

# LIMITARY PROFITABILITY OF FINANCIAL OPERATIONS

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## ABSTRACT

The class of financial operations (named correct financial operations) and the natural characteristic of such operations (limitary profitability) are defined in the paper. The properties of limitary profitability are established. The relations between limitary profitability, Inner Rate of Return and Arrow-Levhari characteristic are examined.

Keywords: correct financial operations, limitary profitability, Inner Rate of Return

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## INTRODUCTION

The legislation of some countries does not suppose loans under too high rate of interest. For example such border is 60 % per year in Canada. The question is clear for simple loans: it is impossible to conclude the treaty in accordance with which, having borrowed 1000\$ on March, 18, 2006, it is possible to demand return 1700\$ on March, 18, 2007. It is the problem how to select admissible loans. It is even impossible in the certain sense: it is proved in [5] the impossibility of financial operations division into admissible and inadmissible for rather natural axiomatic.

Let's introduce the corresponding concepts. We think that all payments to be made at the integer non-negative time moments (years). Any financial operation can be presented as a vector  $C = (c_0, c_1, \dots)$  with finite number of nonzero components and such that there are positive and negative components. We shall consider the financial operation from the point of view of the creditor: negative components correspond to the charge i.e. the sums given in a duty and positive - to returns of funds. The Internal Rate of Return of an operation  $C$  ( $IRR(C)$ ) is a unique root greater 1 (if it is so) of the profitability equation

$$\sum_{i=0}^n c_i x^{n-i} = 0$$

Here  $x=1+r$ ,  $r$  is a rate of interest.  $IRR(C)$  is used usually as the measure of profitability of a financial operation.

The measure similar to Internal Rate of Return for the first time has been defined by I.Fisher [3]. This characteristic has a number of natural properties; moreover such characteristic is unique [8]. At the same time the Internal Rate of Return is defined not for all financial operations. To establish whether operation has Internal Rate of Return, it is necessary to use some algorithm of polynomial processing. Naturalness of this characteristic raises the certain doubts (even in case of existence). Let's consider the following example.

EXAMPLE 1. Let  $C=(-1, 2, -1, 1, -1, 1, -1, 1)$ . This operation has the Internal Rate of Return equal 1.64 (one can check it up). At the same time, if to interrupt operation after the first return of funds then the income of the creditor will be essentially higher (=2). Such situation allows bypassing the law: the parties register the contract and cancel it by mutual consent after the second payment.

Some characteristics of operations applicable to wider class of operations are determined in a number of papers (for example, in [1,7,8]). The other approach is of-

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ferred in the paper. We select the class of financial operations (correct operations) to which can be applied some natural characteristic named "limitary profitability". This class isn't an expansion of a class of operations for which *IRR* exists; the opposite statement is fair also. The testing of operation correctness is elementary. Limitary profitability coincides with *IRR* for standard operations when all credit payments precede all returns of funds (I.Fisher has considered only such operations).

## CORRECT FINANCIAL OPERATIONS

The simplest financial operation is a transaction – a cash flow  $T(i, a, b)$  which is characterized by three numbers: the non-negative integer  $i$ , and two real  $0 \leq a \leq b$ . We admit zero transactions for convenience. The nonzero transaction betokens what the debt  $a$  in the moment  $i$ , is repaid by sum  $b$  in the moment  $i+1$ . The transaction can be represented as a vector  $(0, \dots, 0, -a, b, 0, \dots)$  where the number  $-a$  is in the  $i$ -th position.

DEFINITION 1. We name financial operation CORRECT if it can be represented as the sum of consecutive transactions, i.e. as a series of year loans.

Thus, the correct financial operation looks like

$$C = \sum_{i=u}^{v-1} T(i, a_i, b_i)$$

where  $u$  and  $v$  are the initial and the finishing moments of the operation,  $u$  always coincides with the moment of the first payment and  $v$  can be more than the moment of the last payment. It follows that  $c_u = -a_u$ ,  $c_v = b_v$ ,  $c_i = b_{i-1} - a_i$  for  $u < i < v$ . Nonzero correct financial operations can be described as follows.

PROPOSITION 1. The validity of inequalities

$$c_u < 0, \quad \sum_{i=k}^v c_i \geq 0 \text{ for } k=u, u+1, \dots, v$$

is necessary and sufficient for correctness of financial operation  $C$ . We assume that  $c_i = 0$  when  $i < u$ .

PROOF. Let financial operation  $C$  be correct. Then

$$c_u = -a_u < 0, \quad \sum_{i=k}^v c_i = \sum_{i=k}^v (b_{i-1} - a_i) + b_v = b_{k-1} + \sum_{i=k}^v (b_i - a_i) \geq 0.$$

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Let now the inequalities  $c_u < 0$ ,  $\sum_{i=k}^v c_i \geq 0$  for  $k=u, u+1, \dots, v$  be valid. We prove the correctness of financial operation  $C$  by induction on operation duration. The statement is evident for two-year operations. We assume it is so for the operations with duration shorter than  $s+1$ . Let's consider operation with duration  $s+1$ . If  $\sum_{i=u+2}^v c_i = 0$  then  $c_{u+1} \geq -c_u > 0$  according to operation properties. If  $c_{u+2} = \dots = c_v = 0$  then result is evidence. If it not so then among numbers  $c_{u+2}, \dots, c_v$  exist negative and let  $w$  be the least index of such numbers. Operation is the sum of two ones with smaller durations; the first includes payments with indexes from  $u$  to  $w-1$ , the second – from  $w$  to  $v$ . The second operation inherits the described properties of initial operation, the necessary properties of the first one follow from the nonnegativity of payments with indexes from  $u+2$  to  $w-1$  and inequalities  $c_{u+1} \geq -c_u > 0$ . For each of them the assumption of an induction is fair, zero transaction is used as jointing.

Let now  $\sum_{i=u+2}^v c_i > 0$ . Let's consider two cases. If  $c_{u+1} \geq -c_u$  then we'll choose positive number  $\varepsilon < \sum_{i=u+2}^v c_i$  and assume

$$C = T(u, -c_u, c_{u+1} + \varepsilon) + (0, \dots, 0, -\varepsilon, c_{u+2}, \dots, c_v, 0, \dots).$$

The second item is the financial operation of duration  $s$  with the necessary properties. It follows out of the properties of correct operation. If  $c_{u+1} < -c_u$  then we assume

$$C = T(u, -c_u, -c_u) + (0, \dots, 0, c_u + c_{u+1}, c_{u+2}, \dots, c_v, 0, \dots).$$

The second item is the operation with smaller duration and necessary properties. The proposition is proved.

The set of correct financial operations has structure of a convex cone, i.e. is addition and multiplication by positive numbers closed, it follows out of definition of correct financial operation (or of the proposition 1). At the same time the class of the operations having *IRR*, does not possess such structure: the sum of the operations with *IRR*, may be without *IRR*.

EXAMPLE 2. We assume that the profitability polynomial of first operation  $P_1$  is increasing and has properties  $P_1(2)=1$ ,  $P_1(3)=2$ ,  $P_1(1)<0$  and the profitability poly-

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nomial of the second operation  $P_2(x)$  is increasing in  $x > 4$  and has properties  $P_2(2) = -1$ ,  $P_2(3) = -2$ ,  $P_2(4) = 0$ ,  $P_2(x) < 0$  at  $x < 4$ . It is obvious that polynomial  $P_1 + P_2$  has more than one root in  $[1, \infty)$ .

Let's define gluing (concatenation) of correct operations. Let  $C, D$  - correct operations with the zero initial moment. Operation  $C * D^{(u)}$  is defined as follows. The initial moment of operation  $C * D^{(u)}$  equals to  $u$ ,  $C * D^{(u)}_i = C_{i-u}$  for  $i \leq k+u-1$  where  $k$  is a duration of operation  $C$ ,  $C * D^{(u)}_i = D_{i-u-k}$  for  $k+u \leq i \leq s+k+u-1$  where  $s$  is duration of operation  $D$  i.e. operation  $D$  is carried out directly after operation  $C$ . How it follows out of the proposition 1, the concatenation of correct financial operations is correct financial operation. This property isn't carried out for the operations having  $IRR$  - the example can be constructed by modifying an example 2 as follows. Let's in addition demand that polynomial  $P_2(x)$  has divisor  $x^k$  where  $k$  is suitable natural number. Then concatenation of operations with characteristic polynomials  $P_2(x)$  and  $P_2(x)/x^k$ , each of which has  $IRR$ , hasn't  $IRR$ . The Correct financial operation can be represented as the sum of transactions by different ways. For example,  $(-1, 2, 6, 0 \dots) = T(0, 1, 3) + T(1, 1, 6) = T(0, 1, 4) + T(1, 2, 6)$ . We name representation correct if it doesn't include transactions of kind  $T(k, 0, b)$  where  $b > 0$ .

**PROPOSITION 2.** Any correct financial operation has correct representation.

Really, let financial operation can be represented as some sum of transactions and among them there are transactions of kind  $T(k, 0, b)$  ( $b > 0$ ). The first transaction isn't such. Let  $T(k, 0, b)$  be the last such transaction. We replace transactions  $T(k-1, a_{k-1}, b_{k-1})$ ,  $T(k, 0, b)$  on  $T(k-1, a_{k-1}, b_{k-1} + \varepsilon)$ ,  $T(k, \varepsilon, b)$  where  $\varepsilon \in (0, b)$ . We'll receive correct representation by continuing of this process.

### LIMITARY PROFITABILITY

Profitability of transaction  $T(k, a, b)$  when  $0 < a$  is equal  $b/a \geq 1$ .

**DEFINITION 2.** LIMITARY PROFITABILITY of the nonzero correct financial operation  $C$  is the number

$$LP(C) = \inf \{ \max(b_i/a_i) \},$$

where  $C = \sum_{i=u}^{v-1} T(i, a_i, b_i)$ , the maximum is calculated on the set of nonzero transactions, infimum – on the set of correct representations  $C$ .

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The existence of an infimum doesn't directly follow from the definition as the set of allowable values of parameters  $a$ ,  $b$  isn't compact. We'll prove that, nevertheless, the infimum is achievable.

**PROPOSITION 3.** Any correct financial operation  $C$  can be represented as the sum of transactions some of which have profitability equal to  $LP(C)$ .

**PROOF.** Let's consider the correct representations  $S_j$  of operation  $C$  as the sums of transactions  $\sum_{i=u}^{v-1} T(i, a_{j,i}, b_{j,i})$  with maximum profitability not surpassing  $LP(C)+1/j$

for  $j=1,2,\dots$ . Let's check up that inequalities  $a_{j,i} \leq M_i$  are valid for some numbers  $M_i$  ( $i=u, \dots, v$ ). Let  $k=LP(C)+1$ . We have  $a_{j,u} = -c_u$  for all operations. Then  $b_{j,u} \leq ka_{j,u} = kc_u$ . Further  $a_{j,u+1} = b_{j,u} - c_{u+1}$ , i.e.  $a_{j,u+1} \leq b_{j,u} + |c_{u+1}| \leq k|c_u| + |c_{u+1}|$ . Thus  $b_{j,u+1} \leq ka_{j,u+1}$ . We receive the necessary estimates by continuing. It is possible to extract the subsequence from sequence of representations  $S_j$ , with the properties  $a_{j,i} \rightarrow a_{0,i}$ ,  $b_{j,i} \rightarrow b_{0,i}$  (we suppose for simplicity that the subsequence coincides with sequence  $S_j$ ). Let us prove that

$\sum_{i=u}^{v-1} T(i, a_{0,i}, b_{0,i})$  is a correct representation of the operation  $C$ . Since  $c_{i+1} = b_{j,i} - a_{j,i+1}$

then  $c_{i+1} = b_{0,i} - a_{0,i+1}$ ; since  $b_{j,i} \leq ka_{j,i}$  then  $b_{0,i} \leq ka_{0,i}$ . It means, that transaction  $T(i, a_{0,i}, b_{0,i})$  either is zero, or both payments are positive. If transaction  $T(i, a_{0,i}, b_{0,i})$  is nonzero then transactions  $T(i, a_{j,i}, b_{j,i})$  are nonzero too for sufficiently large  $i$ . But then  $b_{0,i}/a_{0,i} = \lim b_{j,i}/a_{j,i}$  and since  $b_{j,i}/a_{j,i} \leq LP(C)+1/j$  then  $b_{0,i}/a_{0,i} \leq LP(C)$ .  $LP(C)$  is the infimum of the maximal profitabilities of transactions - summands of operation  $C$ , whence some of relations  $b_{0,i}/a_{0,i}$  equal to  $LP(C)$ . The proposition is proved.

The algorithm of the limitary profitability approximate calculation is based on

the following idea. If  $C = \sum_{i=u}^{v-1} T(i, a_i, b_i)$  then as was marked  $c_u = -a_u$ ,  $c_v = b_v$ ,  $c_i = b_{i-1} - a_i$

for  $u < i < v$ . Hence  $b_{i-1} = c_i + a_i$  for  $u < i < v$ . If  $x$  is the maximal profitability of nonzero transactions then the inequalities  $1 \leq (c_i + a_i)/a_{i-1} \leq x$  should be carried out for  $u < i < v$ ,  $1 \leq c_v/a_{v-1} \leq x$ . Thus, limitary profitability of financial operation is the infimum of the set of such numbers  $x$  that the system of linear inequalities

$$c_u = -a_u, 0 \leq a_{i-1} \leq c_i + a_i \leq x a_{i-1} \text{ for } u < i < v, 0 \leq a_{v-1} \leq c_v \leq x a_{v-1} \quad (*)$$

is compatible.

Let's notice that such transformation allows including zero transactions in consideration too. Effective algorithms of checking of compatibility of systems of linear inequalities (for example, Chernikova's algorithm [6]) are developed. It allows to organize a dichotomizing procedure: chose some initial value  $x$  for which the system is

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obviously compatible, then check up value  $(1+x)/2$  etc. There is one more useful representation of limitary profitability.

PROPOSITION 4. The limitary profitability of a correct financial operation coincides with the maximal root of all polynomials  $Q_k(x) = \sum_{i=0}^k c_i x^{k-i}$  ( $k=1,2,\dots,n$ ).

PROOF. The coefficient  $c_0$  of polynomials  $Q_k(x)$  is negative. So all polynomials are negative for numbers  $x$  exceeding the maximal of roots.  $Q_n(1) \geq 0$  by the proposition 1 i.e. the set of polynomial's roots is nonempty on the set  $[1, \infty)$ . Let the system (\*) is compatible for some value  $x \geq 1$ . Multiplying the first equality by  $x$  and adding to the inequality for  $i=1$ , we receive the inequality  $c_0 x + c_1 + a_1 \leq 0$ , i.e.  $Q_1(x) = c_0 x + c_1 \leq -a_1 \leq 0$ ; multiplying this inequality by  $x$  and adding the product to the following inequality of system (\*) we receive the inequality  $Q_2(x) \leq -a_2 \leq 0$ . We receive the inequalities  $Q_k(x) \leq -a_k \leq 0$  at  $k \leq n-1$  by continuing this process. At last multiplying last of these inequalities by  $x$  and adding the product to the last inequality of system (\*), we receive inequality  $Q_n(x) \leq 0$ . Thus  $LP(C)$  isn't less than the maximal root of polynomials  $Q_k(x)$ .

Let now  $Q_k(x) \leq 0$  for some  $x$  and all  $k$ . If we assume  $a_k = -Q_k(x)$  for  $k=1, \dots, n-1$  one can check directly that system (\*) is valid. The proposition is proved.

### SOME PROPERTIES OF LIMITARY PROFITABILITY

The following statement is obvious

PROPOSITION 5. Limitary profitability of correct financial operation doesn't change at time shift.

We consider the natural finite - open topology on the set of financial operations following [6]. The sequence of operations  $C_n$  converges to operation  $C_0$  if durations of all operations are limited by some number  $N$  and  $C_{n,i} \rightarrow C_{0,i}$  for all  $i$ . It is obvious that the closure of the cone of correct financial operations in the described topology consists of vectors with finite number of nonzero components with the property  $\sum_{i=k}^{\infty} c_i \geq 0$  ( $k=0,1,2,\dots$ ). Unfortunately limitary profitability isn't continuous functional on the set of correct operations. The following statement is valid.

PROPOSITION 6. If  $C_n, C_0$  are correct financial operations and  $C_n \rightarrow C_0$  in finite - open topology then  $LP(C_0) \leq \liminf LP(C_n)$ .

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REMARKS. 1. It is possible that  $\lim LP(C_n)$  doesn't exist under conditions of proposition 6. For example if  $C_n=(-1/n, (2+(-1)^n)/n, -1, 1)$  then  $C_0=(0,0, -1, 1)$  but it is simple to check (and by the proposition 11) that  $LP(C_n)=2+(-1)^n$ .

2. It is possible that  $\lim LP(C_n)$  exists but doesn't coincide with  $LP(C_0)$ . For example if  $C_n=(-1/n, 2/n, -1, 1)$ .

PROOF. Let  $A=\lim \inf LP(C_n)$ . Then the inequality  $LP(C_n)\leq A+\varepsilon$  is valid for arbitrary  $\varepsilon>0$  and infinite many terms of sequence  $C_n$ . The system (\*) for corresponding terms of sequence is given by

$$c_{n,u}=-a_{n,u}, 0\leq a_{n,i-1}\leq c_{n,i}+a_{n,i}\leq(A+\varepsilon)a_{n,i-1} \text{ for } u<i<v, 0\leq a_{n,v-1}\leq c_{n,v}\leq(A+\varepsilon)a_{n,v-1}.$$

Here  $v$  is the maximum of operations  $C_n$  durations. By convergence values  $c_{n,i}$  are uniformly limitary and then values  $a_{n,i}$  are limited also. Passing on to a subsequence if necessary it is possible to think that  $a_{n,i}\rightarrow a_{0,i}$ . Passage to the limit in a corresponding subsystem of inequalities leads to the result

$$c_{0,u}=-a_{0,u}, 0\leq a_{0,i-1}\leq c_{0,i}+a_{0,i}\leq(A+\varepsilon)a_{0,i-1} \text{ for } u<i<v, 0\leq a_{0,v-1}\leq c_{0,v}\leq(A+\varepsilon)a_{0,v-1}.$$

It means that  $LP(C_0)\leq A+\varepsilon$ . Thus we receive the inequality  $LP(C_0)\leq A=\lim \inf LP(C_n)$  by virtue of  $\varepsilon$  arbitrariness. The proposition is proved.

Proposition implies that the family of correct operations with the limitary profitability not surpassing some number  $A$  is closed, i.e. inadmissible operation by the law cannot be made allowable by small changes.

Some natural properties which are desirable for financial operations are described in [6, 8]. Let's prove that  $LP$  has some of them.

PROPOSITION 7. If all payments of an operation  $C$  are not less than the appropriate payments of an operation  $D$  then  $LP(C)\geq LP(D)$ . I.e. the limitary profitability is monotonous.

PROOF. The inequalities  $\sum_{i=0}^k c_i x^{k-i} \geq \sum_{i=0}^k d_i x^{k-i}$  are valid for  $k=1,2,\dots,n; x\geq 0$ . The result follows directly from the proposition 4.

REMARK. If some payments  $C$  are more than corresponding payments  $D$  then the strict inequality between limitary profitabilities not necessarily carries out. For example  $LP(-1,10,-1,1)=LP(-1,10,-1,2)=10$ . It can be checked directly and is the sequence of proposition 12.

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PROPOSITION 8. If  $C$  and  $D$  are correct financial operations and  $c_i = d_i - p$ ,  $c_{i+1} = d_{i+1} + p$  where  $p \geq 0$ ,  $c_j = d_j$  for  $j \neq i, i+1$  then  $LP(D) \geq LP(C)$ . I.e. carrying of payments on earlier moments cannot reduce to falling of limitary profitability of the operation.

PROOF. For  $x = LP(C)$  the inequalities (\*)

$c_u = -a_u$ ,  $0 \leq a_{k-1} \leq c_k + a_k \leq x a_{k-1}$  for  $u < k < v$ ,  $0 \leq a_{v-1} \leq c_v \leq x a_{v-1}$  hold. The solution of similar system for operation  $D$  (for the same  $x$ ) can be obtained from the solution of previous system if  $a_i$  change on  $a_i + p$ , the necessary proposition whence follows.

REMARK. It is possible that  $p > 0$  but  $LP(C) = LP(D)$ . For example  $LP(-1, 10, -1, 1) = LP(-1, 10, -2, 2) = 10$ .

PROPOSITION 9. If  $C$  and  $D$  are correct financial operations and an initial part of operation  $D$  coincides with  $C$  then  $LP(C) \leq LP(D)$ .

It is immediate consequence of proposition 4.

The next proposition concerns to degenerate finance operations.

PROPOSITION 10. Limitary profitability of nonzero correct financial operation with the zero sum of payments is equal 1 and vice versa (it is quite naturally).

Really, if  $\sum_{i=u}^v c_i = 0$ ,  $c_u < 0$  then

$C = T(u, -c_u, -c_u) + T(u+1, -c_u - c_{u+1}, -c_u - c_{u+1}) + \dots + T(v-1, -\sum_{i=u}^{v-1} c_i, -\sum_{i=u}^{v-1} c_i)$ . All sums  $\sum_{i=u}^k c_i$

are nonpositive and  $c_v = -\sum_{i=u}^{v-1} c_i$  as come from correct operation properties. Thus all

terms are correct and its profitability equals 1. The inverse statement is evidence.

PROPOSITION 11. LP of financial operation  $C$  is unaffected after concatenation with operation  $D$  which limitary profitability equals 1.

PROOF. Proposition 10 implies that the components sum of the operation  $D$  equals 0. Sum of the inequalities  $0 \leq a_{i-1} \leq d_i + a_i \leq x a_{i-1}$  ( $i = k, k+1, \dots, v-1$ ),  $0 \leq a_{v-1} \leq d_v \leq x a_{v-1}$

is  $0 \leq \sum_{i=k-1}^v a_i \leq \sum_{i=k}^v a_i + \sum_{i=k}^v d_i$  then  $a_{k-1} \leq 0$ , i.e.  $a_{k-1} = 0$ . Thus

$LP(C * D) = \max\{LP(C), LP(D)\} = LP(C)$ . The proposition is proved. Thus may be regarded that the duration of the operation is arbitrary large - it doesn't influence limitary profitability.

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The following statement is the strengthening of the proposition 11.

PROPOSITION 12. If  $C, D$  are the correct financial operations and  $LP(C) \geq LP(D)$  then  $LP(C*D) = LP(C)$ .

PROOF. If  $LP(D) = 1$  then the statement coincides with the proposition 11. Let's believe therefore that  $LP(C) \geq LP(D) > 1$ . Let's designate as  $C^a, D^{-b}$ , where  $a, b$  are positive numbers, the operations differing from  $C$  ( $D$ ) accordingly by increase of the last component  $C$  by  $a$  (by decrease of the first component  $D$  by  $b$ ). It's clear that sufficiently smallness of number  $b$  is necessary and sufficient for operation  $D^{-b}$  correctness. The pair  $(a, b)$  is the gluing transaction at concatenation.  $LP(C*D) = \inf_{0 < a \leq b} \max \{ LP(C^a), LP(D^{-b}), b/a \}$  on  $LP$  definition. Then  $LP(C*D) = \inf_{0 < a \leq b} \max \{ LP(C^a), b/a \}$  while  $LP(C^a) \geq LP(C)$ ,  $LP(D^{-b}) \leq LP(D)$  (this is the consequence of proposition 7). From here  $LP(C*D) \geq LP(C^a) \geq LP(C)$ . On the other hand if  $a=b=0$  then  $LP(C*D) \leq LP(C)$  by  $LP$  definition. The proposition is proved.

Let's quote some properties of limitary profitability connected to algebraic operations.

The following property is evidence.

PROPOSITION 13.  $LP$  is the relative characteristic, i.e.  $LP(\lambda C) = LP(C)$  for  $\lambda > 0$ .

PROPOSITION 14. The estimate  $LP(C+D) \leq \max \{ LP(C), LP(D) \}$  is valid for arbitrary correct operations  $C, D$ .

PROOF. All polynomials  $\sum_{i=0}^k c_i x^{k-i}, \sum_{i=0}^k d_i x^{k-i}$  are negative for arbitrary  $x > \max \{ LP(C), LP(D) \}$ . Then values  $\sum_{i=0}^k (c_i + d_i) x^{k-i}$  are negative for such  $x$ . Then values  $\sum_{i=0}^k (c_i + d_i) x^{k-i}$  are negative for such  $x$ . It means that  $LP(C+D) \leq x$  and hence  $LP(C+D) \leq \max \{ LP(C), LP(D) \}$ .

We don't know any nontrivial lower estimate of  $LP(C+D)$ . The estimate  $LP(C+D) \geq \min \{ LP(C), LP(D) \}$  is false. For example if  $C = (-3, -5, 6, 7, 2)$  and  $D = (-2, 5, -6, -4, 10)$  then  $LP(C) = 1.35$ ,  $LP(D) = 2.5$ ,  $LP(C+D) = 1.34$ . Such lower estimate is correct for  $IRR$ .

## RELATIONS BETWEEN LIMITARY PROFITABILITY AND $IRR$

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A correct financial operation can not have the Inner Rate of Return. For example profitability equation of the correct financial operation  $(-1, 8, -20, 25, -54, 72)$  has the roots 2, 3, 4.

On the other hand incorrect financial operation may have the Inner Rate of Return. For example  $(2-x)(x^2-2x+3)$  is profitability polynomial of incorrect financial operation  $(-1, 4, -7, 6)$ .

The direct sequence of the proposition 4 is

PROPOSITION 15.  $LP(C)$  is not less than the minimal greater 1 root of the profitability equation of a correct financial operation.

In particular if correct financial operation has  $IRR$  then  $LP(C) \geq IRR(C)$  but these values may be different. For example if  $C=(-1, 2, -1, 1, -1, 1, -1, 1)$  then  $IRR(C)=1.64$ ,  $LP(C)=2$ . Thus, use of the entered parameter leads to more cautious credit politics.

Financial operation is named standard operation if the first  $s$  components are nonpositive and the subsequent are nonnegative for some  $s$ . Obviously standard operation is correct if the components sum is non-negative and the first nonzero component is negative. The profitability polynomial has a unique positive root in this case under Cartesian theorem [4], is non-negative at  $r=1$  and the leading coefficient is negative. Thus, the root of a profitability polynomial is not less 1, i.e. the Internal Rate of Return exists.

PROPOSITION 16.  $LP(C)=IRR(C)$  for correct standard financial operation.

PROOF. If  $k \geq s$  then  $\sum_{i=0}^{k+1} c_i x^{k+1-i} \geq x \sum_{i=0}^k c_i x^{k-i}$ . From here the polynomial for  $k=n$

has the maximal of their roots under polynomials properties. The statement follows from the proposition 4.

### ARROW-LEVHARI CHARACTERISTIC

The construction leading to some characteristic of the financial operation is described in [1].

Let  $P_k$  ( $k=1, 2, \dots, n$ ) be profitability functions of the truncated operations, i.e.  $P_k(x) = \sum_{i=0}^k c_i x^{-i}$  and  $\psi(x) = \max_k P_k(x)$ . Here as in introduction  $x=1+r$ ,  $r$  is a rate of interest. The basic result of the paper [1] is

PROPOSITION 17. Function  $\psi(x)$  is nonincreasing on the set  $[1, \infty)$ .

This implies

# LIMITARY PROFITABILITY OF FINANCIAL OPERATIONS

PROPOSITION 18. The equation  $\psi(x)=0$  has the unique root on the set  $[1,\infty)$ .

Let's designate this root as  $AL(C)$ . It is easy to check up validity of the following statement.

PROPOSITION 19.  $AL(C)$  coincides with the maximal root of the set of functions  $P_k(x) = \sum_{i=0}^k c_i x^{-i}$  ( $k=1,2, \dots, n$ ).

Propositions 4 and 19 imply that  $AL(C)=LP(C)$ . Let's notice that in [1] a financial sense of  $AL(C)$  isn't formulated.

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