DURATION OF LIFE INSURANCE LIABILITIES AND ASSET LIABILITY MANAGEMENT

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ABSTRACT

The scope of this paper is to analyse duration as a risk measure of life insurance liabilities from traditional life insurance products using a simple model to assess the problem. First, the liabilities are defined. Then the concept of Macauley duration as a measure for interest rate risk with respect to life insurance liabilities is derived. This concept is discussed with respect to its usefulness for asset liability management of life insurance companies.

1. INTRODUCTION

The term risk and its measurement have been widely discussed with respect to both assets and liabilities. An important feature concerning assets is their interest rate risk, while the risk most commonly associated with life insurance liabilities is dependent on the quality of the contracts in force. The issue of life insurance liabilities being exposed to interest rate risk is very often associated with particular interest sensitive products only. Therefore, in many European countries where most contracts in force are traditional life insurance contracts, managers do not pay attention to the interest rate risk of their insurance liabilities. As a consequence, asset liability management is not standard throughout the life insurance industry. This may change in the future for the following reason:

The growth of the life insurance industry as a whole depends on the consumers' preferences and risk aversion. A consumer sets aside money as a provision for future needs. There are two main reasons for that: one reason might be to make a provision for his offspring in case the consumer cannot provide for it himself, the other is to provide for his old age to secure a minimum standard of living. The former need can only be satisfied by an insurance product. The latter is a

classical savings motive and this need can potentially be met by any savings product. Most classical life insurance products are designed to allow for a demand driven by both motives. The consumer buying such a product is not completely convinced that he will survive a certain period but has a sufficiently high subjective probability of surviving to allow for a savings component. The importance a consumer assigns to provision for old age depends on the level of his subjective probability of surviving. The higher the latter the closer is his decision for a traditional life product to a decision for a savings product.

All products suitable to establish a provision for old age compete with each other for the consumers' savings money(1). Therefore, most traditional life insurance products are in competition with bonds and other savings products. Although they are not complete substitutes, i.e. homogeneous goods, they may be substituted dependant on their prices, i.e. the interest rate paid on the product. The more the consumer perceives insurance products and savings products as homogeneous the more interest rate sensitive is the demand for either of the products.

There is some evidence, at least in Switzerland, that the perception of life insurance products has changed in that respect. The change has even been accelerated since every major Swiss bank has entered life insurance business, either by establishing a subsidiary or by co-operating with a life insurance company. Moreover, the financial press and consumer organisations are beginning to compare insurance products with various savings products offered by banks providing the consumer with helpful information. This clearly raises competition between the banking and the life insurance industry.

To test for the interest sensitivity of life insurance products we run a regression of the Swiss market policy surrender rate on itself lagged one period, the one period lagged premium growth rate and the Swiss bank bond rate.

The result of the analysis given in table 1 shows statistically highly significant values for all three variables (t-values in brackets). The coefficient estimate of the Swiss bank bond interest rate in particular yields a value of 0.062. This implies that a one percent point increase of the interest rate will result in a 6.2 percent point rise of the policy surrender rate. In absolute terms this amounts to nearly 1 billion Swiss francs

(1)This approach for determining the market for a particular product is due to the theory of a gap in the chain of substitution developed by J. and E.A.G. Robinson.

(figures of 1989). This indicates that the market perceives bonds and life insurance products as close substitutes.

Table 1: regression analysis

dependent variable	independent variables	coefficient
surrender rate	surrender rate lagged one period	-0.451 (3.36)
	premium growth rate lagged one period	-1.109 (2.38)
	Swiss bank bond interest rate	0.062 (6.29)

Sample range: 19501989

R2: 43.20%

D.W.: 2.08

Data source: BPV, SNB

The movement towards a more interest driven decision by potential clients has two immediate consequences in practice. First of all, great attention has to be given to the yield of the product when pricing it. Second, the risk management has to comprise the management of interest rate risk on both the asset and the liability side. This paper is concerned with the interest rate risk assessment on the liability side.

In the literature on financial theory experts suggest to use duration as a measure of the interest rate sensitivity for assets and liabilities and to match them accordingly[(2)]. The idea is appealing. The concept of duration is very intuitive and fairly easy to calculate. This makes it an outstanding candidate for a risk management tool. But duration, a dynamic measure by nature, is calculated at a particular point in time and therefore cannot be more than a "snap shot" of the company's exposure to interest rate risk. This is clearly a deficit but not a default. Particularly within asset liability management of life insurance companies it could be useful as a guide for investment decisions which have to be taken with view to the liability side. Initially, the underlying assumptions where too strong for the measure being a general risk management

[(2)]See for example Messmore, T.E. (1990), Toevs, A.L. (1985), Leibowitz, M.L. (1987), Reitano, R.R. (1990).

tool. During the last decade several successful attempts have been made to relax them and allow for the tool to be more widely used[3].

Determined to take expert advice one tries to use the duration concept for the life insurance liabilities. But little support can be expected from the literature. Duration of securities has been extensively discussed. But the topic of the calculation and interpretation of a life insurance liabilities' duration has not been widely discussed. While Choie (1992) shows the impact of the choice of the discount factor for an immunisation portfolio, the question about the economically appropriate discount factor for the calculation of the present value of the liability remains. Leibowitz (1987) addresses the problem of defining a liability return. He deliberately does not discuss the question of an economic definition of the liability. Is the present value of the liability, determined by discounting the "returns", a meaningful measure for the value of the liability? Which variable on the liability side corresponds to the cash flows from a fixed income portfolio? And most important, how is the fact to be treated that duration of insurance liabilities is bound to be the expected value of a stochastic variable? Leibowitz avoids discussing this problem by calculating a retrospective duration for the period between 1980 and 1986 and comparing it to the duration of Salomon Brothers Broad Market Index over the same period.

Far from being able to give an answer to all the questions raised above, this paper attempts to further explore duration as a risk measure of life insurance liabilities. In order to show the significance of interest rate risk, we concentrate on traditional life products which are usually classified as being not interest rate sensitive. First, we define the value of the liability. Then we calculate its sensitivity to changes in interest rates without considering the options attached to the liability. Finally we discuss the results with respect to its impact for using duration as a risk measure.

2. Life insurance liabilities

Life insurance liabilities are determined by the contract with the client. The contract specifies the premium payable to the company, the sum payable to the beneficiary in case of the insured's death and the conditions for the surrender of the policy before termination. It

[3]See for example Bierwag, G.O. et. al. (1990), Choie, K. (1992), Reitano, R.R. (1990).

also specifies the calculation of the sum payable to the insured at the termination of the policy. It is obvious that the liability, i.e. the money the insurance company owes the insured, varies with the time to termination, the probability of the death of the insured, and the probability of the surrender of the policy. The value of the liability is therefore the expected present value of the payments until termination of the contract.

The value of the liability is comparable to the value of a bond. There is an option tied to it, namely the American type put option written to the insured to surrender the policy any time at a predetermined value. So the liability resembles a bond with a put option. Even when those similarities have been considered, there remains the difference that cash flows are stochastic, which is not the case with a straight bond. Moreover, the amount payable at maturity date is generally not determined since it depends on the bonuses allocated to the insured throughout the time the contract was in force, whereas the amount payable to a bondholder is the face value determined at issuing date.

3. The model

In our model the insured primarily wants to provide for his old age signing an endowment insurance contract. While the contract is in force, say T_n periods, he pays a constant premium (P_n) consisting of an amount for savings (E_n) and a risk premium (R_n) at the beginning of each period. For the sake of convenience we assume that loadings for costs and profit are zero and that the beginning of the insurance period is the same for all contracts.

The risk premium is the amount of money paid to the pool for bearing the default risk. It is calculated such that for any point in time, $k = 1, \ldots, \max T_n$, the pool covers the difference between the money accumulated up to that point by insureds who die in the respective period and the sum the beneficiaries are to receive by contract, $S_{n,d}$.

$$[1] \qquad \sum_{n=1}^{N_k} R_n \geq \sum_{n=1}^{N_k} \delta \left(S_{n,d} - \sum_{t=1}^{k} E_n \Big((1+r)(1+b_t) \Big)^t \right)$$
$$\forall\, k = 1, \ldots, \max_n (T_n)$$

By N_k we denote the number of contracts in force at the beginning of a period $t = k$. $S_{n,d}$ are the sums payable due to death. δ is equal to 1

if the insured has died during the period considered and zero otherwise. b_t denotes the bonus rate, r denotes the mathematical interest rate.

Due to [1] we do not have to consider R_n since the long-term liabilities are solely driven by the savings process.

Claims are always paid at the end of a period. The sum payable at the end of the contract is the sum of the savings amount, the interest return and the bonuses. The bonuses are paid each period t to compensate the insured for a low mathematical interest rate. So the bonus results in an increase in the interest rate close to the market rate of interest (i_t). Since we want to consider the sensitivity of the liabilities to a change in the market interest rate, we can assume without loss in generality that the bonus rate makes up for the whole difference to it. Thus for every contract $n \in \{1, \ldots, N_k\}$

$$E_n(1+r)(1+b_t) = E_n(1+i_t) \quad \forall\, t\colon i_t \geq r\,. \tag{2}$$

If a policy is surrendered or if the insured dies, the sum to be paid additionally to the money out of the pool is the savings amount and the interest return on the basis of the market rate i_t up to then.

$$S_{n,k} = \sum_{t=1}^{k} E_n(1+i_t)^t \tag{3}$$

At each point in time the company knows the past interest rate path, whereas for the future development of the interest rate i_t it assumes that the present interest rate is preserved.

At each period the death risk of an insured d_n is the same and equal to the probability of death used for calculating the risk premium. The probability for a policy surrender is denoted by c_n. Then the liability from a contract n at the end of the period k amounts to[4]:

$$V_{n,k} = S_{n,k} + E_n \sum_{t=0}^{T_n-(k+1)} \left[\frac{(1-d_n)(1-c_n)}{(1+i_k)}\right]^t \tag{4}$$

[4]Note that this formula is a consequence of the assumption that the surrender value of a policy and the payment in case of death net of the pool payment are equal. Thus for each period:

$$d_n S_{n,k} + (1-d_n)c_n S_{n,k} + (1-d_n)(1-c_n)S_{n,k} = S_{n,k}\,.$$

For further details see appendix A.

This expression is rather intuitive. The value of the liability equals the money accumulated up to k and the expected present value of future payments from period $(k+1)$ until the termination of the contract. The formula illustrates the fact that at each point in time a life insurance company knows the amount of money it owes due to past premiums received, the interest guaranteed and bonuses distributed. It is not certain about the time of the cash out and, in due course, of the total amount of money accumulated on behalf of the insured up to that event. If the insured would either die during the next period or surrender the policy, then the first term in [4] along with the premium volume of the same period would have to be paid. This amount is the minimum liability, the base, which definitely will have to be paid.

From this formula the liability for the portfolio of policies is readily derived. The liability is the sum over the liabilities from all contracts in force at the end of the period k (which is equal to the contracts at the beginning of the period $k+1$):

$$[5] \quad V_k = \sum_{n=1}^{N_{k+1}} V_{n,k} = \sum_{n=1}^{N_{k+1}} \left(S_{n,k} + E_n \sum_{t=0}^{T_n-(k+1)} \left(\frac{(1-d_n)(1-c_n)}{(1+i_k)} \right)^t \right)$$

3.1. Duration of life insurance liabilities

3.1.1. Definition

Duration is an approximation for the interest sensitivity of a bond. The estimated change in the bond value is usually expressed in percent of the bond value. The standard Macauley duration is based on deterministic cash flows. The only random variable is the market interest rate. This cannot be assumed for life insurance liabilities. As defined above, the liability is the expected present value of future payments. The payments are random variables with a distribution function depending on the random variables "surrender of policies" and "death within the term of the contract". Thus the change of the payments due to a change in interest rates is again a random variable with the same distribution as the random variable "payments" itself.

Given this structure of the problem, an expression for the duration of the liability equivalent to the standard Macauley duration for bonds is the expected change in the present value of future payments due to a change of the interest rate in percent of the expected payments, i.e. the liability.

In a first step the duration for interest independent surrender probabilities is given. Thus cash flows are interest independent. Denote the payments by A. The realisations are the $S_{n,k}$ for $k = 1 \ldots T-k$. Denote the change in future payments by A', which is the first derivative of the liability with respect to it over the total expected present value of payments. Its realisations are the changes in $S_{n,k}$ as it changes. Then the duration of the liability is defined as:

$$[6] \qquad D_L = E\left(\frac{A'}{E(A)}\right) = \frac{E(A')}{E(A)}$$

Using [5] and noting that the cash flows are the expected premium payments, the formula for duration is as follows:

$$[7] \qquad D_L = -\left(\frac{\sum_{n=1}^{N_{k+1}} E_n \cdot \sum_{t=0}^{T_n-(k+1)} \frac{t\left((1-d_n)(1-c_n)\right)^t}{(1+i_k)^{t+1}}}{V_k}\right).$$

The duration is negative. This implies that a positive change in the market interest rate results in a decrease of the insurance liabilities.

The duration as defined above raises two questions with respect to asset liability management. First, what are the consequences of the fact that the duration is the expectation of a random variable? Second, given the expression as in [7], what are the properties of the duration for the liabilities?

3.1.2. Duration as the expected value of a random variable

Assume the target of asset liability management in a life insurance company was the immunisation of the surplus on the base of matching durations of assets (bonds) and liabilities appropriately. Assume further that this has been achieved. Then there is nevertheless a risk of mismatch, since the random variable "payments" has a variance and so has the change in payments due to a change in interest. Since E(A) is a constant, the variance of $A'/(E(A))$ is calculated as follows:

$$[8] \qquad \mathrm{Var}\left(\frac{A'}{E(A)}\right) = \frac{E\left((A')^2\right)}{\left(E(A)\right)^2} - D_L^2$$

The risk of mismatch equals [8]. This risk is not dependent on interest sensitivity but depends on the age structure of the liability

portfolio and the surrender probability. Thus it cannot be influenced by measures taken with respect to interest sensitivity.

The following example gives an idea of possible effects.

Calculation of duration and variance

N_{k+1}	i_k	k
10	6.92%	1992

age at entry & sex	term	d_n	c_n	$S_{n,k}$	E_n	T_n	$T_n - k$
17m	35	0.23%	3.00%	8'300.48	250	2007	15
25w	30	0.18%	3.00%	30'699.07	1'000	2003	11
31m	29	0.51%	3.00%	32'676.82	600	1994	2
35m	20	0.41%	3.00%	3'226.31	200	2000	8
36w	24	0.31%	3.00%	8'300.48	250	1996	4
39m	21	0.65%	3.00%	22'649.39	800	1995	3
40w	15	0.25%	3.00%	1'496.88	700	2005	13
45w	15	0.40%	3.00%	25'561.36	2'000	1997	5
50m	12	1.10%	3.00%	1'069.20	1'000	2003	11
60m	5	1.98%	3.00%	2'217.25	500	1993	1
S_k				136'197.24			

$E(A)$	171'486.90
$E((A')2)$	2'591'406'331.29
$E(A')$	(105,251.58)
$E(A'/E(A))$	(0.61)
$\mathrm{Var}(A'/E(A))$	(0.29)
Var in %	47.02%

As can be seen, D_L is smaller than one and negative. The variance of the payment changes is comparatively high with 0.29 or 47.02% of the expected value. Of course, the variance need not be that high, but it may, and thus the duration of a straight bond (D_B) is not directly comparable to the duration of life insurance liabilities. In particular, matching of D_B and D_L, even if it is done with perfection, does not do the job. The mismatch risk remains due to the probabilities of death and surrender respectively.

3.1.3. Properties of the liability duration

What are the properties of the duration measure as defined in (7)? Note that for a mature portfolio of insurance contracts the absolute value of the duration in most cases will be less than one, since the denominator includes the sum over all $S_{n,k}$, the amount of money accumulated over the time the contracts have been in force, whereas the numerator consists of the weighted present value of the premium volume only. Still, whether the duration is less than one also depends on the distribution of the Tn and its maximum value.

The absolute value of the duration for straight bonds, however, in most cases exceeds one. Whenever[5]

$$[9] \qquad (T_n - k) \leq \frac{(1+i_k)^{T_n-k} - (1+i_k)}{i} \implies D_B \geq 1\,.$$

This condition holds as $(T_n - k) \to \infty$. It does not hold for small $(T_n - k)$. The minimum term to maturity for which [9] holds is dependent on the interest rate. Now consider that the duration of a portfolio is the weighted sum of the duration of each single bond and thus usually clearly exceeds one. Since both the duration of bonds and the duration of liabilities are negative they react in the same direction towards a change in interest rates. If the interest rate rises, the liabilities decrease less than the interest rate change, whereas the value of the bond portfolio decreases in multiples of the interest rate change. What are the implications for asset liability management?

Assume that the target of a company with a fixed income portfolio is an immunisation of surplus. The bond duration necessary and sufficient is equal to[6]

$$[10] \qquad D_B = D_L\left(\frac{L}{B}\right).$$

If $D_L \leq 1$ and $D_B \geq 1$, [10] requires the duration of liabilities to exceed the ratio of bonds over liabilities as usually the quotient of liabilities over bonds is smaller than one. If $D_L \leq 1$ as is often the case, then immunisation requires the liabilities to exceed the fixed income portfolio, which is neither desirable not in the long run possible. If

[5] See appendix B.
[6] Messmore (1990), S. 20.

$D_B = 1$ and $D_L < 1$ or $D_B > 1$ and $D_L \leq 1$ immunisation of surplus is impossible. Then the duration of surplus can only be minimised by setting $D_B = D_L$. The solution $D_B = D_L = 1$ implies $L = B$ which is the trivial solution since by assumption then the surplus is zero.

3.2. The assumptions

The assumption that market rates are being granted to the insured has the consequence that the American type put option of the insured has no value since it will never be exercised. The same holds if the interest rate granted is a fixed percentage of the market rate. A rational consumer signing such a contract in the first place will accept this discount throughout the time the contract is in force. If he surrenders this is not for interest considerations. The situation changes if the interest rate granted follows the market rate with a lag. Then the option has a value which is dependent on the lag assumptions and the interest rate process assumed. If an interest rate close to the market rate at the beginning of a contract is guaranteed throughout the term of the contract the put option also has a value.

So the product considered and the payment assumptions influence the results. Single premium contracts are less interest sensitive than contracts with regular premium payments. This is due to the absence of reinvestment risk. The analysis will also differ when bonuses are not granted per year but at the end of a contract based on market conditions ruling then. This results in S_k also being interest sensitive and thus increases duration.

We consider it most important to relax the assumptions such that the surrender option can be evaluated. The surrenders are of major importance since they may be influenced by the insurance company either through the surrender conditions and/or through the services offered to the customers.

4. Consequences

The results of the paper depend crucially on interpretation and treatment of duration as approximation of interest rate sensitivity. Accordingly, the duration of liabilities turns out to be very low since interest on past premium payments is locked in at the prevailing market rate of the respective period of payment. As shown, duration matching

therefore requires a low duration of the asset portfolio. This can be achieved by short term investments such as money market instruments and bonds with short maturities.

Another method of matching asset and liability side is to change the liability portfolio by introducing products with longer durations. Means to achieve that are e.g. guaranteed investment contracts or contracts where bonuses are allocated at termination of the contracts only. However, this implies that the put option "surrender" has to be designed carefully and to be priced appropriately not to introduce additional risk to the company.

Appendix A

The liability from one contract at t = k is derived as follows[7]:

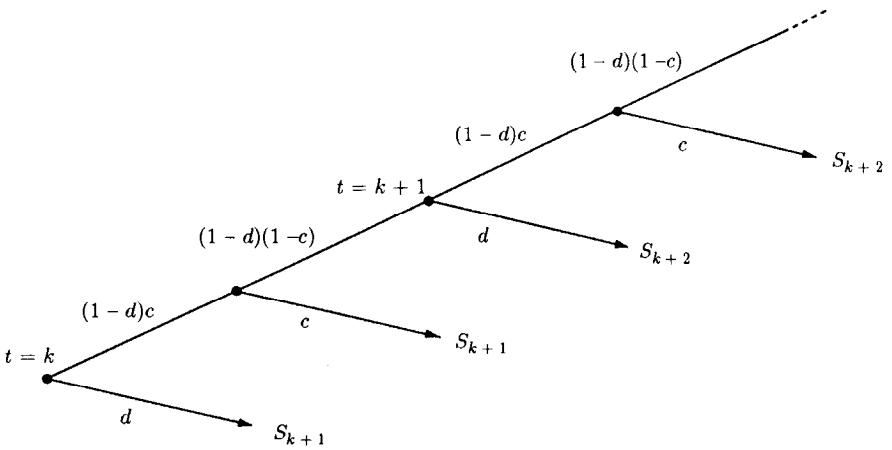

Fig. 1 The probability distribution of S_{k+t}

The probability distribution is given by picture 1. S_k is defined as in [3]. Any S_{k+t} is derived as follows:

$$[11] \qquad S_{k+t} = (S_{k+(t-1)} + E) \cdot (1 + i_k) \qquad \forall\, t = 1, \ldots, (T-k)\,.$$

[7]For the sake of convenience the index n is omitted, although one contract only is analysed throughout appendix A.

The liability is defined as the expected present value of payments. Therefore

$$V_k = \sum_{t=1}^{T-k} \frac{\left((1-d)(1-c)\right)^{t-1}\left(d+(1-d)c\right)\cdot S_{k+t}}{(1+i_k)^t} + \tag{12}$$
$$+ \left(\frac{(1-d)(1-c)}{(1+i_k)}\right)^{T-k} \cdot S_T$$

This equals:

$$V_k = \sum_{t=1}^{T-(k+1)} \frac{\left((1-d)(1-c)\right)^{t-1}\left(d+(1-d)c\right)\cdot S_{k+t}}{(1+i_k)^t} + \tag{13}$$
$$+ \frac{\left((1-d)(1-c)\right)^{T-(k+1)}}{(1+i_k)^{T-k}} \cdot S_T$$

Using [11] and working backwards this can be transformed to:

$$V_k = \frac{S_{k+1}}{(1+i_k)} + E \cdot \sum_{t=1}^{T-(k+1)} \left(\frac{(1-d)(1-c)}{(1+i_k)}\right)^t \tag{14}$$

Using again [11] this yields [4].

Appendix B

Let the term to maturity $(T_n - k)$ of a straight bond equal to n. Macauley duration for a straight bond is given by

$$D_B = \frac{\left(C\sum_{t=1}^{n} \frac{t}{(1+i)^t} + \frac{n\cdot F}{(1+i)^n}\right)}{\left(C\sum_{t=1}^{n} \frac{1}{(1+i)^t} + \frac{n\cdot F}{(1+i)^n}\right)} \tag{15}$$

where F is the face value. Set

$$\sum_{t=1}^{n} \frac{1}{(1+i)^t} = \sum_{t=1}^{n} q^t = s_n \quad \text{nad} \quad \sum_{t=1}^{n} \frac{t}{(1+i)^t} = \sum_{t=1}^{n} tq^t = r_n\,. \tag{16}$$

Calculating s_n and r_n $D_B \geq 1$ can be written as

$$C\left(\frac{(n+1)q^{n+1} - s_n}{(q-1)} - s_n\right) \geq q^n \cdot F(1-n) \tag{17}$$

which is equivalent to

$$C\left(\frac{nq^2-(n+1)q+q^{2-n}}{(q-1)^2}\right) \geq F\cdot(1-n) \tag{18}$$

F is always positive. $(1-n)$ is negative for $n>1$. Thus [18] holds independent of the exact numerical values if $n>1$ and the left hand side of [18] positive. This is the case if the numerator is positive. Using some algebra this condition can be transformed to

$$n \leq \frac{q^{n-1}-1}{q^{n-1}(q-1)} \tag{19}$$

Using the definition of q and transforming yields the desired result.

Bibliography

(1) G.O. Bierwag, C.J. Corrado, G.G. Kaufman, *Computing durations for bond portfolios*, in: The Journal of Portfolio Management, Fall 1990, 51-55.

(2) K.S. Choie, *Caveats in immunisation of pension liabilities*, in: The Journal of Portfolio Management, Winter 1992, 54-69.

(3) M.L. Leibowitz, *Liability returns: A new look at asset allocation*, in: The Journal of Portfolio Management, Winter 1987, 11-18.

(4) T.E. Messmore, *The duration of surplus*, in: The Journal of Portfolio Management, Winter 1990, 19-22.

(5) R.R. Reitano, *Non-parallel yield curve shifts and durational leverage*, in: The Journal of Portfolio Management, Summer 1990, 62-67.

(6) J. Robinson, *The Theory of Imperfect Competition*, London, 1993.

(7) E.A.G. Robinson, *Monopoly*, New York/London, 1949.

(8) Society of Actuaries, *The Valuation Actuary Handbook*, (1987).

(9) A.L. Toevs, *Interest rate risk and uncertain lives*, in: The Journal of Portfolio Management, Spring 1985, 45-56.