Economic Capital versus Regulatory capital for market risk in banking and insurance sectors: Basel II experience and the challenge for Solvnecy II

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ABSTRACT

This paper discusses market risk measurement for regulatory capital requirement purposes in the insurance sector. We present different aspects of market risk in the banking and insurance sectors and we discuss current capital regulations for market risk in both financial services. Today, regulatory approaches for setting capital charges for market risk seem to be converging across financial sectors to the Economic Capital approach. In this context, the Value at Risk (VaR) has become the standard measure to quantify market risk. Although, the VaR is already widespread in banks, this method has not yet become a standard risk measurement tool in the insurance industry. Therefore, we propose necessary adaptations of VaR measure for insurance business specifications. Finally, we compare market risk estimations in insurance by four broad standardized approaches and by assessing three main VaR methods as well as judging the accuracy of all estimations by backtesting program. The article aims to contribute to the current debate concerning the development of a general framework for capital requirements in the insurance sector, including the new EU prudential system.

Keywords: market risk measurement, Regulatory capital in banking and insurance sectors, Economic Capital, Value at Risk

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1. INTRODUCTION

Financial institutions' activities entail a variety of risks. One of the most important category of hazard is a market risk, defined as the risk that the value of an investment may decline due to economic changes or other events that impact market factors (e.g. stock prices, interest rates or foreign exchange rates). Market risk is typically measured using the Value at Risk (VaR) methodology.

In order to provide evidence of safety, firms have to maintain a minimum amount of capital as a buffer against potential losses from their business activities, or potential market losses. The literature distinguishes Economic Capital from regulatory capital. The first is based on calculations that are specific to the company's risk, while regulatory formulas are based on industry averages that may or may not be suitable to any particular company. Moreover, Economic Capital can be used for internal corporate risk management goals as well as for regulatory purposes. This article focuses on Economic Capital estimations for market risk in two main financial institutions: banks and insurers. For banks, the New Basel Accord has provided increased incentives for developing and managing their internal capital on an economic basis. Basel II encourages bankers to use Economic Capital for both: internal management and regulatory capital requirements. VaR is defined as the main tool for the calculation of market risk measurement. In the insurance sector, regulatory capital and Economic Capital diverge. Market risk is calculated via financial ratios integrated to Risk Based Capital formulas, moreover insurance undertakings can calculate their own market risks in internal aims. Nevertheless, VaR has not yet become a standard risk measurement tool in the insurance industry. This situation will be changed in Europe in the comming years, with the introduction of the new prudential system. Solvency II will probably encourage insurers to use a one-year-VaR tool (via internal models) for capital requirement calculations. The European debate is in progress and, today, final decisions are not yet known.

The reflection on the role of market risk in the insurance sector has been dominant for a long time in other geographical zones. Many national insurance prudential systems presently include a market risk element and, for this reason, they are worthy of study. Even when they do not, as in the banking sector, enforce insurers to use VaR methodology for regulatory capital requirements, insurance undertakings can still use VaR tool for internal risk management purposes. However, VaR models can not be used in the insurance industry without modifications and they demand necessary adoptions for business particularities.

The aim of this article is to study market risk measurement in the insurance sector and its estimations for regulatory purposes via standardized approaches and VaR methodology. As, current practices in the European Commission are in favour of using an Economic Capital calculated via VaR methodology for both internal management and regulatory capital purposes (so called Solvency Capital Requirements calculated by internal models), this research will compare Economic Capital calculations for market risk via VaR with existing market risk regulatory capital requirements (RBC formulas for market risk modules) in the biggest insurance markets. Solvency II reform derives many advantages from its predecessor in the banking sector. We dispose today a good experience and fresh knowledge of Basel II Accord in the matter of risk management and control. This analyze will be thus naturally provided in comparison with Basel II amendments and solutions.

In spite of the large quantity of the literature concerning various aspects of the VAR, deeply studding its application in short term trading framework, we can name only a few articles treating for the VaR concept in the insurance field. Here the most important ones are: Albert at al. (1996), Ufer (1996), Panning (1999), Dowd at al. (2001) and Fedor at al. (2006).

The rest of this paper is organized into five sections: Section 2 introduces the VaR definition and methodology, currently in use in the banking sector. Section 3 discusses market risk concepts and its applications in the banking and insurance industries. Next, we present capital regulations for market risk in both sectors: Economic Capital estimations via VaR methodology (introduced in banking business by Basel New Accord amendment) and four existing standardized formulas for market risk regulatory capital requirements in the insurance sector. In Section 4 we propose changes in VaR methodology which are necessary to adapt the concept as an internal tool for Economic Capital market risk measurement in the insurance business. Section 5 empirically compares capital charges for market risk, calculated for five sample investment portfolios, via standardized models (four RBC formulas) and VaR techniques (Economic Capital). The last Section concludes and gives a brief outlook for future research.

2. VALUE AT RISK

In this section, we briefly review the Value at Risk approach, as has been traditionally used in the banking sector. First, we define VaR concept, next, we discuss VaR algorithm, finally, we describe three most popular VaR models in the banking sector.

2.1 Vale at Risk definition

Let V_{t+h} be the future (random) value of a portfolio of financial positions at time $t+h$. Let denote V_t the (known) value of corresponding portfolio at date of estimation. The change in market value of a portfolio over a time horizon *h* is given by

$$\Delta V = V_{t+h} - V_t \tag{2.1}$$

Value at Risk of a portfolio is the possible maximum loss, noted $VaR_h(q)$, over a given time horizon *h* with probability $(1-q)$. The well-known formal definition of a portfolio VaR is

$$P[\Delta V \leq VaR_h(q)] = 1-q \tag{2.2}$$

and therefore

$$VaR_h(q) = R_h^{-1}(1-q) \tag{2.3}$$

where R_h^{-1} is the inverse of the distribution function of random variables ΔV, also called P&L distribution function[1]. Therefore, the VaR estimations depend on R_h distribution.

[1] Profit and Loss distribution function

2.2 Value at Risk measure (algorithm)

Although, the VaR is an easy and intuitive concept, its measurement is a challenging statistical problem. In this paragraph, we discuss a process which is common to all VaR calculations. This algorithm is composed of three procedures:

- measure of portfolio exposure: mapping of all financial positions present in investment portfolio to risk factors
- measure of uncertainty: characterization of the probability distribution of risk factors variations
- computation of the VaR for investment portfolio.

2.2.1 Portfolio exposure measure procedure

First, we describe portfolio exposure by a mapping procedure (representation of investment portfolio positions by risk factors). Assume that investment portfolio is composed from *m* financial positions. Let us define $v_{m,t+h}$ as the future random value of financial position at time $t+h$. Let $v_{m,t}$ be a value of corresponding position at date of estimation. Suppose the portfolio has holdings ω_i in *m* financial positions. Then

$$V_t = \omega_1 v_{1,t} + \ldots + \omega_m v_{m,t} \tag{2.4}$$

$$V_{t+h} = \omega_1 v_{1,t+h} + \ldots + \omega_m v_{m,t+h} \tag{2.5}$$

In general, investment portfolios are complex, and their analysis becomes infeasible if we treat directly all financial positions. Thus, a more manageable approach of modelling the portfolio's behaviour is to represent numerous individual positions by a limited number of specific risk factors. They can be defined as fundamental variables of the market (e.g. equity prices, interest rates or foreign exchange rates) which determine (by their modeling) prices of financial positions, and thus of the whole portfolio. Assume that we chose *n* risk factors. In general, the number *n* of risk factors we need to model is substantially less than the number *m* of positions held by the portfolio. Let us define $X_{i,t+h}$ as the future random value of risk factor at time $t+h$ and $X_{i,t}$ as the value of corresponding risk factor at date *t*.

Each asset $v_{m,t}$ or $v_{m,t+h}$ held by the portfolio must be expressed in terms of risk factors. Thus, there must exist pricing formulas (valuation functions) *F* and *G* for each position $v_{m,t}$ or $v_{m,t+h}$ such that $v_{m,t} = F_m(X_{1,t},\ldots,X_{n,t})$ and $v_{m,t+h} = G_m(X_{1,t+h},\ldots,X_{n,t+h})$. According to (2.4) and (2.5), values of the portfolio V_t and V_{t+h} are linear polynomials of positions values $v_{m,t}$ and $v_{m,t+h}$, thus we can express V_t and V_{t+h} in terms of risk factors :

$$V_t = \sum_{i=1}^{m} \omega_i v_{i,t} = \sum_{i=1}^{m} \omega_i F_i(X_{1,t},\ldots,X_{n,t}) \tag{2.6}$$

$$V_{t+h} = \sum_{i=1}^{m} \omega_i v_{i,t+h} = \sum_{i=1}^{m} \omega_i G_i(X_{1,t+h},\ldots,X_{n,t+h}) \tag{2.7}$$

This are a functional relationships that specify the portfolio's market values V_t and V_{t+h} in terms of risk factors $X_{i,t}$ and $X_{i,t+h}$. Shorthand notations for the relationships (2.6) and (2.7) are

$$V_t = f(X_{1,t},...,X_{n,t}) \tag{2.8}$$

$$V_{t+h} = g(X_{1,t+h},...,X_{n,t+h}) \tag{2.9}$$

Relationships (2.8) and (2.9) are called a portfolio mapping and functions f and g are called the portfolio mapping function. Functions f and g can be linear if the model of portfolio positions price's evaluation is linear (e.g. equities positions). However, the evaluation model is not linear for certain categories of assets (e.g. options), therefore, neither function f *or* g are linear any more.

2.2.2 Uncertainty measure procedure

Functions f and g do not estimate portfolio risk because $X_{i,t}$ and $X_{i,t+h}$ do not contain any information relating to market volatility. We obtain the information about this uncertainty by risk factors distribution. Let us define ΔX_i as the change of the risk factor's price over a time horizon h. Thus, the ΔX_i distribution explains market behavior. We can characterize this distribution by historical data related to all risk factors. We must dispose of the date sample of the risk factors variations, called the window of observations.

Let T be the size of the window of observations (length expressed in trading days), K is defined as $K = \left[\frac{T}{h}\right]$ and $\{\Delta X_i\}_{j=1}^{K}$ are the time series of K returns over h days for each risk factor ($i = 1,...,n$). Generally, this financial data is formed on the basis of one-day-variations of risk factors over a past period, consequently we have T-long time series of one-day returns for all risk factors $\{\Delta X_i\}_{j=1}^{T}$. These time series serve to estimate the ΔX_i distribution. The choice of the window of observations is very important since we must have quotations for all risk factors throughout this time.

2.2.3 Transformation procedure

The portfolio mapping functions map *n*-dimensional spaces of risk factors to the one-dimensional spaces of the portfolio's market value. Although, we need to characterise the distribution of ΔV, mapping functions simply give the value of V_t and V_{t+h}. Thus, we need to apply portfolio mapping functions to the entire joint distribution of risk factors, with the aim of obtaining ΔV distribution. Consequently, we define ΔV as a function of risk factors

$$\Delta V = f(\Delta X_1,...,\Delta X_n) \tag{2.10}$$

$$\Delta V = g(\Delta X_1,...,\Delta X_n) \tag{2.11}$$

where $\Delta X_1,...,\Delta X_n$ are variations of portfolio's risk factors over the period *h*.

A transformation procedure combines thus portfolio's exposure (composition) with characterization of ΔX_i distribution in order to describe the ΔV distribution. Next, we find *q*-quantile of the portfolio distribution which is equal to VaR metric. The third procedure estimates portfolio risk.

In brief, we face two problems while calculating VaR. First, we map portfolio positions to the risk factor by *f* and *g* functions which reflect the portfolio's composition. On its own, however, it cannot estimate portfolio risk because (2.8) and (2.9) do not contain any information relating to market volatility. We obtain this information in the risk factors distribution. We characterise the ΔX_i distribution by historic data. We use for this purpose time series of risk factors $\{\Delta X_i\}_{j=1}^{K}$. However, on its own, ΔX_i distribution can not measure portfolio risk because it is independent of the portfolio's composition. Thus, as soon as we have estimated distribution of risk factors, we continue on to the third procedure by converting ΔX_i description into a characterization of ΔV distribution by mapping functions.

We can specify three basic forms of transformation procedures: variance-covariance, Monte Carlo and historical transformations. Traditionally, VaR models - the computation of a VaR measure providing an output of those calculations (which is the VaR metric) - have been categorized according to the transformation procedures they employ. Even, they follow the general structure presented above, they employ different methodologies for transformation procedure. The presentation of three broad approaches to calculating VaR is beyond the scope of this paper and can be found in Fedor at al. (2006).

3. MARKET RISK IN BANKING AND INSURANCE SECTORS

Conventionally, market risk is defined as exposure to the uncertain market value of a portfolio. Usually, the literature specifies four standard market risk factors: equity risk, or the risk that stock prices would change; interest rate risk, potential variations of interest rates; currency risk, possibility of foreign exchange rates changes; and commodity risk, the risk that commodity prices (i.e. grains, metals, etc.) may modify. This common definition of market risk in financial sector, differs between the bank business and the insurance industry. This section presents the disparity in market risk vision between these two sectors[2].

3.1 Market risk in banks and Economic Capital measurement with VaR techniques as internal model tools: Basel II experience

In the banking industry, the market risk is generally combined with "asset liquidity risk" which represents the risk that banks may be unable to unwind a position in a particular financial position at or near its market value because of a lack of depth or disruption in the market for that instrument. This uncertain is one of the most important category of risk facing banks. Consequently, it has been the principal focus of preoccupation among the sector's regulators.

New Basel Accord[3] defined market risk as the risk of losses in on and off-balance-sheet positions arising from movements in market prices, in particular: risks pertaining to interest rate related instruments and equities in the trading book; and foreign exchange risk and commodities risk throughout the bank. Banks have to retain specific amount of capital to protect themselves against these risks. This capital charge may be estimated by standardised methods[4] or by internal models[5]. That is why banks should have internal methodologies that enable them to measure and manage market risks. Basel II enumerates the VaR as one of the most important internal tools (with stress tests and other appropriate risk management techniques) in monitoring market risk exposures and provides a common metric for

[2] we consider that banking conventions are well known in the finance industry because they were widely discussed in literature and studied by research during Basel II implications. Insurance particularities of the market risk vision is presented in a more exhaustive manner because the vision of market risk measurement is today in evolution. Moreover, new European prudential system preparations demand the research on market risk solutions in insurance sector. These questions are nowadays very important . In consequence, we pay more attention to the insurance rules and their particularities.

[3] Amendment to the Capital Accord to incorporate market risks, Bank for International Settlements, updated November 2005

[4] The standardized approach to market risk measurement was proposed by the Basel Committee in April 1993 and updated in January 1996. The European Commission in its Capital Adequacy Directive (CAD) adopted very similar solutions known as the building block approach. The main difference between the Basel Committee's and the European Union's approaches is in the weights for specific risk. The capital charge is 8% (Basel) or 4% (EU) for equities, reduced to 4% (Basel) or 2% (EU) for well diversified portfolios. The overall capital charge for market risk is simply the sum of capital charges for each of the exposures.

[5] The 1996 amendment to the Capital Accord provided for the supervised use of internal models to establish capital charges. Regulators considered that an internal models approach are able to address more comprehensively and dynamically the portfolio of risks and are able to fully capture portfolio diversification effects. The goal was to more closely align the regulatory assessment of risk capital with the risks faced by the bank.

comparing the risk being run by different desks and business lines. The VaR techniques should be integrated, as an internal model, into the bank's Economic Capital assessment, with the goal to serve as a regulatory capital measurement approach for market risk. General market risk is thus a direct function of the output from the internal VaR model initially developed by and for banks.

The Basel Committee on Banking Supervision in Amendment to the Capital Accord to incorporate market risks states rules for market risk measurment. Part B[6] presents principles use of internal models to measure market risk in the banking sector. The document specifies a number of qualitative criteria that banks have to meet before they are permitted to use internal models for capital requirements purposes (models based approach). These criteria concern among others the: specification of market risk factors, quantitative standards and external validation.

Specification of risk factors concerns separately: interest rates, exchange rates, equity prices and commodity prices. For interest rates, there must be a set of risk factors corresponding to interest rates in each currency in which the bank has interest-rate-sensitive on- or off-balance sheet positions. Banks should model the yield curve using one of a number of generally accepted approaches, for example, by estimating forward rates of zero coupon yields. The yield curve should be divided into various maturity segments in order to capture variation in the volatility of rates along the yield curve; there will typically be one risk factor corresponding to each maturity segment. Banks must model the yield curve using a minimum of six risk factors; in general one risk factor is related to each segment of the yield curve. The risk measurement system must incorporate separate risk factors (difference between yield curves movements e.g. governments bonds and swaps) to capture spread risk.

In the case of equity prices, three risk factors specifications are possible. First, concerns capturing the monitoring of market index, expressing market-wide movements in equity prices. Positions in individual securities or in sector indices could be expressed in "beta-equivalents" relative to this market wide index. Second, treats risk factors in similar way, using more detailed risk factors corresponding to various sectors of the overall equity market. Third, the most extensive approach would be to have risk factors corresponding to the volatility of individual equity issues. Commodity prices' risk factors, being specified in the extensive approach should take account of variation in the "convenience yield" between derivatives positions such as forwards, swaps and cash positions in the commodity.

The Basel Committee on Banking Supervision applied minimum quantitative standards for the purpose of calculating market risk capital charge. No particular type of model is prescribed by Basel II Accord; e.g. banks are free to use: variance-covariance matrices, historical simulations, or Monte Carlo simulations models. The window of observations should not be shorter than one year and data sets should be updated no less frequently than once every three months. The VaR must be computed on a daily basis with a 99^{th} percentile

[6] The document splits into parts A and B. Part A of the Amendment describes the standard framework for measuring different market risk components. The minimum capital requirement is expressed in terms of two separately calculated charges (expressed as percentage): one applying to the "specific risk" of each security (an adverse movement in the price of an individual security owing to factors related to the individual issuer), whether it is a short or a long position; the other to the interest rate risk in the portfolio (termed "general market risk") where long and short positions in different securities or instruments can be offset. Capital charges are applied appropriately to the risk level of each category of assets.

confidence level, for a 10 days horizon. Banks may scale up one-day VaR to ten days by the square root of time, commonly with formula $VaR_{10}(99\%) = \sqrt{10} * VaR_1(99\%)$.

From theoretical point of view, the scaling rule need to be lead in more restrictive environment. All time series for risk factors $\{\Delta X_i\}_{j=1}^{T}$, which serve to estimate ΔV distribution, must be not only i.i.d. (as stated in paragraph 2.3) but also normally distributed. This additional restriction for the $\sqrt{h}$ rule can be explained by the subsequent reasoning. Following paragraph 2.3, variations of log risk factors must be i.i.d. It means that $\ln(X_j) - \ln(X_{j-1}) = \ln\left(\frac{X_j}{X_{j-1}}\right) = \varepsilon_j$, where ε_j is standardised residuals and $\varepsilon_j \overset{iid}{\sim} (\mu, \sigma^2)$. Similarly, if we analyse variations of risk factors between time t-h and date t, we have $\ln\left(\frac{X_j}{X_{j-h}}\right) = \sum_{i=0}^{h-1} \varepsilon_{j-i}$, with variance h$\sigma^2$ and standard deviation $\sqrt{h\sigma^2}$. Hence, the $\sqrt{h}$ rule: to convert a one day standard deviation to h-day standard deviation, we can simply scale by $\sqrt{h}$. However, if the rule of square root of time is applicable to a percentile of the distribution of h-day prices variations (and the VaR is a *q*-quantile of ΔV distribution), variations in prices need to be normally and independently distributed (n.i.d.)[7].

Banks, using internal models, calculate capital requirements in accordance to the following formula:

$$Capital\ requirements = Max\left[VaR_{10,t-1}(99\%), (M+m)\frac{1}{60}\sum_{i=1}^{60} VaR_{10,t-1}(99\%)\right] \qquad (3.1)$$

where M is a regulatory capital multiplier that equals 3 and m, depending on the quality of internal model's estimation (backtesting), varies between [0,1]. To prove the predictive nature of the model from subsequent experience, banks are supposed to use validation techniques. Backtesting – comparison of VaR model's outputs (forecasts) with actual outcomes (realizations) - is a regulatory requirement under the Basel Market Risk Amendment, additionally a sliding scale of additional capital requirements is imposed if the model fails to predict the exposure correctly (three zones approach). The Basel Committee on Banking Supervision requires banks to perform backtesting on a quarterly basis using one year (about 250 trading days) of data. This process simply counts the actual number of times in the past year that the loss on the profit and loss account (P&L) exceeded VaR. The formula (3.1) shows the importance of the predictive nature of VaR models which influence the final amount of capital requirements. As regulators do not define the technique of the modeling approach to be used for capital requirements purposes, it is important that the VaR model works as a good predictor. It encourages banks to apply good quality VaR models because more sophisticated techniques lower capital requirement amounts. The crucial role of backtesting for VaR estimation purposes in insurance sector will be discussed in the following sections of this paper.

[7] For further informations, please see Danielsson et Zigrand (2004).

3.2 Market risk and its regulatory approaches to capital in insurance sector: RBC formula

3.2.1 Market risk definition in the insurance sector

A market risk for the insurance company primarily relates to the risk of investment performance, deriving from: market value fluctuations or movements in interest rates, as well as an inappropriate mix of investments, overvaluation of assets or an excessive concentration of any class of asset. A market risk can also arise from the result of the amount and timing of future cash flows from investments differing from thoses estimated, or from a loss of value if the investment becomes worth less than expected. A particular and important example of investment risk is when liabilities (which cannot be reduced) are backed by assets, such as equities, where the market value can fall.

The market risk, just as in banking sector, is thus defined as the risk introduced into insurance company operations through variations in financial markets. These variations are usually measured by changes in interest rates, in equity indices or in prices of various derivative securities. However, its consequences for insurance undertaking's financial wealth differ from negative results in banking sector. The effects of these variations on an insurance company can be quite complex and can arise simultaneously from several sources, eg. company's ability to realize sufficient value from its investments to allow it to satisfy policyholder expectations. Subsequently, these approaches demand asset-liability matching (ALM) risk also be considered.

Insurance companies are far less susceptible to sudden liquidity needs than banks might be. Thus, the market risk in insurance business is rather combined with "asset-liability mismatch risk" than with "asset liquidity risk". This is the main difference between risk vision in both, insurance and banking, sectors. By the way, insurance companies are not facing a systemic risk of the same importance as the bank sector does. There has been no evidence of the failure of an insurance company being a significant source of systemic risk.

Recent works of the International Actuarial Association (IAA) on risk definitions in the insurance sector tended to define a market risk as risks related to the volatility of the market values of assets and liabilities due to future changes in financial variables such as stock prices, interest rates or exchange rates. Consequently, in addition to the volatility of market risk affecting the net market value of the insurer's asset, IAA considers that market risk may also affect the liabilities and net surplus position. IAA divided market risk into the following subcategories: interest rate risk, equity and property risk, currency risk, basis risk, reinvestment risk, concentration risk, off-balance sheet risk and asset-liabilities mismatch risk[8].

[8] for further information, please read "A global framework for Insurer Solvency Assessment" , International Actuarial Association, 2004

3.2.2 Financial policy in insurance business

The definition of market risk in the insurance sector has thus many common points with banking definition. However, it differs. To well understand market risk nature in the insurance business, its financial policy particularities must be briefly presented. The market risk vision in insurance sector depends on asset allocation character: its long term goals shorted however by regulation rules and provoking fixed income instruments purchasing ; "buy and hold" character; importance of liabilities; and trivial liquidity risk.

The objective of the financial management in insurance sector is the portfolio's return optimization, by respecting the regulatory constraints and the engagements represented in liabilities. Insurers are subjected to a double requirement: preservation of the nominal value of their short-term capital and protection of the real value of this capital at long horizon. The conflict between the short-term risk (evaluated for regulatory purposes) and the need for a long period management (sometimes going up to 20 years) imposes, from the beginning, certain number choices of the asset allocation.

As far as financial policy in the insurance industry has a long term vision, in theory, market risk should also be revised in this long perspective. Insurance undertakings have long term engagements in their liabilities. Its durations is equal to many years, sometimes even to 30-40. Financial policy objective is then a long term goal. However, prudential regulations (regulatory capital requirements, technical provisions) oblige insurers to recognize one year vision of their market risk. Moreover, they must annually ensure a flow of incomes to cover operating expenses (overhead) and yields guaranteed to their customers. These annual constraints as well as limitation of solvency rules to one year vision influence financial management and encourage acquisitions of fixed income instruments, more than variable-yield investments (equities). Holding an important portfolio of fixed income assets (bonds) instead of equities positions changes a market risk perception in the insurance business. That is why, one year market risk solvency rules (regulatory capital requirements) in insurance sector are often criticized by practitioners and academics. They pronounce that regulatory rules, that take into account the objective of minimization of insolvency probability in one year horizon, favor bonds detentions in investments portfolios, and in fact, underestimate bonds risk and overestimate equity risk.

Therefore, insurers' definition and the apprehension of the market risk is impacted by the regulation: investors have to reason in nominal terms, at horizon of one year, even they should acquire a long term market risk vision. As consequence, insurers have a number of important percentage of interest rate positions (bonds, loans and deposits) in their investment portfolios, which permit them to respect the regulatory constraints, engagements in liabilities and to avoid short term risk of nominal loss. At the same time, importance of fixed income positions specifies portfolio risk of insurance undertakings.

Insurance financial policy follows generally a "buy and hold" rule. Insurance investor thus buys financial instruments which guarantee an output, enabling him to respect its engagements towards its customers and its shareholders. Its goal is neither the speculation, nor the trading, as in the baking sector. The financial portfolio is more stable, even in long term perspective that in the banking environment - insurers do not have the same reactivity comparing to trading desks. Indeed, a trader can easily releases his positions, which justifies the estimate of short term market risk. Insurers can not.

The obligation to payback insurance policies (e.g. guaranteed benefits), as well as necessity to cover operating expenses (represented as a minimum return constraint) push insurers to fix theirs assets allocation policies with liabilities engagements, by holding equal assets and liabilities durations. In these cases, financial policy and market risk take an “asset-liability dimension”

3.2.3 ALM risk in insurance sector

As underlined above, insurance undertakings are principally facing liabilities risks while banks are mostly confronted with assets risks. This distinction is very general because banks have also liabilities risks for many reasons, e.g. interest rate or foreign currency risk they can be exposed because of its own debt. Insurance companies are also confronted with assets risks, e.g. their performance is affected by financial market fluctuations. However, liabilities risks are the most important hazards they have to face, especially in many insurance branches where asset and liabilities risks are connected, e.g. by contracts with profit participation clauses. In these cases, risk evaluation should be lead at the net level (market value of insurer), including assets and liabilities positions. Taking into consideration the correlation between assets and liabilities is thus crucial in many branches. ALM risk importance depends on a company’s activity character. For life insurers, mismatch of assets and liabilities risk due to cash flow, currency and timing is still accompanied by pure investment risk (asset risk) resulting from inappropriate mix of investments, overvaluation of assets and excessive concentration of assets in investment type products. For non-life undertakings, market risk is characterized as a pure investment performance derives from inappropriate mix of investments, overvaluation of assets and excessive concentration of assets. Asset-liability matching risk is thus not usually a major issue for non-life insurers due to the short duration of its contracts (for long tail business the claims profile may need to be matched).

The example of asset-liability mismatch risk in insurance is an interest rate risk. The impact of interest rate risk on an investment portfolio cannot be considered in isolation to the effect on valuation of liabilities and guarantees. Therefore, it is critical for risk interaction to be properly reflected in the models. This is a significant difference, as banking models tend to focus separately on the key risk areas. Commercial banks also face asset and liability mismatch risk (in fact, more than non-life insurers). Deposits constitute liabilities that may be due in a short term. These financial sources are transformed in loans with longer maturities that make assets hard to recover in a short term. For that reason, liquidity cries can provoke insolvency and lead to failures mechanism in a short term. In insurance, a substantial part of assets could be easily realized and an important part of liabilities is not due in a short term (has a long term “maturity”). Thus, liquidity cries can not precede insolvency in insurance sector and asset liquidity risk is not, as stressed before, a primary preoccupation of control authorities.

3.2.4 Market risk measurement by prudential systems

Even a market risk has an important ALM dimension in many insurance branches, present prudential systems (RBC formulas) do not seem to consider entirely these particularities and impose mostly capital charges in proportion to assets volatility. However, regulators and prudential authorities are more and more taking into consideration the significance of ALM risk as a market risk component in the insurance industry. We can observe from the past that prudential systems have less and less "asset risk vision" and more and more "ALM risk nature". A good example of these modifications is the S&P model that is now changing its character from "pure asset measurement formula" to "asset-liability mismatch control model". The new European prudential system, called Solvency II, will probably fully privilege asset-liability mismatch risk in market risk formula.

Nowadays capital regulations in the insurance sector do not split market risk measurement into regulatory and Economic calculations. The choice of market risk estimations between standardized methodology and internal models will be introduced by Solvency II. Present regulatory capital charges are established by application of minimum capital ratios. These ratios are equivalent to the standardized method in banking system. VaR is not demanded by insurance prudential systems but it can be used by insurers for internal risk management purposes.

We present briefly the four most representative prudential models founded on RBC formula as examples of market risk calculations for regulatory capital purposes in the insurance sector. We focus on market risk (and ALM risk) capital requirements as key factors for this article. We describe the following prudential systems:
- NAIC system, as a first risk based model, being applied on the largest insurance market in the world (USA) for regulatory capital calculations
- S&P model, applied by one of largest rating agency on every insurance markets for company evaluating purposes
- two recently introduced in Europe prudential systems: FSA model, applied on the largest European market (Great Britain) as well as "2002 GDV" model, applied on the second European market (Germany).

American prudential system identifies four risk categories used in determining the capital charges for life insurance companies (C1: Asset Risk, C2: Insurance Risk, C3: Asset-Liability Mismatch Risk, also called interest rate risk and C4: General Business Risk) and Property&Casualty branches (R1-R2: Asset Risk, R3: Credit Risk, and R4-R5: Underwriting risk). Pure market risk varies by credit quality and asset type of invested assets held by the insurer. The model takes into account asset and liability matching risk in the life insurance business: if interest rate risk is above 40% of total capital requirements, scenario analysis is required to calculate ALM risk capital. The mismatch risk is not considered for non-life branches.

NAIC does not use the term "market risk". Indeed, even the minor part of the investment book that is marked to market (equity positions) is done mostly at yearend (at the same frequency as the reporting cycle). But both "asset risk" (C1 element) and especially interest rate risk (C3 component) contain elements of what would be regarded as market risk by regulators in the banking industry. In spirit C1 is closer to the Basel risk-adjusted assets for credit risk.

Today the S&P model recognizes four classes of risk for life insurers (asset, insurance, interest rate and business) and three for non-life undertakings (asset, underwriting premium and underwriting reserve) with no specific treatment for ALM risk.

The FSA prudential system (for non-life and non-profits life insurance companies) distinguishes asset related and insurance related capital requirements for non-life undertakings; and death, health, expense and market risk factors for annuities. UK's system determinate considers resilience reserve reflecting capital (based on ALM stress scenarios) for life products but do not have special treatment for ALM risk. "Mild punishment" of equities investments seems to be caused by the significant percentage of shares in British insurance undertakings portfolios.

The new German prudential system, called 2002 GDV, include several classes of risk for life insurers (investment, pricing and interest rate) and non-life companies (investment, re-insurance credit, premium, loss reserve and life assurance reserve). GDV includes the treatment of business risk within both the GDV Life and Non-life models. Duration mismatch (ALM risk) is captured and reserves are split between short, medium and long (the interest rate risk capital applies only to the liabilities). Moreover, German's model considers what proportion of the bond portfolio is explicitly used to match liability cash flows and excludes these from the bond value volatility calculation (pure market risk factor).

In aiming to compare prudential systems, the description of market risk elements must stay quite general and can not take into account particularities of each model. Table 1 in appendix B shows prudential systems' capital requirements for each asset's class. For bonds, the S&P, 2002 GDV and NAIC models all have factors that decrease with worsening of the credit rating. In opposite, FSA model (for non-life companies) makes no differentiation between any credit ratings and the S&P formula makes no differentiation between bonds rated up to A. For equities, the factor applied by the 2002 GDV model is significantly higher than the factor used by the S&P and NAIC models. For real estate, the factor applied by the S&P model is significantly higher than the factor used by the 2002 GDV and NAIC models. Treating the equities and the real estate, the FSA standard uses different factors for different product types: lower for non-life products and higher for annuities. Additionally, the 2002 GDV, S&P and NAIC models all capture concentration risks. The description of this factor is not useful for the analysis of this paper.

Insurance prudential regimes have chosen different trade-offs between sophistication and simplicity, considering both market and ALM risks in diverse way. In some systems, the mismatch risk is omitted, in others, it is estimated separately for capital requirements, or implicitly covered by market risk. Its role is also different in life insurance branches (very important) and non-life activities (where due to the short term investment policy character of ALM risk its is not significant). Since this article has general and universal character - in aiming to give the possibility to compare regulatory capital charges between different prudential systems and to compare them with Economic Capital estimations - the paper treats only pure market risk, leaving ALM assessment for further works. ALM risk is not included in all regulatory formulas (e.g. non-life NAIC), moreover, it is still treated separately to market risk (as an additional element in life RBC formula) and it is not fully integrated into capital requirement models.

The focus of this paper on pure market risk permits also the oportunity to deeply investigate how to adapt models of Economic Capital measurement for market risk in the banking industry – the Value at Risk – for insurance needs. The VaR techniques have not yet become a standard risk measurement tools in the insurance industry. However, they can be used in the insurance sector for Economic Capital calculations after introducing necessary modifications to the procedure of VaR measurement, presented in paragraph 2.2.

4.VaR MEA SURE IN INSURANCE BUSINESS

This chapter proposes necessary adaptation of VaR measure for insurance specifications. We discuss all three elements of procedure presented in paragraph 2.2.

4.1 Modification of portfolio exposure measure procedure: taking into account changes of portfolio composition

Throughout the rest of the paper, we consider that investor does not change any portfolio's positions over *h* horizon (investor does not sell or buy any assets between date *t* and date *t+h*).

First procedure of VaR measure (paragraph 2.2.1) maps portfolio position to risk factor by the mapping function which reflects the portfolio composition. In the banking sector, due to the short term character of estimations (one day) composition of V_t and V_{t+h} are identical (assuming that investor does not change positions), in consequence, function *f* and *g* remain equal for (2.8) and (2.9). In insurance context, where portfolios include an important percentage of interest rate instruments and VaR estimations have a long term character, therefore V_t and V_{t+h} can not be supposed to be similar.

Proposition 1: The nominal price and quantities of positions in the investment portfolio including interest rate instruments change over VaR estimation horizon (even the market conditions remain unchanged at time t and at time t+h). The difference between V_t *and* V_{t+h} *are caused by the sum of cash flows generated by interest rate positions (in case of bonds, coupons and face values paid at maturities) as well as durations diminution which changes the price of interest rate instruments over horizon h.*

It follows from Proposition 1 that $V_t \neq V_{t+h}$ (if portfolio contains interest rate instruments) even if the investor does not change positions and the market conditions are similar at time *t* and *t+h*. This particularity changes the risk profile of the investment portfolio ($f \neq g$) and has to be taken into consideration. We propose to analyse the situation (the composition) of the interest rate portfolio at the end of the VaR horizon – at time *t+h* thus, when we consider the portfolio mapping function specified in (2.9).

We are supposed to measure the real risk profile of the portfolio. Thus, portfolio behaviour will depend on its structure at date h, represented by (2.9). This statement modifies also VaR estimations by the square root of time rule. Even should we calculate one-day VaR, we should use portfolio representation at date *t+h*.

We can evaluate interest rate position prices at date *t+h* if we know their prices at time *t* (which is known because it is a current price), the zero coupon yield curve and all cash flows generated by bond in the portfolio. Thus, we calculate asset values of each $v_{m,t+h}$ conditional from $v_{m,t}$ (information at time *t*). Next, we find V_{t+h} and function *g* specified in (2.9).

4.2 Sensibility of uncertainty measure procedure: importance of assumptions characterizing risk factors' distributions

According to paragraph 2.2.2, VaR measure depends on characterisation of the ΔX_i distribution. Thus, VaR depends on expected value of variations of risk factors over the period *h*. $E[\Delta X_i]$ expresses trends (drifts) of risk factors' prices in the future. These expected returns must be assumed while VaR is estimating by parametric methods (variance-covariance and Monte Carlo approaches) and they are included in the historic data set distribution when VaR is measured with non parametric models (historical simulations). The forecast of these trends are problematical and there does not exist one incontestable and unique methodology. For short periods, the expected return is very weak. Thus, in the banking sector, where the VaR is often calculated on one day basis, the assumption of a null expected return is being made. In insurance sector, VaR must be calculated for longer horizons for which the portfolio's expected return becomes significant.

While estimating VaR, we can not take into account the expected return based on historical data (time series of $\{\Delta X_i\}_{j=1}^{K}$) because these expected returns change over time and depend on the length of window of observations K. The VaR amount would become subjective and biased. Thus, expected returns must be supposed. We propose two possibilities:

- choice of a nil expected returns for all risk factors : this neutral assumption does not pass any judgment about future (favourable or adverse) changes of the market situation.
- choice of assumed expected returns for risk factors: expected returns should be forecasted by independent experts (this independence would guarantee reliability of estimations), or should be based on the market consensus at the date of calculation (e.g. by using forward rates or informations deduced from options).

The choice of $E[\Delta X_i]$ is very important because it strongly influences the VaR estimations and, consequently, the portfolio risk. It is thus advisable to make it with the greatest prudence. The first option, when $\forall_i E[\Delta X_i] = 0$, underestimates VaR metric in a period of economic growth and, inversely, it overestimate VaR metric when prices decrease. The determination of expected returns for all risk factors $\forall_i E[\Delta X_i] = a_i$ seems to be quite problematical because it depends on subjective preferences and demand solid analysis. All these choices should be done with greatest prudence because they impact directly final VaR calculations.

The second statement corresponding to $E[\Delta X_i]$ concerns the investment portfolio including interest rate positions. In these cases, the distribution of random variables ΔX_i for corresponding interest rate positions can be estimated either by variations of zero coupon bonds' prices or by variations of zero coupon interest rates. This choice is not equivalent.

Proposition 2: Measuring the risk of interest rate positions by interest rates variations or prices variations is not equal:
(i) if we use variations of prices as risk factors for interest rates positions, we underestimate investment portfolio risk
(ii) if we use variations of interest rates as risk factors for interest rates positions, we overestimate the risk of investment portfolio

This choice should be taken with care. Therefore, selection of zero coupon interest rates variations seems to be the preferred solution.

4.3. Selection of transformation procedure: choice of appropriate VaR model due to data sample characteristics

Basel II amendments do not prescribe any particular type of VaR model for regulatory capital calculations and banks can use any method they prefer. In the insurance sector, many VaR models can not be used for long term estimations due to characteristics of historic data. In consequence, many VaR techniques can not be used for Economic Capital measurement, especially for capital charges estimation purposes, because results of the calculations are biased. We propose a procedure for the choice of VaR method (among the three techniques presented in paragraph 2.2) best adapted to the long term character of insurances estimations. The selection of adequate model is based on data set proprieties.

Let us assume that we dispose of the one day variations (daily returns) of all risk factors at the moment of VaR calculations. As underlined in paragraph 2.3, all observed outcomes in time series $\{\Delta X_i\}_{j=1}^{T}$ (where $i = 1,...,n$) must be identically and independently distributed (i.i.d.). If time series are not i.i.d., in general, in the banking sector we use ARCH/GARCH processes that propose a specific parameterisation for the behaviour of risk factors. They allow for time-varying conditional volatility: even the unconditional one day returns are not i.i.d., suitably conditioned returns became normal. ARCH/GARCH models make assumption of i.i.d. standardised residuals (the most generally used distribution is the standard normal) and specify the distribution of residuals[9]. However, these techniques can not be used for regulatory capital calculations in the insurance sector. According to Christoffersen, Diebold and Schuermann (1998) and Christoffesen and Diebold (1997), if the short term application of GARCH models appears efficient, volatility is effectively not forecastable for horizons longer than ten or fifteen trading days (depending on the asset class). More detailed description is beyond the scope of this paper.

The assumption that all time series are i.i.d. allows estimating VaR with methods presented in paragraph 2.3. We can use Monte Carlo simulations (based on the risk factors historical distribution) or two other methods based on $\{\Delta X_i\}_{j=1}^{K}$ where ($i = 1,...,n$). In practice, variance-covariance and historical simulations approaches are inapplicable because time series of one year returns which allows estimation of risk factors distributions are too short (e.g. if we dispose of a ten years data sample, we have ten variables to estimate risk factors distribution). In these cases, the VaR is very sensitive to data set changes. Sometimes, VaR metric can not be calculate (e.g. in the historical simulation approach we can not calculate $VaR_{1year}(99{,}5\%)$ if we have a ten years long window of observations).

If we can formulate an additional assumption that all time series of risk factors are normally and independently distributed (n.i.d.), we can use Monte Carlo simulations based on normal distribution or we can calculate one year VaR by scaling one day VaR with the $\sqrt{h}$ rule (as shown in paragraphs 3.1 and 4.1).

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[9] The definition and study of temporal aggregations of GARCH processes were presented by Drost and Nijman (1993).

5. EMPIRICAL COMPARISION OF CAPITAL REGULATIONS: REGULATORY AND ECONOMIC CAPITAL ESTIMATIONS

Empirical study compares capital charges for market risk calculated for five fictitious investment portfolios by: four standardized methods (as examples of regulatory capital requirements calculations) and three VaR models (as examples of Economic Capital measurement). These tests allow contrast between levels of capital charges and efficiencies (backtesting issues) of all techniques.

5.1 Data characteristics and calculations

Asset allocations in insurance sector are highly varied because companies lead separate and independent financial policies. Moreover, as stressed in paragraphs 3.2.2 and 3.2.3, investment policies depend on liabilities structures specific for each undertaking. Therefore, we can not define one investment portfolio which is representative of the insurance sector, nor a portfolio representative of national insurance market. For that reason, four investment strategies will be tested: one very prudent (portfolio 1: characteristic for USA insurers), two intermediaries (portfolios 2 and 3: distinctive for many national markets), one very aggressive (portfolio 3: close to many English insurers) and one theoretical (portfolio 5: showing equities importance for VaR estimations). These five investment portfolios are presented in Table 2 in Appendix B. They are based on 30 positions: 25 government bonds (rated AAA, thus the credit risk, which might be included for capital requirements calculations inside market risk formula, is null), 4 international indexes of equities and one index of real estate. We prefer to consider indexes instead of equities positions because indexes express global market movement (they do not include the positions specificities). Moreover, these 4 indexes represent situations in the worldwide economy because of their international split: 30% of shares portfolio is represented by European index (MSEUEU Index), 30% by American index (MXUS Index), 10% represents emerging markets (MXEF Index) and 30% global economy changes (MXWO Index). We study portfolio situation at 31/03/2007 (as specified in paragraph 4.1, we analyse the composition of the interest rate portfolio at the end of the VaR horizon – at 31/03/2008).

International Actuarial Association, International Association of Insurance Supervisors and Committee of European Insurance and Occupational Pensions Supervisors principles indicate that market risk can only be measured appropriately if positions' values are measured by a "fair value" approach. We use economic value based measurement in our empiric study. Thus, we do not base our calculations on any accounting principles. No accounting framework (we consider market values for all positions at date of capital charges estimation) guarantees comparison between prudential systems and VaR techniques as well as evaluation of their efficiencies. Existing differences between European and American accounting standards would not allow us to compare results of calculations (as accounting regulations impact portfolio risk vision). Accounting rules are also unclear and erratic in some situations. For example, on several occasions the NAIC has authorized insurance companies to use association values that have been significantly above year-end closing prices in order to prevent technical insolvencies caused by temporarily depressed market prices[10] or, in the wake of the decline in equity markets after September 11, the German regulator allowed

[10] see Troxel and Bouchie (1995)

insurers to create "hidden losses" by postponing write-downs if the market value of their investments fell below the purchase value instead of booking equity losses in the year they occurred [11]. By the way, accounting standards presently converge towards fair value approaches (IFRS and FASB regulations) which match with our article's framework. A more detailed description of accounting rules is beyond the scope of this paper. Finally, we study asset risk (not ALM risk) because four prudential standards still focus on this approach and we consider that the portfolio has no cover (no derivative instruments).

First, we calculate regulatory capital requirements for market risk by four standardized methods (prudential formulas) presented in paragraph 3.2.4: NAIS, S&P, FSA and GDV. We impose capital charges for each category of asset. The capital rates (expressed as a percentage of investment portfolio amount) are presented in Table 1 in appendix B.

Second, we estimate Economic Capital for market risk. As presented in 3.1, in banks Economic Capital is estimating by VaR methodology. We acquire the same idea for the insurance sector. Thus, in this paper, insurance undertaking's Economic Capital for market risk is based on VaR techniques. We assume the 99,5% confidence level for our estimations which is analogous to Solvency II considerations. We also chose a one year estimation horizon due to prudential rules which oblige insurance undertakings to evaluate their risks at least once a year for regulatory capital purposes. One year term is shorter than insurers' investment horizons (multiple years). Consequently, our investigation would be prolonged over multiple years. However, conclusions would remain unchanged.

Therefore, to compare Economic Capital calculations with regulatory capital requirements, the VaR estimations horizon is equal to one year – 262 labor days ($h = \{262\}$). The VaR is measured using several windows of observation. We have four historical observation periods with lengths of one year, two years, five years and ten years; in fact we have sample periods of 262, 523, 1305 and 2610 trading days.

We imitate Basel II risk factors specifications and we choose to monitor 4 international market indexes (expressing worldwide market movements in equity prices for investment portfolios of European and American insurers) as risk factors for equity prices and one index for real estate prices. For interest rates, we select a set of 15 risk factors estimating forward rates of zero coupon yields for different maturities: from 3 and 6 months to 1 year and 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 30 years (I01403M Index, I01406M Index, I01401Y Index, I01402Y Index, I01403Y Index, I01404Y Index, I01405Y Index, I01406Y Index, I01407Y Index, I01408Y Index, I01409Y Index, I01410Y Index, I01415Y Index, I01420Y Index and I01430Y Index). The yield curve is thus divided into 15 maturity segments in order to capture variation in the volatility of rates along the yield curve. According to (2.9) and *Proposition 1*, function g is a linear combination of these 20 risk factors ($n = 20$): $V_{t+262} = g(X_{1,t+262}, X_{2,t+262}, \dots, X_{20,t+262}) = \sum_{i=1}^{20} e_i X_{i,t+262} = \sum_{i=1}^{20} e_i (X_{i,t} + \Delta X_i)$. In accordance with paragraph 4.1, all cash flows from the bond portfolio are redistributed on all zero coupon maturities t and $t+h$. Thus, we can establish d_i and e_i coefficients ($i = 1, \dots, 20$) and N. To simplify the demonstration, we choose the duration neutral mapping method $\Delta V = \sum_{i=1}^{20} e_i \Delta X_i + N + \sum_{i=1}^{20} (e_i - d_i) X_{i,t}$, where N is the cash flows value at the time $t+h$ (these

[11] see Hulverscheidt and Fromme (2003)

cash flows were paid between dates t and $t+h$). We stress that cash flows must balance the treasury of the company but they can also be reinvested directly in new positions. In aiming to be prudent, these cash flows will be considered as remaining in the treasury. In consequence, Σ equals the algebraic sum of cash flows that occurred between times t and $t+h$.

We referee as impossible the choice of one expected return for each risk factor which is accurate for all cases: we make no assumption on a market evolution which, by the way, allows comparison of markets from different geographical zones. All risk factors thus have a zero expected return at h days (see paragraph 4.2): $E[\Delta X_i] = 0$ (and $i = 1,...,20$). Next, we calculate expected portfolio return between dates t and $t+h$. Then $E[\Delta V] = \Sigma + \sum_{i=1}^{20} (e_i - d_i) X_{i,t} \approx 0{,}01\%$ which means that cash flows evaluation by algebraic sum is very prudent.

We estimate VaR with the three models presented in paragraph 2.3:

- variance-covariance approach (based on the normal distribution): one day VaR estimations multiplied by $\sqrt{262}$;
- Monte Carlo method (based on normal distribution);
- historical simulation method, estimating the VaR for one day, then scaling by $\sqrt{262}$.

Therefore, we compare VaR metrics (results) calculated by models using extrapolation of one day VaR by "square root of times" (historical method and analytical method) with random simulations of futures returns (Monte Carlo method); as well as with regulatory capital requirements. All capital charges are represented as a percentage of total investment portfolio amount at 31/03/2007 in Table 3 in Appendix B.

5.2 Regulatory capital requirements results and Economic Capital estimations

We calculate regulatory capital requirements for two sorts of models. The first family is represented by NAIC and S&P systems, created in early 90’s as primary RBC formulas in the insurance sector. Second, FSA and GDV models are two recently introduced prudential systems. European models distinguish market risk in more prudent way.

Economic Capital requirements depend on VaR method. Our empirical research shows that the most pessimist results are given by historic simulations and variance-covariance methods. Both models, using “square root of time rule”, tend to underestimate the VaR metric because the normality assumption of ΔV distribution seems not to be consistent with the behaviour of financial returns. As discussed in paragraph 4.3, we can not use ARCH/GARCH models to correct non-n.i.d. standardised residuals. This misspecification issue is relevant for the VaR metric and is visible especially in the long term perspective. However, the historical simulations model is more biased than variance-covariance method because its estimations are directly based on data set while variance-covariance approach is founded on variance-covariance matrix which diminishes the impact of non-n.i.d. data. By the way, in the banking sector, approximation of n.i.d. data is justified by short estimation horizon. In this context, the VaR model that gives the most accurate Economic Capital estimations is the MC method. Even this technique is based on a normal distribution, its calculations are best approximation because they do not use the square root time rule (MC model results are close to variance-covariance estimations for short data sets).

In Table 4 in Appendix B, we give empirical evidence for *Proposition 2*, proving that using variations of prices as risk factors for interest rates positions tends to underestimate investment portfolio risk as well as applying variations of interest rates as risk factors leads to higher level of Economic Capital.

By their theoretical construction, VaR models charge investment positions proportionally to their volatilities. As discussed in paragraph 3.2.2, many practitioners and academics propose not to base the Economic Capital measurement for market risk on VaR models, pointing that the market risk is not crucial in insurance sector and VaR techniques would push insurers to sell equities positions. Although Economic Capital (e.g. estimated by MC VaR with data sets of 1 year and 2 years) might be close to FSA and GDV requirements, in nearly all occurrences, regulatory requirements underestimate portfolio risk (Economic Capital is higher that regulatory charges). If regulatory capital charges were less restrictive than Economic Capital ones (e.g. the four national systems presented in this article), companies would prefer to use standardized methods. Moreover, regulatory capital formulas give very favourable treatment to interest rate positions (note that some of the interest rate risk must be captured by the additional approaches, e.g. NAIC rules expect separate calculations under C3 interest rate risk component in the context of a particular liability exposure, which is beyond the scope of our empirical study). According to VaR estimations, bonds risk seems to be underestimated by regulatory systems. Any future prudential European model will have to take into consideration these remarks if one of its goals is encouraging insurers to use internal models for capital requirement estimations.

Finally, we stress a modest differences between Economic Capital requirements for Portfolio 1 (characteristic for USA insurers) and Portfolio 2 (representative for many European insurers). In most cases, investors should tend to choose Portfolio 2 which offers higher expected return with comparable level of capital requirements. This remark should be taken into consideration during the discussion on Solvency II project.

5.3 Backtesting results

Empirical results show that capital requirements diverge for standardised methods and even Economic Capital estimations. In the latter case, they depend on the choice of VaR method and its hypotheses (e.g. the size of the window of observations). Therefore, we need to judge the quality of estimation by a backtesting program. The verification of models (both, standardised and VaR) accuracy is based on ex-post comparison of capital charges calculated by each model against real yearly changes in portfolio value over past ten years. Thus, the backtesting will help to form a quantitative opinion (based on our empirical sample) of prudential systems' and VaR efficiency in the insurance sector. We test our calculations only on "Portfolio 5" (composed of 100% equitie positions) because the interest rate positions portfolio would not have the same composition in the past or many bonds did not exist in the past years (we stress that backtesting methodology adapted for insurance regulatory framework, where investors hold an important portfolio of interest rate positions, is a great challenge for future research). Moreover, we have a long data sample for equities positions.

We apply Basel II criteria for testing our results: we compare the models' outputs (forecasts) for each day in the past with actual outcomes (realizations) of equities portfolio. We present backtesting results in the Table 5 in Appendix B as the number of times (in percentage of all tested days) in the past ten years when the loss on the profit and loss account (P&L) exceeded VaR and standardised method estimations (calculated one year ahead).

We note that NAIC, S&P and FSA models have a high failure rate. The first two models do not specify a target confidence level of capital requirements - this level is inherent in the calculation of NAIC factors; in S&P model, the capital requirements are a function of the desired debt rating (100% for a Capital Adequacy Ratio). Contrary, the FSA model fixes a target confidence level at 99,5% and assuming an equivalence with BBB rating, German system's target confidence level is assumed as equivalent to the BBB rating and equals 99,78%. In this context, all four standardised approaches do not offer good quality of estimations. In contrast, Economic Capital has been well estimated with most VaR methods (at least VaR models with 1 year and 2 years data sets). Slightly exceeding the MC method and the deviation in the historic simulations method based on the 1 year window of observations would have been accurate if the assumption of null expected return for risk factors had been changed.

In Figures 1-6 in Appendix B, we represent the evolution of VaR estimations with different windows of observations and regulatory capital charges for NAIC and S&P models (15%) and GDV systems (26,6%), compared with real losses of equity portfolio (one year after capital charges estimations). All results are represented as a percentage of equity portfolio amount at 31/03/2007. This backtesting analyse in time outlook gives interesting remarks. First, the historical simulation techniques present predictable jumps, due to the discreteness of extreme portfolio price variations. For example, while computing the VaR of a portfolio using a rolling window of one year and one return is a large negative number, we can predict that the VaR metric jumps downward, because of this observation. The same effect (reversed) reappears after one year, when the large observation will drop out of the window. This very undesirable characteristic for capital requirements (capital charge should be stable over one year not to depend on date of estimation) can be "smoothed" by the prolongation of the size of data sets. All VaR models are very sensitive to the size of windows of observation. The length of data samples determines the reactivity of VaR estimations to market trends. Indeed, when the size of the window of observations enlarges, VaR metrics vary less in time (the stability in time of the 0,5 % quantiles of ΔV distributions is more important for methods using the matrix of variance-covariance). In contrast, when the data set is short the VaR estimations react actively to market trends. However, Economic Capital calculations with VaR methods need to be stable in the one year outlook in order to be accurate during the entire year and not to fluctuate significantly when changing the date of estimation. In other words, the line, showing the VaR amount's evolution in time, should be a flat.

In conclusion, based on the empirical example, all standardised approaches underestimate portfolio risk (only the 2002 GDV model is very close to its target confidence level) and they do not react to market changes (being by that too pessimist in period of economic wealth). We do not distinguish neither an accurate VaR model: the variance-covariance and historical simulations methods, which scale one-day-VaR on one-year-VaR with the square root of time, have acceptable backtesting failure occurrence but essentially overestimate capital charges and are not "smooth" enough (their VaR metrics are not fairly stable in time); Monte Carlo simulations model has too large degree of backtesting failure over last ten years. We should emphasise that this judgment is based on this papers' empiric sample and can not be generalised.

6. CONCLUSIONS

This paper discusses market risk measurement in the insurance sector. We describe four existing standardised methods and we present VaR methodology for Economic Capital calculations. For this reason, we propose necessary adaptations of the VaR measure - initially developed by and for banks - for insurance particularities. We focus our interest on VaR adjustment for regulatory capital requirements purposes. Finally, we compare accuracies of standardized methodologies and VaR models.

Market risk estimates the uncertainty of future earnings, due to the changes in market conditions. In an insurance company, this risk is introduced through variations in financial markets. It's business character usually involves a mismatch between the time when premium income is received and the time when expenses and policy benefits are paid. Market risk in the insurance sector thus takes an asset-liability mismatch dimension. This concept will probably be accepted by the new EU prudential system. Current works on Solvency II tend to consider market risk as asset-liability mismatch risk. However, the main existing prudential systems still have a pure market risk (asset risk) vision. In order to compare VaR measuring with prudential models' estimations, this article concentrates on investment risk perception of the market risk.

The comparison between standardised methods and VaR models shows that:

- both capital charges estimation approaches (standardised methods and VaR models) demand important capital requirements for volatile financial instruments, especially equities positions, as the source of market risk in an insurance company is not in excessive assets' volatility but in asset and liabilities durations mismatch;
- standardised methods seems to underestimate bonds risk (when compared with VaR results);
- empirical results show that, in nearly all cases, VaR metrics are more important (more pessimistic) that standardised methods calculations. This finding is contradictory to Solvency II goals assuming that internal models will take into consideration diversification advantages and will calculate inferior regulatory capital requirements to standard formulas. Solvency II regulations, as in the baking sector, will push insurers to use internal methodologies. If capital requirements measured with VaR models are higher than standardised methodologies' charges, insurers will not be encouraged to use internal models;
- in all cases, standardised methods are not accurate. Backtesting results show the superiority of VaR estimations with short (1 year or 2 years) windows of observations (the most efficient VaR approaches are the variance-covariance and historic simulations methods with 1 and 2 year data sets). Thus, in order to encourage insurers towards internal models, Basel II solutions concerning the influence of backtesting results on capital requirements calculations should also be applied by Solvency II;
- Economic Capital charges (VaR metrics) vary between Regulatory capital levels: they are stricter in bad economic situation and close to the 2002 GDV requirements, and less restrictive in periods of economic stability, being similar to NAIC, S&P and FSA capital expectations.

Although, VaR models for calculating market risk have proved to be useful to banking regulators, they can not be directly applied in the insurance service. There are similarities in the risks to which banks and insurers are exposed, however some notable differences between these two sectors are also important. Therefore, the VaR concept, applied as Economic Capital measure for market risk in the insurance industry, requires a few remarks:

- regulatory capital requirements calculated with VaR methodology are very sensitive to assumptions. They depend on risk managers' beliefs and suppositions at the moment of calculation. These hypotheses influence significantly VaR estimation due to the long term character of VaR measurement in the insurance sector;
- the VaR measure from banks can not be used directly in the insurance industry: VaR procedures should take into account insurance business's particularities (e.g. changes of interest rate positions' portfolio structure);
- from a theoretical point of view, many VaR models can not be used for regulatory capital requirements estimations in the insurance sector: non-n.i.d. data excludes methods using the "square root of time rule" in a long term context. Our empirical results prove that models using extrapolation of one day VaR with "square root time rule" are more pessimistic on average than MC simulation methods (MC simulation techniques are also biased because they are in general based on a normal distribution). However, empirical evidence shows that models using the extrapolation of one-day-VaR have better backtesting results !;
- the VaR techniques employed to regulatory capital requirement purposes need to be stable over a one year time horizon. The VaR metric estimated on the 1st of January should be stable during the whole year which means that Economic Capital calculated in June and in December (for the same investment portfolio) should correspond to the similar VaR metric. We can "smooth" the VaR in time by the choice of a VaR method (techniques using the variance-covariance matrix of risk factors seem to be more stable in time than methods using directly returned risk factors) or by extension of the size of the window of observations. The arbitrage of this length impacts the level of capital charges and its stability in time: lengthening of the window of observations makes VaR more stable in time (thus less reactive) and more pessimist (guaranteeing better accuracy of backtesting results). Empirical evidence illustrates that the optimal size of this window is around one or two years.

Finally, we stress the importance of backtesting. We believe that backtesting offers the best opportunity for incorporating suitable incentives into the internal models approach in the insurance industry. New approaches to backtesting are still being developed and discussed in the banking sector. Active efforts to improve and refine the methods currently in use are underway, with the goal of distinguishing more sharply between accurate and inaccurate risk models. We contend that the backtest specifications for insurance portfolios and risk particularities are a great challenge for future research.

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APPENDIX A: PROOFS

Throughout the rest of the appendix, we make an assumption that we know financial market conditions (e.g. the yield curve or the discount rate) at time *t* and that these market conditions remain unchanged at time *t* and at time *t+h*.

We begin by proving three useful lemmas.

Lemma 1. If market conditions remain unchanged at time t and at time t+h, the nominal price of zero coupon bond changes with time

Proof of Lemma 1. We analyse the evolution of the zero coupon bond over the time horizon *h*. Let $v_t^{zc}(C,\tau)$ be the time *t* price of a zero coupon bond. The value of this bond is a function of its coupon and maturity date τ (and $t \le \tau$). Thus, $v_t^{zc}(C,\tau) = C\left[1 + r_t(\tau)\right]^{t-\tau}$, where C is the face value and $r_t(\tau)$ is the time *t* rate of interest applicable for period $\tau - t$ (e.g. instantaneous forward interest rate). The price $v_t^{zc}(C,\tau)$ changes over the time horizon *h*:

- if $t+h \ge \tau$, the zero coupon bond is not present any more in the portfolio at time *t+h*, but the holder of this bond has received a cash-flow C at time $t+\tau$
- if $t+h < \tau$, the zero coupon bond is still in the portfolio at time *t+h*, with new theoretical price is $v_{t+h}^{zc}(C,\tau) = C\left[1 + r_{t+h}(\tau)\right]^{t+h-\tau}$. Even if the yield curve does not change between times *t* and *t+h*, the time left until maturity and the yield to maturity rate ($r_{t+h}(\tau) = r_t(\tau - h) \ne r_t(\tau)$ if the yield curve is not flat) have varied. The price of zero coupon bond depends on different variables at times *t* and *t+h*.

Although we make the assumption of stable market conditions, the zero coupon bond's price changes over the time horizon *h*. Hence, $v_t^{zc}(C,\tau) \ne v_{t+h}^{zc}(C,\tau)$, which completes our proof.

Lemma 2. If the investor does not change any position in the portfolio between date t and date t+h and the market conditions remain unchanged at time t and at time t+ h:

(i) nominal prices and quantities of equities and similar positions are the same in time t and in time t+h

(ii) nominal prices and quantities of fixed income positions (interest rate instruments) changes with time

(iii) nominal price and composition of investment portfolio including interest rates positions (bonds, loans and mortgages) changes over time period h

Proof of Lemma 2. First, we show (i). Assume $v_t^{eq}(D_k, r_t)$ as value of equity position at date *t*. *T*he theoretical price of this equity position is the present value of its future dividends (under condition of certainty). Then, $v_t^{eq}(D_k, r_t) = \sum_{k=1}^{a} D_k\,(1+r_t)^{-k}$, where D_k represents future dividends and r_t is a desired rate of return (discount rate) at date *t*. Let

$v_{t+h}^{eq}(D_k, r_{t+h}) = \sum_{k=1}^{a} D_k\,(1+r_{t+h})^{-k}$ be the price of equity position at time *t+h*. If portfolio positions and market conditions are equal at date *t* and *t+h*, then $r_t = r_{t+h}$, hence $v_t^{eq}(D_k, r_t) = v_{t+h}^{eq}(D_k, r_{t+h})$, which completes the first part of the proof.

Second, we show (ii). Let $v_t^{bd}(C,\tau)$ be the time *t* price of a interest rate (fixed income) position. Throughout the rest of the proof, we will consider that all interest rate positions (bonds and any other fixed income instruments) in the insurer's investment portfolio can be represented as a portfolio of zero coupon bonds. If investor holds one bond generating in the future *b* cash flows (the investor receives a coupon C once per period and a face value at the bonds maturity), these cash flows can be regarded separately as *b* distinct zero coupon bonds with face values C_i and maturities τ_i (and $i = 1,...,b$). Then, $v_t^{bd}(C,\tau) = \sum_{i=1}^{b} v_{i,t}^{zc}(C_i,\tau_i) = \sum_{i=1}^{b} C_i\,[1 + r_{i,t}(\tau_i)]^{-\tau_i}$, where C_i is the face value and $r_{i,t}(\tau_i)$ is the time *t* instantaneous rate of interest applicable for periods $\tau_i - t$. Using Lemma 1, we have:

- if $t + h \geq \tau_b$, then the bond is not present any more in the portfolio at the date *t+h*, but the investor has already received *b* cash flows of amounts C_i (representing coupons and face value of the bond) since times $t + \tau_i$ (and $i = 1,...,b$).
- if $t + h < \tau_b$, then the bond is still considered to be in the portfolio at time *t+h*. Then, $\exists\ c \in \{0,...,b-1\}$ such as $\tau_c \leq t + h < \tau_{c+1}$ and $\tau_0 = t$, the holder has received *c* cash-flows C_j since times $t + \tau_j$ (and $j = 1,...,c$). The new market price of the bond is $v_{t+h}^{bd}(C,\tau) = \sum_{i=c+1}^{b} v_{i,t+h}^{zc}(C_i,\tau_i) = \sum_{i=c+1}^{b} C_i\,[1 + r_{t+h}(\tau_i)]^{t+h-\tau_i}$

Market price of the zero coupon bond does not depend on the same variables at dates *t* and *t+h*, even though the yield curve does not change. Hence, if $\sum_{i=1}^{b} v_{i,t}^{zc}(C_i,\tau_i) \neq \sum_{i=1}^{b} v_{i,t+h}^{zc}(C_i,\tau_i)$, then $v_t^{bd}(C,\tau) \neq v_{t+h}^{bd}(C,\tau)$, which completes the third part of our proof.

Third, we show (iii). Let us denote V_t as the market price of an investment portfolio at date *t* and V_{t+h} as the market price of the investment portfolio at date *t+h*. If the portfolio contains interest rate positions then $v_t^{bd}(C,\tau) \neq v_{t+h}^{bd}(C,\tau)$. Hence, $V_t \neq V_{t+h}$, which completes the third part of our proof.

Lemma 3. In the case of an interest rate position, we can measure the risk through price changes or interest rate variations. In a first option, we evaluate changes of prices using interest rate variations; in second case, we calculate interest rate changes using price variations. However, the choice between two options is not equal:
(i) if we use interest rate changes to measure variation of prices, we overestimate the expected value of prices' variations, consequently, we underestimate the risk
(ii) *if we use price changes to measure variation of interest rates, we overestimate the expected value of interest rates' variations, in consequence, we overestimate the risk*

Proof of Lemma 3. First, we show (i). Let us study a zero coupon bond yielding 1 with maturity τ. Let us note $v_t^{zc}(\tau)$ and $v_{t+h}^{zc}(\tau)$ as prices of this zero coupon bond at dates *t* and *t+h*. Consequently, $r_t^{zc}(\tau)$ and $r_{t+h}^{zc}(\tau)$ are interest rates of the zero coupon bond at dates *t* and *t+h*. By definition, we have $v_t^{zc}(\tau)=\left[1+r_t^{zc}(\tau)\right]^{-\tau}$ and $v_{t+h}^{zc}(\tau)=\left[1+r_{t+h}^{zc}(\tau)\right]^{-\tau}$, therefore $r_t^{zc}(\tau)=\left[v_t^{zc}(\tau)\right]^{-\frac{1}{\tau}}-1$ and $r_{t+h}^{zc}(\tau)=\left[v_{t+h}^{zc}(\tau)\right]^{-\frac{1}{\tau}}-1$. Let us define random variables $\Delta v^{zc}(\tau)$ and $\Delta r^{zc}(\tau)$ as follows $\Delta v^{zc}(\tau)=v_{t+h}^{zc}(\tau)-v_t^{zc}(\tau)$ and $\Delta r^{zc}(\tau)=r_{t+h}^{zc}(\tau)-r_t^{zc}(\tau)$. Suppose that interest rates and prices of the zero coupon bond are always strictly positive. Then $\Delta v^{zc}(\tau)=v_{t+h}^{zc}(\tau)-v_t^{zc}(\tau)>-v_t^{zc}(\tau)$ and $\Delta r^{zc}(\tau)=r_{t+h}^{zc}(\tau)-r_t^{zc}(\tau)>-r_t^{zc}(\tau)$.

Let us define function δ on $]-r_t^{zc}(\tau);+\infty[$, as $\delta(x)=\left[1+r_t^{zc}(\tau)+x\right]^{-\tau}-\left[1+r_t^{zc}(\tau)\right]^{-\tau}$. We have then $\Delta v^{zc}(\tau)=\delta\left[\Delta r^{zc}(\tau)\right]$. When we derive function δ at two times, we have the second derivative of function δ as $\dfrac{d^2}{dx^2}\delta(x)=\left(\tau^2+\tau\right)\left(1+r_t^{zc}(\tau)+x\right)^{-\tau-2}>0$, which means that function δ is strictly convex. Using Jensen inequality, we obtain $E\left[\Delta v^{zc}(\tau)\right]=E\left[\delta\left(\Delta r^{zc}(\tau)\right)\right]>\delta\left(E\left[\Delta r^{zc}(\tau)\right]\right)$. Hence, $E\left[\Delta v^{zc}(\tau)\right]>\delta\left(E\left[\Delta r^{zc}(\tau)\right]\right)$, which completes the first part of our proof.

Second, we show (ii). We study the zero coupon bond from (i) and we use the same variables: $v_t^{zc}(\tau)$, $v_{t+h}^{zc}(\tau)$, $r_t^{zc}(\tau)$, $r_{t+h}^{zc}(\tau)$, $\Delta v^{zc}(\tau)$ and $\Delta r^{zc}(\tau)$. Suppose functions γ on $]-v_t^{zc}(\tau);+\infty[$, as $\gamma(x)=\left[v_t^{zc}(\tau)+x\right]^{-\frac{1}{\tau}}-\left[v_t^{zc}(\tau)\right]^{-\frac{1}{\tau}}$. Then, we have $\Delta r^{zc}(\tau)=\gamma\left[\Delta v^{zc}(\tau)\right]$. Let us derive function γ at two times. Therefore, $\dfrac{d^2}{dx^2}\gamma(x)=\dfrac{\tau+1}{\tau^2}(v_t^{zc}(\tau)+x)^{-\frac{1}{\tau}-2}>0$, which means that function γ is strictly convex. Using Jensen inequality, we have $E\left[\Delta r^{zc}(\tau)\right]=E\left[\gamma\left(\Delta v^{zc}(\tau)\right)\right]>\gamma\left(E\left[\Delta v^{zc}(\tau)\right]\right)$. Hence, $E\left[\Delta r^{zc}(\tau)\right]>\gamma\left(E\left[\Delta v^{zc}(\tau)\right]\right)$, which completes the second part of our proof.

Proof of Proposition 1. Let define $X_{i,t}$ as risk factor's value at time *t* and $X_{i,t+h}$ as its value at date *t+h* (and $i=1,...,n$). The change in value of a risk factor over period *h* is then $\Delta X_i = X_{i,t+h} - X_{i,t}$. Note that V_t and V_{t+h} are values of a portfolio of financial positions respectively at time *t* and time *t+h*. Using equations (2.8) and (2.9), we have $V_t = f(X_{1,t},...,X_{n,t})$ and $V_{t+h} = g(X_{1,t+h},...,X_{n,t+h})$. Using (iii) in Lemma 2, we have $V_t \neq V_{t+h}$ if portfolio includes interest rate positions, thus $f \neq g$. Let *N* be the value of the cash flows occurring over the period *h*. No assumption is made concerning the method of *N* estimation (e.g. *N* might be successively reinvested in bonds market or in monetary market). The change in value of portfolio including interest rate positions between times *t* and *t+h* is given by $\Delta V = V_{t+h} + N - V_t$. Therefore, $\Delta V = g(X_{1,t+h},...,X_{n,t+h}) + N - f(X_{1,t},...,X_{n,t})$ and $\Delta V = g(X_{1,t}+\Delta X_1,...,X_{n,t}+\Delta X_n) + N - f(X_{1,t},...,X_{n,t})$. Assume that f and g are linear combinations of risk factors. Then, $V_t = f(X_{1,t},...,X_{n,t}) = \sum_{i=1}^{n} d_i X_{i,t}$ and $V_{t+h} = g(X_{1,t+h},...,X_{n,t+h}) = \sum_{i=1}^{n} e_i X_{i,t+h} = \sum_{i=1}^{n} e_i \left(X_{i,t} + \Delta X_i\right)$ and so $\Delta V = g(X_{1,t},...,X_{n,t}) + g(\Delta X_1,...,\Delta X_n) + N - f(X_{1,t},...,X_{n,t})$. The expected value of the change in price of a portfolio of financial positions over a time horizon *h* is $E[\Delta V] = E[g(X_{1,t},...,X_{n,t})] + E[g(\Delta X_1,...,\Delta X_n)] + E[N] - E[f(X_{1,t},...,X_{n,t})]$ and $E[\Delta V] = g(X_{1,t},...,X_{n,t}) + g(E[\Delta X_1],...,E[\Delta X_n]) + E[N] - f(X_{1,t},...,X_{n,t})$, where $g(X_{1,t},...,X_{n,t}) + E[N] - f(X_{1,t},...,X_{n,t})$ is the expected value of the portfolio prices variations when the variations of the all portfolio's risk factors have a null expected value. Hence, we notice that a portfolio including bonds will never have the same value at times *t* and *t+h* even if market conditions are stable (unchanged), which completes our proof.

Proof of Proposition 2. First, we show (i). Let note ΔV^{bd}, the change in value of a portfolio of interest rate positions over a time horizon *h*. We also define ΔX_i^{zc} as variations of a portfolio's risk factors (zero coupon bonds) over the period *h;* $\Delta v^{zc}(\tau)$ as variations of prices of zero coupon bonds over the period *h;* and $\Delta r^{zc}(\tau)$ as variations of interest rates of zero coupon bonds over the period *h.* According to (2.11), $\Delta V^{bd} = g(\Delta X_1^{zc},...,\Delta X_n^{zc})$. If we estimate risk factors through price variations of zero coupon bonds, then $\Delta X_i^{zc} = \Delta v_i^{zc}(\tau)$ and $i=1,...,n$. Thus, we estimate $\Delta v_i^{zc}(\tau)$ by zero coupons' interest rate variations. Using (i) in Lemma 3, note that we underestimate the risk of risk factors. Hence, we underestimate portfolio risk, which completes the first part of our proof.

Second, we show (ii). Let us consider the investment portfolio of interest rate positions from (i). If we use variations of zero coupon bonds' interest rates to estimate risk factors, then $\Delta X_i^{zc} = \Delta r_i^{zc}(\tau)$ and $i=1,...,n$. Thus, we evaluate $\Delta r_i^{zc}(\tau)$ by variations in zero coupon prices. Using (ii) in Lemma 3, note that we overestimate the risk of risk factors. Hence, we overestimate portfolio risk, which completes the first part of our proof.

APPENDIX B: TABLES AND FIGURES

Table 1: Investment position weights in four standardized methodologies

Market risk weights for financial instruments				
Investment positions	***Standardized models***			
	NAIC (factors)*	**S&P (factors)**	**FSA (factors)****	**GDV (factors)**
Bonds				
AAA	-	0,42%	3,50%	-
AA	0,30%	0,42%	3,50%	0,42%
A	1,00%	0,42%	3,50%	0,42%
BBB	2,00%	3,26%	3,50%	3,26%
BB	4,50%	7,52%	3,50%	7,52%
B	10,00%	13,72%	3,50%	13,72%
CCC	30,00%	20,18%	3,50%	20,18%
Defaut	30,00%	30,00%	3,50%	30,00%
Equities	15,00%	15,00%	16,00%	26,60%
Real estate	10,00%	18,00%	7,50%	10,30%

*NAIC factors are for P&C insurers. For life these are typically higher but are offset by the effect of tax reductions
**UK-FSA factors are applied for non-life products only
Source: Author's compilations

Table 2: Composition of five theoretical portfolios

Compositions of investment portfolios					
Financial positions	***Portfolio 1***	***Portfolio 2***	***Portfolio 3***	***Portfolio 4***	***Portfolio 5***
Equities	4%	20%	30%	60%	100%
Bonds	93%	77%	67%	37%	-
Real estate	3%	3%	3%	3%	-

Table 3: Comparison of capital charges

Estimations of capital requirements		Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5
Regulatory capital						
Standardized methods		***Capital charges (in percentage of investment portfolio amount)***				
NAIC		-0,84%	-3,34%	-4,84%	-9,27%	-15,00%
S&P		-1,47%	-3,91%	-5,33%	-9,67%	-15,00%
FSA		-4,07%	-6,15%	-7,42%	-11,10%	-16,00%
GDV		-1,26%	-5,70%	-8,38%	-16,22%	-26,60%
Economic Capital						
VaR methods	***The size of the window of observations***	***VaR metrics (in percentage of investment portfolio amount)***				
The variance-covariance	*1 year*	-6,25%	-7,16%	-8,55%	-14,60%	-23,40%
	2 years	-6,42%	-6,69%	-7,64%	-12,52%	-20,22%
	5 years	-7,51%	-7,51%	-8,70%	-14,94%	-25,07%
	10 years	-8,03%	-8,13%	-9,32%	-15,47%	-25,72%
Monte Carlo	*1 year*	-5,97%	-6,76%	-8,01%	-13,06%	-21,02%
	2 years	-6,01%	-6,31%	-7,21%	-11,50%	-18,30%
	5 years	-7,15%	-7,05%	-8,26%	-13,58%	-22,73%
	10 years	-7,42%	-7,69%	-8,85%	-13,94%	-23,24%
Historic simulations	*1 year*	-7,09%	-8,97%	-10,65%	-19,25%	-31,34%
	2 years	-7,32%	-7,06%	-8,38%	-14,70%	-25,41%
	5 years	-9,89%	-8,91%	-11,31%	-19,02%	-34,90%
	10 years	-9,91%	-9,40%	-11,39%	-18,62%	-33,24%

Table 4: Estimations of Economic Capital for the portfolio of interest rate positions by prices variations and interest rates variations

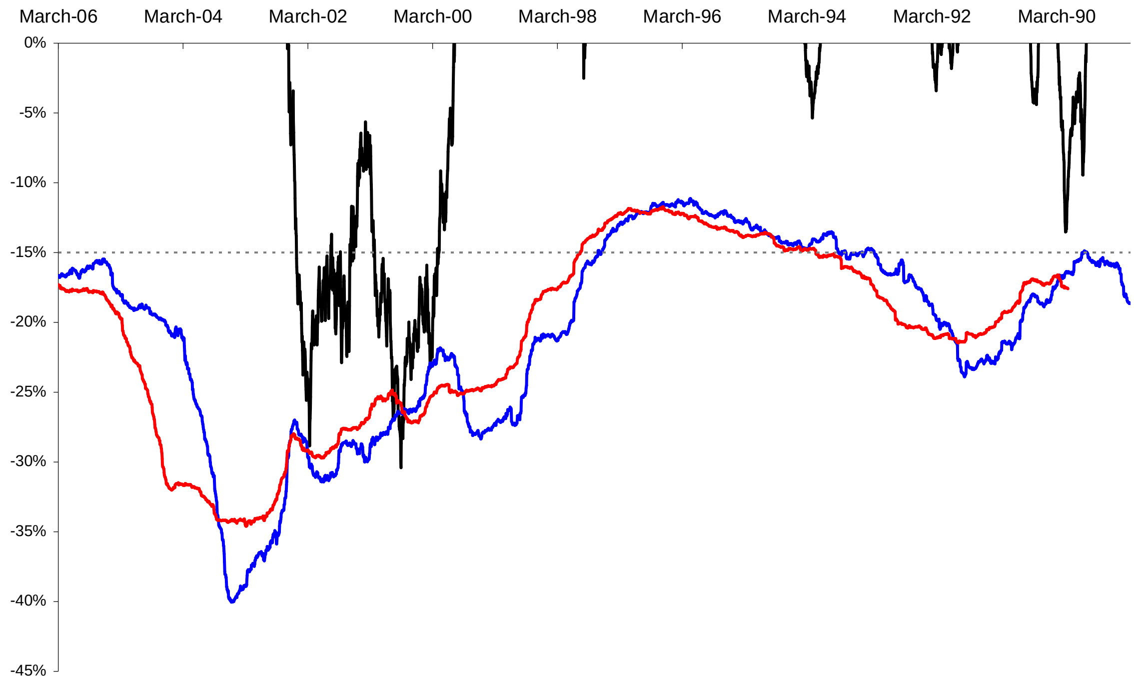
VaR methods	***The size of the window of observations***	***VaR metrics (in percentage of interest rate portfolio amount)***	
		Variations of prices as risk factors	*Variations of interest rates as risk factors*
The variance-covariance	*1 year*	-6,24%	-7,45%
	2 years	-6,57%	-8,77%
	5 years	-7,78%	-10,85%
	10 years	-8,28%	-10,08%
Monte Carlo	*1 year*	-6,02%	-6,63%
	2 years	-6,17%	-7,53%
	5 years	-7,27%	-8,99%
	10 years	-7,68%	-8,56%
Historic simulations	*1 year*	-6,52%	-8,81%
	2 years	-7,30%	-9,80%
	5 years	-9,95%	-11,43%
	10 years	-10,27%	-10,63%

Table 5: Backtesting results

Backtesting results for Portfolio 5		
Regulatory capital		
Standardized methods		***Percentage of exceptions****
NAIC		11,00%
S&P		11,00%
FSA		10,37%
GDV		0,58%
Economic Capital		
VaR methods	***The size of the window of observations***	***Percentage of exceptions****
The variance-covariance	*1 year*	0,51%
	2 years	0,66%
	5 years	3,25%
	10 years	11,43%
Monte Carlo	*1 year*	1,51%
	2 years	1,58%
	5 years	5,54%
	10 years	15,90%
Historic simulations	*1 year*	1,40%
	2 years	0,35%
	5 years	0,87%
	10 years	2,52%

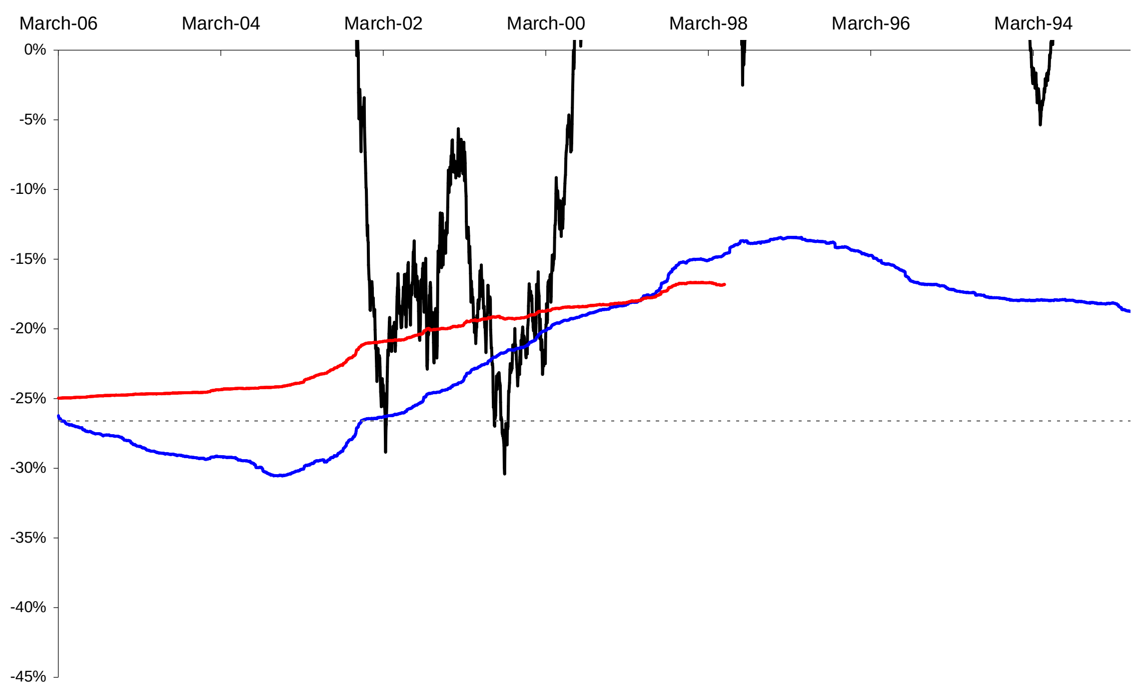
*number of times (in percentage of all tested days) in the past ten years when the loss on the profit and loss account (P&L) exceeded VaR and standardised method estimations (calculated one year ahead).

Figure 1: Backtesting results for variance-covariance approach (with data sets of one year and two years)
Black line: real losses of Portfolio 5 returns at time *t+h* (one year after Economic Capital estimations)
Blue line: Economic Capital estimated by VaR at time *t* (time specified at Y axis) with one year long windows of observations
Red line: Economic Capital estimated by VaR at time *t* (time specified at Y axis) with two year long windows of observations
Grey (dashed) line: Regulatory capital charges for the NAIC model



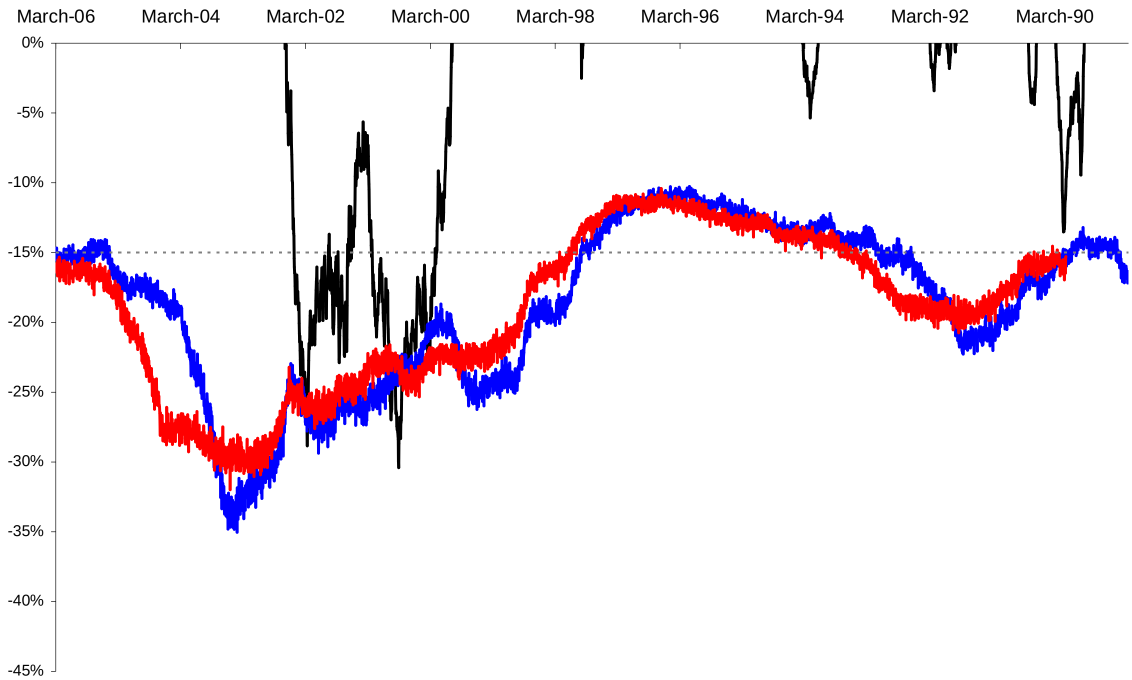
*all results are represented as a percentage of Portfolio 5 amount at 31/03/2007

Figure 2: : Backtesting results for variance-covariance approach (with data sets of five and ten years)
Black line: real losses of Portfolio 5 returns at time t+h (one year after Economic Capital estimations)
Blue line: Economic Capital estimated by VaR at time t (time specified at Y axis) with five year long windows of observations
Red line: Economic Capital estimated by VaR at time t (time specified at Y axis) with ten year long windows of observations
Grey (dashed) line: Regulatory capital charges for the 2002 GDV model



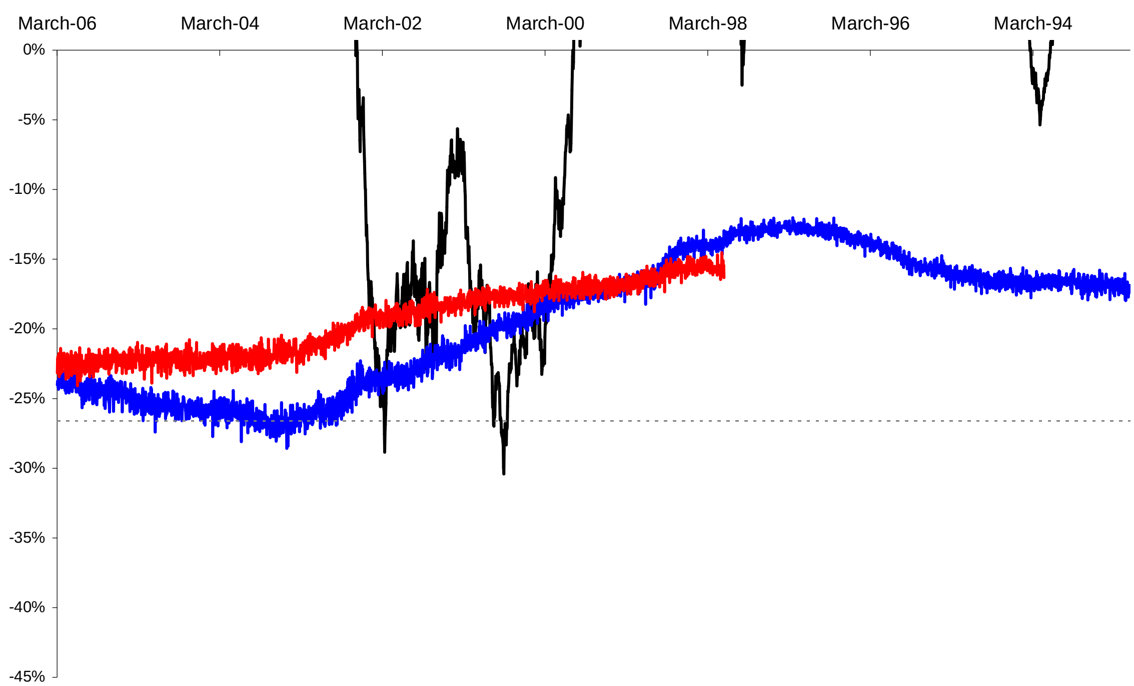
*all results are represented as a percentage of Portfolio 5 amount at 31/03/2007

Figure 3: Backtesting results for Monte Carlo method (with data sets of one year and two years)
Black line: real losses of Portfolio 5 returns at time *t+h* (one year after Economic Capital estimations)
Blue line: Economic Capital estimated by VaR at time *t* (time specified at Y axis) with one year long windows of observations
Red line: Economic Capital estimated by VaR at time *t* (time specified at Y axis) with two year long windows of observations
Grey (dashed) line: Regulatory capital charges for the NAIC model



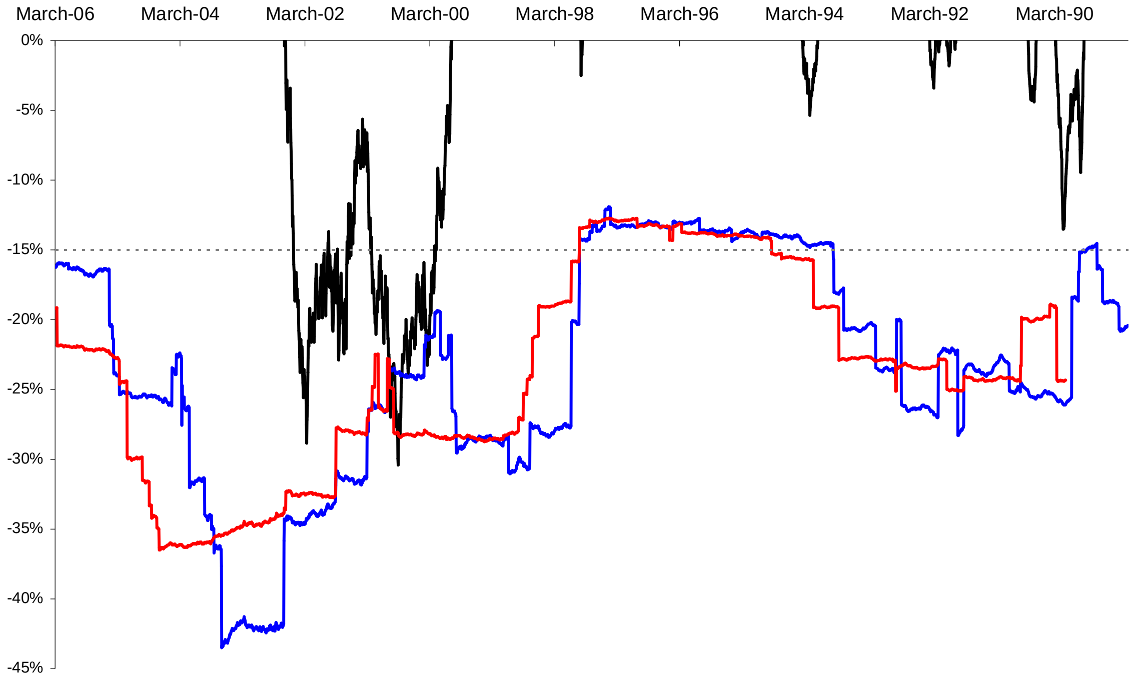
*all results are represented as a percentage of Portfolio 5 amount at 31/03/2007

Figure 4: : Backtesting results for Monte Carlo method (with data sets of five and ten years)
Black line: real losses of Portfolio 5 returns at time t+h (one year after Economic Capital estimations)
Blue line: Economic Capital estimated by VaR at time t (time specified at Y axis) with five year long windows of observations
Red line: Economic Capital estimated by VaR at time t (time specified at Y axis) with ten year long windows of observations
Grey (dashed) line: Regulatory capital charges for the 2002 GDV model



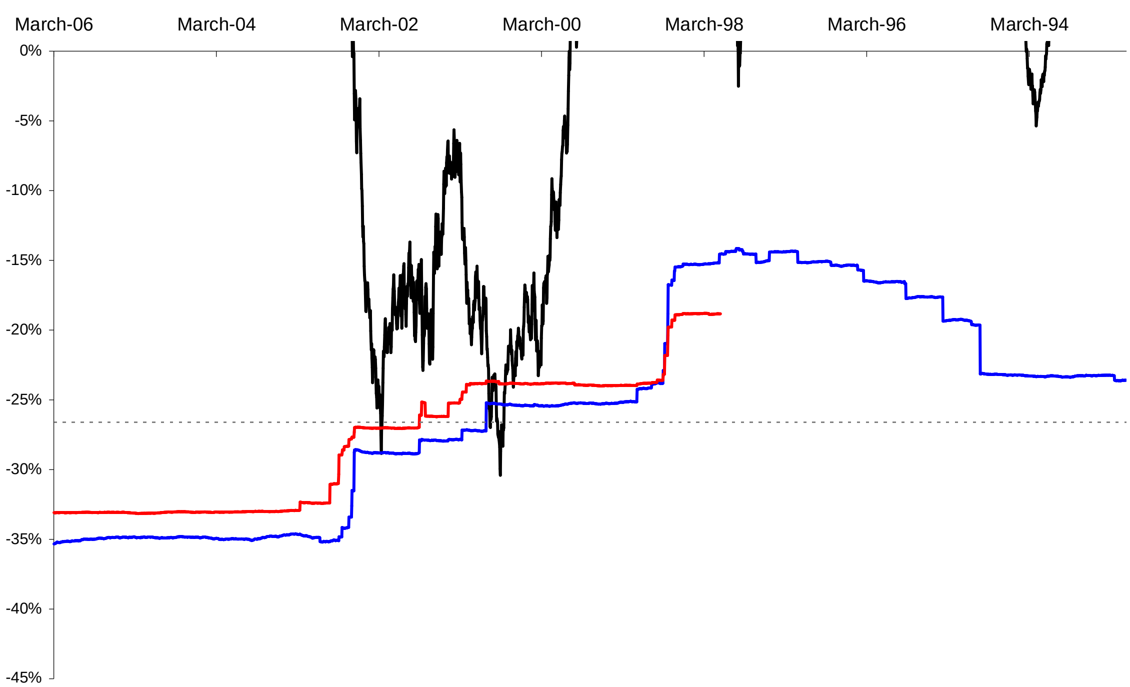
*all results are represented as a percentage of Portfolio 5 amount at 31/03/2007

Figure 5: Backtesting results for historical simulation method (with data sets of one year and two years)
Black line: real losses of Portfolio 5 returns at time *t+h* (one year after Economic Capital estimations)
Blue line: Economic Capital estimated by VaR at time *t* (time specified at Y axis) with one year long windows of observations
Red line: Economic Capital estimated by VaR at time *t* (time specified at Y axis) with two year long windows of observations
Grey (dashed) line: Regulatory capital charges for the NAIC model



*all results are represented as a percentage of Portfolio 5 amount at 31/03/2007

Figure 6: Backtesting results for historical simulation method (with data sets of five and ten years)
Black line: real losses of Portfolio 5 returns at time t+h (one year after Economic Capital estimations)
Blue line: Economic Capital estimated by VaR at time t (time specified at Y axis) with five year long windows of observations
Red line: Economic Capital estimated by VaR at time t (time specified at Y axis) with ten year long windows of observations
Grey (dashed) line: Regulatory capital charges for the 2002 GDV model



*all results are represented as a percentage of Portfolio 5 amount at 31/03/2007