

Valuation of Fixed-Income Securities with Uncertain Cash-Flow

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Abstract

Competitive and regulatory pressures in the sphere of their core business tempt insurance companies to increase their profitability by earning extra income on a remoter playground, on the investment side. One example of such activity is the increased use of callable or puttable bonds and of all kinds of securitized mortgages in the U.S. market. All these securities have one thing in common: the uncertainty of their future cash-flow.

The aim of this paper is to describe a methodology which allows for a simple and proper valuation of these securities. This is achieved by the option-adjusted spread concept (OAS). The OAS takes into consideration the incremental return of a bond compared to the term structure of government bonds as a whole, as opposed to the traditional return measure yield, which is based on a static concept referring to just one point on the yield curve.

Keywords

Interest-rate-contingent cash-flow, term structure of interest rates, implied forwards, binomial tree, option-adjusted spread, prepayment model.

Résumé

Du fait des pressions exercées par la concurrence et les organismes de contrôle dans la sphère de leurs activités centrales, les compagnies d'assurance sont tentées d'accroître leur rentabilité en s'assurant des revenus supplémentaires sur un terrain quelque peu différent, c'est-à-dire dans le domaine de l'investissement. L'utilisation accrue des "callable" et "puttable bonds" ainsi que de toutes sortes de "securitized mortgages" sur le marché nord-américain illustre bien cette activité. Toutes ces valeurs mobilières ont un point commun: le caractère incertain de leur futur cash-flow.

Cet exposé a pour but de présenter une méthode permettant d'évaluer simplement et correctement ces valeurs mobilières. Cela est possible grâce au concept d'"option-adjusted spread" (OAS). L'OAS tient compte du rendement supplémentaire d'une obligation par rapport à la structure du terme des obligations d'État dans leur ensemble, par opposition à la méthode de mesure traditionnelle du rendement à l'aide du taux d'intérêt, laquelle est basée sur un concept statique se référant à un point unique sur la courbe des taux.

Mots clefs

Cash-flow dépendant du taux d'intérêt, structure du terme des taux d'intérêt, "implied forwards", arbre binomial, "option-adjusted spread", modèle de paiement anticipé.

1. Investment principles of insurance companies

The main function of asset management within an insurance company is to produce a sufficient return to support the core business, while at the same time limiting the financial risk to which the company is exposed. The desired risk-return profile is determined to a high degree by the liabilities arising from the insurance business. Risks include market, credit and liquidity risks, to mention only the most important ones. Limiting them is not easily accomplished, otherwise A.M. Best would not state: „Asset management remains an essential element of A.M. Best rating analysis. Going forward, we will place increasing emphasis on a company's ability to manage its portfolio in a dynamic interest rate environment...“.¹⁾ This statement underscores the importance of this issue. Since fixed-income securities play a major role in the asset allocation process for insurance companies, the following considerations will focus on that asset class.

One straightforward approach is to match the liability profile exactly with assets of the appropriate maturities. This process is known as Cash-Flow Matching, which involves very little risk but does not leave much room for extra returns. Deviations from the given maturity of the liabilities open opportunities to earn extra income by taking advantage of the usually positively sloped term structure of interest rates.

In order to limit the risk of failing to meet the liabilities, the Macaulay duration of the assets has to be adjusted constantly to the Macaulay duration of the liability profile (Duration Matching).

Furthermore, the convexity of the assets should be greater than the convexity of the liabilities.²⁾

Another way to improve investment income is to take on more credit risk. For example, in the 5-year maturity range a BBB issue yielded about 100 basis points more than 5-year treasuries. In maturity terms, this corresponds approximately to the yield spread between a 1-year and 5-year treasury. Both figures are as of end of April 1996 from the US market. However, monitoring the credit quality of bond issuers requires substantial resources, which a company might not want to build up and maintain.

A third alternative for boosting asset returns is to buy a relatively new fixed-income security generation. These securities contain, in addition to their traditional fixed-income part, features which make their cash-flow uncertain. In turn, they yield a significantly higher return than older generation fixed-income securities of comparable credit quality.

How to manage these securities in a dynamic interest rate environment? In this paper we will investigate this question, posed by A.M. Best, and explain the option-adjusted spread method by analyzing two examples explicitly. In order to focus on the main issues, a simple probabilistic model has been chosen, but it is clear how to generalize the idea to more general situations.

The paper is organized in the following way: In the next section we will deal with four examples of fixed-income securities with uncertain cash-flow and look at their individual characteristics. In section 3 we lay out

the underlying probability model, which is based on a binomial tree and forward rate analysis. In section 4 we describe the option-adjusted spread approach and apply it to two examples.

2. Examples of fixed-income securities with uncertain cash-flow

1. Callable bond:

The buyer of a callable bond acquires a traditional fixed-income security and at the same time sells a call option to the issuer. This call option entitles the issuer to buy back the bond prior to the stated maturity at a fixed price.

2. Puttable bond:

The buyer of a puttable bond acquires a traditional fixed-income security and a put option at the same time. This put option entitles the bond holder to sell the bond back to the issuer prior to the stated maturity at a fixed price.

3. Various combinations (e.g. sinking-fund bond, bonds securitized from pools of mortgages):

A sinking-fund bond contains a mandatory schedule of nominal amounts which have to be bought back by the issuer in the course of time. The timing and prices of these redemptions are laid out in the prospectus. Besides the mandatory sinks there might also be additional voluntary sinking provisions, for example to double up. This provision gives the issuer the right to redeem twice as much as

specified in the prospectus. This sinking-fund acceleration corresponds to a series of call options owned by the issuer.

Furthermore, the issuer also possesses the open market option. It gives the issuer the right to buy back in the secondary market instead of having to call from all the bond holders. Due to this option, the bond holder cannot be sure that his bond holdings are retired according to the mandatory sink schedule.

There is a great variety of bonds securitized from pools of mortgages, which serve as collateral. One of the simpler structures are pass-throughs, which pass directly through the cash-flow received from the holders of the mortgage to the bond holder after subtracting a servicing fee. The cash-flow consists of principal and interest payments. Since the holders of the mortgage have the right to prepay part or all of their mortgage, there are scheduled and unscheduled principal payments. This prepayment option depends heavily on the interest rate movements and acts like a put option sold to every mortgage holder of the pool.

The common characteristic of these fixed-income securities is an interest rate contingent cash-flow. Depending on the future development of interest rates, the fixed-income security will have a range of possible redemption dates due to prepayment. Further, the cash-flow of bonds securitized from pools of mortgages depends on factors unrelated to interest rates. These factors influencing the prepayment behaviour include turnover rate on existing homes and defaults. Therefore, these securities need specific prepayment models to predict their cash-flow.³⁾

3. Pricing of fixed-income securities
via forward rates and binomial tree

A portfolio manager is offered an annually paying 3-year bond A with a 5 % coupon at 101. The issuer of the bond is entitled to redeem the bond at 100 one year before maturity. Should he buy, given the following term structure of interest rates for government zero bonds:

z_1 , one-year rate:	3 %
z_2 , two-year rate:	3.5 %
z_3 , three-year rate:	4 %
z_4 , four-year rate:	5 %

Instead of buying that bond A, he could also invest in a 1-year zero bond (with the nominal amount of 5 at 4.85), 2-year zero bond (with the nominal amount of 5 at 4.67), and a 3-year zero bond (with the nominal amount of 105 at 93.34)

and thus create a comparable cash-flow at a total cost of 102.86. That means the portfolio manager pays 1.86 less per every nominal amount of 100 than with government zero bonds in exchange for taking the additional risk of being called early and the additional credit/liquidity risk.

In order to judge whether this is sufficient, he has to take into account the option sold to the issuer of the bond. The value will depend heavily on the current market assessment of the further development of rates. These are known as implied forward rates⁴⁾ and can be derived from today's term structure of interest

rates by assuming that they are consistent with the term structure. For example, given the 1-year and 2-year rates z_1 , z_2 , we are able to calculate the 1-year rate one year from today, which we denote by f_1 . Here consistency means that a two-year investment yields the same return as a one-year investment followed by another one-year investment (roll-over). We obtain:

$$(1 + z_1)(1 + f_1) = (1 + z_2)^2 \Rightarrow f_1 = \frac{(1 + z_2)^2}{1 + z_1} - 1$$

and hence

f_1 , 1-year zero bond rate 1 year from today : 4 %

f_2 , 1-year zero bond rate 2 years from today : 5 %

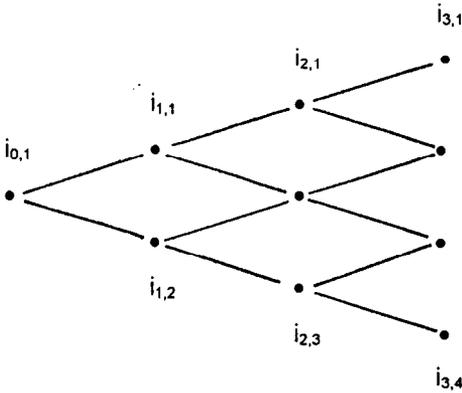
f_3 , 1-year zero bond rate 3 years from today : 8 %.

Forward rates are not predicting future rates. They are by definition break-even rates. If forward rates materialize over time, then all investments with the same time horizon would yield the same return, regardless of the maturities chosen. However, they are an indicator for expensive or cheap areas of the term structure of interest rates. We can also use them to derive fixed-income security prices:

$$\left[\frac{(105 + 5)}{(1 + f_2)} \cdot \frac{1}{1 + f_1} + 5 \right] \cdot \frac{1}{1 + z_1} = 102,86 \quad (I)$$

Using the forward rates, we are able to compute a price indication of the callable bond on the call date: 99.99. Based just on implied forward rates, the option would not contain any value and therefore 102.86 would be the fair value for bond A. However, so far we have ignored an important dimension as far as optionality is concerned: volatility of interest rates.

In order to account for the variability of rates, we introduce different paths of future 1-year rates and assign probabilities to each path. This is accomplished by using the standard approach of a sparse binomial tree:



Let $i_{0,1}$ be the current 1-year rate,
 $i_{1,1}$ be the upper 1-year rate in period 1,
 $i_{1,2}$ be the lower 1-year rate in period 1,
 $i_{2,1}$ be the upper 1-year rate in period 2,
 $i_{2,2}$ be the middle 1-year rate in period 2,
 $i_{2,3}$ be the lower 1-year rate in period 2, ...

The ratios $i_{1,1} : i_{1,2}$, $i_{2,1} : i_{2,2}$, $i_{2,2} : i_{2,3}$, ... are defined by the expected annual yield volatility of forward rates with ⁵⁾

$$\sigma = \frac{\ln(i_{up}) - \ln(i_{dn})}{2\sqrt{\Delta t}} \quad (II)$$

where i_{up} denotes the upper value of the binomial tree, i_{dn} denotes the lower value of the binomial tree and Δt denotes the length of the forward period in years, in our case $\Delta t = 1$.

Given the volatility, the ratio between the i_{up} and i_{dn} is determined by

$$i_{up} : i_{dn} = \exp(2 \sigma)$$

σ	$i_{up} : i_{dn}$
2.5 %	1.05
5 %	1.11
10 %	1.22
14 %	1.32

As most other standard interest rate models do, we assume equal likelihood of all interest rate paths. Furthermore, it is only reasonable to require that the binomial tree of 1-year interest rates reflect the current term structure of interest rates in the following way. For example, for a two-year zero bond the expected present value of its cash-flow given by the binomial tree has to be the same as the price observed in the market:

$$\frac{1}{2(1+z_1)} \left[\frac{1}{1+i_{1,1}} + \frac{1}{1+i_{1,2}} \right] = \frac{1}{(1+z_2)^2} \quad \text{(III)}$$

For a three-year government zero bond this translates into the following equation:

$$\frac{1}{4(1+z_1)} \left[\frac{1}{1+i_{1,1}} \left(\frac{1}{1+i_{2,1}} + \frac{1}{1+i_{2,2}} \right) + \frac{1}{1+i_{1,2}} \left(\frac{1}{1+i_{2,2}} + \frac{1}{1+i_{2,3}} \right) \right] = \frac{1}{(1+z_3)^3} \quad \text{(IV)}$$

and so on.

(II), (III), (IV), ... can be solved uniquely and with an assumed volatility of 14 % give us the following tree:

			12.00 %
		6.53 %	
	4.57 %		9.07 %
3 %		4.92 %	
	3.44 %		6.85 %
		3.71 %	
			5.18 %

Note that for small values of σ the pricing mechanism employed by equation (III), (IV), ... exhibits similarities to the one used in equation (I), as can be seen from the following deliberations.

Equation (III) can equivalently be expressed as

$$\frac{1}{2} \left[\frac{1}{1 + i_{1,1}} + \frac{1}{1 + i_{1,2}} \right] = \frac{1}{1 + f_1} \tag{V}$$

For small values of σ we approximate $\frac{1}{1 + i_{1,1}}$ and $\frac{1}{1 + i_{1,2}}$ by $\frac{1}{1 + f_1}$

in equation (IV), which then reads

$$\frac{1}{4} \left[\left(\frac{1}{1 + i_{2,1}} + \frac{1}{1 + i_{2,2}} \right) + \left(\frac{1}{1 + i_{2,2}} + \frac{1}{1 + i_{2,3}} \right) \right] = \frac{1}{1 + f_2} \tag{VI}$$

We conclude that for every single period the expected present value of a cash-flow occurring in one year is for small values of σ

approximately equal to the value given by the implied forward rates.

We now return to our example and determine the expected present value of bond A as given by the binomial tree. For each path we discount the cash-flow of bond A step by step starting at maturity by the appropriate rate $i_{k,t}$. As a result we obtain a value of 102.54, which tells us how much we would expect to pay for an investment in government zero bonds with the identical cash-flow. In comparison to a government bond investment, bond A offers today an extra return of 1.54.

4. Evaluation via option-adjusted spread

The portfolio manager has another offer: an annually paying 4-year bond B with a 7 % coupon at 105.3. The issuer of bond B is entitled to redeem the bond at 100 one year before maturity. Which one should he go for? There are different maturities and call features to consider.

Using our pricing model from section 3, we obtain an expected present value of 107.34 for bond B. Therefore, in comparison to bond A, bond B offers today an extra return of 0.50. However, so far the different maturities of both bonds have not been taken into account. For this reason we introduce a very helpful approach, which is to look at the incremental return which the portfolio manager can earn in addition to the term structure of government bonds after taking into account the different call schemes. We have seen in the previous chapter that the binomial tree has been constructed to price government securities correctly. To find out the incremental return, we adjust

the interest rate paths of the tree to fit the observed price in the market. This is done by raising or lowering all the $i_{k,t}$ rates of the binomial tree by a constant spread and thus pricing the callable bond in the same manner as described under 3, yet using the following underlying binomial tree:

			12.00 % + s
		6.53 % + s	
	4.57 % + s		9.07 % + s
3 % + s		4.92 % + s	
	3.44 % + s		6.85 % + s
		3.71 % + s	
			5.18 % + s

This spread „s“, which matches the observed market price, is called the option-adjusted spread (OAS) and can be viewed as the yield spread earned on top of the government bond rates. ⁶⁾

For the 3-year bond A the model comes up with an OAS of 61 basis points, for the 4-year bond B with an OAS of 58 basis points.

This result shows that, in comparison to bond B, bond A offers an extra return of 3 basis points on an annual basis after considering the different call features and maturities. Of course, the OAS is not the only factor on which an investment decision should be based. It will also be greatly influenced by the portfolio manager's perception of how interest rates will develop and how

sensitively the bond will react. On the other hand, the OAS offers a proper and simple way to condense the information available in the market to a single figure and to evaluate the cheapness/ richness of securities. It expresses the incremental return of a bond compared to the term structure of government bonds. This becomes extremely valuable, considering the high complexity of the securities described in section 2.

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