HOW TO REFORM THE PORTFOLIO.

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Abstract.

Let's consider the following problem. An investor is financing some investment portfolio and in the same time a new suggestion about financing arises. Partial refuse to finance any of the previous projects is connected with paying the corresponding fine. This situation gives us the number of optimization problems. The optimization criterion here is discounted income.

We should define two values for every financed project: moment and fraction of reduction of financing and part of the project for new projects being accepted.

If borrowing money is impossible the problem is reduced to the serious of linear optimization problems. Numerical experiments showed that usually the optimal strategy for the old projects will be to continue the old portfolio in general or stop the whole portfolio since some moment.

If it is possible to borrow money, the serious of piecewise linear optimization problems appears.

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It was supposed in the paper [1,2] that in the concrete moment the investor has received the number of offers from which he forms according to his financial statement the portfolio which is optimal in the sense of discounted income. However, in reality the suggestions do not appear in one and the same time. After the suggestion has been made the investor can make the decision to reform the portfolio he is investing in the moment. It should be considered that changing the existing contracts can include some penalty sanctions. All the following notations are taken from [1].

Let's assume that the investment project C_i is ($c_{i0},...,c_{in_i}$), where c_{ik} (as in the quoted papers) -money paid by investor (when $c_{ik}<0$) or received by investor (when $c_{ik}>0$) in the k-th moment of time (for example, by the end of some year). Let's assume that the investor can agree to finance some part of the project C_i, which is characterized by x_i from the interval [0,1]. This corresponds to condition 2 from [1]. Let's also assume that the penalty sanctions for the decision to cut financing the project are defined by the function

$$(1 - y_i)x_i\varphi_i(c_{ik},..., c_{in_i}),$$

where y_i – part of investment in the project C_i, which investor is going to continue after moment k. We shall discuss the possible character of the function $\varphi_i(c_{ik},..., c_{in_i})$ below. In the moment let's note that it seems to be natural that the penalty depends only on the part of the project which is left for financing after the investor's decision to cut financing.

Let investor makes the investments in projects $C_1,...,C_n$ in corresponding parts $x_1,...,x_n$ in the moment k and the new suggestion $D = (d_k,...,d_s)$ arises. In these condition the investor should choose whether he will include part z of the project D in his portfolio or not. Given that his resources are limited it might follow that some existing parts of the projects C_i being invested will decrease with the payment of the corresponding fines. Here we should consider that different projects can be changed in different time moments - from k to the end of corresponding projects validity.

Without loss of generality we can consider k=0 and all x_i=1. It means that we consider the situation to be accomplished - the portfolio is formed and the way it was done has no importance for the paper. Let's assume that the investor has the starting

capital F_0 and there will be no additional money attraction (financing condition A from the paper [1]). As the criterion of portfolio quality let's choose discounted income with the interest rate i_B as it was done before.

We need to define time moments $k_1,...,k_n \geq 0$ and rates $y_1,...,y_n, z \in [0,1]$ such that the decision to decrease the investment of the project C_i in the moment k_i to the rate y_i with the payment of corresponding penalties and rate z of financing the project D firstly will not lead to ruining at any time moment and secondly will provide the maximum discounted income in the moment n.

Here are the corresponding formulas.

The discounted (to the moment 0) investor's capital in the moment t consists of the following values:

- starting capital F_0,

- discounted incomes and payments according to the projects whose financing has not been changed by the moment t:

$$\sum_{j=0}^{t} \sum_{s:k_s \geq t} c_{sj}(1+i)^{-j}$$

- discounted penalties according to the projects whose rate of financing has been changed by the moment t:

$$\sum_{s:t \geq k_s} (1-y_s)\varphi_s(c_{k_s},...,c_n)(1+i)^{-k_s}$$

- discounted summary payments according to the project whose financing rate has been decreased by the moment t:

$$\sum_{s:t \geq k_s} \left(\sum_{j=0}^{k_s-1} c_{sj}(1+i)^{-j} + \sum_{j=k_s}^{t} y_s c_{sj}(1+i)^{-j} \right)$$

- discounted summary payments according to the project D $z\sum_{j=k}^{t} d_j(1+i)^{-j}$

Lets denote the sum of all of these numbers by $B(k_1,..., k_n; y_1,...,y_n, z, t)$. The condition of unruining in the moment t means that $B(k_1,...,k_n; y_1,...,y_n, z; t) \geq 0$. Given these conditions it is necessary to achieve the maximum of the value $B(k_1,...,k_n; y_1,...,y_n, z; m)$. Here m is the maximum of the final moments of all considered projects.

The case when reforming the portfolio is not profitable corresponds to arbitrary values of k_i and $y_i=1$, $z=0$. It is possible when it is profitable to add some part of the project D to the existing portfolio. Then $y_i=1$ и $z>0$. The optimization problem formulated is not standard in the sense that some variables (y_i,z) are continual and some of them $\{k_i\}$ are discrete and these two groups are interdependent.

Lets plan the ways of numerical solution of this problem. Since the problem formulated is linear correspondingly to variables y_i and z independently from the penalty function, we can solve the corresponding set of linear programming problems with all the possible sets $\{k_i\}$ and to choose one which maximizes the goal function.

Now we shall describe the group of penalty functions $\varphi_i(c_{ik_i},...,c_{in_i})$, which can reasonably be used on practice. It seems to be reasonable that penalty function consists from two parts: one of whose is defined only by the moment of financing reduction and is equal to $u_1(n_i-k_i)$, where u_1 depends only on the total volume of financing of the project. The second one is defined by the size of under financed part of the project if this part is against the investor and is equal to $-u_2\left(\sum\limits_{j=k_i}^{n_i} c_{ij}\right)^{-}$.

The formulation given is quite difficult. Lets see the setting of the problem in the simplest case when the starting portfolio consists only of one project $C=(c_0,c_1,...,c_n)$ and new suggested project $D=(d_0, d_1,...,d_n)$ has the same validity period as C.

Lets define the function $B(k,x,y;t)$ for natural k, $t\le n$ and real $y,x\in[0,1]$ in the following way.

For $t<k$
$$B(k,x,y;t) = F_0+\sum_{j=1}^{t} c_j(1+i)^{-j}+y\sum_{j=1}^{t} d_j(1+i)^{-j};$$

for $t\ge k$
$$B(k,x,y;t) = F_0 + \sum_{j=1}^{k} c_j(1+i)^{-j} + x\sum_{j=k+1}^{t} c_j(1+i)^{-j} +$$

$$+y\sum_{j=1}^{t} d_j(1+i)^{-j} + (1-x)\varphi(c_k,...,c_n)(1+i)^{-k}.$$

We need to find k,y,x such that $B(k,x,y;t) \ge 0$ for all t and the value of $B(k,x,y;n)$ is maximum.

Similarly the problem of optimal reforming of the investment portfolio given that it is possible to borrow money with the interest rate i_b (condition B from [1]) is formulated. Lets formulate this problem for the portfolio which consists from one project only. The goal function $B(k,x,y;t)$ is defined by induction the following way.

$B(k,x,y;0) = F_0 + c_0 + yd_0$

for $0 < t < k$;

$B(k,x,y;t) = B(k,x,y;t-1)(1 + i_B) + c_t + yd_t$, if $B(k,y,z;t-1) \geq 0$;

$B(k,x,y;t) = B(k,x,y;t-1)(1 + i_3) + c_t + yd_t$, if $B(k,y,z;t-1) < 0$;

$B(k,x,y;k) = B(k,x,y;k-1)(1 + i_B) + c_k + yd_k + (1-x)\varphi(c_k,...,c_n)$, if $B(k,y,z;k-1) \geq 0$;

$B(k,x,y;k) = B(k,x,y;k-1)(1 + i_3) + c_k + yd_k + (1-x)\varphi(c_k,...,c_n)$, if $B(k,y,z;k-1) < 0$;

for $t > k$

$B(k,x,y;t) = B(k,x,y;t-1)(1 + i_B) + xc_t + yd_t$, if $B(k,y,z;t-1) \geq 0$;

$B(k,x,y;t) = B(k,x,y;t-1)(1 + i_3) + xc_t + yd_t$, if $B(k,y,z;t-1) < 0$.

We need to find natural k and real $x,y \in [0,1]$ such that $B(k,x,y;n)$ is maximum. Here n is the moment of the both projects end.

Here are the results of numerical solution of some modeled problems.

Lets remainder (by the moment of new suggestion) of the project C is (50, -100, 50, -200, 200), the project D is (-70, -20, 100, 100, -30, 100).

Lets the starting capital $F_0 = 30$, bank rates $i_B = 0.2$; $i_3 = 0.5$ (for borrowing money).

Penalty function (with the structure described above) parameters are $u_1 = 20$, $u_2 = 0.5$.

Given these condition the optimal behavior of the investor is full financing of the new project and reduction of financing of project C to the rate of 0.3 in the 3rd time moment. This decision will lead to the discounted income of 20.47.

Lets given that the projects and bank rates are the same, penalty function parameters are different: $u_1 = 15$, $u_2 = 0.7$.

Then the optimal behavior of the investor is full financing of the new project and stop the financing of project C in the 2rd time moment. This decision will lead to the discounted income of 24.32.

It is natural that the profit goes up with reduction of penalty conditions (u_1 parameter usually affects more than u_2).

Lets see the examples of calculations when borrowing money is impossible.

Lets remainder of the old project is (50, -100, 50, 30), the new project is (-120, -140, 300, 50). We assume that $F_0=300$, $i_B=0.5$, $u_2=0.5$.

Calculations were done for different u_1. It was found out that the structure of an optimal portfolio is quite sensible to changing u_1.

For $u_1=9$ the optimum is to stop financing of the first projects in the first time moment and full financing the second project (profit is 16.47).

For $u_1=9.2$ the old project needs to be reduced in the first moment to 42% and the new projects - to be accepted totally (profit is 16.08).

For $u_1=10$ the optimum is to continue financing the old project while the simultaneous financing of the new one at 87% (profit is 16.06).

Numerical solutions show that while the contents of portfolio is sensitive to parameter u_1, profit of the optimal portfolio is not sensitive to this parameter. On the other hand, it was found out that in the typical situation the optimal decision can be either staying without changing an old portfolio at all or terminating financing in it from some moment. These observation help to form the simple numerical schemes of solving the formulated problem.

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