Managing Economic Capital within the Provider and Consumer Framework

Per Simon Voldsgaard cand. act.
Sarah Rasmussen cand. act.

Sparinvest, Denmark
Hvidesten
Kingosvej 1
DK-2630 Taastrup

Email:
psv@sparinvest.dk sra@sparinvest.dk

Abstract:

The complex challenges brought on by Solvency II creates an increasing need for a concise and transparent methodology. Our contribution is the introduction of a framework that identifies and decomposes the factors influencing the demand of economic capital into providers and consumers. The advantage of the methodology is that if enables us to asses the aggregated demand of economic capital through analysis of the effects of diversification, reinsurance, and embedded value. As a result, the framework serves as a link between the complex analysis within the model and information that is an adequate and natural part of the strategic and decision processes. The methodology is useful for decision making within the insurance sector because of the clarity of the information it produces. In our experience, it has been met with enthusiasm.

Keywords: Demand of economic capital, Pricing, Reinsurance, Diversification, Premium Quality, Strategy

Managing Economic Capital within the Provider and Consumer Framework

1. Introduction

The Solvency II legislation has forced many insurance companies to begin assessing their businesses differently than they have previously. Many have already realised the advantages to internal capital models, advantages that reach beyond determining an appropriate level of solvency. The models give an insight into the business' overall condition.

At times, it can prove challenging to transform the complex analysis behind capital models into clear, concise information that can be used by decision makers. The framework presented in this article can serve as a catalyst in the information generating process. The framework is called the Provider and Consumer Framework. It is used to form a link between complex statistical analysis and the information necessary in strategic decision making processes. The strength of this framework is that it maintains its simplicity while conveying a high level of information.

This article focuses on the capabilities of this particular framework. There is particular emphasis on how the results can be presented in a highly informative manner. The lines of business utilised in this article are of a more generic form and limited time has been spent on modelling realistic lines. The reason for this is that our primary focus is the framework and not the modelling of business lines. The generic form of the business lines also allows the reader to easily insert his own models into the framework.

Furthermore, this article does not focus on the process of allocating economic capital to specific lines of business. There have been a number of innovative articles concerning the allocation of economic capital to specific lines of business, see for example [5]. The majority of the methods discussed in these papers could be implemented after using the P & C Framework. Therefore, it seems natural to allow the individual to follow his own intuition on this subject.

2. Measuring the Demand of Economic Capital

The demand of economic capital will vary depending on the measure that is used to find it. Therefore, the calculation of the demand of economic capital should be accompanied by the measure which was used to calculate it.

There are many measures currently in use with this type of modelling. At this point in time, we believe it is unwise to restrict oneself to one measure. In our experience, different measures can support one another. One measure may reveal information that the other cannot and vice versa. In some cases one measure may reveal information about another measure. Therefore, at this point in time, the authors recommend using as many measures as one is comfortable with.

Naturally, the P & C Framework can be utilised in combination with many different measures and underlying models. In this article a measure called the annual conditional ruin probability is used. The 1 year annual conditional ruin probability is essentially the Value at Risk, see [2]. Thus, a 1 year annual conditional ruin probability of 1% would coincide with a VaR of 99%. However, when a multiple year annual conditional ruin probability is used this relationship becomes more complex. The 5 year annual conditional ruin probability of 1% will give the demand of economic capital necessary to maintain 5 years of business, given that the company still has sufficient funds at the end of each business year, with a 99% certainty.

In the following Demand of Economic Capital will be referred to as DEC, and the Annual Conditional Ruin Probability will be referred to as the ACRP.

3. The P & C Framework

The purpose of this article is to present a new framework for the determining of the demand of economic capital. The P & C framework will allow for a deeper understanding of the different risk components within an insurance company. The framework is accompanied by a figure which is used to display the changes in DEC as a result of the providers. The framework can also be helpful when identifying high risk strategies and developing guidelines for allocation of the DEC to types of risk.

Determining the DEC can be an intricate process. The P and C framework splits the world of capital into two groups, Providers and Consumers. The providers provide a type of reduction with respect to the DEC. They provide some capital to compensate for the capital demand the consumers create, thus lowering the necessary capital. The consumers are exactly what their name suggests. They consume or create a need for capital.

Examples of some providers and consumers can be seen on the chart below:

Providers	Consumers
Own capital resources	Necessary capital (premium= equivalence
Diversification	Premium)
Reinsurance	Operational risk
Embedded capital	
Risk Management	

Here, the necessary capital refers to the capital needed to cover claims less the equivalence premium. Naturally, there are more consumers one could include, however, in the interest of maintaining simplicity, the focus will be on a single consumer, necessary capital.

Each provider lowers the DEC by some amount; the goal is to find out how much. The diversification, in this example, is the effect between three independent lines of liability. In practice there will be lines of business that are correlated. Of course it is more helpful, with respect to the DEC that the lines of business are negatively correlated or have a negative tail correlation, such that one line may cover the loss on the other. This may not be the case, but in general, having several lines of liability will create a diversification effect, which can lower the capital demand. The advantage of diversification can be seen in that the relative standard deviation decreases when an extra liability is added. There is also a diversification effect in having more than one policy holder. The risk is in effect spread amongst policy holders. This particular type of diversification within a single line of business, is already built into the model and will not be referred to as a providing diversification, but rather as an internal reducer of the consumer.

Reinsurance contracts can be found in many forms and combinations all of which will affect the DEC differently. Ideally, reinsurance covers should reduce the DEC. It is possible that investigating the effects of various reinsurance treaties could aid in the selection of a reinsurance contract.

The embedded capital provider is simply an expression for the effect a higher premium has on the DEC. It is reasonable to expect that the DEC is lower when a company takes higher premium. Recall that old solvency regulations were based on a percentage of the premium, thus a higher premium would not decrease the capital requirement, quite the opposite.

Risk management is a rather broad term but can include management of operational risks, paradigm changes, etc.

3.1 The P &C Framework in action

The strengths of the P & C framework are most clearly illustrated through a demonstration. The following is a simple example demonstrating the framework.

Consider a newly started company with three lines of business. For simplicity, it is assumed that all claims are settled within the claim year such that there is no need for reserves and the various uncertainty considerations that accompany them.

The three lines of business are based on a claim size distribution and a claim number distribution. The claim size distribution for all three lines is a Pareto distribution. The claim number distribution is a Poisson distribution. As mentioned earlier, the lines of business are simple in order to illustrate the example.

The first measure used in this example is a 1 year ACRP of 1%. The raw capital needed to sustain this ACRP is found using the P & C Framework. The term, raw capital, should be understood as the amount in excess of the equivalence premium that is necessary to cover the claims to maintain a given ACRP.

3.2 Simulation of the Empirical Profit-Loss Distribution

The purpose of the simulation is to determine the DEC associated with a predetermined ACRP and to examine how the providers influence this capital. The DEC can be found using the profit-loss distribution of the model. An empirical profit-loss distribution was created using crude Monte Carlo methods, see [1]. The simulation was based on 50.000 claim years which were ordered to produce an empirical distribution. This empirical distribution is then used to determine the capital needed to maintain a 1 year ACRP of 1%.

In the simulations, there are three lines of business, A, B, and C. Each line has its own claim size distribution and number of claims distribution. For each claim year the number of claims, n_j, is determined by a Poisson distribution with parameters 20, 30, and 20 respectively.

Distributions			
	Pareto		Poission
	α	β	λ
Line A	2	12.000	20
Line B	4	36.000	30
Line C	2	12.000	20

An inverse Pareto distribution function was used to simulate the n_j claims. Naturally, other distributions can be used to simulate claim sizes, but here, the Pareto distribution used to generate more heavy-tailed claims.

There are many different parameterizations of the Pareto distribution. The parameterization used here is as follows:

$$F(x) = 1-\beta^{\alpha} (\beta + x)^{-\alpha}, \quad \alpha > 0$$

The claim sizes are created by defining the inverse Pareto distribution, $F^{-1}(u)$, for all three lines of business. A uniformly distributed random number, U, between 0 and 1, is generated. This number is used as the input value in the inverse Pareto function thus producing a claim.

$$(\text{Poisson } (\lambda)) \rightarrow n_j$$

$$(U \sim \text{unif } (0, 1)) \rightarrow F^{-1}(U) = X_{ij}$$

The claims from year j are summed to give the total claim amount for the year, X_j. The total claim amount for the year is subtracted from the equivalence premium to show the loss or profit from that year without the benefit of providers.

$$\sum_{i=1}^{nj} X_{ij} = X_{.j}$$

$$\text{Equivalence premium} = \lambda \times \mathbf{E}(X)$$
$$\text{DEC year } j = \text{Equivalence premium} - X_j$$

The embedded value and reinsurance providers are applied within the claim year to each line of business individually. In the case of an excess of loss or proportional reinsurance contract, the

contract is applied to the individual claims. A catastrophe cover would be applied only to catastrophe claims, but these types of claims are not discussed in this paper.

The examples in this paper use an excess of loss reinsurance contract. The excess of loss contract has a threshold roughly based on the 90% quantile of each claim size distribution. For each claim the amount above this threshold is removed, thus the maximum size of a claim becomes the threshold. If the claim is below the threshold then the entire claim is left for the direct insurer to cover. The reinsured claims are summed to produce a total claim amount with reinsurance. The reinsurance premium and the total reinsured claims are subtracted from the equivalence premium and the associated DEC is recorded as seen in the column DEC with reinsurance in the table below.

$$\text{Total reinsured claims} = \sum_{i=1}^{nj} \min(X_{ij}, 25.000)$$

$$\text{DEC with reinsurance} = \sum_{i=1}^{nj} \min(X_{ij}, 25.000) - \text{equivalence prm} + \text{reinsurance prm}$$

An example of how the claims are generated and adjusted for an excess of loss reinsurance contract, as well as, the total DEC capital with the without reinsurance, can be seen in the table below. A positive DEC capital indicates the amount of capital needed in addition to premiums already taken, while a negative value would indicate a profit in that year.

	n_j	x_{ij}	X_{ij} w. reinsurance	equivalence premium	raw DEC year j	reinsurance premium	DEC w. reinsurance year j
Line A	1	28.000	25.000				
	2	25.000	25.000				
	⋮						
	26	1.500	1.500				
total	**26**	**775.000**	**560.000**	**240.000**	**535.000**	**100.000**	**420.000**
Line B	1	29.000	28.000				
	2	2.300	2.300				
	⋮						
	31	15.000	15.000				
total	**31**	**690.000**	**605.000**	**360.000**	**330.000**	**125.000**	**370.000**
Line C	1	9.000	9.000				
	2	29.000	25.000				
	⋮						
	18	12.300	12.300				
total	**18**	**620.000**	**465.000**	**240.000**	**380.000**	**100.000**	**325.000**

The DEC with embedded value is found by subtracting the total claims within a claim year from the premium with embedded value. A combination of DEC with embedded value and reinsurance is found by subtracting the reinsurance premium and the total reinsured claims for the premium with embedded value.

$$\text{DEC with embedded value} = \sum_{i=1}^{nj} X_{ij} - \text{embedded prm}$$

$$\text{DEC with embedded value and reinsurance} = \sum_{i=1}^{nj} X_{ij} - \text{embedded prm} + \text{reinsurance prm}$$

The addition of embedded value is illustrated in the table below.

	$X_{.j}$	equivalence premium	raw DEC year j	premium embedded value	DEC w. emb. Value year j	DEC w. reinsurance year j	DEC w. emb value & re-insurance year j
Line A	775.000	240.000	535.000	360.000	415.000	450.000	330.000
Line B	860.000	360.000	500.000	460.000	400.000	386.000	286.000
Line C	620.000	240.000	380.000	340.000	280.000	325.000	225.000

The process is repeated for 50.000 claim years. The results of the claim years are organized in a matrix. The total claim amounts are calculated from the individual claims. Each claim year occupies a row in the matrix. The row contains the necessary capital for each line of business individually, the capital demand for each line of business with reinsurance, with embedded value, with reinsurance and embedded value, and finally the combined results of all three lines of business for each of these four groups. Thus, the matrix has 16 columns and 50.000 rows. The headers of the columns can be seen below.

	No providers			Diversification	Reinsurance			Reinsurance & Diversification
line	A	B	C	total	A	B	C	total
column	1	2	3	4	5	6	7	8

	embedded Value			Embedded Value & Reinsurance	Reinsurance & Embedded Value			All providers
line	A	B	C	total	A	B	C	total
column	9	10	11	12	13	14	15	16

To create the empirical distribution each column in the matrix is sorted individually so that the worst results for each line of business are at the bottom and the best are at the top. The totals over all three lines of business are also sorted individually to create the diversification effect. The result in the total column will be lower than the sum of the three individual lines because the columns are sorted individually. The idea is based on the principle that when the lines of business are allowed to diversify on another, all lines will not necessarily have their worst result at once. Where as, if one looks at the line individually, one would analyze that lines individual worst result. Thus, a combination of the worst individual results in three lines would be a greater loss than the worst combined result within a claim year.

The matrix can now be seen as an empirical distribution where it is possible to read the desired quantile from the associated row. For example the capital needed to maintain a 1% ACRP for one year can be found for the three lines of business without providers, or any combination of providers, by reading row 40.500.

3.3 The 1 year ACRP for three independent lines of business

The providers shown in this example are diversification, reinsurance, and embedded value. The diversification effect has been found between three independent lines of liability. The reinsurance contract is an excess of loss contract. The excess limit for each line of business was determined

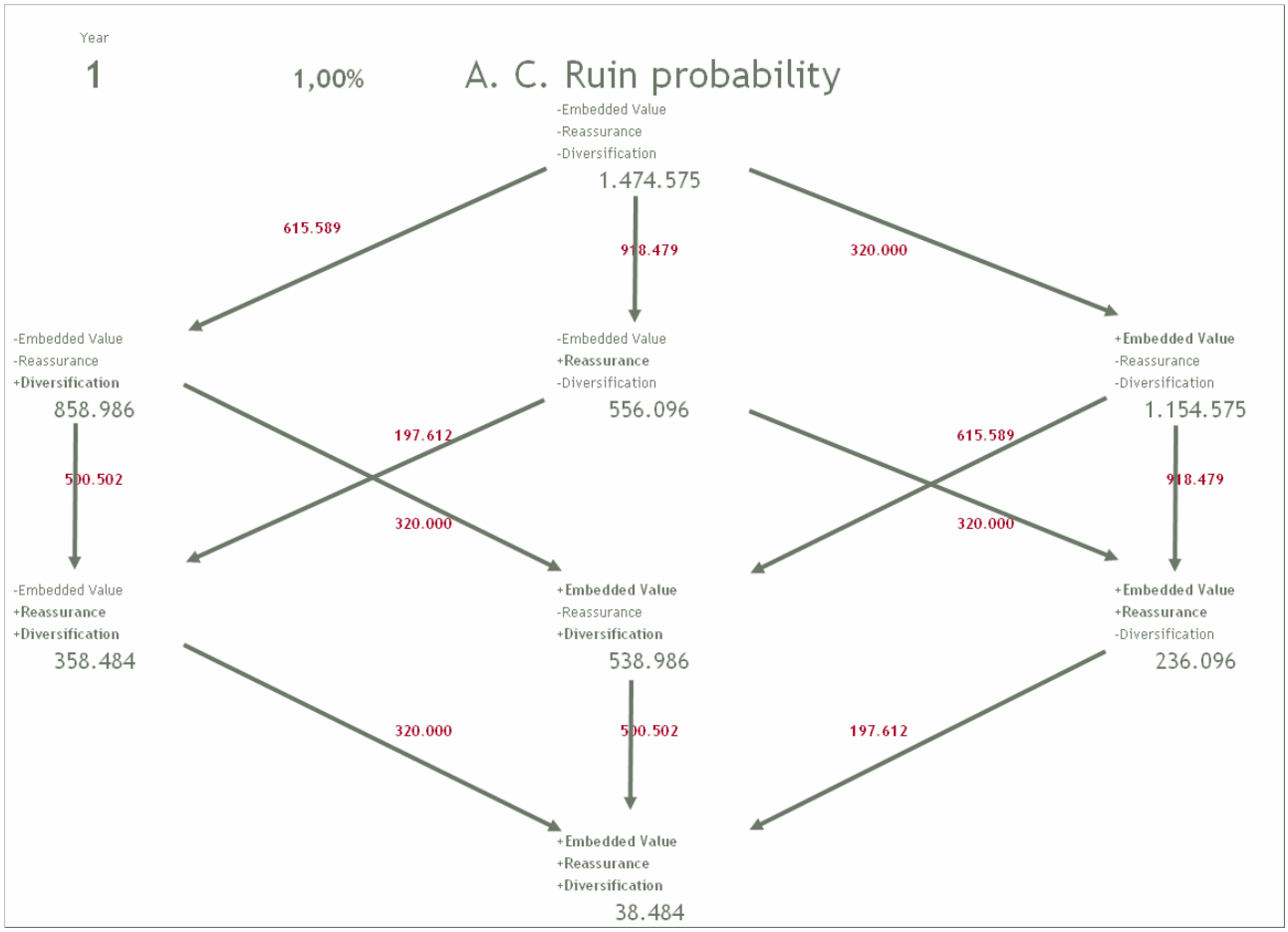
using the 90% quantile of the claim size distributions. The reinsurance premium is equal to the expected coverage times a 25% loading factor. A summary of the parameters and assumptions for the model can be seen in the table below.

	distributions				embedded value		excess of loss reinsurance	
	pareto		poisson	premium	premium	embedded	90%	reinsurance premium
	alpha	beta		=EX		value	quantile	=expected coverage*1,25
line A	2	12.000	20	240.000	360.000	120.000	25.000	100.000
line B	4	36.000	30	360.000	460.000	100.000	28.000	125.000
line C	2	12.000	20	240.000	340.000	100.000	25.000	100.000

Each provider will have a different effect on the DEC. The capital reduction a provider can contribute will also be effected by the reductions of other providers. When two providers are used in combination their collective reduction can be less than the sum of their individual reductions. However, it is important to note that the final result, after all providers have been applied, is the same.

The following figure shows how each provider contributes to the total reduction of the DEC. The figure uses plus and minus symbols to indicate which providers are applied at a given point in the diagram.

The P&C Diamond



1. The DEC without the benefit of providers is shown at the top of the diamond, while the capital after all three providers is shown at the bottom. The monetary decrease in capital demand is indicated on each step.

The P & C framework clearly spells out the influence of each provider so that it may be considered alone or in combination with other providers.

The diagram visibly demonstrates that the order in which the providers are applied has an influence on the reduction of the provider. This issue must be addressed. Specifically, one should be aware of how to exploit this effect when using the diamond as a basis for tactical decisions. A more detailed discussion of this topic will follow in section 4.

In general, providers decrease the DEC most when they are applied early in the diamond. Since the diamond is progressive, the further down the in the chain, the less there is to contribute. Thus, although a provider may appear to reduce the DEC an insignificant amount when applied last, especially if many providers are applied, it is clear from when the provider is applied first that it is actually much more significant than originally assumed.
Embedded value is interesting with this particular measure, because it contributes the same amount of capital regardless of the order in which it is applied. Under this measure, embedded value is independent of the other providers, and therefore, it has no impact on the DEC reducing abilities of

the other providers. This is clearly seen by comparing the effect of a provider before embedded value and after embedded value.

The independence of the embedded value provider can be seen clearly in the diamond. Consider the reduction of reinsurance when it is the first provider applied.

$$- - - \longrightarrow - + - \quad = 918.479$$

This reduction is identical to the reduction the reinsurance provider will allow when it follows the embedded value provider.

$$- - + \longrightarrow - + + \quad = 918.479$$

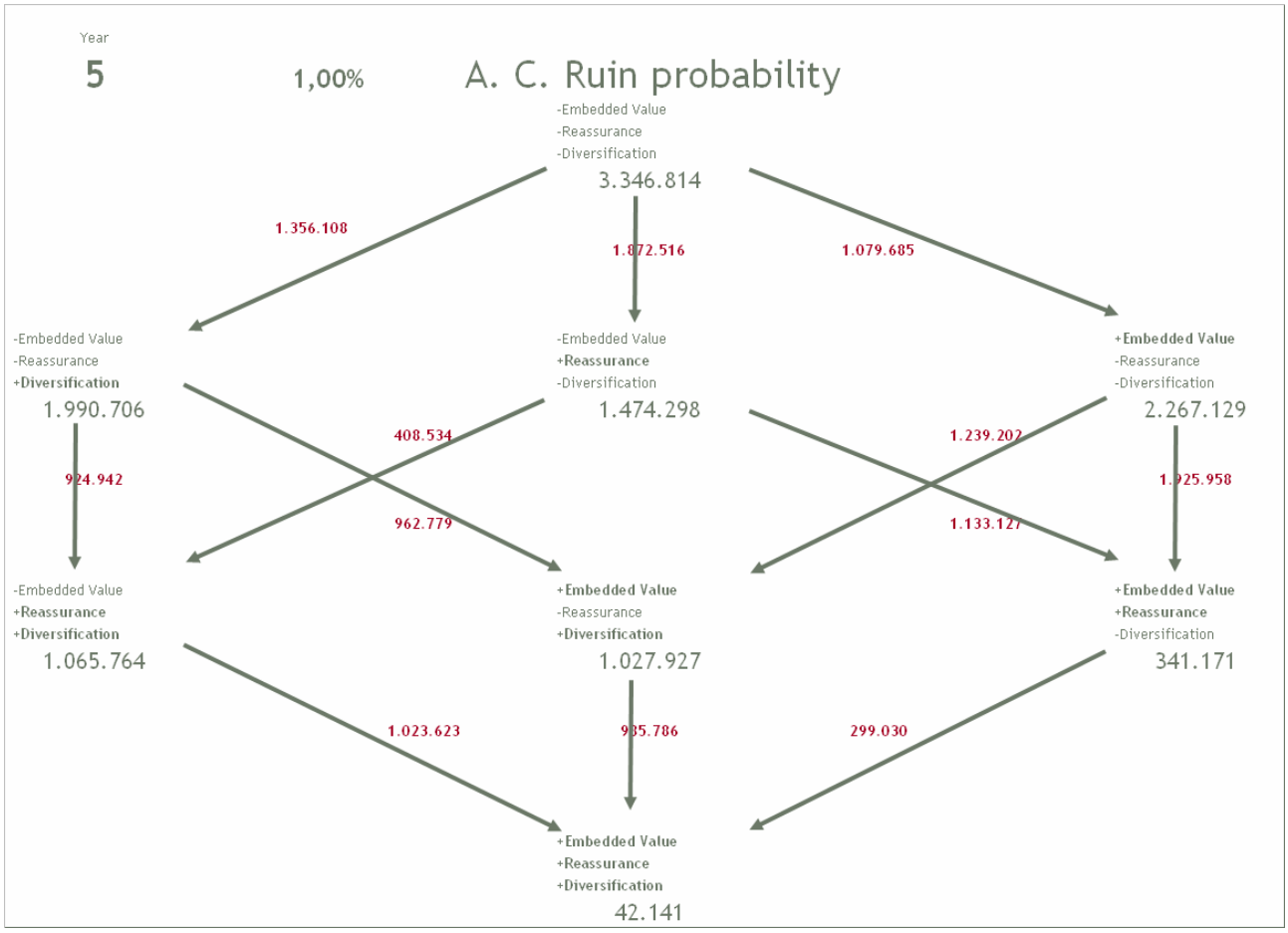
A similar pattern can be seen with the diversification provider. The independence of the embedded value provider can be contributed to the fact that this is a 1 year measure.

Embedded value phenomenon is emphasized because the independence seen under this measure will disappear when a 2 or 5 year ACRP is used. The explanation for this difference is quite simple. The first year embedded value is exactly the difference between the equivalence premium and the larger premium which is collected from the policyholders. This difference can be used to supplement capital. However, the embedded value for the 2^{nd}, 3^{rd}, 4^{th} and 5^{th}, should be considered stochastic and will be dependent on the other providers, as they have an effect on the distribution.

The providers can be ordered in a number of ways and the reasoning behind the ordering can be approached from many vantage points. One approach is to determine the order which will be the most efficient and logical progression in practice. There are some providers that naturally fall into place, while others demand more attention when determining their position. The P & C framework can be used as a tool to get an explicit picture of the influence of the order. It allows one to form opinions on the topic that are based on analytical facts. The figure is also useful when comparing two different reinsurance strategies, because the effect of the reinsurance treaty and the dependence on other providers is explicit.

3.4 The DEC under a 5 year ACRP

Consider another measure, the **5 year ACRP**. The following diagram shows the effect the providers have on the DEC when a 5 year ACRP of 1% is used. The parameters describing the distributions of the three lines of business, embedded value, and reinsurance cover remain unchanged.



The raw consumption of DEC is greater when a 5 year ACRP is used, than with the 1 year ACRP. This is quite logical, in that the capital needed to keep the company afloat for 5 years will be larger than the capital required to maintain the business for just 1 year. However, it is noteworthy that the DEC did not increase more significantly.

The embedded value provider is no longer independent of the other providers. Its reduction is effected by the providers it is used in combination with. Percentage wise it has become a more significant provider which speaks to this provider's long term importance. The P & C framework also shows that, unlike with the 1 year ACRP, the capital reduction of the reinsurance and diversification providers are now affected by the embedded value provider.

The P & C framework allows for a direct comparison of the two measures. In this way one can investigate if one measure can reveal something about the other. Clearly, the measure used is not without importance for the result. It is important to a clear idea as to how the providers affect the business in both the long and short term. Some providers have a greater influence in the long term than they do in the short term. The deeper the understanding of the measure, the better the understanding of the capital demand. The P & C framework allows for clear cut comparisons to be made because all the details are spelled out.

4. Strategic considerations about DEC

The P and C framework gives an excellent basis for determining how each provider impacts the DEC. As each step is spelled out, the details and effects are clear. However, it may not be prudent to use the full capital reduction of all providers. Some providers are sensitive to changes in the market, paradigm changes, etc. Providers need to be evaluated with care.

First of all, embedded value can be dependent on market conditions. It may be possible to charge a high premium for certain contracts, particularly when the type of insurance is new, but lucrative contracts will attract competition and prices may fall. A fluctuation of this type will have an effect on the DEC over time. It can, in critical cases, result in the company shifting from solvent to insolvent. If a large part of a company's capital is provided by the reduction coming from embedded value, and this capital has a significant impact on the company's solvency, it can be questioned if relying so heavily on this provider is a prudent long-term strategy.

The capital provided from reinsurance can also be dependent on marked conditions. Due to the changing capacity on the reinsurance marked, it is prone to sizeable price fluctuations. These fluctuations can put a pressure on embedded value, thus create an incentive for the restructuring of reinsurance coverers. The DEC should allow for some flexibility with regard to reinsurance coverers. If such flexibility is not present the company may be forced to decrease their underwriting results, reconsider their prices, or even, close areas of their business.

The effect of the diversification provider might also be affected by marked conditions. Competition on one or more lines can result in a shift of the diversification effect between lines. As with the previous provider, this may force the company to choose between, decreasing their underwriting results, etc.

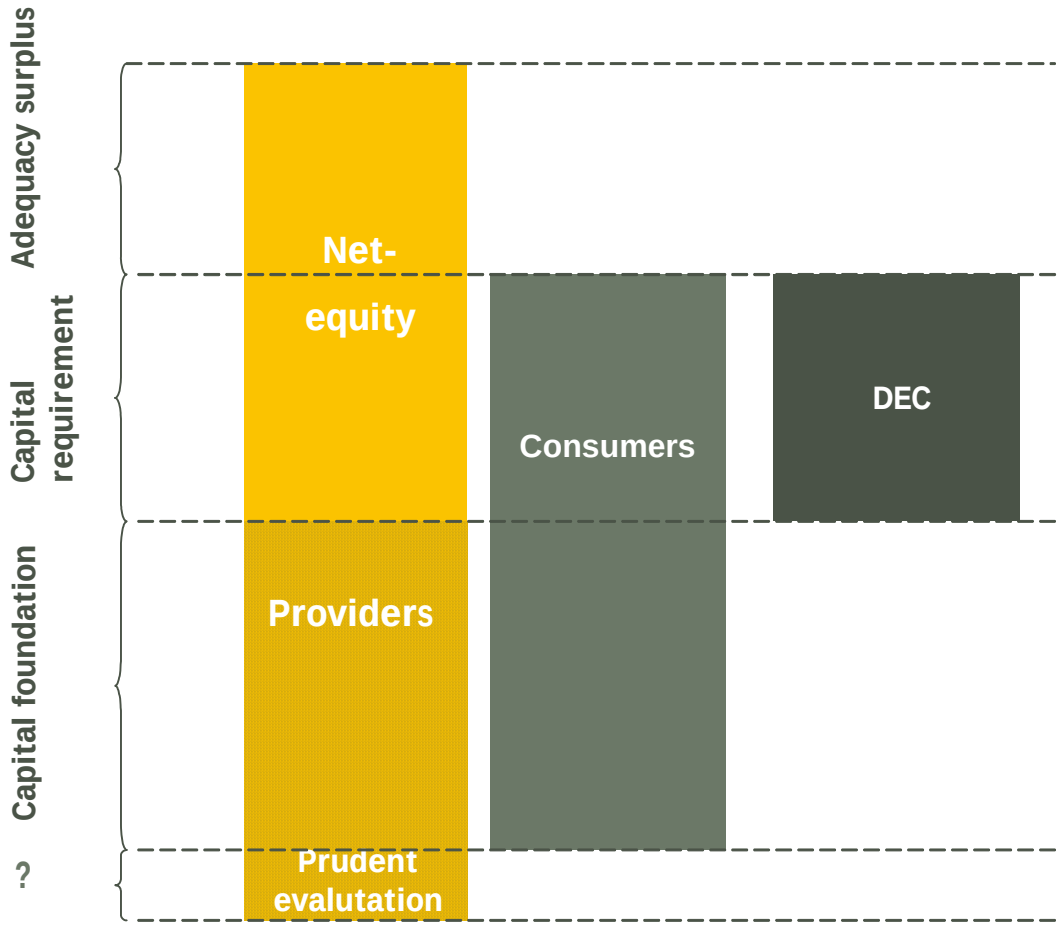
The P & C frame obviously gives rise to many strategic questions through the investigation and evaluation of the providers. These questions are important, and it is positive that they have been brought to light. Furthermore, the explicitness of the P & C framework makes it possible to recalculate the DEC if a provider is not permitted to give its full reduction. The P & C framework allows decision makers to order the providers according to their consistency as providers. In this way the more 'risky' providers are applied last where their impact is lowest. These types of considerations can be informative when balancing solvency and prudence.

It is natural that issues of this character influence the decision making processes within a company. Ultimately, these discussions can be beneficial in determining if a new business plan is careful and prudent or risky and radical.

5. The structure of DEC

Once the less certain providers have been evaluated and adjusted accordingly, the structure of the DEC or capital base, can be described as follows:

DEC structure:



The objective for many companies will be to ensure that the foundation under the net equity is solid. This foundation is composed of the providers and the additional capital needed as a result of the prudent evaluation of the providers. This why it is important to consider the scenarios that could cause the influences of ones providers to change.

So what is capital? Is it just net equity?

Ultimately, it is the board of directors who will determine what they will accept as capital, and the P & C framework is a useful tool in the explanation and decision making process. Since it is the board that will choose the strategic plans for the business, the author has refrained from answering many of the strategic questions posed earlier.

The example in this article was kept very simple, using only a few providers. However, in practice the framework can handle much more complex inputs. The construction of the P & C framework allows for a deeper understanding of the business. A deeper understanding will lead to fact based decisions and limit the risk of allowing the measure along to drive strategic decisions.

Undoubtedly, the DEC and strategic considerations will differ greatly depending on the providers chosen and the extent to which their reductions are applied, but it is possible to compare companies by considering their capital base. The DEC becomes more comparable when the P & C framework is utilized, because the providers and consumers are clearly specified, and the reduction each can provide is clear. Strategic decisions can be considered on a different level when various combinations of providers and their impact can be compared more easily.

6. Conclusion

The P and C Framework can serve as a link between a complex data analysis and a higher level of information. It can be a strong tool for decision makers because it allows for clear cut comparisons between different strategic choices.

The P and C Framework can also prove beneficial in many internal decisions. The company can learn a great deal from comparing the DEC using various providers. This gives the decision makers a deeper understanding of how different providers affect the business. It is also easily seen how weighting the different providers according to their reliability will effect the DEC. The Framework is also useful when considering several reinsurance contracts since the effect each has on the DEC can been seen explicitly.

Another strength of this framework is that it can be applied using a many different measures. In particular a company is able to choose both long and short term measures. The knowledge that can be gained by considering both the company's current situation and the long term view is very valuable. The decision makers have a much better basis on which to base their short term and long term strategies.

References

[1] Glasserman, Paul, 2004, *Monte Carlo Methods in Financial Engineering*, Springer.

[2] McNeil, Alexander J., F.R, E,P., 2005, *Quantative Risk Management,* Princeton Series in Finance.

[3] Meyers, Glenn G., 2001, An Analysis of the Underwriting Risk for DFA Insurance Company, paper presented to the CAS 2001 Dynamic Financial Analysis Conference.

[4] Mueller, Hubert, 2004, Recent Market Developments and Current Trends, (http://www.fenews.com/fen37/topics_act_analysis_topics_act_analysis.html).

[5] Perch Nielsen, Jens, M.P., P.R., 2005, Capital Allocation for Insurance Companies: Issues and Methods, (http://www.math.ku.dk/~rolf/MumfordPerchPoulsen.doc).

[6] The Danish Financial Supervisory Authority's Note of Guidence regarding the implementation of Solvency II Pillar 2 into the Danish Accounting Standards, 2006, vej nr. 10116, (Vejledning for forsikringsvirksomheder i henhold til § 71, stk. 1, nr. 1-8 i lov om finansiel virksomhed)