THEORETICAL CONSIDERATIONS OF THE EFFECT OF FEDERAL INCOME TAXES ON INVESTMENT INCOME IN PROPERTY-LIABILITY RATEMAKING

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ABSTRACT

Although the use of investment income in ratemaking for property-liability insurance has received much theoretical discussion since the mid-1970's, little attention has been paid to the role of federal income taxes in ratemaking. The pricing models of Fairley, Hill and Modigliani and Myers and Cohn all incorporated federal income taxes as an accounting item, a necessary cost of doing business. This paper establishes a theoretical foundation for the proper handling of taxes through the Myers-Cohn pricing model and the Myers Theorem. Explicit derivations of the CAPM betas for both the tax and the after-tax returns in terms of the asset portfolio beta are included. In particular, it is shown that the after-tax beta is not equal to one minus the tax rate times the investment return beta.

1. INTRODUCTION

Since the mid-1970s much theoretical discussion has occurred on the role of investment income in property-liability ratemaking. Often cited in this regard are the articles by Querin and Waters (1975), Biger and Kahane (1978), Fairley (1979), Hill (1979), and the follow-up advances by Myers and Cohn (1987), Hill and Modigliani (1987), and Kraus and Ross (1982). Surveys of the general developments of this period can be found in Cummins (1990a, 1990b and 1991) and Derrig (1990).

Interest in this subject was not restricted to academicians. Actuarial notice of the topic resulted in a full Call Paper Program on total returns for insurance companies at the 1979 meeting of the Casualty Actuarial Society. Applications of the theoretical models, primarily those derived from the Capital Asset Pricing Model (CAPM), were made for both the automobile and workers' compensation lines of property-
liability insurance in several states, most notably in Massachusetts. Derrig (1987) provides a developmental history of the important issues in Massachusetts for the period 1976-1984. Some of the empirical results of the Massachusetts experiment through 1990 can be found in Derrig (1992).

Regulators, aware of the pressures of inflation on costs and the availability of relatively high current market yields for investments in the late 70's and early 80's, began to pay close attention to the question of the inclusion of investment income in ratemaking. Williams (1983) included a survey of current methods in the New York Excess Profits Study. Explicit recognition of investment income in ratemaking also emerged in the prior approval states of Florida, Minnesota, North Carolina and Texas. Finally, the National Association of Insurance Commissioners (NAIC) undertook an extensive review of the question. This prompted a voluminous compilation of material and recommendations by an Advisory Committee chaired by Richard Haayen, President of Allstate Insurance Company (1983)(1). A final report was adopted by an NAIC committee in June of 1984.

Throughout this period little, if any, attention was paid to the crucial role that federal income taxes would play in any ratemaking model. The simple no-tax, no-risk, one-period result of an underwriting profit provision equal to minus the risk-free rate appeared in both the Querin and Waters and Biger and Kahane articles. Their result led to the surprising observation that the fair premium, or underwriting profit provision, was independent of the amount of supporting surplus(2) (Biger and Kahane, p. 124), but that actual results do seem to depend upon a solvency constraint (Querin and Waters, pp. 438-439).

The potential importance of the omission of both risk and taxes was recognized immediately in the comments by Brennan (1975) on the Querin and Waters paper. On taxes, Brennan noted at pp. 446-447:

I think that one still might find that equilibrium expected underwriting losses were less than predicted by the QW model which focuses on the similarity of insurance companies to investment companies while neglecting a significant difference,

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(1) The Advisory Committee Report comprises three volumes which can be characterized as a majority report, several minority reports, and a volume of technical appendices.

(2) The nomenclature in this area may be forever hopelessly confused by the intermingling of economic and accounting definitions. For this paper, capital, surplus, polyholders' surplus, capital and surplus, and net worth all mean the economic value of equity invested in the insurance enterprise.
namely that insurance companies must pay taxes on their investment income. So, in opting to become an insurance company rather than an investment company a significant tax burden is accepted. Equilibrium considerations would suggest then that it is only reasonable to expect some countervailing advantage in the form of the ability to borrow at preferential rates from policyholders.

Hill also observed the independence of the profit margin from surplus requirements but went on to derive a simple but necessary dependence in a Capital Asset Pricing Model (CAPM) framework with corporate taxes on investment income\(^\text{(3)}\) (Hill, pp. 180–181). Fairley’s CAPM profit margin equation with taxes essentially recognizes the same phenomenon (Fairley, p. 202, eq. 11a). Fairley begins to reveal the potentially large effects from the estimation of an appropriate tax rate in a long cryptic footnote. He estimates that there would be an increase in the required profit margin of about three percent when the model insurer was assumed to invest only in U.S. government bonds\(^\text{(4)}\), taxable at the full corporate rate of 46 percent, rather than investing in a tax–preferred diversified portfolio, taxable, when combined with underwriting income, at an average rate of about 20 percent.

Hill and Modigliani spell out more clearly the sources of investment income taxes, the potential tax shield of underwriting losses, and the magnitudes of the effects of both on the underwriting profit provision. At the same time they modified the one–tax rate Fairley profit equation to incorporate separate tax rates for investment and underwriting. This came as a result of litigation of the 1980 Massachusetts Private Passenger Automobile Rate Case in which the appropriate tax rate emerged as the principal issue\(^\text{(5)}\). Kraus and Ross developed a sophisticated insurance pricing model incorporating inflation and risk but assumed a world without taxes. It remained for Myers and Cohn to elegantly reveal the essential value of the tax considerations. Their basic equation, derived

\(^\text{(3)}\)Hill’s first pricing equation with taxes [Eq. 19] ignored the taxes on underwriting income and, therefore, in terms of underwriting losses, the potential tax shield available to investment income. Hill’s more general equation [Eq. 21] contained both tax terms but only an approximate solution was presented [Eq. 24].

\(^\text{(4)}\)This model insurer was called the “regulatory standard company” by the Massachusetts Division of Insurance on the grounds that any insurer can do at least as well in anticipated investment returns as the model company that invests only in relatively riskless government bonds. (Derrig, 1990, pp. 7–9).

from a balance sheet net present value principle, was:

\[ PV \text{ (Premium)} = PV \text{ (Losses and Expenses)} + PV \text{ (Underwriting Taxes)} + \]

\[ PV \text{ (Investment Income Taxes on Surplus)} + PV \text{ (Investment Income Taxes on Operations)} \]

The Myers-Cohn equation leads to an understanding of the precise economic reason for the aforementioned independence/dependence of the profit margin on surplus requirements; namely, that the investor expects pre-tax investment income on surplus, as in the case of an investment trust, plus an appropriate profit on the risky underwriting process. Therefore, the present value of the tax burden imposed on the investor's equity must be transferred to the policyholder though the premium charged in order to preserve the value of the investor's equity at the time of investment.\(^6\) By bearing the present value of the tax burden on investment income derived from both surplus and policyholder funds, the policyholder effectively provides a preferential lending rate to the insurer, much as Brennan expected should be the case.

Although the general multi-period framework of the theory of accounting for taxes was well established by Myers and Cohn, the theoretical and empirical support for the magnitude of the appropriate tax rate was not. The absence of a definitive treatment of taxes ultimately led to the 1983 adoption in Massachusetts for private passenger automobile insurance of a "realistic" investment tax rate of about 28 percent to be offset by an underwriting loss tax shield at the 46 percent marginal corporate tax rate. This was accomplished largely by following the Hill and Modigliani methods while ignoring their avoidance of unusable tax credits.\(^7\)

As the 1983 Massachusetts auto decision demonstrated, the choice of the effective tax rates on investment and underwriting income is a major parametric issue in the implementation of theoretical underwriting profit models. Table 1 shows a comparison between the overall profit

\(^6\)If no insurance operation were present and the investment income on the company's portfolio were taxed as a corporation rather than as a mutual fund, the value of the invested equity would be immediately decreased by the present value of the corporate tax burden. See Section 2 below.

\(^7\)The method consists of using recently observed insurer asset distributions and CAPM expected yields, with an adjustment to tax-exemple yields, together with current tax rates by asset type.
margins which arose from the proposed continued use of the marginal corporate investment tax rate of 46 percent\(^{(8)}\) and the tax rate of 28 percent adopted by the Commissioner. There is a 5.5 percent difference in premium.

### Table 1. Massachusetts Private Passenger Automobile

<table>
<thead>
<tr>
<th>Policy Year</th>
<th>Investment Tax Rate</th>
<th>Underwriting Tax Rate</th>
<th>Overall Profit Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983 (Proposed)</td>
<td>46%</td>
<td>46%</td>
<td>-2.2%</td>
</tr>
<tr>
<td>1983 (Decision)</td>
<td>28%</td>
<td>46%</td>
<td>-7.7%</td>
</tr>
</tbody>
</table>

Considering that the magnitude of the total expected operating profit, after investment income and taxes, was approximately 2 percent of premiums\(^{(9)}\), the investment tax rate choice becomes critically important.

In this paper the author reports on recent developments concerning the appropriate treatment of federal income taxes on investment income\(^{(10)}\). Myers' Theorem (1984) that the risk-adjusted present value of the investment tax burden depends only on the risk-free interest rate and the effective investment tax rate of an assumed investment portfolio, is presented in Section 2. Next, the general framework for estimating an appropriate effective investment tax rate is discussed in Section 3. Section 4 concludes the theoretical discussion.

2. **The Present Value of the Investment Tax Liability**

The general net present value model of Myers and Cohn calls for the inclusion of the present value of the investment tax liability on all

\(^{(8)}\)As in the Fairly footnote, this investment tax rate was appropriate for the so-called regulatory standard insurer that invested only in Treasury securities.  
\(^{(9)}\)A risk loading of slightly under 2% emerged from the use of an underwriting beta of -.16 combined with a market risk premium of 9% in the Massachusetts CAPM profit model.  
\(^{(10)}\)For purposes of this paper, it is assumed that the underwriting tax rate will be the marginal corporate rate. This is tantamount to assuming that any tax credits that arise from the underwriting operation can be used immediately to offset taxable income from some insurer source. The alternative is beyond the scope of this paper.
assets invested in support of the policy\(^{(11)}\).

These investments are normally made in a variety of risky bond and stock portfolios. The U.S. tax code currently\(^{(12)}\) provides for an income tax rate of approximately 34 percent on all investment income but allows various deductions based upon the source of the income. For example, some income from tax-exempt bonds is fully deductible, while 70 percent of dividend income from stocks is deductible. These deductions from taxable investment income lead to nominal tax rates between zero (all tax-exempt bonds purchased prior to August, 1986) and 34 percent (all fully taxable Treasury Bills and Bonds) depending upon the composition of the portfolio.

Historical statutory underwriting, investment and federal income tax results are shown in Table 2. While statutory accounting does not allow detailed analyses of the finer points of corporate tax accounting (the effects of consolidation, for example), the data in Table 2 and 3 reinforce the importance of the federal tax component, especially subsequent to the Tax Reform Act of 1986. That Act fundamentally changed the tax system applicable to property-liability insurers. As a result, the net tax credit of $5.5 billion in 1980-86 has been replaced by a tax liability of $17.4 billion in 1987-91 (see Figure 1). More importantly for ratemaking purposes, the explicit tax liability\(^{(13)}\) has reached about two percent of premium in 1991.

Fortunately, one key simplification for ratemaking is possible. As shown directly by Myers in 1984, the present value of the investment tax burden depends upon the tax rate but is independent of the riskiness of the portfolio. This result is implicit, if somewhat obscured, in the derivations of the tax terms in Fairley and Hill. Details of Myers' Theorem and proof are given below.

\(^{(11)}\)Myers and Cohn used per period supporting assets roughly equal to the value of total reserves plus surplus proportional to outstanding loss and expense liabilities, all at present value (MC, Appendix).

\(^{(12)}\)An extensive revision of the tax code, as it applies to property-casualty insurers, was made by the Tax Reform Act of 1986. The marginal corporate tax rate was reduced from 46 to 34 percent effective July 1, 1987. The Act also redefined the scope of taxable income for property-liability insurers in significant ways. The net effect of TRA to raise the required underwriting profit margins (Massachusetts Rating Bureau, 1987).

\(^{(13)}\)The explicit tax liability arises from taxes actually incurred; the implicit tax liability appropriate for ratemaking also includes items like the foregone yields implicit in tax-exempt investments.
Table 2. PROPERTY-CASUALTY ORGANIZATIONS INDUSTRY COMPOSITE
Statutory Underwriting, Investment and Tax Results Since 1980

<table>
<thead>
<tr>
<th>Year</th>
<th>Net Underwriting Income</th>
<th>Net Investment Income</th>
<th>Pretax Operating Income</th>
<th>Implicit Tax Rate*</th>
<th>Implicit Investment Tax Rate**</th>
<th>Net Operating Income</th>
<th>Realized Capital Gain/Loss</th>
<th>Unrealized Capital Gain/Loss</th>
<th>Other Gain/Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>-3,334</td>
<td>11,063</td>
<td>7,729</td>
<td>7.4%</td>
<td>18.5%</td>
<td>7,115</td>
<td>533</td>
<td>4,274</td>
<td>275</td>
</tr>
<tr>
<td>1981</td>
<td>-6,288</td>
<td>13,248</td>
<td>6,960</td>
<td>0.7%</td>
<td>21.7%</td>
<td>6,911</td>
<td>276</td>
<td>-2,666</td>
<td>467</td>
</tr>
<tr>
<td>1982</td>
<td>-10,290</td>
<td>14,907</td>
<td>4,617</td>
<td>-13.1%</td>
<td>26.2%</td>
<td>5,294</td>
<td>572</td>
<td>2,908</td>
<td>276</td>
</tr>
<tr>
<td>1983</td>
<td>-13,322</td>
<td>15,973</td>
<td>2,651</td>
<td>-25.1%</td>
<td>27.3%</td>
<td>3,847</td>
<td>2,110</td>
<td>1,358</td>
<td>511</td>
</tr>
<tr>
<td>1984</td>
<td>-21,268</td>
<td>17,660</td>
<td>-3,609</td>
<td>NOL</td>
<td>39.2%</td>
<td>1,942</td>
<td>3,063</td>
<td>-2,848</td>
<td>-132</td>
</tr>
<tr>
<td>1985</td>
<td>-25,288</td>
<td>19,508</td>
<td>-5,780</td>
<td>659.9%</td>
<td>38.7%</td>
<td>-3,822</td>
<td>5,483</td>
<td>5,228</td>
<td>-214</td>
</tr>
<tr>
<td>1986</td>
<td>-16,613</td>
<td>21,924</td>
<td>5,312</td>
<td>-4.9%</td>
<td>24.5%</td>
<td>5,911</td>
<td>6,874</td>
<td>2,027</td>
<td>-474</td>
</tr>
</tbody>
</table>

Tax Reform Act of 1986

<table>
<thead>
<tr>
<th>Year</th>
<th>Net Underwriting Income</th>
<th>Net Investment Income</th>
<th>Pretax Operating Income</th>
<th>Implicit Tax Rate*</th>
<th>Implicit Investment Tax Rate**</th>
<th>Net Operating Income</th>
<th>Realized Capital Gain/Loss</th>
<th>Unrealized Capital Gain/Loss</th>
<th>Other Gain/Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987</td>
<td>-10,620</td>
<td>23,960</td>
<td>13,340</td>
<td>20.1%</td>
<td>27.8%</td>
<td>9,993</td>
<td>3,335</td>
<td>-3,026</td>
<td>-87</td>
</tr>
<tr>
<td>1988</td>
<td>-11,800</td>
<td>27,723</td>
<td>13,923</td>
<td>19.7%</td>
<td>23.2%</td>
<td>12,256</td>
<td>2,725</td>
<td>2,703</td>
<td>-265</td>
</tr>
<tr>
<td>1989</td>
<td>-21,093</td>
<td>31,207</td>
<td>10,114</td>
<td>19.9%</td>
<td>28.2%</td>
<td>7,170</td>
<td>4,849</td>
<td>8,035</td>
<td>72</td>
</tr>
<tr>
<td>1990</td>
<td>-21,856</td>
<td>32,901</td>
<td>11,046</td>
<td>22.0%</td>
<td>29.3%</td>
<td>7,987</td>
<td>2,880</td>
<td>-5,116</td>
<td>-333</td>
</tr>
<tr>
<td>1991</td>
<td>-20,930</td>
<td>34,247</td>
<td>13,317</td>
<td>24.3%</td>
<td>29.5%</td>
<td>8,921</td>
<td>4,806</td>
<td>13,427</td>
<td>-476</td>
</tr>
</tbody>
</table>
80-86 | -96,403                | 114,283               | 17,880                 | -14.8%            | 29.2%                         | 23,315             | 18,911                      | 10,241                     | 708           |
87-91 | -86,299                | 150,038               | 63,740                 | 21.2%             | 28.1%                         | 46,326             | 18,396                      | 16,022                     | -1,091        |
80-91 | -182,702               | 264,321               | 81,620                 | 10.1%             | 28.6%                         | 69,641             | 37,307                      | 26,303                     | -383          |

Note: * Income Tax/[Pretax Operating Income + Realized Capital Gains].
Table 3. PROPERTY-CASUALTY ORGANIZATIONS INDUSTRY COMPOSITE
Statutory Percentages of Premiums

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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>95,702</td>
<td>-3.48%</td>
<td>11.56%</td>
<td>8.08%</td>
<td>0.64%</td>
<td>7.44%</td>
<td>0.56%</td>
<td>4.47%</td>
<td>0.29%</td>
</tr>
<tr>
<td>1981</td>
<td>99,373</td>
<td>-6.33%</td>
<td>13.33%</td>
<td>7.00%</td>
<td>0.05%</td>
<td>6.95%</td>
<td>0.28%</td>
<td>-2.68%</td>
<td>0.47%</td>
</tr>
<tr>
<td>1982</td>
<td>104,038</td>
<td>-9.89%</td>
<td>14.33%</td>
<td>4.44%</td>
<td>-0.65%</td>
<td>5.09%</td>
<td>0.55%</td>
<td>2.80%</td>
<td>0.27%</td>
</tr>
<tr>
<td>1983</td>
<td>109,247</td>
<td>-12.19%</td>
<td>14.62%</td>
<td>2.43%</td>
<td>-1.09%</td>
<td>3.52%</td>
<td>1.93%</td>
<td>1.24%</td>
<td>0.47%</td>
</tr>
<tr>
<td>1984</td>
<td>118,591</td>
<td>-17.93%</td>
<td>14.89%</td>
<td>-3.04%</td>
<td>-1.41%</td>
<td>-1.64%</td>
<td>2.58%</td>
<td>-2.40%</td>
<td>-0.11%</td>
</tr>
<tr>
<td>1985</td>
<td>144,860</td>
<td>-17.46%</td>
<td>13.47%</td>
<td>-3.99%</td>
<td>-1.35%</td>
<td>-2.64%</td>
<td>3.79%</td>
<td>3.61%</td>
<td>-0.15%</td>
</tr>
<tr>
<td>1986</td>
<td>176,993</td>
<td>-9.39%</td>
<td>12.39%</td>
<td>3.00%</td>
<td>-0.34%</td>
<td>3.34%</td>
<td>3.88%</td>
<td>1.15%</td>
<td>-0.27%</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1987</td>
<td>193,689</td>
<td>-5.48%</td>
<td>12.37%</td>
<td>6.89%</td>
<td>1.73%</td>
<td>5.16%</td>
<td>1.72%</td>
<td>-1.56%</td>
</tr>
<tr>
<td></td>
<td>1988</td>
<td>202,285</td>
<td>-5.83%</td>
<td>13.70%</td>
<td>7.87%</td>
<td>1.81%</td>
<td>6.06%</td>
<td>1.35%</td>
<td>1.34%</td>
</tr>
<tr>
<td></td>
<td>1989</td>
<td>208,834</td>
<td>-10.10%</td>
<td>14.94%</td>
<td>4.84%</td>
<td>1.41%</td>
<td>3.43%</td>
<td>2.23%</td>
<td>3.86%</td>
</tr>
<tr>
<td></td>
<td>1990</td>
<td>218,100</td>
<td>-10.02%</td>
<td>15.09%</td>
<td>5.06%</td>
<td>1.40%</td>
<td>3.66%</td>
<td>1.32%</td>
<td>-2.35%</td>
</tr>
<tr>
<td></td>
<td>1991</td>
<td>223,243</td>
<td>-9.38%</td>
<td>15.34%</td>
<td>5.97%</td>
<td>1.97%</td>
<td>4.00%</td>
<td>2.15%</td>
<td>6.01%</td>
</tr>
<tr>
<td></td>
<td>80-86</td>
<td>848,804</td>
<td>-11.36%</td>
<td>13.46%</td>
<td>2.11%</td>
<td>-0.64%</td>
<td>2.75%</td>
<td>2.23%</td>
<td>1.21%</td>
</tr>
<tr>
<td></td>
<td>87-91</td>
<td>1,046,151</td>
<td>-8.25%</td>
<td>14.34%</td>
<td>6.09%</td>
<td>1.66%</td>
<td>4.43%</td>
<td>1.76%</td>
<td>1.53%</td>
</tr>
<tr>
<td></td>
<td>80-91</td>
<td>1,894,955</td>
<td>-9.64%</td>
<td>13.95%</td>
<td>4.31%</td>
<td>0.63%</td>
<td>3.68%</td>
<td>1.97%</td>
<td>1.39%</td>
</tr>
</tbody>
</table>

Assume a corporation holds a portfolio yielding one-period investment returns and is subject to a tax liability on the realized income. Further, assume a simple CAPM market where the effective tax rate $T$ is known with certainty. Myers' theorem says that, within the CAPM framework, the risk-adjusted present value of the tax liability on investment income from a risky portfolio held by a corporation is

$$
PV(T\tilde{r}_A) = \frac{T r_f}{1 + r_f}
$$

where

- $T$ is the effective tax rate on investment income,
- $r_f$ is the risk-free rate of return
- $\tilde{r}_A$ is the rate of return on the risky portfolio, and

Following Myers (1984), suppose $\$1$ is invested at the beginning of the period in a risky asset or portfolio with a beta of $\beta_A$ and an expected rate of return consistent with the capital asset pricing model,

$$
r_A = r_f + \beta_A (r_m - r_f)
$$

The corporation must pay taxes at the rate $T$ on investment income at the end of the period. The net present value (NPV) of this investment
is:

\[ [4] \text{NPV} = (\$1 \text{ invested}) + PV(\text{pre-tax return}) - PV(\text{tax liability}) \]
\[ = -1 + PV(1 + \tilde{r}_A) - PV(T\tilde{r}_A) \]

where the tilde (\(\sim\)) indicates the actual return, which is not known when the investment is made. The NPV of the first two terms is zero, because the expected payoff \(1 + r_A\) is discounted at the same risky rate \(r_A\).

\[ [5] \]
\[ PV(1 + \tilde{r}_A) - \frac{1 + r_A}{1 + r_A} - 1. \]

This simply assumes an efficient market, in which the extra return offered by a risky asset just compensates for its risk. Therefore, combining [4] and [5], and noting that, in an efficient market, present value (PV) is a linear function

\[ NPV = -PV(T\tilde{r}_A) = -T PV(\tilde{r}_A) \]
\[ = -T PV((1 + \tilde{r}_A) - 1) \]
\[ = -T[PV(1 + \tilde{r}_A) - PV(1)] \]

but \(PV(1 + \tilde{r}_A) = 1\), as in [5], and \(PV(1) = 1/(1 + r_f)\), since the present value of a safe future dollar, the allowed deduction of the original investment from the total return, is found by discounting at the risk-free rate. Thus, it follows from [6] that

\[ [7] \]
\[ NPV = -T \left(1 - \frac{1}{1 + r_f}\right) = -\frac{Tr_f}{1 + r_f}. \]

In other words, the present value of the tax liability on the risky return \(r_A\) is calculated as if that return were risk free. The present value of the tax liability on \(r_A\) does not depend on \(\beta_A\).

An alternative proof of the Myers Theorem using the “certainty equivalent” form of the CAPM (Brealey and Myers, p. 198) is due to Richard Honomoff (1985). That form of the one-period CAPM can be stated as follows.

\[ PV(CF_1) = \frac{E(CF_1) - \lambda \text{COV} (CF_1, 1 + \tilde{r}_m)}{1 + r_f} \]
Theoretical considerations of the effect of, etc.

where

\[ E(\overline{CF}_1) = \text{expected value of cash flow in period 1} \]
\[ \text{COV} (\overline{CF}_1, 1 + \overline{r}_m) = \text{covariance between cash flow and} \]
\[ \lambda = \frac{[E(\overline{r}_m) - r_f]}{\sigma_m^2} \]
\[ \sigma_m^2 = \text{variance of return on the market}. \]

For Myers' present value of the tax on a random return \((\overline{r}_A)\), the cash flow per dollar invested is

\[ \overline{CF}_1 = T\overline{r}_A = T[r_f + \beta_A(r_m - r_f)] + \tilde{\epsilon} \]

where

\( T \) = tax rate
\( \beta_A \) = asset portfolio beta
\( \tilde{\epsilon} \) = a zero mean random variable with zero covariance with the market. Combining [9] into [8], the certainty equivalent CAPM, gives

\[ PV(T\overline{r}_A) = \frac{E[T(\overline{r}_f + \beta_A(r_m - r_f) + \tilde{\epsilon})]}{1 + r_f} + \lambda \text{COV} [T\overline{r}_f + \beta_A(r_m - r_f) + \tilde{\epsilon}, 1 + \overline{r}_m] \]

\[ = \frac{T[r_f + \beta_A(r_m - r_f)] - \lambda \beta_A \sigma_m^2}{1 + r_f} . \]

But \( \lambda = \frac{r_m - r_f}{\sigma_m^2} \), so that

\[ PV(T\overline{r}_A) = \frac{T\overline{r}_f}{1 + r_f} , \text{ as Myers showed}. \]

The above result does not mean that the tax liability is risk free; the tax is risky. The beta of the tax can be calculated and the same present value can then be obtained by discounting the tax at a risk-adjusted rate.

Let \( \beta_{\text{TAX}} \) = the beta of the tax payment. Since \( T\overline{r}_A = T((1 + \overline{r}_A) - 1) \), and since linearity of \( PV \) yields \( PV(T\overline{r}_A) = PV(T(1 + \overline{r}_A)) - PV(T \cdot 1) \),
it follows that

\[ \beta_{\text{tax}} PV(T\bar{r}_A) = \beta_A PV(T(1 + \bar{r}_A)) - \beta_1 PV(T \cdot 1) \]

so, \[ \beta_{\text{tax}} \frac{Trf}{1 + rf} = \beta_A T \], or

\[ \beta_{\text{tax}} = \beta_A \frac{1 + rf}{rf} . \]

The risk-adjusted rate for the forecasted tax \( Tr_A \) is thus

\[ r_{\text{tax}} = rf + \frac{1 + rf}{rf} \beta_A(r_m - rf) . \]

It follows that

\[ PV(T\bar{r}_A) = \frac{T\bar{r}_A}{1 + rf + \frac{1 + rf}{rf} \beta_A(r_m - rf)} \]

\[ = \frac{TrA(rf/(1 + rf))}{rf + \beta_A(r_m - rf)} = \frac{Trf}{1 + rf} . \]

Equation [12] shows that the tax liability is indeed risky: in fact, unless \( \beta_A = 0 \), \( \beta_{\text{tax}} > \beta_A \), making the tax liability riskier than the pre-tax return. This is due to the fact that the tax essentially acts like a levered investment. On the other hand, the after-tax return is less risky than the pre-tax return. It is worth noting the following corollary to Myers’ Theorem.

If a corporation invests in a portfolio then the present value of its after-tax return on that portfolio is given by:

\[ PV(1 + (1 - T)\bar{r}_A) = \frac{1 + (1 - T)rf}{1 + rf} . \]

Since the total return expected on the investment is given by \( 1 + r_A \), the inclusion of taxes on the return \( r_A \) decomposes \( 1 + r_A \) into

\[ 1 + r_A = 1 + (1 - T)r_A + Tr_A . \]

Taking present values and applying the theorem gives

\[ PV(1 + \bar{r}_A) = PV(1 + (1 - T)\bar{r}_A) + PV(T\bar{r}_A) , \quad \text{or} \]

\[ PV(1 + (1 - T)\bar{r}_A) = 1 - \frac{Trf}{1 + rf} = \frac{1 + (1 - T)rf}{1 + rf} . \]
Just as in the case of the tax liability, the present value of the after-tax risky return does not depend on the beta. Likewise note that the less risky beta of the after-tax return is given by

\[
\beta_{\text{after-tax}} = \frac{(1 - T)(1 + r_f)}{1 + (1 - T)r_f} \beta_A \leq \beta_A
\]

with equality only if \( \beta_A = 0 \) or \( T = 0 \).

It must be remembered that the after-tax return is also on a levered investment of \( 1 - Tr_f/1 + rf \) but that leverage still produces returns that are generally less risky than the return on the portfolio as a whole. At this point it is also interesting to note that, precisely because of the leveraged investment, [17] implies

\[
\beta_{\text{after-tax}} \neq (1 - T)\beta_{\text{pre-tax}}
\]

with the exception of the trivial cases of \( T = 0 \) or \( \beta_A = 0 \).

It may also be useful to see why this result works in the Myers–Cohn present value model. For that purpose, following Myers, consider the one-period insurance balance sheet at the time a policy is issued. Using the notation of Myers–Cohn, the balance sheet would show the format in Table 4.

### Table 4.

**Balance Sheet (Market Values)**

<table>
<thead>
<tr>
<th>Asset value (= premium plus equity invested)</th>
<th>Present value of expected losses and loss expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Value</td>
<td>Total Value</td>
</tr>
<tr>
<td><strong>Balance Sheet Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>( P )</td>
<td>( L/(1 + r_L) )</td>
</tr>
<tr>
<td>( S )</td>
<td>( T(P/(1 + r_f) - L/(1 + r_L)) )</td>
</tr>
<tr>
<td></td>
<td>( T PV ) (Taxable Investment Income)</td>
</tr>
<tr>
<td></td>
<td>( S )</td>
</tr>
<tr>
<td>Total Value</td>
<td>Total Value</td>
</tr>
</tbody>
</table>
The equity invested is equated on the balance sheet with the supporting surplus S. The after-tax present value of investing surplus and premium is

\[
PV = [(S + P)(1 + \hat{r}_A) - T(S + P)((1 + \hat{r}_A) - 1)] = \\
= PV[(1 - T)(S + P)(1 + \hat{r}_A) + T(S + P)] = \\
= (1 - T)(S + P) + T(S + P)/(1 + r_f) = \\
= (S + P) - Tr_f(S + P)/(1 + r_f).
\]

Thus, the present value of the tax on the investment income from any portfolio choice for the investment balance \((S + P)\) is

\[
P(V \text{ (Investment Tax)}) = (S + P)Tr_f/(1 + r_f).
\]

Once the Myers theorem is known, the present value of the investment tax term can be evaluated using an estimate of the risk-free rate and an effective tax rate for an appropriate investment portfolio. We next turn to the establishment of a framework in which to consider the question of an effective tax rate.

3. A FRAMEWORK FOR THE APPROPRIATE EFFECTIVE TAX RATE

The effective tax rate for the investment income on an asset is, by definition, one minus the ratio of the after-tax yield on the asset to the yield on a fully taxable asset of comparable risk and maturity. For example, for tax-exempt bonds,

\[
\text{Effective Tax Rate} = 1 - (r_{EXEMPT}/r_{CBOND})
\]

where CBOND is a fully taxable (corporate) bond that has the same risk and maturity as the exempt bond\(^{(14)}\).

Another example would be a preferred stock. In that case,

\[
\text{Effective Tax Rate} = (1 - .898r_{PREFSTK}/r_{CBOND})
\]

\(^{(14)}\)Since tax-exempt yields respond directly to real or perceived changes in the tax codes while taxables may not respond at all, there may be no such comparable securities.
Theoretical considerations of the effect of etc.

where CBOND is a fully taxable corporate bond with the same risk and maturity characteristics as PREFSTK, the preferred stock\(^{(15)}\).

The Myers theorem can be interpreted as follows. The present value of the tax liability on investment income from a risky portfolio held by a corporation is independent of the risk level of the portfolio and depends only on the effective portfolio tax rate and the risk-free rate of interest. The insurance ratemaking question ultimately comes down to two choices: which portfolio of assets should be used and how can the effective tax rate of that portfolio be calculated.

Under the Myers–Cohn model, equation (1) above, rates are reduced by an estimate of the anticipated after-tax income that may be earned from the investment of premiums from the time premiums are received until the ultimate payment of losses or expenses. The net present value model assumes that the sum of this anticipated investment income plus premiums will be sufficient to pay all claims and expenses and provide reasonable profits to the insurers. A postulate of this procedure is that the policyholder purchases insurance to eliminate underwriting risk, not to assume investment risk.

One technique to eliminate investment risk is to use the yields on a portfolio of Treasury securities, whose maturities are matched to the expected loss payment patterns. This was the investment portfolio of the regulatory standard company introduced in 1976 by Stone (Derrig (1990), pp. 7-9). The use of Treasury securities is assumed to eliminate call and default risk, while the matching of maturities\(^{(16)}\) is assumed to eliminate holding period or interest rate risk\(^{(17)}\). If this regulatory standard company portfolio is assumed for modeling purposes, then the effective investment tax rate calculation is simple. All income from

\(^{(15)}\)Given the nature of preferred stocks, a truly comparable corporate bond may be in reality impossible to find. These simple examples assume no capital gains or pro-rata. The tax rate on preferred stocks is assumed to be 34 percent after the exclusion of 70 percent of the dividend and is thus .102.

\(^{(16)}\)Maturity matching was a first approximation to the more correct duration matching.

\(^{(17)}\)In this context “holding period risk” refers to the risks associated with holding long maturity bonds to meet short maturity liabilities. One aspect of such risk is the risk of price fluctuations in such bonds as interest rates change. This is sometimes called price risk. Another aspect is the risk of not being able to reinvest the interest on the bonds at the same rate as the original bond. This is sometimes called reinvestment risk. The two types of risk are characteristic of fixed income investments, and together they are sometimes called interest rate risk. See Reilly and Sidhy (26, p. 64) for a clear summary.
Treasury securities is fully taxable at the 46 percent marginal corporate rate.

Another technique is to assume some portfolio of risky assets. The Myers theorem allows present value calculations to use the risk-free equivalent yield on that portfolio, but the theorem will not provide a usable tax rate. In a world without taxes, most sensible asset pricing theories will demand that the present value of any investment adjusted for risk will be the same. This is the principle used in the proof of the Myers theorem.

In a world with taxes, there is a question as to whether a true tax advantage exists, in a mixed portfolio, when all differences in risk are properly accounted for. Stated differently, the question, as Myers posed it in 1981, is whether some other portfolio with lower tax rates is actually superior in all relevant aspects to the regulatory standard portfolio, so that it confers additional value, before the fact, on a company holding such a portfolio. This question reduces to whether we can construct, in theory, a portfolio with tax preferences that produce an effective tax rate lower than 34 percent, which also eliminates all investment risk, including risk arising from the tax code itself. If no such portfolio exists, the 34 percent marginal corporate rate must be used as the effective tax rate. If some portfolio with higher after-tax, after-risk adjustment yields can be found, then a tax rate lower than 34 percent can be used. The Tax Reform Act of 1986 plays a pivotal role in the evaluation of tax preferences. According to Walker (1991, p. 78–82), although the explicit tax rate on U.S. property-liability insurers rose from zero in pre-Reform years to about 20% in 1987–88, the effective tax rate adjusted for tax preferences in tax-exempt bonds, preferred and common stocks rose from an average of about 29% pre-Reform to 34.5% post-Reform. Walker’s results point toward an effective investment tax rate for ratemaking (at least post-Reform) of the

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(18) Rational investors seek after-tax yields. They therefore seek tax-advantaged securities. Investor demand should drive up the prices of such securities and thus decrease their yields until the yields are in equilibrium with the yields on equivalently risky securities with high statutory tax rates. The existence of true tax advantages implies that investors are not rational, or that the market is not efficient, or that it is segmented in a way that permits tax advantages to persist.

(19) Walker’s results for 1987 and 1988 average 35.0% and 34.3%, respectively.

(20) Adjusting only for implicit taxes on tax-exempt bonds in 1988, Walker finds an effective tax rate of 33.7%. No data are presented beyond 1988.

(21) Of course, the underwriting tax rate is 34%, which leads to an overall effective tax rate of 34% as well.
marginal rate of 34%. We look to the capital markets, especially the tax-exempt bond market, for a portfolio which might, after adjusting for all risk, produce higher after-tax yields and thus have a true tax advantage over a portfolio of Treasury securities. If such an advantage does indeed exist, the additional question of the amount of the advantage which should be passed onto policyholders in the fair premium must also be confronted. This empirical question will be dealt with in a subsequent paper.

4. SUMMARY AND CONCLUSION

The previous sections have reported on the recent developments which affect the estimate of the federal investment income tax burden appropriate for inclusion in the calculation of an economically derived fair premium. The major development is the Myers Theorem; i.e., the fact that the present value of that tax burden is independent of the riskiness of the portfolio but depends upon the effective tax rate of the portfolio. The second development is the application of the Myers Theorem to calculate the betas of both the tax and the after-tax returns. In particular, it has been shown that the beta of the after-tax return is not one minus the beta of the pre-tax return.

Using the theoretical foundation of the Myers Theorem and its corollaries, the combined theoretical–empirical question of the appropriate effective tax rate is discussed in Part Two of this paper. Two important aspects of that discussion are the concept of risk, as it applies to tax-favored investments, and immunization or duration as it applies to the policyholder flows. Both of these concepts require an examination of the U.S. market for tax-exempt bonds as the primary prototype of a tax-favored capital market.

A final observation is in order. It is one thing to seek refinements of financial models which have significant practical effects. It is quite another thing, however, to begin to believe that simplified financial models can fully reflect the markets. Derrig (1987, 1992)) has shown that it is more important to estimate accurately the magnitudes of the loss and expense flows in calculating the underwriting profit provisions than it is to be absolutely precise in calculating parametric inputs to the profit models. Nevertheless, as we have shown here, the choice of the federal investment income tax rate within seemingly plausible ranges can, by itself, determine whether the insurer can expect to make a profit or a loss by issuing a policy.
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