. The per-incident typhoon scale (loss frequency) conforms to a logarithmic normal distribution. The distribution function $F(x)$ can be expressed as follows.

$$F(x) = \int_{0.001}^x \frac{1}{\sqrt{2\pi}\sigma(x-0.001)} \exp\left\{-\frac{1}{2\sigma^2}[\log(x-0.001)-\mu]^2\right\} dx \quad (x > 0.001) \quad (IV.2.1)$$

Where $\mu = -5.3327$ and $\sigma = 2.2558$

(2) Typhoon Modeling

The symbols below are defined as follows.

n_i : number of typhoon occurring in the i^{th} year ($i = 1, 2, \dots, 10,000$)

$$v = \sum_{i=1}^{10000} n_i$$

l_{ij} : loss frequency of the j^{th} typhoon ($j = 1, 2, \dots, n_i; n_i \neq 0$) in the i^{th} year

L_{ij} : direct insurance claims for the j^{th} typhoon in the i^{th} year

(i) Annual incidence

The 10,000 integers from 1 to 10,000 are randomly assigned one per year. The integer assigned to the i^{th} year is represented by α_i . When

$$P\{N \leq n\} = \begin{cases} \sum_{k=0}^n e^{-\lambda} \frac{\lambda^k}{k!} (\lambda = 2) & n \geq 0 \\ 0 & n < 0 \end{cases}, \tag{IV.2.2}$$

the number of typhoons n_i occurring in the i^{th} year is set as the integer

$$P\{N \leq n_i - 1\} < \frac{\alpha_i - 0.5}{10000} \leq P\{N \leq n_i\} \tag{IV.2.3}$$

Accordingly,

$$v = \sum_{i=1}^{10000} n_i = 20,000.$$

(ii) Typhoon - loss frequency

The v integers from 1 to v ($v = 20,000$, from (i) above) are arranged in random order.

When β_{ij} is defined as the $\sum_{k=1}^{i-1} n_k + j^{th}$ integer (the j^{th} integer when $i = 1$), the loss frequency of the j^{th} typhoon occurring in the i^{th} year l_{ij} is set as follows.

$$l_{ij} = F^{-1}\left(\frac{\beta_{ij} - 0.5}{v}\right) \tag{IV.2.4}$$

($F^{-1}(y)$ is the inverse function of the distribution function to which the typhoon - loss frequency conforms.)

(iii) Typhoon - loss claims

Claims paid for the j^{th} typhoon occurring in the i^{th} year L_{ij} is set by multiplying l_{ij} above by the amount of retained claims. That is,

$$L_{ij} = l_{ij} \times 25,000/1,000 \text{ (billions of yen)}. \tag{IV.2.5}$$

3. Setting Reinsurance Coverage

In the simulation, we use excess of loss cove reinsurance (ELC reinsurance). The symbols below are defined as follows and set using the techniques described in “How to Cope with Storms” (25th ICA, 1995).

X : Value set as the excess point (billions of yen)

Y : Value set as the cover limit (billions of yen)

M_{ij} : Claims recovered from reinsurance for the j^{th} typhoon occurring in the i^{th} year (direct insurance claims L_{ij})

M_i : Claims recovered from reinsurance in the i^{th} year

ϵ : The pure insurance premium component of the reinsurance premium when the excess point is X and the coverage limit is Y

Λ : Reinsurance premium loading rate; Λ is set at 0.7

E : Reinsurance premium when the reinsurance premium's risk premium component is ϵ and loading rate is Λ ; $E = \epsilon / (1 - \Lambda)$ (IV.2.6)

For this simulation, X and Y were set at 6 and 28, respectively, in which $\epsilon = 0.812$ and $E = 2.707$.

4. CAT Bond Settings

The CAT bond terms used in this simulation are set as follows.

(1) Issue Terms

The symbols pertaining to the issue terms are defined as follows.

B : Amount of bond issues (billions of yen)

M : Duration of catastrophe risk coverage (years)

$n(n \geq m)$: Maturity (years)

R_{ij} : Risk index value resulting from losses from the j^{th} typhoon occurring in the i^{th} year (billions of yen) (see 4. (6) below)

t : Number of triggers

T_k : Trigger for the k^{th} , progressing in ascending order of monetary value ($k=1, 2, 3, \dots, t-1$)

α_k : Rate of principal forfeited when $T_k < R_{ij} \leq T_{k+1}$

w_{ij} : Rate of principal forfeited due to damage from the j^{th} typhoon occurring in the i^{th} year

r : Base interest rate

Δr : Premium interest rate

(2) Bond - Issue Amount

The bond-issue amount B is set at the same value as the reinsurance coverage limit, which is included in the profit/loss comparison (see IV.4.16).

(3) The Catastrophe Risk Coverage Term and Maturity

For this simulation, the catastrophe risk coverage term m and the bonds' maturity n are set at $m = n$

However, this simulation assumes that another issue of CAT bonds with identical coverage is issued instantaneously when the bonds' catastrophe coverage term ends, either upon its expiration or on forfeiture of principal due to a catastrophic incident. Hence, the simulation is unaffected by the duration at which m and n are set.

(4) The Principal Forfeiture Rate

The risk index for the j^{th} typhoon occurring in the i^{th} year is defined as R_{ij} (The setting of R_{ij} is discussed in detail in (6) Setting the Risk Index below.). The CAT bonds' principal forfeiture rate is set on a graduated basis, which varies with the value of the risk index R_{ij} .

For this paper, the principal forfeiture rate w_{ij} corresponding to trigger number t is determined as follows.

$$w_{ij} = \begin{cases} 1 & (R_{ij} > T_t) \\ \alpha_k & (T_k < R_{ij} \leq T_{k+1}, k = 1, 2, 3, \dots, t-1) \\ 0 & (R_{ij} \leq T_1) \end{cases} \quad (\text{IV.4.1})$$

Methods for setting the trigger T_k and the principal forfeiture rate α_k are discussed in 4(7) below.

(5) Interest Rates

The base interest rate r is determined independent of the risk coverage, and is thus excluded from cash-flow comparisons in this simulation. The premium interest rate Δr varies depending on risk-coverage condition settings. In this simulation, the premium interest rate is determined by equating premium receipts and claims payments, so that the cash-flow comparison between CAT bonds and reinsurance is evaluated fairly. The premium interest rate is calculated from the total forfeited bond principal and the total amount of premium interest paid over 10,000 years, based on the 10,000 years of typhoon data used in the simulation (see (i) and (ii) below).

To simplify the model, we assume that interest $\Delta r \cdot B$ is paid once annually at mid-year (without regard to the timing of the CAT bond issuance), and we do not calculate the present value of future cash flows.

(i) Total Amount of Principal Forfeited

The amount of principal forfeited, due to the j^{th} typhoon occurring in the i^{th} year, can be expressed as follows, based on formula (IV.4.1).

$$N_{ij} = w_{ij}B \tag{IV.4.2}$$

Accordingly, the amount of principal forfeited in the i^{th} year N_i can be expressed

$$N_i = \sum_j N_{ij} = \sum_j w_{ij}B \tag{IV.4.3}$$

(ii) Premium Interest Rate

The portion of the premium interest rate Δr pertaining only to catastrophic loss is designated Δr_1 (for an insurance premium, Δr_1 corresponds to the pure premium component only).

According to the principal of the equivalence of premium receipts and claims payments,

$$\sum_i^{10000} N_i = \Delta r_1 \cdot B \cdot \sum_i^{10000} 1 \tag{IV.4.4}$$

Thus,

$$\Delta r_1 = \frac{\sum_i^{10000} N_i}{10,000B} = \frac{\sum_i^{10000} \sum_j w_{ij}B}{10,000B} = \frac{\sum_i^{10000} \sum_j w_{ij}}{10,000} \tag{IV.4.5}$$

Δr_1 represents only the portion of the interest rate pertaining to catastrophic loss. For a fair comparison of cash-flow between CAT bonds and reinsurance, the premium interest rate Δr is calculated by applying the loading rate used when calculating the reinsurance premium. Accordingly,

$$\Delta r = \Delta r_1 / (1 - \Lambda) \tag{IV.4.6}$$

(6) Risk Index (Defining of Catastrophe)

As illustrated by the three definitions of loss occurrence in III.4 above, there exist several concepts of risk indices. In this simulation, R_{ij} is set as the risk index value. The parameters μ and σ for are the same for both R_{ij} and the amount of direct insurance claims paid L_{ij} . And R_{ij} has a correlation ρ to L_{ij} .

The method of setting the risk index value R_{ij} is explained below.

First, v ($v = 20,000$) values of x_i are prepared to obtain a series of uniform random numbers separated by equal intervals.

$$x_i = \frac{i - 0.5}{v} \quad (i = 1, 2, \dots, v) \quad (IV.4.7)$$

Next, v values of y_i conforming to a standard normal distribution are generated.

$$y_i = f^{-1}(x_i), \text{ where} \quad (IV.4.8)$$

$$f^{-1} \text{ is the inverse function of } f(x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \quad (IV.4.9)$$

Two arbitrary sets of random numbers a_i ($i=1,2,\dots,v$) and b_i ($i=1,2,\dots,v$) are generated and arranged in correspondence with y_i , as follows.

$$(y_1, a_1, b_1)$$

$$(y_2, a_2, b_2)$$

...

$$(y_v, a_v, b_v)$$

For random numbers y_i, a_i values rearranged in ascending order are designated

$$z_{a1}, z_{a2}, \dots, z_{av}$$

For random numbers y_i, b_i values rearranged in ascending order are designated

$$z_{b1}, z_{b2}, \dots, z_{bv}$$

The following equation is used to obtain a standard normal distribution of random numbers with correlation ρ .

$$w_i = \rho \cdot z_{ai} + \sqrt{1 - \rho^2} \cdot z_{bi} \quad (IV.4.10)$$

The loss frequency $l_{\rho i}$, which has a correlation of ρ to l_i , the loss frequency for the i^{th} typhoon, is obtained from the following equation.

$$l_i = \exp(\sigma \cdot z_{ai} + \mu) + 0.001 \quad (IV.4.11)$$

$$l_{\rho i} = \exp(\sigma \cdot w_{ai} + \mu) + 0.001 \quad (IV.4.12)$$

Above, μ and σ are parameters of the lognormal distribution $LN(\mu, \sigma)$ to which the typhoon loss frequencies conform.

Based on the proceeding, the amount of direct insurance claim payments L_{ij} corresponding to l_{ij} , the loss frequency for the i^{th} typhoon occurring in the j^{th} year, is

$$L_{ij} = l_{ij} \times 25,000 / 1,000 \text{ (billions of yen)} \quad (IV.4.13)$$

Hence, the risk index value R_{ij} with a correlation of ρ to L_{ij} , the amount of direct

insurance claims paid (parameters μ and σ are equal), can be expressed as follows.

$$R_{ij} = I_{\rho ij} \times 25,000 / 1,000 \text{ (billions of yen)} \quad (\text{IV.4.14})$$

If $\rho = 1$, then the risk index value R_{ij} = the amount of direct insurance claim payments L_{ij} .

(7) CAT Bond Risk Coverage

For a fair comparison of cash-flow between CAT bonds and reinsurance, the CAT bond risk coverage and cost are set, respectively, at the same values as the risk coverage and cost of reinsurance, so that:

$$B = Y \quad (\text{IV.4.15})$$

and

$$\Delta r \cdot B = E \quad (\text{IV.4.16})$$

From equations (IV.4.15) and (IV.4.16),

$$\Delta r = \frac{E}{B} = \frac{E}{Y}$$

Additionally, from equations (IV.2.6), (IV.4.5) and (IV.4.6),

$$\frac{1}{1 - \Lambda} \cdot \frac{\sum_i \sum_j^{10000} w_{ij}}{10,000} = \frac{1}{Y} \cdot \frac{\epsilon}{1 - \Lambda}$$

Accordingly,

$$\frac{\sum_i \sum_j^{10000} w_{ij}}{10,000} = \frac{\epsilon}{Y} \quad (\text{IV.4.17})$$

From the above, w_{ij} is determine by the trigger T_k and the principal forfeiture rate α_k , which are set to satisfy equations (IV.4.17).

5. Simulation

Based on the above, we use the model to conduct simulations based on randomly generated typhoon - loss data for 10,000 years.

(1) Scenario Perspectives

We formulate scenarios from perspectives a, b, c, and d below in order to conduct a simulated cash-flow comparison of CAT bonds and reinsurance, as explained at the beginning of section IV.

Case a : Risk coverage is identical for both CAT bonds and reinsurance (the number of

trigger is 1 and percentage of principal forfeited is 100%)

Case b : The number of triggers is increased

Case c : The trigger is lowered without changing the bond cost

Case d : Risk coverage is the same, but the risk index's correlation to direct insurance claims varies

(2) Simulation Results

a. Simulation with Risk coverage is identical for both CAT bonds and reinsurance (the number of trigger is 1)

we compare CAT bonds to reinsurance in the case of the number of trigger $t = 1$.

w_{ij} can be expressed as follows:

$$w_{ij} = \begin{cases} 1 & (R_{ij} > T_1) \\ 0 & (R_{ij} \leq T_1) \end{cases}$$

Where, value set as the excess point X and the cover limit Y were set at 6 and 28, respectively, as described in IV.3.

To simplify the model, the risk index R_{ij} is set at the same value as the amount of claim payment L_{ij} . The resulting values for T_1 , which is set to satisfy equation (IV.4.7), is 16.05.

The relation between typhoon losses L and the amount forfeited/recovered c is as shown in Figure IV-1.

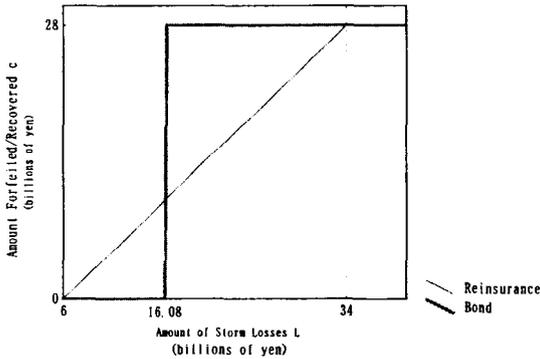


Figure IV-1: Relation between Amount of Typhoon Losses L and Amount Forfeited/Recovered c

If only one typhoon occurs in a given year, annual cash flow S is calculated by the following formula in accordance with Table II-1. S is correspondent to (n) in the table.

When CAT bonds are used,

Annual cash flows $S =$ direct insurance premium income (40) – claims for ordinary losses (15)

$$\begin{aligned}
 & - \text{net operating expense (16) - typhoon losses (L)} \\
 & - \text{cost (0.812)/(1 - loading rate (0.7)) + principal forfeited (c)} \\
 & \doteq 6.3 - L + c \\
 & = \begin{cases} 6.3 - L & (L \leq 16.08) \\ 34.3 - L & (L > 16.08) \end{cases}
 \end{aligned}$$

Similarly, when reinsurance is used,

$$S = \begin{cases} 6.3 - L & (L \leq 6) \\ 0.3 & (6 < L \leq 34) \\ 34.3 - L & (L > 34) \end{cases}$$

Annual cash flows S are shown in Figure IV-2. Additionally, the number of typhoons by amount of losses (billions of yen) is shown as a bar graph in the same figure. Note that as though the number of typhoons with the amount of losses is less than 1 billion yen is 16,455, it omitted from the figure.

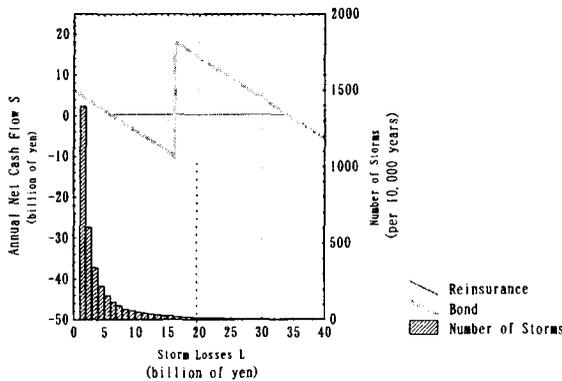


Figure IV-2: Typhoon Losses L and Annual Cash Flow S , and Number of Typhoon by Amount of Losses (per 10,000 years)

In the following simulation, we assume that typhoons occur several times in a year, and take into considers restoration and additional premiums for reinsurance.

The simulation results are shown in Figure IV-3.

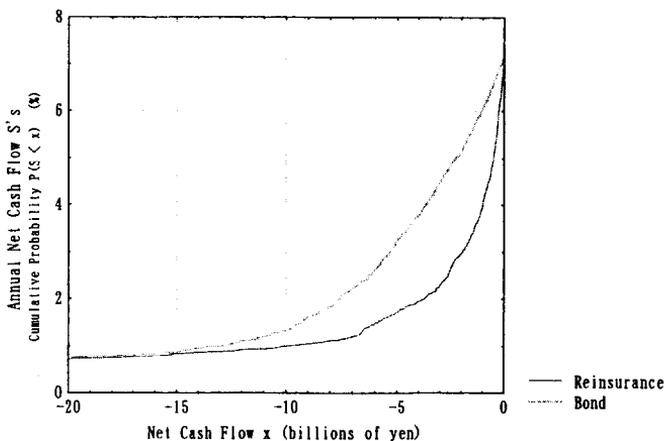


Figure IV-3: Simulation Results

In this simulation, the cumulative probability $P(S < X)$ for bond is never lower than that for reinsurance, indicating that reinsurance is always more effective than bond. In Figure IV-5, we find a substantial recovery surplus with bond once typhoon losses exceed the trigger of 16.06. However, this surplus is apparently offset by the recovery shortfall that arises prior to the trigger of 16.06, because typhoon frequency is inherently low.

b. Simulation with Increased Triggers

What happens to net cash flow when the number of triggers increases? As shown below, we increased the number of triggers to four. We kept the increments between triggers equal and the cost of bonds equal to that of reinsurance, as in the case above.

Bond (a): $t=2$ in equation IV.4.1

Bond (b): $t=3$ in equation IV.4.1

Bond (c): $t=4$ in equation IV.4.1

Bond (d): $t=5$ in equation IV.4.1

The resulting values for parameters (T_k, α_k) , which are set to satisfy equation (IV.4.17) for each bond, are shown in Figure IV-4. The relationship between the amount of typhoon loss when only one typhoon occurs and the annual net cash flow S is shown in Figure IV-5 for bonds 1 through 4.

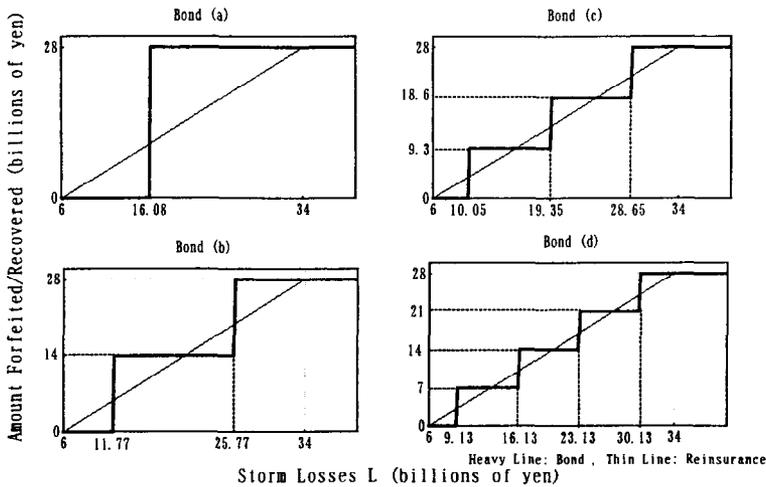


Figure IV-4: Relationship between Typhoon Losses L and Amount Forfeited/Recovered c

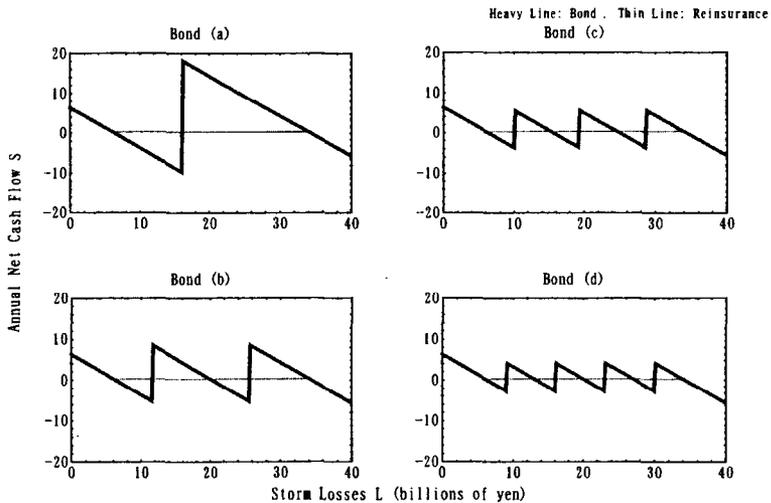


Figure IV-5: Relationship between Typhoon Losses L and Annual Net Cash Flow S

Simulation results are shown in Figure IV-6. Due to the increase in the number of triggers, the curves representing cumulative probability for the bonds draw progressively closer to the reinsurance curve, while net cash flow gains stability. Under the conditions set for this simulation, a bond with even four triggers may be as effective as reinsurance.

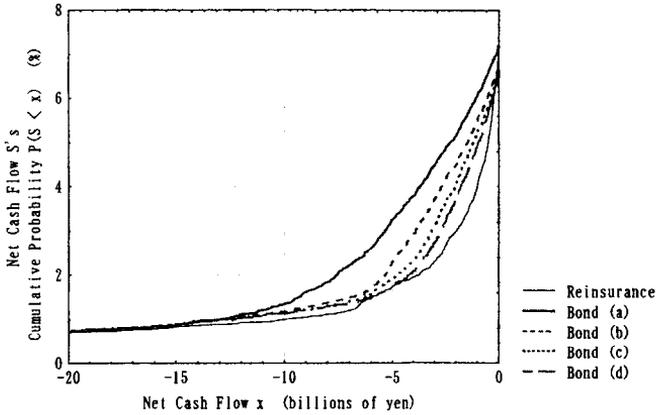


Figure IV-6: Simulation with Increased Triggers

c. Simulation with Lowered Triggers

We now examine the effect on net cash flow when the trigger is lowered, while cost remains constant. The principal amount forfeited is fixed at 28 (Figures IV-7 and IV-8).

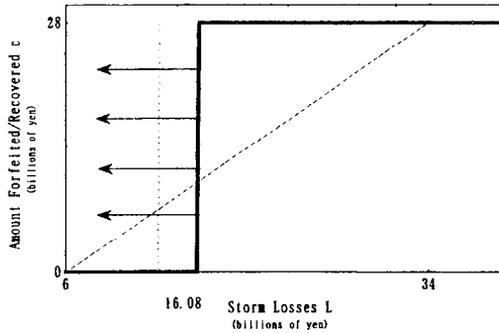


Figure IV-7: Relationship between L and c when Trigger is Lowered

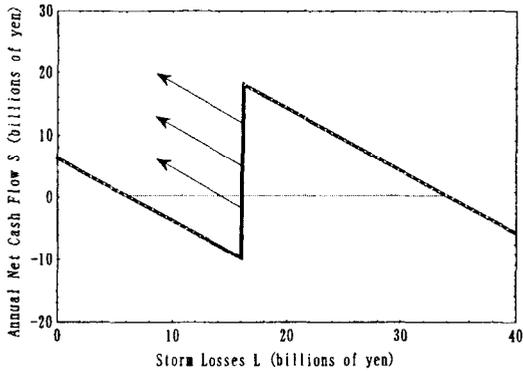


Figure IV-8: Relationship between L and S when Trigger is Lowered

The simulation results are shown in Figure IV-9.

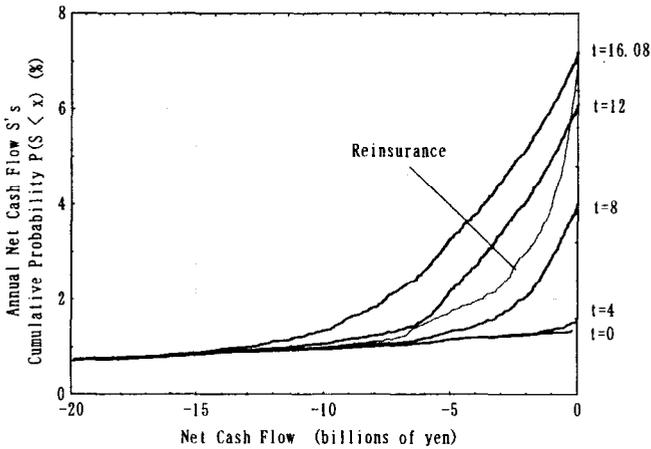


Figure IV-9: Simulation Results

We can see that lowering the trigger improves net cash flow. In this case, the cumulative probability of the bonds falls below that for reinsurance once the trigger falls to 8.

When the trigger is lowered, actual cost (i.e., the return to investors) must increase (Table IV-2). However, if the premium interest rate is high enough (in this case 2.9%) relative to market interest rates, lowering the trigger while keeping cost constant (i.e., keeping the premium interest rate at 2.9%) is also a viable option.

Table IV-2: Trigger vs. Cost and Premium Interest Rate

Trigger (billions of yen)	Cost (billions of yen)	Premium Interest Rate (%)
16.08	0.812	2.900
12	1.117	3.990
8	1.702	6.080
4	3.814	11.370
0	24.212	86.470

d. Simulation with Correlation Coefficient Variation

If the risk index that serves as the activation condition for the trigger is set independent of typhoon losses, it is virtually impossible to obtain a correlation of 1 between the two. Hence, we conducted a net cash flow simulation for Bond (a) in case b above, with the correlation coefficient set at 0.9, 0.7, 0.5, and 0.

Figure IV-10 shows the distribution before and after a change from pairs of normal random numbers (X, Y) with a correlation coefficient ρ to pairs consisting of values for typhoon losses and the risk index. The first 1,000 of 20,000 typhoons are plotted. The white dots represent typhoons for which the amount forfeited/recovered is the same for both reinsurance and bonds. The black dots represent typhoons for which we find discrepancies in recovery amount between reinsurance and bonds.

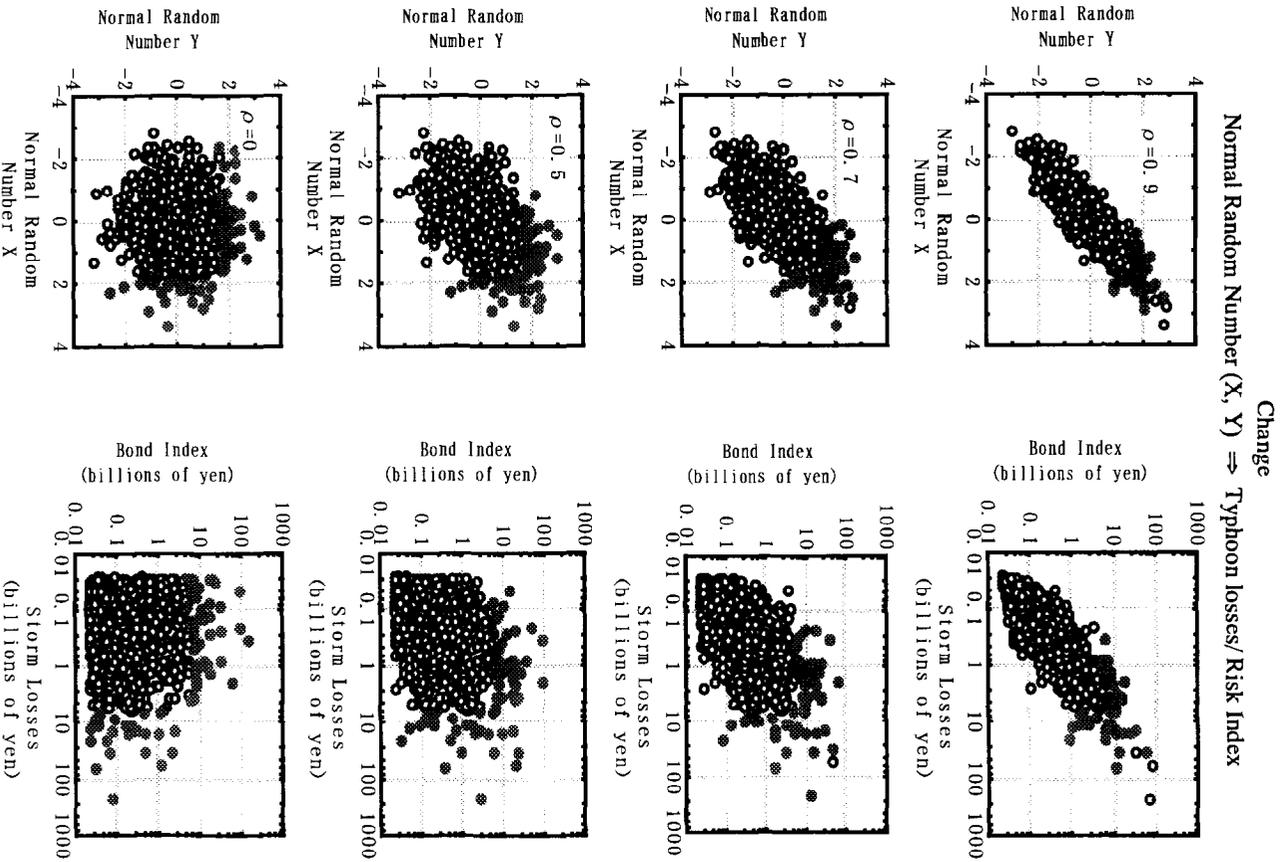


Figure IV-10: Appearance of Distribution when the Correlation Coefficient is Varied

Simulation results are shown in Figure IV-11. We can see that as correlation declines the number of cases in which losses are incurred without forfeiture of principal increases, with consequent reductions in net cash flow.

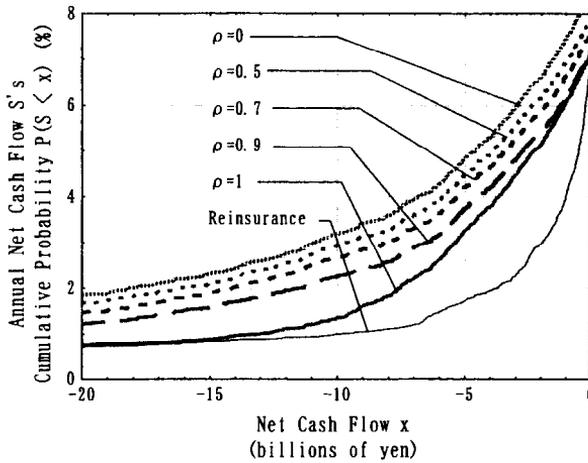


Figure IV-11: Simulation with Correlation Coefficient Variation

By adjusting the trigger or the amount of principal forfeited, is it possible to increase the effectiveness of bonds to a level equivalent to reinsurance or equivalent to cases in which the correlation is 1?

To answer this question, we conducted simulations with the correlation coefficient ρ set at 0.9 and costs held constant. We lowered the trigger to 6 and increased the amount of principal forfeited from 28 to 50, 100, and 150. The resulting simulation results are shown in Figure IV-12.

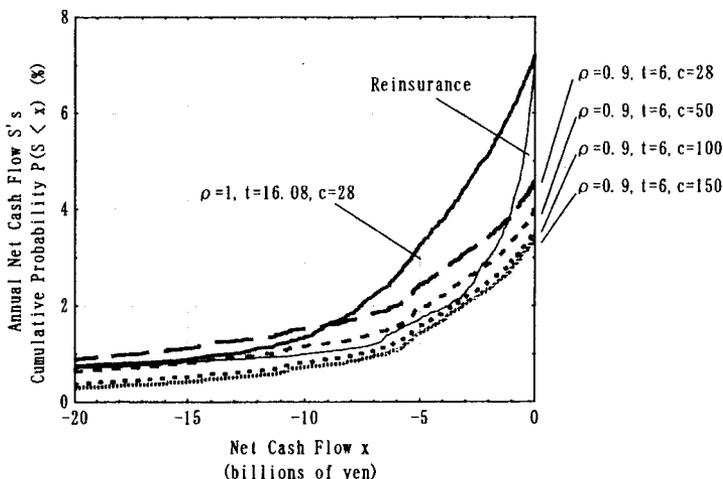


Figure IV-12: Results of Varying the Amount of Principal Forfeited c

We can see that when the amount forfeited increases to 50, the curve moves closer to the curve for which

$\rho=1$. When amount forfeited rises to 100, the curve falls below the reinsurance curve.

When cost (i.e., return to investors) is held constant while the amount of principal forfeited (i.e., amount of bond issue) increases, the premium interest rate falls, as shown in Table IV-3. Hence, when implementing this approach, one must consider whether the market will accept it.

Table IV-3: Amount of Principal Forfeited and Premium Interest Rate

Amount Forfeited c (billions of yen)	Premium Interest Rate (%)
28	2.900
50	1.624
100	0.812
150	0.541

(3) Conclusions

With reinsurance, insurers are able to recover 100% of their losses in the range between the excess point and the excess point plus the coverage limit. However, if the bond cost is set equal to the reinsurance cost and the amount forfeited/recovered is kept the same for both the bonds and reinsurance, the trigger must be raised above the excess point. If the trigger is set equal to the excess point, the amount of bond principal forfeited must be set lower than the reinsurance recovery amount. Whatever the case, a recovery shortfall or surplus will occur. Bonds, therefore, cannot serve as a perfect substitute for reinsurance.

However, if a bond is structured to approximate a reinsurance scheme by adding more triggers on a graduated basis, it is possible to stabilize net cash flow nearly as effectively as with reinsurance.

In addition, when it is possible to offer a nominal interest rate (corresponding to the risk component) offering sufficiently attractive yields, in comparison to market interest rates, bonds can be made to serve as substitutes for reinsurance by adjusting the trigger(s) and/or amount forfeited, while keeping the cost constant.

If the trigger is linked to a public index (risk index) independent of insurance claims payable, the risk index must be highly correlated to claims payable. But even without adequate correlation, by adjusting the trigger(s) and/or amount forfeited as described above, one can achieve results largely identical to cases having a correlation of 1, which are in turn identical to reinsurance cases.

We conclude that bonds can substitute for reinsurance if properly structured. To do so, we need to:

- (1) formulate a model of estimated losses,
- (2) calculate the cost of the bonds and reinsurance,
- (3) ascertain financial market demand and interest rate trends; and
- (4) select a risk index with a high correlation to losses.

V. Closing

This paper provides results obtained when we ran simulations based on the hypothetical direct issue of bonds by insurers. Our purpose was to verify the effectiveness of bonds on the basis of set conditions and a model. If such methods are to be placed into practice, factors such as differences in tax laws and accounting practices will require realistic parameters. Additionally, when we pose questions such as the following, differences will likely arise with respect to the concept of internal accumulation of reserves: In comparison to reinsurance, will direct issuance of the bonds be accompanied by a transfer of risk? How will the effects on solvency change? Are the proceeds of a bond offering treated as a liability or as owner equity? When issuing CAT bonds or otherwise securitizing insurance, we must ensure that we thoroughly understand various aspects of this approach in order to capitalize upon its advantages and disadvantages.

In this paper, we make use of the simulation method described in "How to Cope with Storms" (1995), a paper presented at the 25th ICA. We would like to express our thanks to the authors of the paper for permission to do so.

In closing, we express our hope that this paper will contribute to continued advances in the study of typhoon models.

