

Managers do not Lean Back: A Profit Testing Model that Evaluates Scenario Dependent Management Decisions

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Abstract

It is argued that actuaries should be aware of the needs of the management of insurance companies. We, actuaries, must understand that managers do not always consider our beautiful actuarial models useful tools. Managers do not lean back, they react to the changing world. Models should therefore incorporate management decisions which are dependent on scenario and moment in time.

A description of a stochastic profit testing model which allows for dynamic management decisions is given in this paper. The working of the model is illustrated for the management decision with respect to the rate of dividend paid out. The paper concludes with other examples and some topics for further investigation.

Résumé

Il est avancé que les actuaires doivent se réaliser quels sont les exigences du management des compagnies d'assurance. Nous, les actuaires, doivent nous réaliser que les gérants ne considèrent pas toujours que nos beaux modèles actuariels seraient des instruments utiles. Les chefs d'entreprises ne sont pas passifs mais réagissent à un environnement toujours changeant. Les modèles doivent donc contenir les décisions de la gérance qui dépendent d'un scénario et du temps.

Dans cette conférence, on donne un modèle d'essai des profits sur les principes stochastiques, qui prends de telles décisions du management en considération. Le mécanisme du modèle est illustré pour les décisions de la gérance visant au pourcentage de dividende à payer. La conférence se termine avec d'autres exemples et quelques points très intéressants pour une investigation plus approfondie.

Keywords

Profit testing, dynamic decision rules, ruin.

1 Introduction

- 1.1 Most, if not all, profit testing models I saw disregard reactions of the management of insurance companies to a changing world. This does not add to the popularity of an actuary. Again and again actuaries should give themselves notice of the requirements of the management.
- 1.2 Three examples out of many why static models are not satisfactory to managers:
- ▲ If interest rates decline, investment portfolios are restructured.
 - ▲ In case of a stop/loss reinsurance a non stochastic model does not take into consideration any pay-in from the reinsurance contract because mortality experience is as expected. (The same is true for any type of non linear reinsurance cover). Now it seems if reinsurance only costs money.
 - ▲ Dividend payments to shareholders depend on the height of the profits. The higher the profits, the higher the dividend percentage.
- 1.3 Managers do not lean back to wait and see. Managers manage. A management tool, like a profit testing model, should allow management and therefor management decisions should be incorporated. When these decisions are incorporated, it models the world much more realistic and creates acceptance by managers.
- 1.4 The managers I spoke were quite unhappy with traditional deterministic profit testing and asset liability models because they do not model the risky world in a realistic way. The same managers are already more happy with a stochastic model, because it models the world more realistic. However, they

are still under the impression that such models are of only limited use because throughout the period, depending on the actuarial and economical surroundings, they react to changing circumstances. Having seen these models managers already have doubts when you want some of their time to explain another (stochastic) model, which incorporates their behaviour. I must admit that I understand their doubts. However, the law of the growing insight teaches us that improvement is always possible. Today's techniques allow management decisions which are dependent on scenario and time. This not only allows for management decisions, but also makes it possible to evaluate the decisions and to find 'optimal' solutions for the structuring of investment portfolios. Optimal between inverted commas because I know that a model always provides us with only a flavour of what is really happening in the world. Nevertheless when the flavour is strong enough, it may help to improve decisions.

- 1.5 In this paper I will briefly present a stochastic model which takes management decisions dependent on scenario and time in consideration. I have tried to give a description of the model with a limited use of mathematics. For those readers who are interested, some of the mathematics used in the model can be found in Appendix I. Possible implementations of decision rules are also given and discussed briefly. Three examples of the implementation of a decision rule in the model are worked out in more detail. The paper concludes with topics for further investigation.

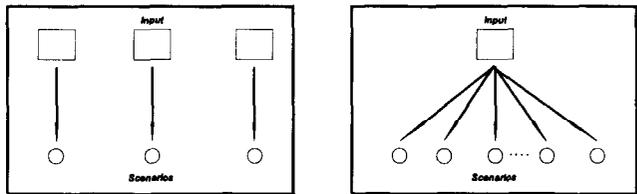
2 A stochastic profit testing model

2.1 Scenario analysis

2.1.1 An actuary has to cope with the unpredictability of the future. Many futures are possible. Every possible future can be regarded as a scenario.

2.1.2 Definition: *A scenario is a description of possible developments of the future consistent with a set of assumptions².*

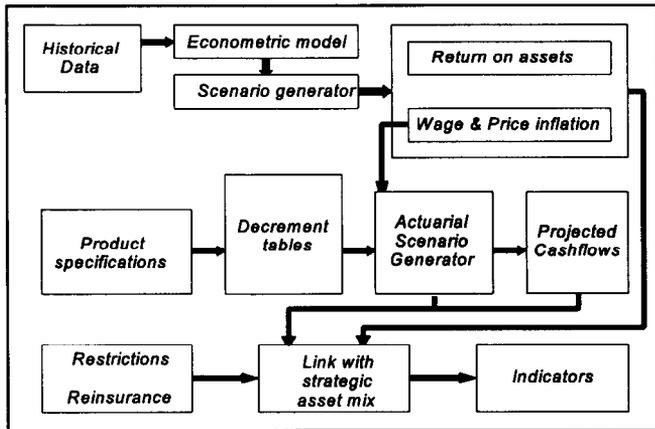
2.1.3 The following chart is a visualisation of the definition.



2.1.4 In the deterministic approach a scenario is the input for the model. In the stochastic approach a two stage process can be identified. The first stage is the generation of a predetermined number of scenarios (eg. 100 or 1,000) based on a set of assumptions. The second stage models the future in case of each of the generated scenarios.

2.2 Model overview

2.2.1 The following chart gives an overview of the functionalities of our stochastic profit testing model.



2.2.2 The top layer in the chart is the econometric module. Based on historic data, relations between economic factors are investigated and projections (= economic scenarios) are generated by means of simulation. Vector Autoregressive Models are used to model the time series. The interdependencies of the time series are investigated by means of co-integration which results in adopting a Vector Error Correction model³.

2.2.3 The practical result of using a Vector Error Correction model is that short term fluctuations are modelled whilst the long term relationship, which exists between some of the economic variables, is not broken. The co-integration of variables results

in an adaptation process which precludes that differences from the long term relationship become too large.

- 2.2.4 The second layer in the chart is the actuarial module⁴. Each source for decrement is modelled as a Binomial (N, q) distribution. The combination of multiple decrement sources result in a Multinomial $(N, q_1, q_2, \dots, q_k)$ distribution⁵.
- 2.2.5 Actuarial scenarios are simulated by drawing from an Uniform $[0, 1]$ distribution and comparing the outcome with the multiple decrement probability. If the outcome is greater, the status of the policy will be unchanged. If the outcome is smaller the model determines, using the size of the probabilities and the order of decrement, what the new status of the policy will be and makes calculations of the resulting financial effects.
- 2.2.6 Clearly the economic module must have an impact on the actuarial module. The costs involved in the liabilities are usually linked with inflation. In case of profit sharing based on the current yield on bonds, this yield is necessary as input in order to be able to make the necessary calculations related to the contractual profit sharing.
- 2.2.7 The bottom layer gives an impression of the links between the modules and the resulting indicators. In this part of the model management decision rules can be implemented.

2.3 Indicators

2.3.1 Financial decisions are made by carefully weighing up the risk against return. Everybody is free to define his own risk/return indicators. In our standard approach we use the net present value as the return indicator. For a discussion on alternative return indicators, I refer to the, in my view, good paper written by MOURIK, T.J. (1995).

2.3.2 There are several possibilities to measure risk. Examples are:

- ▲ standard deviation;
- ▲ funding ratio;
- ▲ probability of ruin.

Everyone should consider the various possibilities carefully and select the risk indicator he feels the most comfortable with. In practice all risk indicators have advantages and disadvantages and frequently more than one risk indicator is evaluated.

2.3.3 In our standard approach we state that there are two important aspects of risk: the probability of having a funding ratio below one and the size of the underfunding. Both aspects can be integrated in the risk measure of ruin when ruin is defined as:

Ruin is the situation where in a scenario a funding ratio below one occurs for x years in succession and when in one of those years the funding ratio is below a certain critical funding ratio.

2.3.4 In practice we often use the rule that when speaking of ruin the funding ratio is below one for three years in succession and in

one of those years the funding ratio is even below 0.8. Of course is it possible and advisable to 'play around' with the number of years and the critical funding ratio⁶.

2.4 Management Decision Rules

- 2.4.1 I have said it before. The world changes continuously and the manager does not lean back. In particular managers react to changes in the environment. The reaction differs depending on the type, size and moment of change. As in every scenario 'the past' is known, the model is able to analyse the data of 'the past'. The past is placed between inverted commas because we are talking about a moment in the future when the model looks back. So however, we are still talking about future events.
- 2.4.2 Now it is possible to instruct the model to react to certain data patterns like the manager would do. This means that we have to interview the manager and try to find out how he would react under certain circumstances.
- 2.4.3 In paragraph 1.2 I have already given three examples of imaginable management reactions. In this paragraph I will give for each example one possibility to implement a management decision rule in the model just for illustration. I leave it to the interested reader to change the rules to his own taste and to find many more examples.

- 1st Example : If interest rates decline, investment portfolio's are restructured.
- Possible rule : If the interest rate in year $t-1$ is higher than the interest rate in year t and the interest rate in year $t-1$ is higher than the average long term interest rate plus 1.5% and the decline is 0.5% or more, short term bonds should be sold and long term bonds for a maximum of 5% of the total portfolio value⁷ should be bought.
- 2nd Example : In case of stop-loss reinsurance a non stochastic model does not take into consideration any pay-in from the reinsurance contract because mortality experience is as expected. (The same is true for any other type of non linear reinsurance cover). Now it seems if reinsurance only costs money.
- Possible rule : If the sum of the capitals at risk with respect to insured lives which died in year t exceeds the retention limit, a cash inflow (from the reinsurance contract) is generated for the exceeding amount.
- 3rd Example : Dividend payments to shareholders depend on the height of the profits. The higher the profits, the higher the dividend percentage will be.

Possible rule : The dividend pay-out rate is the average dividend pay-out rate, but increases when the funding ratio exceeds the normative funding ratio.

This 3rd Example is worked out further in the next chapter.

3 Example of dynamic dividend pay-out rate

- 3.1 In most deterministic models I saw all cashflows occurring in a year are discounted back to time $t=0$ and added up in order to get the Net Present Value. To put it another way all profits and losses are taken at once. The probability of ruin, that cannot be derived from a deterministic model, must be very high if not one (depending on the definition of ruin). This because no surplus is left which can function as a cushion. In any year when the experience is worse than expected, the funding ratio drops to a definition below one. At the end of the year profits as well as losses are taken, which means that the funding ratio is yearly set to exactly one.
- 3.2 Every manager knows it is wise to leave part of the profits in the company as long as the policy exists. This creates a cushion which can be used in minor years. Some deterministic models offer the feature of taking only part of the profit as dividend. This is more realistic and results in a lower Net Present Value, because profits are taken in a later stage. These models do not show what one gets in return by doing so. Stochastic models show us that the probability of ruin comes down rapidly when the profit taking is postponed. This gives information to managers: 'What return do I have to give up in order to reduce risk?' Or to put it another way: 'What risk do I have to take in order to obtain my desired return?'
- 3.3 In order to get a flavour of the information our stochastic model generates a table with output for a specific situation, which is described in more detail in Appendix II, is set out below.

Dividend pay-out	Net Present Value (in DFL 1,000)	Probability of ruin
40%	[2,865 ; 3,826 ; 4,787]	0.01
60%	[3,164 ; 4,215 ; 5,266]	0.18
80%	[3,406 ; 4,490 ; 5,574]	0.58
100%	[3,501 ; 4,610 ; 5,719]	0.71

3.4 From the tabel it can be seen that information is provided on return and risk. The figures in the column Net Present Value represent a 95%-confidence interval. The probability of ruin is based upon occurring funding ratios below one for three years in succession and in one of those years the funding ratio is even below 0.8. It is clear that the probability of ruin declines rapidly when the taken profit rate comes down to a more or less generally accepted level of 40%. The reason that the probability of ruin, in case of a 100% dividend pay-out, is still below one can be explained from the adopted definition of ruin. In every scenario funding ratios below one do occur, but not always for three years in succession.

3.5 In the example of the previous paragraph the level of dividend pay-out through time is still static. The level is fixed to the percentage, as shown in the table, for each year. JONG, C.A. DE (1995) gives us an example of a more dynamic approach by the following management rule:

$$dpo_t = bdpo + \gamma \cdot (fr_t - bfr)$$

in which:

dpo_t = dividend pay-out at time t;

$bdpo$ = basic dividend pay-out level;

γ = adjustment parameter ($0 \leq \gamma \leq 1$);

fr_t = funding ratio at time t;

bfr = basic funding ratio (minimum accepted level)

3.6 When using these techniques I advise to play with the parameters in order to get a feeling of their sensitivity. On the other hand do not play too much, it might happen that because of the amount of pages produced, you are not longer able to decide which results are acceptable and reflect your own or your company's feelings about risk and return the most.

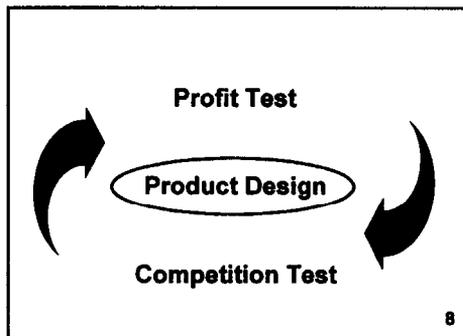
4 Topics for further investigation

- 4.1 Economic models always cause fights (luckily most of the time only verbal combats) between the developers. Our VECTOR ERROR CORRECTION model developed by my colleague C.A. de Jong is on theoretical grounds superior to many other approaches. However, it would be nice if the empirical proof could be given. I understand that my colleague is thinking of setting up a comparison between his model and the popular Wilkie-model.
- 4.2 The table in paragraph 3.3 shows a reduction of the probability of ruin in case a lower dividend pay-out ratio is adopted. At the same time the net present value comes down. This is because the discount rate remains unchanged. In case of a reduced risk the providers of capital will accept a lower risk premium. As far as I know this is a topic which has not been studied in great detail yet but is of great importance. I agree with MOURIK, T.J. (1995), page 112 where he states that the embedded value should be unchanged in case valuation systems within a company change. This implies that only real cash flows should be discounted.
- 4.3 Recently I compared the results of three different profit testing models using exactly the same input. The results were totally different. Luckily, sensitivity analysis showed that the impact of changes in the input led up to changes in the result in the same direction and with about the same magnitude. I think that it would be good when the actuarial profession would investigate the different approaches and give suggestions or guidelines. So many details can be incorporated in a different way. In my view it would be good to describe a kind of generally accepted profit testing model. Since it is not acceptable that the results of different models on the

basis of the same input give rise to different judgements of the product under consideration.

5 Concluding remarks

- 5.1 Stochastic models are to be preferred above deterministic models. Every management decision consists of the weighing of return against risk. Risk can only be modelled using a stochastic model.
- 5.2 The stochastics of both liabilities and assets should be considered. This means that not only a stochastic model is required to generate economic scenarios, but also to generate scenarios with respect to the portfolio of insured lives.
- 5.3 Deterministic models are still very useful. In my own practice I have



adopted a two-stage rocket approach in the development of a new life insurance product. The first stage is to get insight in the profitability and the competitiveness of the product.⁸

In this stage a deterministic model is very useful because it is relatively simple and very fast. The second stage is the risk analysis. When there is a good idea on how the product should look, detailed insight in the risk profile is needed. It is clear that I myself can no longer imagine to approach this problem without a dynamic stochastic model which incorporates management decisions rules.

5.4 With the present computer power available a full stochastic evaluation only takes a few minutes. This in order to make the necessary calculations. The interpretation of the results will take longer. However, this is where the most interesting part of the job starts and where the actuary can show his skills.

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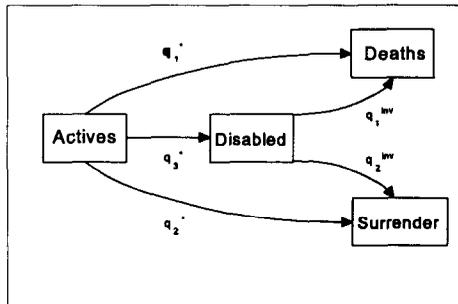
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Notes

1. Acknowledgement: I am grateful to all the colleagues who have helped me in writing this paper. Especially I want to thank Ronald Doornbos for his comments. However, any errors or omissions remain entirely my own responsibility.
2. BUNN, D.W. / SALO, A.A. (1993)
3. JONG, C.A. DE (1995)
4. DOORNBOS, R. (1995)
5. Usually the independent probabilities q should be derived from the known dependent probabilities q^* . The following chart illustrates these relationships in case of three decrement sources.



6. The standard definition of ruin in text books on general insurance (e.g. GOOVAERTS et al. (1990), page 57) is the occurrence of a funding ratio below one for the first time. Our definition has more flexibility built in.
7. This rule can mathematically be described by means of indicator functions:

$$\Delta Astb_t = - I_{\{i_{t-1} - i_t > 0.005\}} \cdot I_{\{i_{t-1} > \bar{i} + 0.015\}} \cdot \text{MIN}\{ 0.05 \cdot A_t, Astb_t \}$$

in which:

$$Astb_t = \text{Amount of Assets invested in Short Term Bonds at time } t;$$

$$A_t = \text{Amount of total Assets at time } t.$$

8. For an evaluation on how your new product is likely to score relative to your competitors a multicriteria model can be very useful. I refer to VALKENBURG (1995) for a further discussion on multicriteria models in the insurance industry

Appendix I

The presented model is built to evaluate the profitability of an insurance product in its isolation. In the presented version no allowance is made for the fact that shareholders have to pay for losses. In line with this approach attention is only paid to profit before tax. The model as presented has and is still being evolved. Retirements have been added, as well as tax as shareholders items.

The profit testing model is extensively described in DOORNBOS, R. (1995) and JONG, C.A. DE (1995). The mathematics of the model described here is from the doctoral essay of C.A. de Jong, who developed this part of the model at our office. For more background, for example in relation to the the mathematics of the Vector Error Correction model used I refer to the two mentioned essays.

$$\max \frac{1}{S} \sum_{s=1}^S \left[\sum_{t=1}^T W_{ts} + (B_{Ts} - V_{Ts}) \right] - C_1 \quad (\text{I.1})$$

$$\text{s.f. } \hat{r}_{ts} = \sum_{i=1}^n x_{it} r_{its} \quad t=1, \dots, T; s=1, \dots, S \quad (\text{I.2})$$

$$\tilde{B}_{ts} = (1 + \hat{r}_{ts})(B_{t-1s} + P_{ts}) \quad t=1, \dots, T; s=1, \dots, S \quad (\text{I.3})$$

$$w_{ts}^i = (B_{t-1s} + P_{ts}) \hat{r}_{ts} - [(V_{t-1s} + P_{ts}) c_r - U_{ts}] \quad t=1, \dots, T; s=1, \dots, S \quad (\text{I.4})$$

$$\tilde{w}_{ts} = k \left[\max(0, w_{ts}^i) + \max(0, w_{ts}^d) + w_{ts}^a \right] - \min(0, w_{ts}^k) \quad t=1, \dots, T; s=1, \dots, S \quad (\text{I.5})$$

$$W_{ts} = k \left[w_{ts}^i + w_{ts}^d + w_{ts}^a \right] + (1 - k) \left[w_{ts}^i I_{\{w_{ts}^i < 0\}} + w_{ts}^d I_{\{w_{ts}^d < 0\}} \right] + w_{ts}^k \quad t=1, \dots, T; s=1, \dots, S \quad (\text{I.6})$$

$$B_{ts} = \tilde{B}_{ts} - U_{ts} - \tilde{w}_{ts} \quad t=1, \dots, T; s=1, \dots, S \quad (\text{I.7})$$

$$d_{ts} = \frac{\tilde{B}_{ts} - U_{ts}}{V_{ts}} \quad t=1, \dots, T; s=1, \dots, S \quad (\text{I.8})$$

$$D_{ts} = I_{\{d_{ts} < c_d \vee d_{t-1s} < c_d \vee \dots \vee d_{t-i+1s} < c_d\}} I_{\{d_{ts} < 1\}} I_{\{d_{t-1s} < 1\}} \dots I_{\{d_{t-i+1s} < 1\}} (1 - D_{t-1s}) + D_{t-1s} \quad t=1, \dots, T; s=1, \dots, S \quad (\text{I.9})$$

$$\frac{1}{S} \sum_{s=1}^S D_{Ts} \leq \epsilon \quad (\text{I.10})$$

$$x_{it} \geq 0 \quad i=1, \dots, n; t=1, \dots, T \quad (\text{I.11})$$

$$\sum_{i=1}^n x_{it} = 1 \quad t=1, \dots, T \quad (\text{I.12})$$

The objective function (I.1) indicates that the expected profit over the scenarios should be maximized. However, one can also operate with a fixed investment mix without optimizing. This is done in the example based on 80% fixed interest and 20% stocks. The profit is set up from the results on mortality, buying off (surrender), costs and interest (see (I.6)). Upon maturity all remaining (insurance) policies should be paid out.

Constants

- S : the number of scenarios used.
- T : the term of insurances taken out.
- n : the number of investment categories.

Exogenous variables

- C_1 : the first costs.
- c_r : the used arithmetic interest.
- c_d : the crucial degree of cover.
- l : the term of underabsorption experienced as crucial.
- ϵ : the maximum granted chance on ruin.
- V_{ts} : the actuarial reserve at the end of year t in scenario s .
- P_{ts} : sum of premiums received (including insurance premiums) at the beginning of year t in scenario s .
- w_{ts}^d : result on mortality in year t of scenario s .
- w_{ts}^k : result on costs in year t of scenario s .
- w_{ts}^a : result of surrender in year t of scenario s .
- k : the part of positive results withdrawn from the fortune built up for the product, $k \in [0,1]$.
- U_{ts} : payments at the end of year t of scenario s .
- r_{its} : total return of investment object i in year t of scenario s .

Endogenous variables

- B_{ts} : nett invested fortune at the end of year t of scenario s (after deduction of payments and the dividend pay-out).
- \tilde{B}_{ts} : gross invested fortune at the end of year t of scenario s (before deduction of payments and the dividend pay-out)
- I_s : indicator function indicating whether in a scenario the first costs can be recovered.
- w_{ts}^i : result on investment in year t of scenario s .
- \tilde{w}_{ts} : sum of the results at the expense of the invested fortune in year t of scenario s .
- W_{ts} : sum of the results in year t of scenario s .
- d_{ts} : degree of cover in year t of scenario s .
- D_{ts} : indicates whether until year t of scenario s ruin (drama) already took place.
- \hat{r}_{ts} : weighted average of the returns of the n investment categories in year t of scenario s .
- z_s : time of recovery in scenario s .
- μ_z : expected time of recovery.

Decision variables

- x_{it} : fraction invested in investment category i in year t .

Appendix II

This Appendix describes the product and assumptions used in the example of paragraph 3.

Insurance type	:	Endowment, annual premiums
Profit sharing	:	Interest surplus ---> increase of sum insured

Commission structure

▲ at sale	:	3.5% over insured capital sum
▲ bonus	:	0.2% over insured capital sum
▲ annual	:	2.0% over gross premium

Cost loadings

▲ first costs	:	4.0% over insured capital sum (incl. commission at sale)
▲ admin costs	:	0.2% over insured capital sum
▲ costs collecting premiums	:	2.0% over gross premium

Mortality

▲ table	:	Entire population Males 1985-1990 as published by the Dutch Institute of Actuaries
▲ age rating	:	males: 0 females: -5

Interest rates

- ▲ discount rate tariff : 4%
- ▲ yield on shareholders' equity : 11%

Reserving methods

- ▲ for tax purposes : nett
- ▲ commercial : lineair amortisation in 8 years of commission at sale
- ▲ at surrender : 4% Zillmer

Portfolio characteristics

- ▲ 3200 policies to sell in the first year

Gross premium age / duration	1,600	5,000	10,000
50 / 10	0	400	400
40 / 20	800	400	400
30 / 30	800	0	0

Assumptions real costs

- ▲ first costs : 750 (+ commission)
- ▲ ongoing costs : 100 (+ commission)
- ▲ inflation : 3%/year (mean)
- ▲ efficiency result : 1%/year

Assumptions investments

- ▲ fixed interest : 7%/year (mean)
- ▲ shares : 11%/year (mean)
- ▲ asset mix : 80% fixed interest, 20% shares

Taxation

- ▲ company tax : 35%

Assumed decrements

- ▲ mortality : 70% of tariff mortality
- ▲ surrender : 3%/year (first 10 years)
1%/year (after 10 years)

