

The Towers Perrin Global Capital Market Scenario Generation System

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Abstract

Financial management requires a systematic approach for generating scenarios of future capital markets. Today's global environment demands that the scenarios link the economies of individual countries within a common framework. We describe a global scenario system, developed by Towers Perrin, based on a cascading set of stochastic differential equations. The system applies to financial systems for pension plans and insurance companies throughout the world. A case study illustrates the process.

Résumé

La gestion financière demande une approche systématique pour générer des scénarios pour les marchés futurs des capitaux. L'environnement global de nos jours demande que les scénarios relient les économies des pays dans une approche commune. Nous décrivons un modèle de scénario global développé par Towers Perrin basé sur une cascade d'équation différentielles aléatoires. Le modèle s'applique aux portefeuilles de pension et d'assurance dans le monde entier. Un exemple est utilisé pour illustrer le modèle.

Keywords

Asset-liability management, economic forecasting, pension planning, insurance industry.

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1.0 Introduction

Towers Perrin, one of the world's largest actuarial consulting companies, employs a capital market scenario generation system, called CAP:Link, for helping its clients in understanding the risks and opportunities relating to capital market investments. The system produces a representative set of individual simulations – typically 500 to 1000. Each scenario contains key economic variables such as price and wage inflation, interest rates at different maturities (real and nominal), stock dividend yields and growth rates, and exchange rates through each year for a period of up to 20 years. We model returns on asset classes and liability projections consistent with the underlying economic factors, especially interest rates. The economic variables are simultaneously determined for multiple economies within a common global framework. Long-term asset and liability management is the primary application.

A variety of stochastic optimization models exist for multi-period financial analysis. Prominent examples include: Berger and Mulvey (1995), Boender (1995), Cariño *et al.* (1994), Dempster (1996) and Wilkie (1995). These systems are designed around a set of simplifying assumptions that depend upon the target application. Stochastic financial simulations fall within three broad categories:

Prediction - These systems forecast short-term market movements. It is not necessary to achieve perfect accuracy, only enough to realize an expected profit. Hedge funds and other leveraged traders often employ computerized prediction systems. The desired output is a single scenario for next period's economic variables. Prediction becomes more challenging as the length of the planning period increases since markets adapt to changing conditions. Most trading systems consider short-term horizons --- hours or days. Hence, validity can be ascertained by historical back-testing the recommended investment strategies.

Pricing - Derivative securities often possess complex price relationships with capital market variables, such as the pattern of interest rates movements. By capturing this in-

formation via models where the end-of-period outcome of the various alternatives can be compared through a set of scenarios, we can check for consistency in pricing. It often applies in the creation of custom deals wherein the parties exchange specific market risks. The arbitrage-free condition is important since the existence of arbitrage opportunities suggests inconsistency with traded securities.

Risk Analysis - This activity evaluates the potential rewards and risks of various investment and liability management strategies. By managing investment decisions and considering liability issues as part of an integrative financial picture, significant risks can be avoided and opportunities for enhanced return created by accepting risks which may be more severe to other investors – due to a different liability structure. Viewing investment choices from the perspective of their ability to meet specified liability objectives changes the relative riskiness of investment alternatives. As an example, long term bonds which may be relatively risky to some short term investors are attractive to a pension plan with a specific long-term and fixed horizon. In contrast, when the liability structure depends upon inflation, index linked bonds becomes a conservative asset category. Alternatives like cash may be risky in this context because the inflation adjusted value at a long term horizon is uncertain.

Several obstacles stand in the way of successful applications of asset and liability management. First, industries, such as insurance and banks, have been controlled by regulations and legal restrictions, causing difficulty in modeling the dynamic aspects of the key economic variables with reference to the market values of assets and liabilities. As regulations and rules change across the globe, however, economic issues gain in importance, and as a consequence, the ability to model the stochastic elements improves. A second receding obstacle is the computational resources needed to solve the multi-period financial planning model. Today, powerful workstations, PCs, and efficient algorithms are available for solving financial optimization problems; for example, see Mulvey, Armstrong and Rothberg (1995).

CAP:Link, developed primarily for asset liability management (ALM), entails an ongoing process of information gathering, evaluation and action in order to maximize the organization's wealth over time. Asset and liability mixtures complicate the situation. It is common to either over emphasize the liability matching investment alternatives (via immunization or similar approaches) or to focus on expected return and volatility of asset return. The investor must balance expected asset return and confidence in meeting liability obligations. A case study (section 4) offers more ideas on asset liability management and CAP:Link's role in financial planning. In brief, CAP:Link portrays the relationships between modeled variables, their interactions through time, and the ranges and distributions of outcomes in a consistent manner. Investment strategies and assumption setting can be evaluated by means of representative scenarios.

The rest of the paper is organized as follows. The next section describes the overall structure of global CAP:Link. We emphasize the relationship of the single country modules within a global setting. Much experience has been gained by implementing CAP:Link in over 14 countries throughout North America, Europe and Asia. The global design links the single country modules in a consistent fashion. In addition, we model currencies between all pairs of countries, as described in section 2.4. Section 3 takes up some implementation issues, including the critical assumption setting and calibration elements. We present a case study in section 4 in which the scenarios form the basis for an analysis of a large pension plan. We show the process for evaluating the a pension plan's health from several perspectives. Last, in section 5, we mention ideas for future work.

2.0 Model Structure

The global CAP:Link system forms a linked network of single country modules. Figure 1 illustrates the overall structure for four countries. The three major economic powers -- the United States, Germany, and Japan -- occupy a central role, with the remaining countries designated as home or other countries. We assume that the other countries are affected by, but do not impact, the economies of the three major countries. The basic

stochastic differential equations are identical in each country, although the parameters reflect unique characteristics of each particular economy. Notice the direction of Figure 1 arcs. Additional countries can be readily included in the framework by increasing the number of other countries.

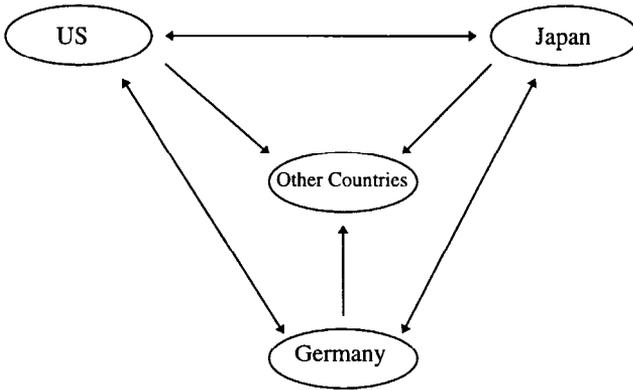


Figure 1: Triad of cornerstone countries and other countries. Direction of arcs shows the flow of information in the model.

Within each country, the basic economic structure is illustrated in Figure 2. Variables at the top of the structure influence those below, but not vice-versa. This approach eases the task of calibrating parameters. The ordering does not reflect causality between economic variables, but rather captures significant co-movements. Linkages across countries occur at various levels of the model -- for example, interest rates and stock returns. These connections will be discussed later. Roughly, the economic conditions in a single country are more or less affected by those of its neighboring countries and by its trading partners. The degree of interaction depends upon the country under review.

The structure is based on a cascade format -- each sub-module within the system is possibly affected by modules above and equal to that module. Briefly, the first level consists of short and long interest rates, and price inflation. Interest rates are a key attribute in modeling asset returns and especially in coordinating the linkages between asset re-

turns and liability investments. To calculate a pension plan's surplus, we must be able to discount the projected liability cash flows at a discount rate which is consistent with bond returns. Also, since dynamic relationships are essential in risk analysis, the interest rate model forms a critical element.

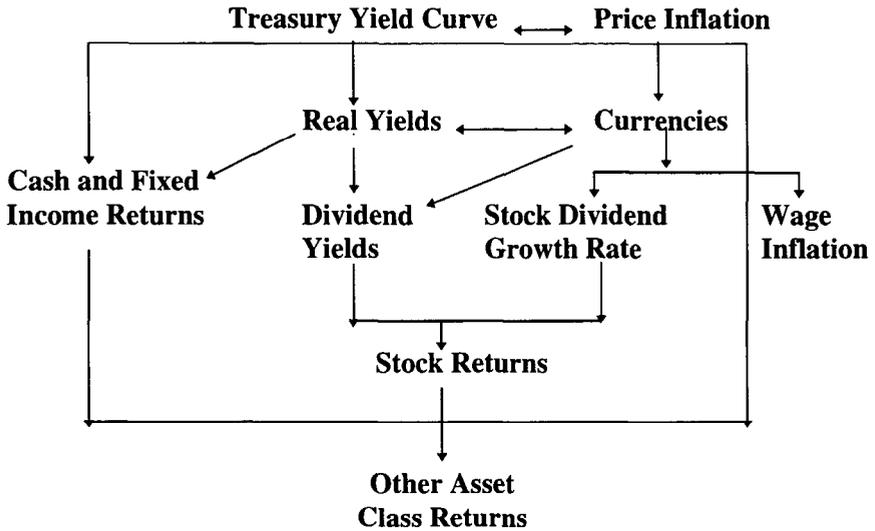


Figure 2: The cascade CAP:Link structure within a single country. Each country in Global CAP:Link depicts a common heritage.

The second level entails real yields, currency exchange rates and wage inflation. At the third level, we focus on the components of equity returns: dividend yields and dividend growth. Returns for the remaining asset classes form the next level, with fixed income assets reflecting the term structure of interest rates and other mechanisms as discussed below. Each economic variable is projected by means of a stochastic differential equation -- relating the variable through time and with the stochastic elements of the equation and, of course, to other variables and factors at the same or higher levels in the cascade. See Mulvey (1996) for a discussion of the single country design.

2.1 Interest Rate Generation

The path of spot interest rates for non-callable government obligations sits at the top of the cascade. Yield curve values are determined in a sequential fashion. First, the short and long spot rates are computed by a variant of the two factor Brennan Schwartz approach (1982). At its simplest within a country, we assume that long and short interest rates link together through a correlated white noise term and by means of a stabilizing term which keeps the spread between the short and long rates under control. In addition, we link the white noise terms across selected countries. The requisite equations are listed below. All elements are indexed as vectors across the target countries.

$$\text{Short rate: } dr_t = f_1(r_u - r_t) dt + f_2(r_u, r_t, l_u, l_t, p_u, p_t) dt + f_3(r_t) dZ_1 \quad (1)$$

$$\text{Long rate: } dl_t = f_4(l_u - l_t) dt + f_5(r_u, r_t, l_u, l_t, p_u, p_t) dt + f_6(r_t) dZ_2 \quad (2)$$

where r_u is the normative level of short interest rates,

r_t is the level of interest rates at time t ,

l_u is the normative level of long interest rates,

l_t is the level of rates at time t ,

p_u is the normative level of inflation and p_t is the level at time t , and

f_1, \dots, f_6 are vector functions that depend upon various economic factors up to period t .

The random coefficient vectors -- dZ_1 and dZ_2 -- depict correlated Wiener terms.

These two vector diffusion equations provide the building blocks for the remaining spot interest rates, and the full yield curve. At any point t , the mid-rate is a function of the short and long rates. Other points on the spot rate curve are computed by smoothing a double exponential equation for the spot rate of interest.

2.2 Inflation Generation

The price inflation routine lies aside the interest rate model in the cascade structure. Price inflation at period t depends upon price inflation in previous time periods and on the current yield curve. Again, the economic time series is modeled as a diffusion process. Controls are placed on internal parameters and interest rates -- the yield curve. The

key diffusion equations are shown below. As before, these equations index over the individual countries (as vectors):

$$\text{Price inflation: } dp_t = f_7 dr_t + f_8(p_w, p_b, r_w, r_b, l_w, l_b) dt + f_9(v_t) dZ_3 \quad (3)$$

$$\text{Stochastic volatility: } d v_t = f_{10}(v_u - v_t) dt + f_{11}(v_t) dZ_4 \quad (4)$$

where v_u is the normative level of inflation volatility and v_t is the level at time t , and f_7, \dots, f_{11} are vector functions that depend upon various economic factors up to period t .

The random coefficient vectors -- dZ_3 and dZ_4 -- depict white noise terms consisting of parts reflecting local and global effects. In the US and other economies, price inflation is more volatile than interest rates. Part of the explanation is the lack of a traded security representing inflation¹. Also, the *volatility* of inflation persists. Once inflation volatility increases, for example, through a economic shock such as the oil crisis, it remains high for some time before settling down to a more normal level. Modeling stochastic volatility requires a second diffusion equation (4).

Next in the cascade is wage inflation. This parameter connects to price inflation in a lagged and smoothed fashion. Wages are relatively slow to react to changes in price inflation, but they inevitably follow over time. Pension plan liabilities are directly tied to wage inflation.

2.3 Real Yields

Real interest rates are defined in two distinct ways. First, we equate real interest rates with the spread of nominal interest rates to current inflation. Some economies provide explicit real return bonds as traded securities issued by government bodies, for instance, index linked bonds in the UK and Canada. These securities provide a specified return over inflation for a fixed period. Real returns in a net of inflation framework are equivalent to nominal yields in a total return context. Regulatory bodies in some countries set valuation by discounting with real yield rates. Real yields relate to the movement of interest rates, current inflation, as well as expectations for future inflation. The equation for long term real yields is shown below:

$$dk_t = f_{12}(k_w, k_b, l_w, l_b, p_w, p_t)dl + f_{13}(p_w, p_b, k_w, k_b, l_w, l_t)dt + f_{14}(k_t)dZ_5 \quad (5)$$

where k_w is the normative level of real yields and k_t is the level at time t , and

f_{12}, \dots, f_{14} are vector functions that depend upon various economic factors up to time t .

The random coefficient vector dZ_5 depict correlated Wiener terms. To derive the complete real yield curve, we calculate a short-term real yield based on short nominal rates and current inflation -- providing two points on the curve. Remaining points are determined by interpolating the structure of the nominal interest rate curve.

2.4 Currencies

Several complication issues arise when modeling currency exchange rates. First, currencies must enforce the arbitrage free condition among spot exchange rates and among forward rates with differential interest rates. The second issue involves symmetry and numeraire independence; we must create a structure in which the distribution of currency returns from country A to B has the same distribution as returns from B to A. Both issues limit the form of the currency exchange models, especially when integrating three or more currencies. To avoid these problems, we focus on the strength of each country's currency. Exchange rate follows as the ratio between the strengths of any two counties. See Mahieu and Schotman (1994) for a similar approach to currencies. The absolute strength of any currency is a notional concept; the relative levels reflect the difference in the exchange rates.

The exchange rate equation between country i and j becomes the ratio: s_i / s_j , where s_i and s_j reflects the strength of currency i and j , respectively.

The equation describing the development of the individual currency strengths follows:

$$ds_t = f_{15}(r_w, r_b, p_w, p_b, pp_t)dt + f_{16}dZ_6 \quad (6)$$

where

pp_t is the average cost of goods in foreign countries relative to domestic cost and

f_{15} and f_{16} are vector functions that depend upon various economic factors up to period t .

The random coefficient vector dZ_6 depicts correlated Wiener terms. Addressing relative currency strengths involves not only current economic conditions but also historical trends. For example, an important long-term relationship is purchase power parity (PPP). The pp_t term depicts the pattern of inflation and currency movements in the projection up to time t . Purchase power parity states that exchange rates should keep pace with price inflation. Currency is an intermediary for the ultimate exchange of goods and services. The inflation differential between two economies approximates the change in the relative cost of goods in each country; currency being conduit for the ultimate goods and services having equal value. (Of course, transportation costs and other issues complicate the issue.) Currency movements should offset relative changes in inflation based on trading arguments. Empirical tests have validated the purchase power conditions over long time frames. Depending on the countries, reversion periods range from 2 years to 12 years. Two issues are important for risk analysis -- the strength of the relationship within individual scenarios and the strength of the relationship when considered across multiple simulations. CAP:Link includes both of these issues.

The system reflects a number of other currency issues relating to interest rates, real interest rates as well as other economic factors. For example, global investors are interested in maximizing their risk adjusted rewards. Over the past 25 years, a desirable currency strategy has been to place assets in countries that possess relatively high real interest rates. Currencies with high real interest rates rose as compared with those currencies with lower real interest rates. Under this theory, investments move to countries with high real interest rates -- causing the currency to strengthen. These concepts can be readily modeled within the CAP:Link structure.

2.5 Stock Returns

We divide stock returns into its two elements: dividends and capital appreciation. Dividend yields equals the ratio of the annual rate of dividend payment divided by price.

Given the initial dividend yield, an arbitrary price level can be set (say 100) which specifies the current dividend payment level. Dividend growth through the period determines the income as well as the end of period dividend rate. By separating the base components, we can accurately depict cash income. Also, we found that for modeling purposes, due to their temporal stability, the decomposed structure provides more accurate linkages to the key economic factors -- interest rates and inflation levels; see Poterba and Summers (1988).

The dividend growth equation is shown below:

$$dg_{ct} = f_{17}(p_w, p_t, g_u, g_t) dt + f_{18}(g_t) dZ_7 \quad (7)$$

where

g_u is the normative level of dividend growth and g_t is the level at time t , and f_{17}, \dots, f_{18} are vector functions that depend upon various economic factors up to period t .

The random coefficient vector dZ_7 depicts correlated Wiener terms. We take advantage of the relatively strong relationship between dividend growth rates and inflation.

The dividend yield equation is shown below:

$$dy_t = f_{19}dr_t + f_{20}dl_t + f_{21}ds/s_t + f_{22}(y_u, y_b, r_w, r_b, k_w, k_t) dt + f_{23}(g_t) dZ_8 \quad (8)$$

where

y_u is the normative level of dividend yield and y_t is the level at time t , f_{19}, \dots, f_{23} are vector functions that depend upon various economic factors up to period t .

The random coefficient vector dZ_8 depicts correlated Wiener terms. Dividend yield depends upon the movement of interest rate levels and currency exchange rates, among other factors. Also, there are longer-term relationships between the absolute level of real yields and a mean reverting tendency for dividend yields. These equations are first developed for the large capitalization markets: the US S&P 500, Germany's DAX index, and Japan's TOPIX indices. Next, we calculate stock returns for the remaining countries.

2.6 Asset Class Construction

The cascade structure provides a framework for building generic asset classes. The fundamental asset classes are already defined: cash, government bonds and large capitalized equities. Other fixed income assets are modeled via the appropriate spreads over government rates. CAP:Link addresses spreads and the underlying interest relationships of a given fixed income security. This approach extends to real return (or index linked) bonds based on the real yield curves. Given this methodology, we can calculate foreign asset returns by combining the foreign country local return with the currency exchange rate to calculate a total return in the currency of choice.

In addition, we model a number of asset classes outside the fundamental group. Prominent examples include: real estate; venture capital; alternative equity market segments such as small capitalization stocks; emerging market investments; catastrophe related securities; and derivatives. CAP:Link addresses these asset classes by providing tools to describe levels of volatility, serial correlation, relationships to interest rates and inflation, correlations to other asset classes, and relative return expectations. The tools represent our view of the dominant characteristics important to ALM -- that is, interest rate and inflation relationships, diversification potential over different time horizons, and range of potential tradeoffs of risk and expected rewards.

2.7 Alternative Approaches

The classic alternative to stochastic differential equations underlying CAP:Link is the mean-covariance model. This model assumes either normal or lognormal time independent distributions. The user specifies expected values and variances of each asset category as well as covariances for all pairs of assets. The approach often appears in finance textbooks and software packages but suffers in realism for ALM since it avoids features such as mean reversion in interest rates. A second alternative is vector autoregressive (VAR). This approach forms a rolling regression analysis in which the independent variables -- interest rates, inflation, asset returns, and liability returns in previ-

ous periods -- determine the next period values for the economic variables as well as the asset returns and liabilities. Because of the unstructured nature of the process, VAR has considerable adaptability to changing economic conditions. It has been employed in Cariño *et al.* (1994) and others. The approach has proven effective for prediction and pricing applications. At times, however, VAR may produce divergent results for long-term risk analysis. In order to overcome this problem, VAR can be coupled with equilibrium conditions as discussed by Boender (1995). A third alternative has been developed by Wilkie (1995). The approach possesses a cascade structure similar to CAP:Link. However, there are substantial differences in the form of the equations, in the calibration process, and in the ordering of the variables in the cascade.

3.0 Implementation Details

Setting the parameters for the stochastic differential equations (1) through (8) presents a formidable task with conflicting objectives. We separate the process into two steps. In the first, called *assumption setting*, we determine the expected level of returns for asset classes and economic variables. The second, called *calibration*, calculates the parameter coefficients for the relationship and distribution characteristics, such as correlations, standard deviations, ranges, rates of convergence, spreads, and other statistics.

3.1 Assumption Setting

For our discussion, assumption setting refers to coefficients that affect risk premiums and expected rewards. Towers Perrin assesses a return premium appropriate to each element of risk. The first block focuses on inflation. The second determines the expected return on cash net of inflation. The third encompasses the spread of long term interest rates over short term interest rates. Next, corporate bonds require a spread for quality risk. Large cap stocks assess a spread of expected equity returns over a bond universe fund. Capital market line relationships (comparing expected return to volatility) are considered for consistency. This approach depends upon the assumption that each element of expected return should be appropriate to compensate for a risky characteristic. Determining the relative premiums is based on dissecting the historical re-

turns into expectation elements (such as the initial level of interest rates) and valuation change elements (such as capital gains/losses due to changes in interest rates). An additional component relates initial condition values to normative assumptions.

Assumption setting is an iterative process: examining the reasonableness of the inputs and the results, analyzing the sensitivity of the recommendations, and modifying the parameters until the patterns are deemed acceptable by the economic staff.

3.2 Calibration Procedure

Calibration describes the elements relating to risks characteristics, such as distributional spreads. This step presents a more challenging technical task than assumption setting. Target values are determined for identified important relationship and distributional characteristics of the model based on an analysis of historical values and expert judgment. Calibration parameters are adjusted until the simulated statistics become sufficiently close to the target values. Performing this task uncovers new characteristics. Non-convex optimization tools can automate the calibration procedure (Mulvey, Rosenbaum and Shetty 1996), but in the end judgment is required to accept a satisfactory outcome or demand further investigation of a perplexing element.

3.3 Sampling Procedures

The stochastic differential equations (1) through (8) form the basis for generating scenarios – the main CAP:Link output. We employ a sampling procedure based on variance reduction methods. For example, we have found antithetic variates to be effective for many applications. Each scenario depicts a single realization of the stochastic equations, by sampling from the white noise terms, over the planning horizon. The number of scenarios depends upon the targeted application and the risk aversion of the investor. For example, a long-term planning horizon coupled with dynamic investment strategies, such as portfolio insurance, necessitates a relatively large set of scenarios to provide accurate estimates of risks. On the other hand, stable investment strategies, such as dy-

namically balanced, involve small market impact costs under volatile conditions and therefore require fewer scenarios to produce acceptable risk estimates.

3.4 Precision Tests

A critical issue involves reliability of the model's recommendations, especially relating to risks and rewards. Accordingly, we conduct stress tests of the proposed strategies by means of out-of-sample analyses. First, we generate a new set of scenarios based on further sampling as described in the previous section, or by modifying the parameters of the stochastic equations and re-generating the results. The precision of the risk measures and valid confidence limits are estimated by simulating the investment/liability strategies with the new data. The second risk estimates are compared with the original. If the two values are close, we deem the precision tests to be a success. Otherwise, we can employ a larger set for the optimization stage of ALM system.

By employing precision tests, we have found that 500 scenarios is adequate for many financial planning purposes. The number of scenarios will need to be substantially increased, however, when the investor displays extreme risk averse.

4.0 Illustration of Pension Planning

Consider the case of a defined benefit pension plan in the US -- the NEWCO company -- which is in the process of developing an investment strategy for its portfolio managers. We will discuss the planning process and demonstrate sample forms of analysis. The approach applies to other long term financial investment decisions (insurance companies, endowment funds, personal financial planning) with differing emphases (such as funding decisions).

The process consists of three stages. First, we determine the objectives for the pension plan and carry out the assumption setting tasks. In the second, we analyze risks and rewards for alternative investment strategies. The final stage involves decision making and implementing the asset and liability allocation strategy. In practice, the process of-

ten entails a cycle -- observations made from the analysis lead to modifications in the objectives and assumptions. The goal is to understand the financial dynamics, not to run the data through a standardized process and produce a single "answer".

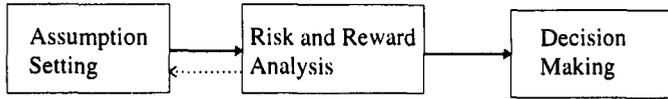


Figure 3: The three stages for pension planning. The process entails a combination of feedback and revision in order to become comfortable with the recommendations.

Stage 1 : Objective and Assumption Setting

Setting goals for a defined benefit pension plan is often difficult due to the presence of multiple parties possessing differing interests: beneficiaries, with benefit security or even benefit enhancement; and plan sponsors, with cost levels and volatility. Taken from the sponsors' view, there are alternative bases on which to measure cost: contributions, expense levels for accounting, long term economic cost, impact on corporate income statement, etc. Differing actuarial bases are possible. Issues of time frame (1 year, 3 years, or longer) and risk measurements (standard deviation, volatility, or chances of failing to meet some minimum acceptable target) add complexity. To approach the problem, we focus on the major objectives and stakeholders. Having made a list of these items, we render projections in order to identify areas of concern. For our illustration, we examine projections of contributions and timings, funded ratio levels, and pension expense under current funding and investment policy.

Generating estimates of the future viability and surplus of the pension plan requires four elements: the scenario generation program; a package for projecting the liability cash-flows based on the plan's actuarial rulesⁱⁱ; a multi-period investment and contribution management system (possibly with an optimization component)ⁱⁱⁱ; and a system for calculating financial and accounting statistics. The four components must fit together with regard to structure and dependence on key assumptions.

We apply the Towers Perrin's system to assist NEWCO. The resulting estimates for the range of contributions over the next ten years appear in Figure 4. This projection shows that the likelihood of making contributions in any one of the next ten years is less than 50%. Since contributions are linked temporally, we must examine the distribution of cumulative contributions through the decade. Because contributions are strongly affected by funding policy, however, these numbers can be difficult to evaluate^{iv}. In some cases, inadequate cash flows increase the importance of the contributions.

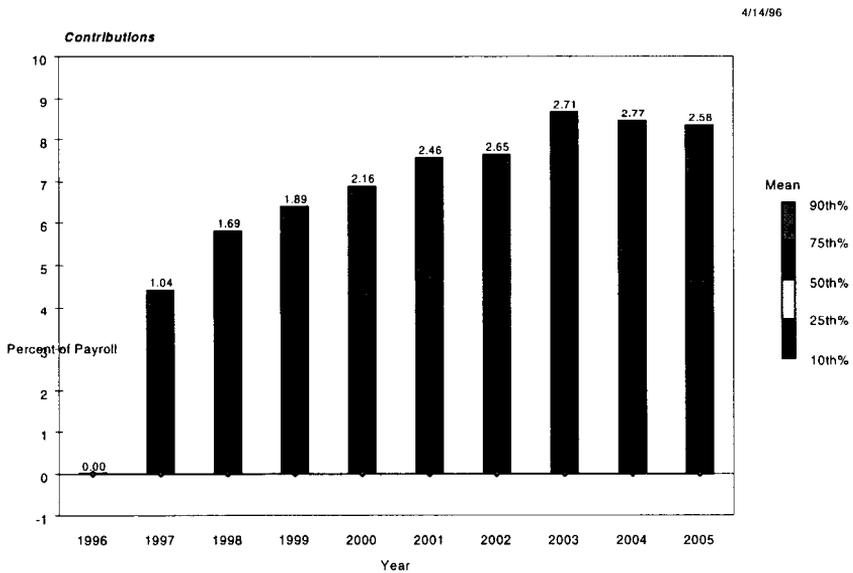


Figure 4: Distribution of contributions. The probability of a contribution in any year is less than 50%.

The next issue to consider is the company's expenses. In the US, pension expenses depend upon accounting and other regulatory rules such as FAS 87, which requires plans to discount their liabilities at a close to market rate. Figure 5 depicts the range of possible expenses for NEWCO over the next ten years. We see that they are roughly 1-2% of the company's payroll. However, there is considerable uncertainty, given the range of investment returns and the company's contribution policy during the ensuing period. In

fact, there is a reasonable chance that expenses will be negative – indicating income to the firm.

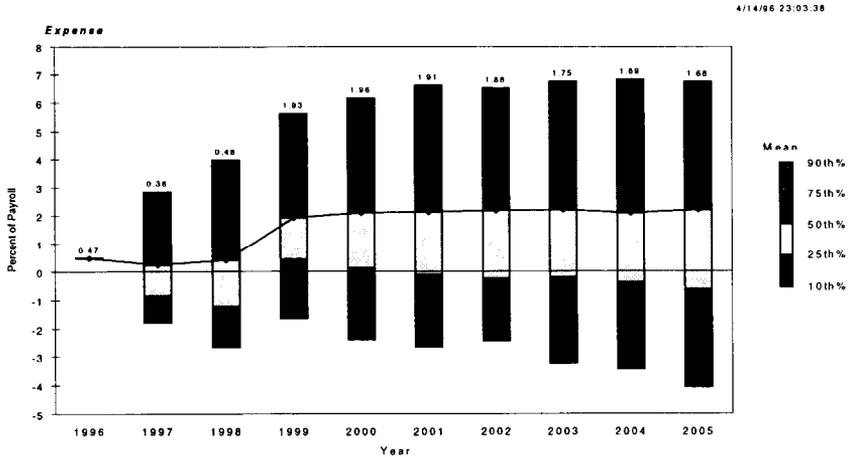


Figure 5: Distribution of expenses as a percentage of payroll. Expenses can become negative in future years indicating income to the firm.

As a consequence of the investment asset mix and the contribution policy, we can observe the health of the pension plan surplus over the 10 year planning horizon. One view of the surplus is the Accumulated Benefit Obligation (ABO) – roughly the amount of money needed to pay off the current promised liabilities at present value. Figure 6 shows the range of possible ABO numbers. Here, the 100% value indicates a fully funded pension plan on a ABO basis, or there is just enough asset value to compensate for liabilities at present value (discounting at current government non-callable rates). Amounts above 100% are in surplus, whereas amounts below 100% indicate a deficit. A company's required contribution depends upon the value of the ABO numbers.

Projecting the impact of today's decisions on a company's future wealth make sense for long term investors. A multiyear simulation tool based on discrete scenarios is essential. To make these projections, we must address the interaction of economic variables such as interest rates and inflation levels with capital market returns. Also, the individ-

ual patterns impact the financial results, such as pension expenses. In addition to the economic and capital market simulations, as provided by CAP:Link, we must convert this information into the relevant financial statistics. The calculation ought to be straightforward, but in practice many regulatory, timing, and other technical issues must be addressed. All financial projections presented have been produced by Towers Perrin's FIN:Link system.

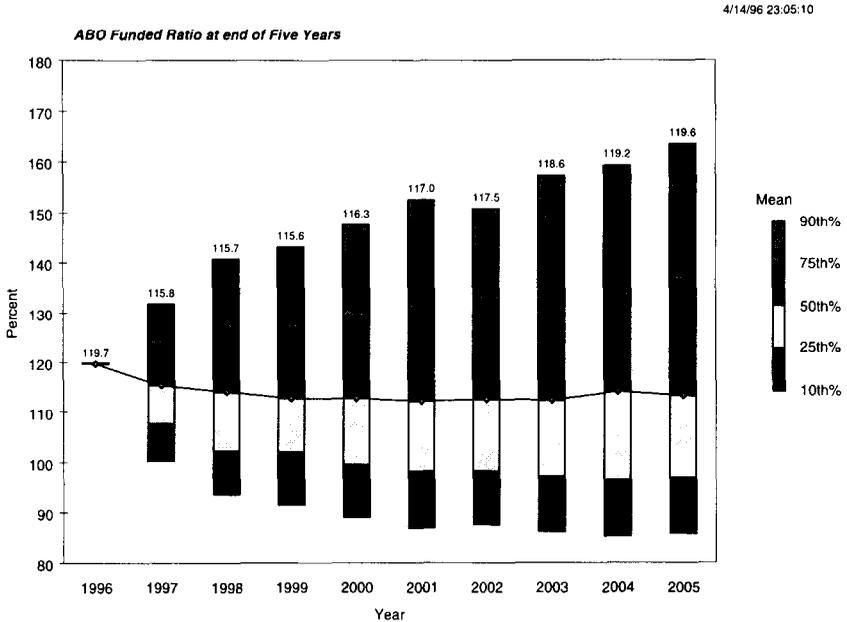


Figure 6: Distribution of ABO funded ratios over the next 10 years. The NEWCO pension plan is likely to be in surplus during the period, given the proposed contribution policy.

Contribution, expense, and other projections depend upon calibration assumptions. Similar to the objective setting process it is useful to examine the projections to assess their reasonableness and also to provide more insight into the drivers behind the financial projections.

For NEWCO, we reflect the economic variables at the beginning of 1996 -- adjusting the assumptions to an "equilibrium" condition, where long term expectations are set equal to current conditions and the spread between expected returns of various asset classes equal the normative assumptions. The normative approach helps develop broad policy guidelines. Alternatively, the assumptions could equal the company's best long-term estimates. This approach is suitable where consideration of strategic shifts in asset allocation are under review. Of course, consideration should be given to whether fund managers will be capable of performing ongoing analysis and adjustments to investments to reflect changing conditions.

Graphs showing the projections of inflation and interest rates (Figures 7, 8, and 9) -- the primary liability drivers -- are presented below along with the range of returns for each of the asset classes.

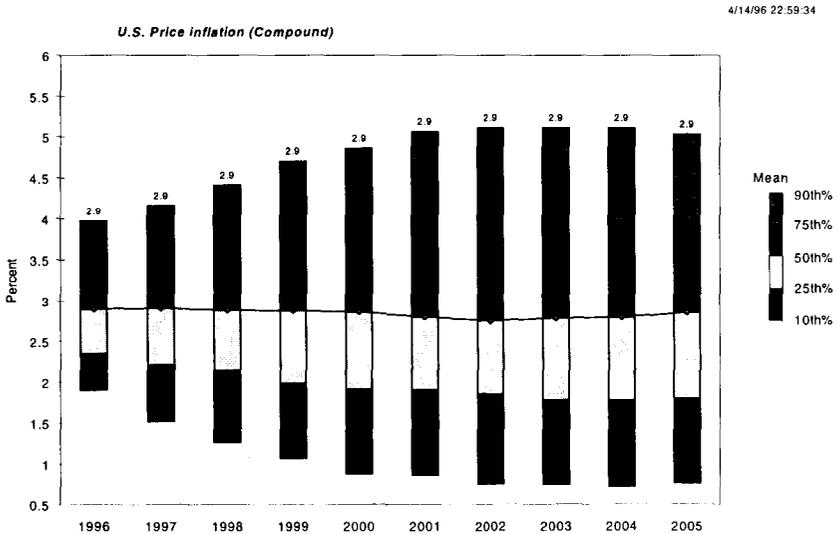


Figure 7: Distribution of estimated price inflation in US. These ranges reflect historical inflation in the US over the past 40 years.

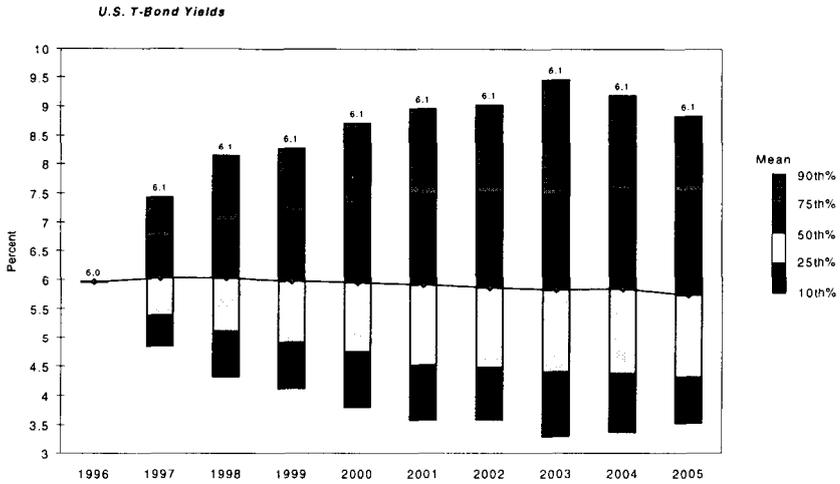


Figure 8: Distribution of estimated US T-bond yields. These results assume that long term interest rates equal the starting interest rates (1 January 1996 in this case).

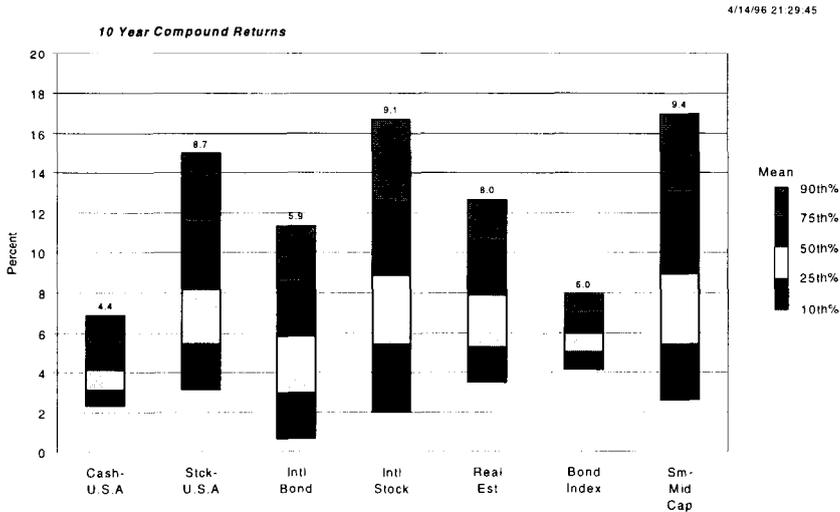


Figure 9: Distribution of ten year compounded returns. Cash becomes dominated by both stock and bond indices as the horizon lengths.

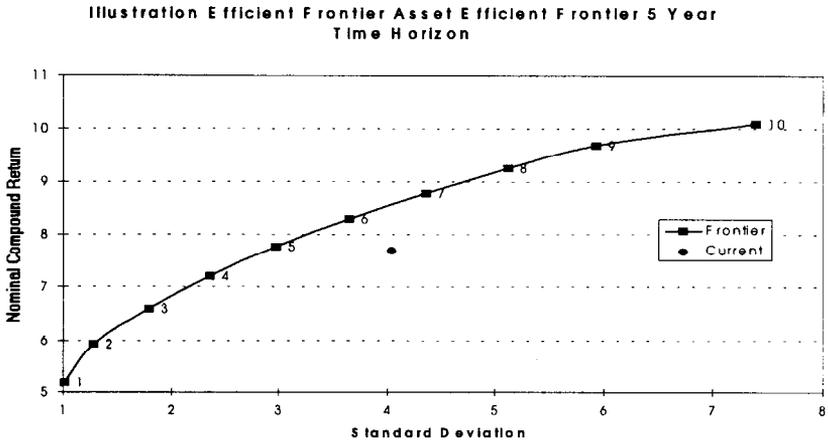
After completing this step, we pinpoint a specific concern -- funded levels falling below the 90% ratio of assets to the present value of liabilities. In an actual assignment, several elements for further investigation emerge. For example, the time horizon is significant. It is appropriate to investigate both the immediate term (1-3 years) and the longer term (5+ years). The former is critical – due to the nature of evaluating management performance by stockholders and other stakeholders. The long term must be considered as well: pension plan sponsors have a fiduciary responsibility to see that the company's promised benefits will be realized. Again a real world case would involve multiple time horizons, perhaps with differing objective elements: a short time horizon for funded status, and a longer one for pension expense levels. In this illustration, we focus on a five year time horizon.

Stage 2 : Analysis of Risks and Rewards

Efficient frontiers can be developed at various time horizons. Traditional efficient frontiers plot expected investment return versus the volatility of asset return over a single period – an asset only concept; see Konno *et al.* (1993) and Kroll *et al.* (1984). It is useful to understand the recommendations of the “standard single period asset only approach” so that they can be compared with a comprehensive ALM approach. To address standard practice, we calculate an asset-only frontier with a five year time horizon (Figure 10). Additional efficient frontiers should reflect differing elements of the objectives identified in Stage 1. Thus, we select average funded ratio at the end of five years as the reward measure and first-order downside risk with a 90% target as the risk measure (Figure 11). First-order downside risk equals the average shortfall below the target averaged over all scenarios (including those where there is no shortfall). For reference, second-order downside risk equals the average squared shortfall. First-order downside risk breaks into the product of two intuitive risk measures: the probability of shortfall and the average shortfall when one exists.

The scope of a risk analysis can be portrayed on a risk ladder (Mulvey 1996) with five rungs: 1) total integrative risk management; 2) dynamic asset-liability; 3) static asset-liability; 4) static asset only (a.k.a. Markowitz mean variance portfolios); and 5) single

security risks. Most current risk analyses are conducted at rungs 3 or 4. However, there are several groups that are pushing the analysis to higher, more comprehensive levels on the risk ladder. In this context, there are several criteria for measuring risk, including variance for symmetric outcomes, semi-variance and downside risks for asymmetric outcomes, and von Neumann Morganstern expected utilities. Ultimately, an organization ought to be concerned with risks to its net worth – at the top of the risk ladder.

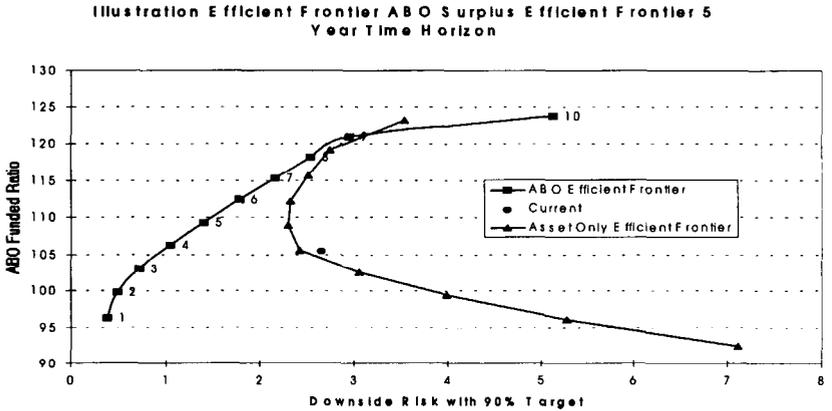


Asset Mix %	1	2	3	4	5	6	7	8	9	10	Current
Cash-U.S.A	67.0	46.0	25.6	8.7	0.0	0.0	0.0	0.0	0.0	0.0	5.0
Stck-U.S.A	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0
Intl Bond	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Intl Stock	0.0	3.9	7.5	11.0	15.5	20.5	25.4	30.0	30.0	30.0	0.0
Real Est	0.0	6.8	14.0	20.0	20.0	20.0	20.0	20.0	20.0	0.0	0.0
Bond Index	9.2	25.9	42.9	52.4	51.1	39.5	27.9	16.0	4.5	0.0	45.0
Treas Bond	23.8	14.4	4.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Sm-Mid Cap	0.0	3.0	5.4	7.9	13.3	20.0	26.7	34.0	45.5	70.0	0.0
Average	5.2	5.9	6.6	7.2	7.8	8.3	8.8	9.2	9.7	10.1	7.7
St. Dev.	1.0	1.3	1.8	2.4	3.0	3.6	4.4	5.1	5.9	7.4	4.0

Figure 10: An asset-only efficient frontier. Cash and bonds are considered the more conservative category for short term investors without liabilities.

We might consider several efficient frontier calculations with a variety of risk and reward measures, some which might be a weighted combination of differing measures. The goal is to find a set of candidate portfolios based on the efficient frontiers which might best meet the competing considerations. In theory, this step could be done by constructing the “correct” risk and reward measures and performing the efficient frontier calculation on these objectives. In practice, however, the correct measures become

known as the analysis progresses. Employing candidate portfolios help draw out this knowledge. In this illustration we limit the discussion to the two frontiers described above. Calculating efficient frontiers over multiple time periods requires non-linear stochastic optimization programs; see Mulvey, Armstrong and Rothberg (1995)⁶. Additionally, because of the non convexity due to the dynamic aspects of multi-period ALM and the lack of performance guarantees, the investor must be careful to access if a global optimal solution has been found. This consideration argues for the candidate portfolio approach. Based on this analysis, we focus on promising areas on the frontiers. For comparison, portfolios recommended by the asset-only efficient frontier have been plotted on the funded ratio efficient frontier. Differences are quite dramatic -- the low-risk portfolios from the asset-only efficient frontier are highly risky from a funded ratio perspective at the five year point⁶ (see Figure 11).



Asset Mix % :	1	2	3	4	5	6	7	8	9	10	Current
Cash-U.S.A	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0
Stock-U.S.A	8.9	6.3	8.1	5.2	1.1	0.0	0.0	0.0	0.0	0.0	50.0
Intl Bond	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Intl Stock	1.2	6.5	10.8	14.2	17.6	22.1	24.2	26.1	29.6	0.0	0.0
Real Est	0.0	3.1	2.5	6.4	10.8	12.8	14.6	18.0	16.5	0.0	0.0
Bond Index	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	45.0
Treat Bond	83.6	71.6	60.6	49.3	38.5	28.3	18.4	8.1	0.0	0.0	0.0
Sm-Mid Cap Stock	6.2	12.5	18.0	24.8	32.1	36.8	42.8	47.8	53.9	100.0	0.0
Average	96.3	99.7	103.0	106.2	109.3	112.4	115.3	118.2	121.0	123.7	105.4
Downside Risk	0.4	0.5	0.7	1.0	1.4	1.8	2.2	2.5	3.0	5.1	2.7
Prob. under 90%	13.6	14.8	15.0	17.4	18.6	20.0	20.4	20.8	21.6	29.4	27.8
Avg. Shortfall	2.8	3.3	4.8	6.0	7.5	8.9	10.6	12.3	13.7	17.4	9.5

Figure 11: An alternative efficient frontier with downside ABO risk. Treasury bonds are considered the least risky asset based on the investor's liability structure.

By selecting portfolios from the most attractive regions of the efficient frontiers we develop a list of candidate portfolios. NEWCO's efficient frontier in Figure 11 suggests substantial investments in small capitalization and international stocks and real estate. In practice, these recommendation might be limited for liquidity and prudence issues. Constraints on the optimization could be readily applied and the optimization re-run to generate a revised investment strategy.

Stage 3 : Decision Making and Implementation

We have chosen two candidate portfolios for comparing the distributions of contributions, expense, and funded ratio. In addition, it is useful to consider the differences in potential performance relative to NEWCO's current position. For alternatives 1 and 2, contributions and expenses are reduced, both on absolute range and as relative improvements over the current allocation (Figures 12 and 13). In this case, we see that although the first alternative has the potential for higher absolute levels of expense, the distribution of improvement from the current portfolio is much more attractive than for the second. Alternative 1 outperforms the current policy approximately 70% of the time, while alternative 2 outperforms the current policy about 50% of the time (Figure 14). From this perspective, alternative 1 appears superior to the current policy as well as alternative 2. Similarly, alternative 1 has the highest ABO funding ratio on average (Figure 15).

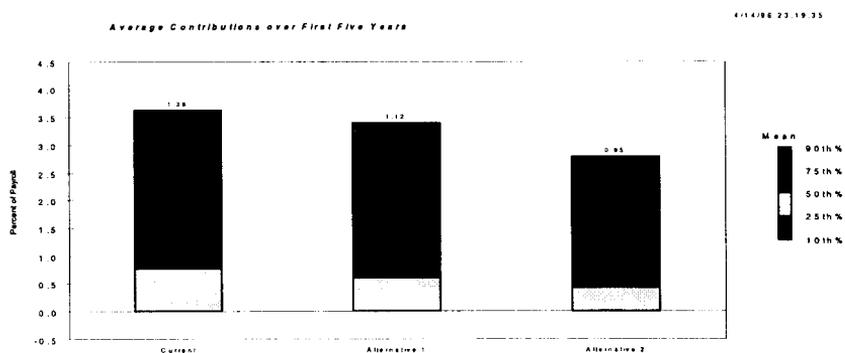


Figure 12: Distribution of contribution over 5 years. Both alternative 1 and 2 reduce contribution over the 5 years.

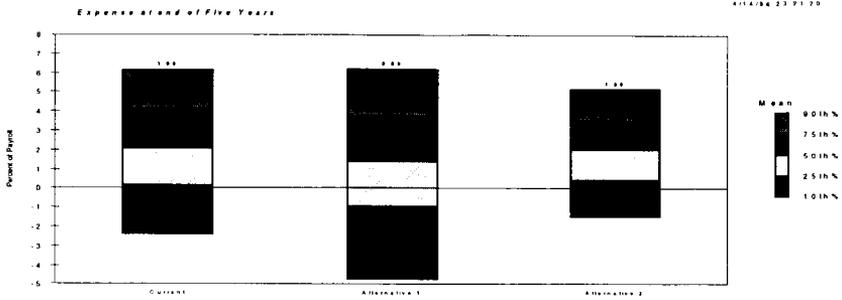


Figure 13: Distribution of expenses at end of 5 years. Alternative 1 provides lower average expenses from the current strategy, while alternative 2 provides lower expenses at the 90% confidence level.

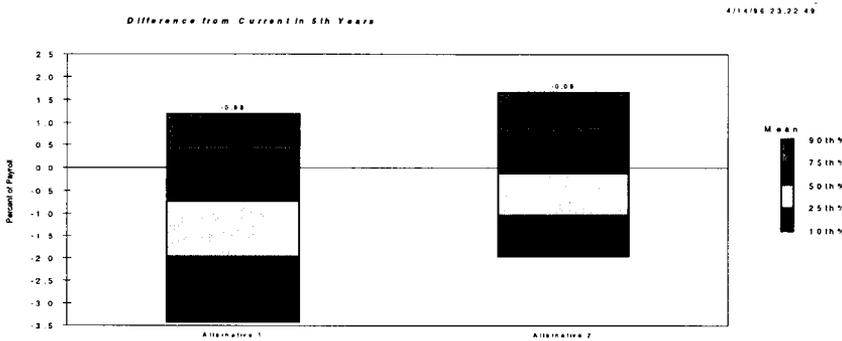


Figure 14: Distribution of expenses as a percentage of payroll as compared with the current policy on a scenario by scenario basis.

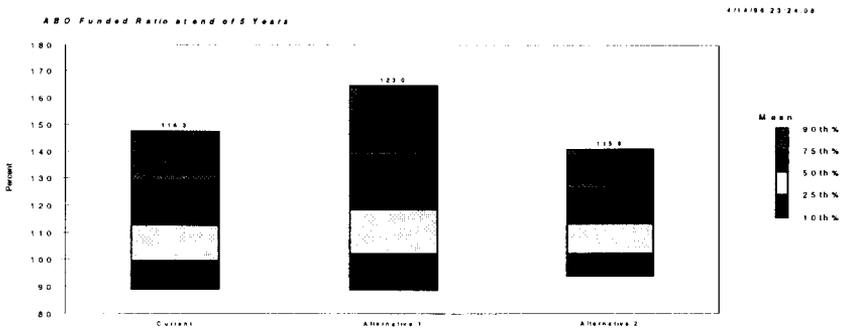


Figure 15: Comparison of ABO ratios for three alternatives. The average ABO is highest on average for alternative 1.

Once an allocation strategy is found, it is essential to carry out a sensitivity analysis. If a higher allocation to equity is suggested over the current asset allocation, for example, we recommend that the investor employ assumptions which reduce this spread. This approach applies to all asset classes targeted for increase in allocation. Observations made at this stage may lead back to an earlier stage. Simulation lends itself to pinpointing the cause of a particular result. Recommendations can be segmented by inflation, interest rates, equity returns, contribution levels, or other proposed causes. The analysis leads to an improved understanding of the investment dynamics. Combining this with other factors, such as liquidity, perceptions, transition problems, and implementation capability produce a decision.

The analysis may also suggest modifying the pension plan design, contribution policy, or asset and liability smoothing in order to improve overall financial performance. In addition, long term ALM systems can address more focused questions, such as whether and how much currency hedging is appropriate, how unexpected inflation would effect the contributions and other measurements, and how short-term funding decisions will impact longer-term risks.

5.0 Conclusions and Future Directions

Towers Perrin's system provides an internally consistent approach for generating capital market scenarios over long-term horizons. It captures an extensive range of market relationships -- making it an effective tool for risk analysis. There are several directions for future research. First, the system easily extends for new asset categories. For example, combining the purchase of equity and selling an off-setting call option -- buy-writing -- can be modeled as a separate asset category. This strategy is effective in volatile and driftless markets. The options can be valued in the usual manner via arbitrage free approaches since the risk free rates, volatility and other factors are known within a scenario. Buy-writing may be attractive for risk averse investors since downside risks are considerably reduced over a pure equity category. Another promising asset category involves securitizing catastrophic (CAT) insurance, as implemented by the

Chicago Board of Trade. Here, risks are uncorrelated with the returns of other asset categories. Investigating the pros and cons of these and other asset categories is made possible due to the system's flexibility.

Another extension is to employ the currency exchange rates for hedging strategies. Remember that local returns for countries outside the home country are inaccessible due to uncertain currency movements and the lack of equilibrium with respect to implied forward rates (Brinson 1993). Since the system defines currency as a separate economic factor, we can evaluate cross hedging strategies. See Sorensen (1993) for an example application. Next, we can assist corporations in setting organizational goals. For instance, we could evaluate alternative contribution policies. By analyzing policies under stressful conditions such as a recession, we can pinpoint total risks – rather than sub-optimizing the risks for individual components – such as portfolios in a single country.

In conclusion, multi-stage financial models cannot replace an understanding of capital markets. Rather, they aid in analyzing the competing issues of risks and rewards over time. As such, the capital market projections must provide a representative range of plausible scenarios. The global CAP:Link provides this information in a practical and internally consistent fashion.

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ⁱ The US government will begin selling index linked bonds in late 1996. Thus, estimates of inflationary expectations will be available as a traded security.

ⁱⁱ Towers Perrin's system is called VALCAST.

ⁱⁱⁱ Towers Perrin's system, OPT:Link, is based on Leon Lasdon's GRG package (1978).

^{iv} Contribution in the US is based on a complex set of regulatory rules and employee preferences.

^v Towers Perrin's OPT:Link system solved these efficient frontier calculations.

^{vi} We have discovered numerous similar situations in practice.