RISK TOLERANCE LEVELS FOR INSURANCE COMPANIES

1. Anna Battauz
Department of Finance and Department of Decision Sciences, Bocconi University
Via Sarfatti 25, 20136 Milano
anna.battauz@unibocconi.it

2. Marzia De Donno
Department of Decision Sciences, Bocconi University
Via Sarfatti 25, 20136 Milano
marzia.dedonno@unibocconi.it

3. Alessandro Sbuelz*
Department of Economics and SAFE Center, University of Verona
(*corresponding author)
Palazzo Giusti, Via Giardino Giusti 2, 37129 Verona, Italy
alessandro.sbuelz@univr.it

4. Marco Tolotti
Department of Finance, Bocconi University
Via Sarfatti 25, 20136 Milano
marco.tolotti@unibocconi.it

ABSTRACT
We describe two procedures that assist insurance firms in determining shareholders' risk tolerance thresholds and in using such thresholds within the decision-making process. The first procedure is based on parsimonious measures of the risk/return tradeoff such as the Sharpe Ratio; the second procedure makes a direct use of expected utility theory.

KEYWORDS
Shareholders' risk appetite, insurance firms.

JEL Classification: G22 - Insurance; Insurance Companies; G11 - Portfolio Choice; Investment Decisions; G32 - Financing Policy; Financial Risk and Risk Management; Capital and Ownership Structure.

1. INTRODUCTION

Following the standards recently laid down by the Italian supervisory body for insurance companies (ISVAP), the Board of Directors is responsible for the risk management system. Among other things, this implies that it must determine risk tolerance thresholds and ensure that top-level executives take on risk exposures attuned to those thresholds. Under the assumption of full alignment of Board's and shareholders' interests, we describe two possible procedures that help determining shareholders' risk tolerance thresholds and aid executives in undertaking new risky projects.
Section 2 describes the notation employed. Section 3 explains the first procedure of eliciting shareholders' attitude towards risk. Section 4 gives the details of the second procedure of educing risk tolerance levels. Section 5 concludes.

2. NOTATION

We denote with A the market value of an insurance firm's assets and with L the market value of its liabilities. L comprises the value of senior debt and the reserves set aside to shield the expected losses from the insurance business. The firm's total equity capital includes common stock and hybrid/junior debt. Its future value is given by the difference $\widetilde{A}-\widetilde{L}$. The standard deviation of $\widetilde{A}-\widetilde{L}$ is s and quantifies equity capital's risk.
The absorbed equity capital is denoted by $K(A,L,s)$. It is the sum of the minimum required capital and of the additional solvency capital. Y is the residual equity capital after absorption, hence $Y = A - K(A,L,s) - L$. We denote by $\widetilde{R}$ the annual earnings after interests and taxes (EAIT), and by R the expected EAIT, $E(\widetilde{R})$.
The cost of equity capital per annum is denoted with k, the market price of common stock with p, and the payout ratio based on the firm's dividend policy with π.

2.1 THE INTERPLAY BETWEEN s (QUANTITY OF RISK) AND $K(A,L,s)$ (ABSORBED CAPITAL)

The volatility s is conditional upon the current market values of assets and liabilities:

$$s = \left(\mathrm{var}\left(\widetilde{A}-\widetilde{L} \mid A,L\right)\right)^{\frac{1}{2}}$$

If the internal risk management model is based on Value at Risk (VaR), we can reasonably assume that the absorbed capital by $K(A,L,s)$ conforms to the following definition:

$K(A,L,s) =$ minimum required capital + additional solvency capital =   = max (minimum required capital, $VaR_{1-\alpha}$)

where $VaR_{1-\alpha}$ is the (potential) decrease of future equity capital that solves the equation

$$\Pr\left(\widetilde{A}-\widetilde{L} \geq A - L - VaR_{1-\alpha} \mid A,L\right) = 1-\alpha$$

with α small and selected by the firm's risk management taskforce. VaR is a well-known measure of maximum loss. It is widely applied in finance for quantitative risk management for many types of risks (for example, see Jorion (2005)).

We denote $\Pr(\cdot \mid A, L)$ the objective probability measure conditional upon the current market values of assets and liabilities.

2.2 THE BALANCE SHEET

The balance sheet in the absence and in the presence of a new risky project is as follows.

Capital structure without the project

A	Y $K(A,L,s)$ L

Capital structure with the project

A'	Y' $K(A',L',s')$ L'

3. THE SHARPE RATIO APPROACH FOR MEASURING THE ATTITUDE TOWARD RISK

The first procedure for eliciting shareholders' attitude towards risk employs the Sharpe ratio to put a figure on the incentive implicit in the decision of undertaking a given benchmark project. We first derive the implied expected EAIT and then we quantify shareholders' risk appetite by means of the implied Sharpe ratio.

The cost of equity capital can be computed from Gordon's Growth Model adjusted for the efficiency impact of changes in the absorption ratio. For more details we refer the reader to Brealey and Myers (2003), who offer a thorough review of the techniques for corporate securities valuation. Among these, the Gordon's Formula stands out for its neat and parsimonious blend of the pricing primitives. Accordingly, firm's share price is

$$p' = \frac{\pi \vartheta' \hat{R}' / N'}{k' - \hat{g}'}$$

where the change in the absorption ratio is denoted by

$$\vartheta' = \frac{K(A,L,s)}{K(A',L',s')} \frac{A' - L'}{A - L}$$

N' is the number of shares, $\hat{R}'$ and $\hat{g}'$ are the levels of R' and g' as forecasted by internal and external experts. The levels of $K(A',L',s')$ and p' embed the final absorption of equity capital and the market reaction due to a possible adjustment of the credit rating assigned by a rating agency to the insurance firm after the project has been undertaken.

Shareholders' stimulus to go for the project is strong enough if the implied expectation for its EAIT, R'^{*}, is sufficiently high:

$$\frac{\pi \vartheta' R'^{*}}{Y' + K(A',L',s')} = \pi \vartheta' \frac{R'^{*}}{A' - L'} \geq k' - \hat{g}'.$$

The minimum expected \ EAIT implicit in the undertaken project is

$$R'^{*}_{\min} = \frac{k' - \hat{g}'}{\pi \vartheta'} A' - L'.$$

The expected implied total return,

$$\frac{\pi\vartheta' R'^{*}}{A'-L'}+\hat{g}'.$$

can be expressed as the sum of the risk-free rate r_f, and of the percentage premium r'^{*} that, by embarking on the project, shareholders <u>subjectively</u> believe to collect as a compensation for the new level of risk s':

$$\frac{\pi\vartheta' R'^{*}}{Y'+K(A',L',s')}+\hat{g}'=r_f+r'^{*}.$$

Notice that if ϑ is smaller than 1, the risk entailed by the project causes a greater absorption of equity capital and, hence, a penalty for the new expected return (`leverage effect').
The Sharpe Ratio is

$$S'^{*}=\frac{r'^{*}}{s'},$$

so that the implicit percentage risk premium can be expressed as follows:

$$r'^{*}=\underbrace{S'^{*}}_{\substack{\text{Subjective assesment of the}\\ \text{reward per unit of risk}}}\times\underbrace{s'}_{\substack{\text{Quantity}\\ \text{of risk}}}.$$

The implied Sharpe ratio S'^{*} quantifies *the implied level of risk appetite* as it is a neat measure of the risk tolerance implicit in accepting the risk entailed by the new project. Shareholders accept the new project, thus displaying confidence in receiving a total premium high enough to compensate for the new risk level s'.

3.1 A CRITERION FOR DECISION MAKING

S'^{*} can be used as a criterion for future investment opportunities. Suppose shareholders have to decide whether or not to invest in a novel project (the items related to the novel project will be marked with $''$). Shareholders are strongly egged on accepting the novel project if the estimated risk appetite S'^{*} is high enough to digest the novel quantity of risk.

$$\boxed{\begin{array}{c}\underbrace{r_f+\underbrace{\frac{\vartheta''}{\vartheta'}}_{\substack{\text{Novel adjustment}\\ \text{for absorption}}}\times\underbrace{S'^{*}}_{\text{Implied risk appetite}}\times\underbrace{s''}_{\substack{\text{Novel quantity}\\ \text{of risk}}}>\underbrace{k^{*}}_{\substack{\text{Current cost}\\ \text{of equity capital}}}.}\\ \Downarrow\\ \text{accept the new project}\end{array}}$$

3.2 EXAMPLE

We exemplify the Sharpe ratio approach by focusing on a firm monitored at three successive dates. The firm's original capital structure is observed at date 0. Shareholders decide to undertake a project at date 1 and the implied risk appetite is educed from the effects of such a decision. The implied risk appetite is used to assess a prospective project at date 2.
The original capital structure is characterized by a market value of assets equal to 100 and by a market value of equity capital equal to 50.

Capital structure at date 0

$A = 100$	$Y+K(A,L,s) = 50$ $L = 50$

Equity capital is divided into N=100 shares whose price is p=50/100=0.5. The firm's dividend policy is set by the payout ratio $\pi = 0.4$. Analysts appraise firm's growth, its equity capital's risk, and its expected EAIT at $\hat{g} = 3\%$, $s = 15\%$ and $\hat{R} = 10$ respectively. It follows that the cost of equity capital is 11%:

$$k = 11\% \Leftrightarrow \frac{100-50}{100} = p = \frac{1}{\underbrace{100}_{N}} \times \frac{\overbrace{0.4}^{\pi} \times \overbrace{10}^{\hat{R}}}{k - \underbrace{0.03}_{\hat{g}}}.$$

The equity capital that the firm is obliged to take aside for solvency issues is

$$K(100,50,0.15) = 11.665.$$

Such a computation for the absorbed capital comes from the following assumption: conditionally upon the current market values A and L, $\widetilde{A} - \widetilde{L}$ is distributed as a Gamma random variable with mean $A - L$ and standard deviation $s(A-L)$ (for more details on the Gamma distribution see the Appendix). The non-negative support of the Gamma random variable fits shareholders' limited liability.

Notice that a refined assumption for the mean would be $E\left[\widetilde{A} - \widetilde{L} - (A-L) \mid A, L\right] = (A-L) \cdot k$. However, the more involved nature of the ensuing calculations would jeopardize example's fluency.

The original situation can be abridged in the following table.

Detailed capital structure at date 0 (*k* = 11%)

$A = 100$	$Y = 38.335$ $K(A,L,s) = 11.665$ $L = 50$

At date 1, shareholders undertake a new benchmark project that has the following impact on the market-consensus levels of firm's fundamentals: $\hat{g}' = 4\%$, $\hat{R}' = 10$ $s' = 20\%$. The project improves firm's growth but makes equity more risky. The new capital for project's commencement is raised via a seasoned equity offering (30 new shares are issued) and the market value of equity capital swells up to 70. The resulting cost of equity capital is

$$k' = 8.375\% \Leftrightarrow \frac{120-50}{100+30} = p' = \frac{1}{100+30} \times \frac{0.4 \cdot \left(\frac{K(100,50,0.15)}{K(120,50,0.20)} \frac{70}{50} \right) \cdot 10}{k' - 0.04},$$

where the change in the absorption ratio is

$$\left(\frac{K(100,50,0.15)}{K(120,50,0.20)} \frac{70}{50} \right) = 0.7656.$$

The following table depicts the firm's balance-sheet outcome:

Detailed capital structure at date 1 (k' = 8.375%)

$A' = 100+20$	$Y' = 48.67$ $K(120,50,0.20) = 21.33$ $L' = 50$

If the risk free rate is $r_f = 5\%$ then the implied Sharpe ratio is

$$S'^{*} = \frac{1}{\underbrace{0.2}_{s'}} \times \left(\frac{\overbrace{0.40}^{\pi} \cdot \overbrace{0.7656}^{\text{Change in absorption ratio}} \cdot \overbrace{10}^{R'^{*}_{\min}}}{\underbrace{70}_{A'-L'}} + \underbrace{0.04}_{\hat{g}'} - \underbrace{0.05}_{r_f} \right) = 0.16874 .$$

Shareholders accept the new project, thus exhibiting confidence in receiving a premium per unit risk of at least 0.16874. This completes the calibration of shareholders' risk attitude.

At date 2, shareholders face a novel project that will push up equity capital's risk to 30%. Given the implied level of their risk tolerance, shareholders should undertake the project if its implied return on equity capital,

$$0.05 + \frac{\vartheta''}{\vartheta'} 0.16874 \times 0.30 = 9.608\%,$$

is greater than firm's current cost of equity capital. The adjustment in absorption rate is conservatively set to account for a project's zero impact on the market values of firm's securities:

$$\frac{\vartheta''}{\vartheta'} = \frac{K(100{,}50{,}0.30)}{K(120{,}50{,}0.20)} / \frac{K(100{,}50{,}0.15)}{K(120{,}50{,}0.20)} = 0.91017.$$

4. THE EXPECTED UTILITY APPROACH FOR ELICITING THE LEVEL OF RISK TOLERANCE

The second procedure for bringing forth the representative shareholder's tolerance towards risk employs the Optimality Condition for shareholder's portfolio holdings and the market value of firm's equity capital. Cochrane (2001) is one of the classic references to see how such first order conditions emerge in the context of consumption-based models.

The representative shareholder makes the rational decision of retaining full control of firm's equity capital through time. Optimality of her portfolio decisions implies that she is indifferent between (a) keeping all the firm's shares and (b) divesting ε units of them for immediate consumption:

$$u_c(c) \cdot \varepsilon (A - L) - \sum_{h=1}^{\infty} E\left[\beta^h u_c(\tilde{c}_h) \cdot \varepsilon \pi \tilde{R}_h \mid c, R\right] = 0 \quad (\textit{Optimality Condition}),$$

where we assume that her preferences are represented by time-additive expected utility, β is her intertemporal discount rate, $u(\cdot)$ is the concave utility she derives from yearly consumption, $u_x(x)$ is u's first derivative, and $c(\tilde{c}_h)$ stands for yearly current (date-h) consumption. It follows that her subjective assessment of the equity capital value must be in line with the market-consensus one:

$$\sum_{h=1}^{\infty} E\left[\beta^h \frac{u_c(\tilde{c}_h)}{u_c(c)} \cdot \pi \tilde{R}_h \mid c, R \right] = A - L,$$

where $\beta^h \dfrac{u_c(\tilde{c}_h)}{u_c(c)}$ is shareholder's intertemporal marginal rate of substitution between current consumption and date-h consumption. If $u(\cdot)$ takes the form of power utility,

$$u(x) = \frac{x^{1-\gamma} - 1}{1-\gamma} \mathrm{I}_{\{x>0\}},\ \gamma \geq 0, \qquad \lim_{\gamma \to 1} u(x) = \ln(x),$$

then the Arrow-Pratt measure of relative risk aversion is: $-x \dfrac{u_{xx}(x)}{u_x(x)} = \gamma.$

As the following figure highlights, the relative risk aversion is related to the curvature of the utility from yearly consumption.

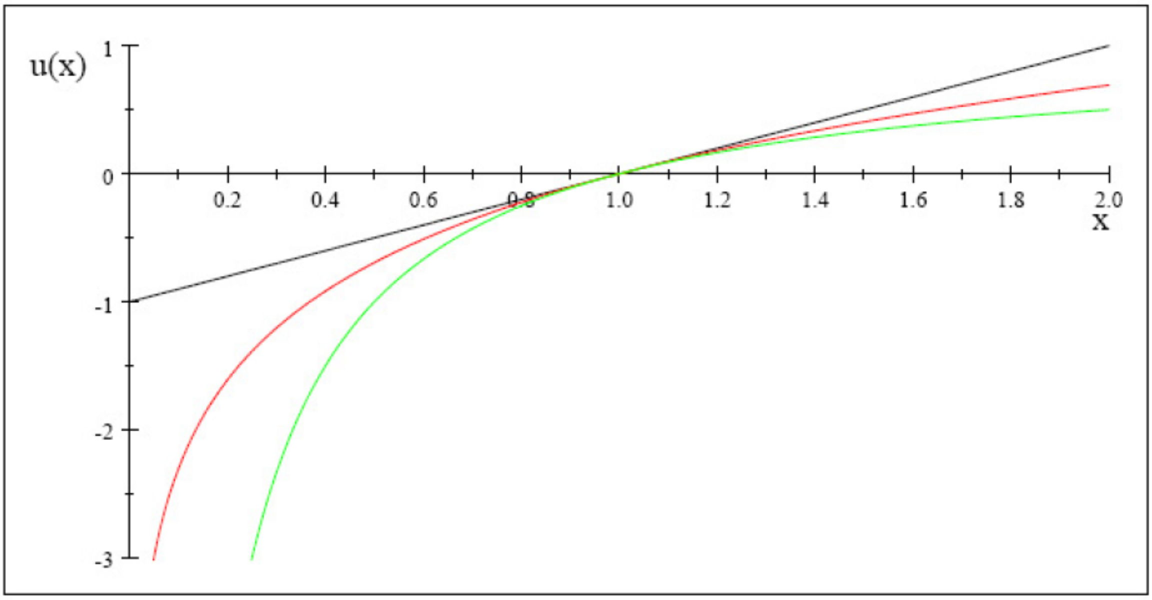

Figure 1: $\gamma = 0$ **(risk neutrality, black),** $\gamma = 1$ **(log utility, red),** $\gamma = 2$ **(green).**

Once the risk aversion γ has been estimated from past equity prices via the Optimality Condition, the merits of a new project can be assessed on the basis of the welfare improvements induced by project's impact on shareholder's consumption stream and on firm's EAIT stream.

4.1 SHAREHOLDER'S SUBJECTIVE VALUATION OF EQUITY CAPITAL IN CLOSED FORM

We assume that $\widetilde{R}_h$ and $\widetilde{c}_h$ are log-normally distributed with mean and variance as follows:

$$\mu_{\widetilde{R}_h} = R(1+g_R)^h,$$
$$\mu_{\widetilde{c}_h} = W(1+g_c)^h,$$
$$\sigma^2{}_{\widetilde{R}_h} = h\sigma_R{}^2,$$
$$\sigma^2{}_{\widetilde{c}_h} = h\sigma_c{}^2.$$

Their correlation is conveniently stated after a log transformation:

$$\rho = corr(\ln \widetilde{R}_h, \ln \widetilde{c}_h).$$

Such assumptions lead to the following structure for shareholder's subjective assessment of the equity capital value:

$$\sum_{h=1}^{\infty} \beta^h E\left[\exp\left\{-\gamma\left(\ln\tilde{c}_h - \ln c\right) + \ln\tilde{R}_h\right\}\pi \mid c, R\right] =$$

$$= \pi R \sum_{h=1}^{\infty} \beta^h \left[\frac{1+g_R}{(1+g_c)^\gamma}\right]^h \left(\frac{h\sigma_c^{\,2}}{W^2(1+g_c)^{2h}}+1\right)^{\frac{1}{2}(\gamma+\gamma^2)} \times$$

$$\times \exp\left\{-\gamma\rho\left[\ln\left(\frac{h\sigma_R^{\,2}}{R^2(1+g_R)^{2h}}+1\right)\ln\left(\frac{h\sigma_c^{\,2}}{W^2(1+g_c)^{2h}}+1\right)\right]^{\frac{1}{2}}\right\}.$$

The infinite sum can be proved to converge for $\frac{\beta(1+g_R)}{(1+g_c)^\gamma} < 1$.

Notice that it does not depend on the level of current consumption. This is not surprising since what matters for shareholder's value assessment is consumption's rate of substitution rather than consumption’s levels.

4.2 EXAMPLE

We exemplify the expected utility approach by focusing on two successive dates. The market value of firm's equity capital is observed at date 0 and employed to calibrate the risk aversion parameter γ. The implied risk appetite is used to assess a prospective project at date 1. The intertemporal discount rate is fixed at $\beta = 0.9$. The firm's payout ratio is $\pi = 40\%$.

At date 0, the outlook for shareholder's consumption stream and for firm's cash flows is as follows: $R = 10$, $g_R = 4\%, g_c = 1\%, \sigma_R^{\,2} = 10\%, \sigma_C^{\,2} = 1\%, \rho = 30\%$.

These parameter values grant infinite sum's convergence for $\gamma \geq 0$. Given such an outlook for consumption and cashflows, the following table reports the risk aversion levels implied by several market values of equity capital (centered around 50). The table visualizes the intuitive inverse relationship between risk aversion levels and market prices (*coeteris paribus*, the more conservative is the shareholder, the lower is her subjective assessment of the equity capital value).

Equity capital’s market value (*A* - *L*)	Implied relative risk aversion (γ)
48	1.3976
49	1.2387
50	1.0867
51	0.9406
52	0.7998

At date 1, a prospective project is considered. If undertaken, it would change the outlook as follows: R=10, $g_R = 5\%, g_c = 1.25\%, \sigma_R^{\,2} = 15\%, \sigma_C^{\,2} = 1.25\%, \rho = 30\%$.

Given γ = 1.0867, these parameter values imply convergence of the infinite sum. Shareholder's subjective assessment of the equity capital value with the project in place would be 55.096. Any contemporaneous market value below that figure would strongly motivate the representative shareholder to press for such a project.

5. CONCLUSIONS

We have examined two techniques whose objective is to provide support to insurance companies in calibrating shareholders' risk appetite levels and in employing the calibrated levels within the decision-making process. The first technique is based on a mean-variance performance gauge like the Sharpe Ratio, whereas the second technique takes direct advantage of expected utility theory.

APPENDIX: THE GAMMA DISTRIBUTION

The Gamma distribution is defined for $x > 0, a > 0, b > 0,$ by the integral

$$F(x; a, b) = \frac{1}{b^a \Gamma(a)} \int_0^x u^{a-1} e^{-\frac{u}{b}} du$$

where $\Gamma(t) = \int_0^\infty e^{-u} u^{t-1} du,$ is the Gamma function. The parameters a, b are called shape parameter and scale parameter, respectively. The mean of this distribution is ab and the variance is ab^2. In our context, $a = \frac{1}{s^2}, \quad b = As^2 - Ls^2$.

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