

Measurement of Risk

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Abstract

Various measurements of risk have been proposed for the appreciation of the results of stochastic cashflow tests regarding life assurance companies and pension funds. The risk which these measurements deal with can be broadly categorised into two types; volatility of the financial outcome and the risk of insolvency. This paper discusses the criteria for choosing the measurement of risk referring to a collection of works written by actuaries and economists.

Résumé

Pour apprécier les résultats des prévisions stochastiques du cash-flow à l'égard des compagnies d'assurance sur la vie et des caisses de pensions, différents dispositifs de mesure des risques ont été proposés. Ces derniers tendent généralement à évaluer deux catégories de risques: variabilité des résultats financiers et probabilité de ruine. Le présent article examine les critères de choix concernant les instruments de mesure des risques en se référant à une série de travaux d'actuaristes et économistes.

Keywords

Risk, matching.

1.Introduction

1.1. There have been many works which deals with stochastic cashflow projections. After calculating future cashflows, it is necessary to evaluate the results in some ways. A typical method of measurement may be the use of a set which measures the reward and the risk.

This paper discusses the choice of the measurement of risk for financial institutions which take long term liabilities, in particular, pension funds and life assurance companies.

1.2. In this paper, firstly, various measurements of risk appeared in past works are reviewed, and an analysis is given in section 2. Section 3 discusses the use of cashflow projections to seek appropriate choice of measurement of risk. Section 4 gives a brief discussion on possible objectives of pension funds and proprietary life assurance companies. In section 5, the concept of 'matching' is analyzed in the light of discussion on risk in previous sections, and finally, section 6 gives a conclusion.

This paper is based on part of Chapter 5 of Fujiki(1994), with thorough review and expansion of the contents.

2.Measurement of risk

2.1. There have been many works on the management of assets and liabilities applying the idea of stochastic cashflow projections. In those works, the results of the cashflow projections are measured in some

ways. In particular, various measurements of risk have been introduced in those works. We shall start the discussion by revisiting some of those works related to this topic.

2.2. Markowitz(1952) identified risk as the variability of financial outcomes, and adopted the standard deviation of the residual assets as the measurement of risk.

Markowitz(1959) gave a more generalised discussion on risk, and listed alternative measurements; semi-variance, expected value of loss, expected absolute deviation, probability of loss and the maximum loss. Also, he further developed the discussion using utility functions. However, Modern Portfolio Theory has been developed using the standard deviation of the residual assets (Moore(1972 and 1984) and Cummins(1990)), and other measurements of risk have not been discussed so fully. Markowitz(1987) argued that the use of mean-variance is justifiable

- as a quadratic approximation to utility functions,
- because of the ease of performing the numerical and theoretical calculations, and
- because of its objectivity.

2.3. Wise(1984a,b and 1987a,b) and Wilkie(1985) developed the P-E-V space model, which deals with the portfolio selection problem by looking at the initial amount of the assets, P, the expected value of the residual assets, E, and their standard deviations, V. This model is pri-

marily based on the approach which measures risk by a standard deviation of residual assets after meeting all the liability outgoes. Risk of insolvency was treated as one of boundary conditions.

2.4. Sherris(1992) developed a framework based on the idea of maximising the utility of residual assets, and analyzed the choice of portfolio. In the paper, the P-E-V model above has been explained as a special case of his framework with the utility function $U(S) = -\exp\{-S/r\}$, with S : the residual assets and r : the risk tolerance of the fund. The risk dealt with by this utility function is the risk of volatility because the function is concave. A further extension of the framework to allow rebalancing of the portfolio has been briefly discussed. However, he argued that the amount of the initial asset is in practice more likely to be determined so as to limit the risk of insolvency within a specified level, and expressed some reservation on the P-E-V model.

Haberman(1993) applied the standard deviations of the contribution rate and the funding level together as the measurements of risk in stochastic projections in order to obtain optimum funding strategies.

2.5. Daykin et al(1987) took the probability of ruin as a measurement of risk for a general insurance company.

Clarkson(1989 and 1990) identified the risk as the expected value of $W(x-L)$, where,

W : severity of loss

- L: minimum achievement required to make $W = 0$
- x: the amount of the residual asset.

Clarkson(1995) emphasized the importance of downside risk, and introduced the concept of 'solvency frontier'.

2.6. Smink(1991) used two measurements of risk; the standard deviation and the below target standard deviation. Similarly, Booth(1995) applied semi-variance, which measures the likelihood of under-performance, and variance for pension funds.

2.7. As we have seen, one approach may be to define the risk as the volatility of financial outcomes. The measurement for volatility may be a standard deviation, the difference between the upper and lower quartile results, and so on. Here, the financial outcome may be defined as the residue after discharging all the liabilities, the value of assets after a specified time, or the difference between the value of assets and the value of liabilities at a specified time.

An example of such an approach is seen in Markowitz(1952, 1959 and 1987). Wise(1984a,b and 1987a,b) and Wilkie(1985) treated volatility as the main risk, while risk of insolvency was treated as one of boundary conditions. Sherris(1992) is based on a similar view, but with some more weight on the risk of insolvency. Haberman(1993) also used variability as the measurement of risk.

2.8. On the other hand, another type of approach defines the risk as the probability of insolvency, which can be the probability that the value of the asset becomes less than the value of liabilities, the probability of asset becomes 0 before discharging all the liabilities.

Daykin et al(1987) and Clarkson(1989, 1990 and 1995) may be the examples of this approach.

2.9. Some papers (Smink(1991), and Booth(1995)) applied two types of measurements without showing any preference.

2.10. In the examples above, risk is defined by only two distinctive approaches. One approach is to regard the volatility of financial outcomes as risk, and the other regards the probability of failing to achieve a certain target as risk. Risk of insolvency is included in the latter.

2.11. We shall briefly examine the relationship between these types of measurements of risk. As a representative of the former, we shall take the standard deviation of surplus after paying out all the liability outgoes, and the probability of ruin as that of the latter.

2.12. Now, where the mean is equal to or more than 0 and the probability distribution follows the shape of the normal distribution, the more the standard deviation is, the higher the probability of ruin is. If the standard deviation is 0, the probability of ruin would become 0.

In particular, where mean is 0, the two measurements may give similar evaluation of risk. (they could be used as an approximation to the other with some limitations.)

An approach in asset liability management may divide the assets into two parts; one directly corresponding to liabilities and free assets. The first part (we shall call it as 'tied assets'), can be determined so that the mean of the ultimate surplus is 0.

If we consider only this part, two measurements of risk may give similar results. Here, investment policy of free-assets may be judged separately.

2.13. However, if mean is very large, a high standard deviation could be accompanied with almost nil probability of ruin. On the other hand, if mean is 0, in order to limit the probability of ruin to a certain level, the standard deviation must remain at a low level.

Similarly, a low probability of ruin does not necessarily indicate a low standard deviation because of possible existence of surpluses.

In these cases, two measurements have different meanings. In practice, some margins can be added to the tied assets so as to add some security within that part, and the expected surplus may be more than 0.

Also, risk may not necessarily be measured for the tied part only. For instance, the level of risk of insolvency as a whole can be very impor-

tant, too.

Furthermore, the probability distribution of the ultimate surplus may not be symmetric around mean, and the two measurements can show different level of risk even if the mean is 0.

Therefore, the two measurements can give totally different evaluation regarding the level of risk, and it is important to choose an appropriate measurement of risk because different measurements could lead to totally different conclusions.

2.14. There can be different approaches in recognising risk. For example, failing to maintain liquidity can be an important risk if assets available to a financial entity have limited marketability. However, in the following discussion in this paper, we shall limit our scope into the two approaches which are discussed in sections 2.10.-2.13..

3.Use of Cashflow Projections

3.1. Now, the question is how the measurement can be chosen. To answer this question, we need to consider the reasons for performing cashflow projections. The purposes depend on who is using the projections and on the nature of the financial entity. Here, the nature may be characterised by the legal and tax position of the financial entity, the nature of liabilities it holds, and so on.

If managers of a financial entity are the users of projection results, man-

agers' purpose of using the results is to achieve the objectives for which the financial entity is established and statutory requirements which the entity must satisfy. (The objectives of different financial entities are discussed further in section 4.)

3.2. Where there is a choice of actions for the management of a financial entity, a judgment may be made after considering the likely consequences of each possible action. If future consequences are subject to probabilities of future events, stochastic cashflow projections may be performed. (or, deterministic projections as proxy to stochastic ones.)

After performing projections, some evaluation process must be followed to draw a conclusion on the matter. Because the decisions are made to achieve the objectives of the financial entity, the evaluation should be made in the light of the objectives. Therefore, the evaluation process should assess how closely a particular cashflow pattern satisfies the criteria, which reflect the objectives appropriately.

3.3. One way to check the result is how far the objective is achieved, or how much the result contributes to the achievement of the objectives. For example, if the objective is to maximise the earnings of a company, the result may be measured by the earnings, and the criteria is if the earnings is above the target figure, or which choice gives better earnings. (usually represented by the mean of the projected earnings.)

3.4. At the same time, there can be defensive objectives (targets). For

example, a minimum to be achieved under whatever circumstances may be set as a target, and the maximum tolerable level of the probability of obtaining the result under the target may be specified. Then, the risk in the projection results may be measured in the light of this criterion.

Volatility of results itself could be an undesirable characteristics, and the volatility of the projection results may be chosen as a down-side measurement. Volatility may also be a proxy to the likelihood of ruin. Should other conditions are the same, the more volatile the results are, the more risk of ruin there is.

3.5. To evaluate a particular pattern of cashflow, the criteria for judgment should be expressed in mathematical formulae so that they can be directly applied to the results.

At the same time, criteria should be practicable. If the method is too complicated, it may be impossible to derive any conclusions owing to, for example, insoluble equations.

Furthermore, simplicity of criteria helps understanding of the evaluation process. This can be very important in a practical context to persuade the various parties involved and to obtain support from them on the conclusion drawn from the investigation.

These requirements, practicability and simplicity, may conflict against the requirement to reflect the objectives closely. In a sense, setting cri-

teria is to find a reconciliatory answer between these two.

3.6. In section 3.1., the need to obey statutory requirements was mentioned, but here, the supervisory authority itself can also be regarded as the user of cashflow projections, and they perform projections according to their own needs.

The purposes of using cashflow projections by supervisory authorities can be different. They have own purposes; e.g. to protect policy holders of life assurance companies, shareholders (current and potential), or members of pension funds, and they may regard the downside risk as most important than the managers may feel.

Further, supervisory authorities may prefer formulated approaches in performing projections and in evaluating the results, so that the approach could be disclosed as part of regulations. Once such formulae is set, this becomes a requirement (or one of objectives) to the managers.

Other parties who have interest in the financial entity may also have different purposes. For example, potential acquirers of a company may use projections to assess future profitability of the target.

Because of limited space, we need to halt discussion on supervisory and other objectives here, and concentrate on the case of managers of financial entities.

4. Objectives of Financial Entity

4.1. We shall now consider what can be the objectives for which different financial entities are established. Objectives can vary from one entity to another. Here, we shall consider two cases; pension funds and life assurance companies.

4.2. In the case of a pension fund, assets are held to pay benefits, and scope for holding free assets might be limited. In particular, if tax privileges are given to pension funds, punitive taxation might be applied to that part.

Who manages the pension fund may depend on regulations, and would differ in each country. In some cases, trustees manage the fund, while, in other cases, pension funds are run by the sponsoring employers. The objectives to be pursued may be different for trustees and sponsoring employers.

Also, the objectives depends on legal and tax frameworks, which can be different in each country. Furthermore, the objectives would differ by the nature of the benefits provided by the scheme; particularly, defined contribution schemes and defined benefit schemes may be run totally differently.

Here, we shall take an example of trustees of a defined benefit scheme. We assume that they are responsible to protect the members' rights within the prescribed rules, while the sponsoring employer can termi-

nate the scheme unilaterally.

4.3. For the trustees, there can be objectives to be achieved as below;

stable (predictable) contribution
discontinuance solvency, and
on-going solvency.

Haberman(1994) pointed out the first two as objectives. Particularly, the stability of contribution was discussed fully. Now, we shall examine each possible objectives in turn.

4.4. A pension fund is established in order to provide pension and other benefits to the existing and future members. Continuation of the fund depends on the ability and willingness to pay contributions by the sponsoring employer, who may prefer smooth contribution patterns from year to year. In this sense, stabilising the contribution rates (or to avoid sudden rises in contribution rates without disturbing the payments of promised benefits) is an important objective.

However, it must be noted that, in the case of defined benefit schemes, the real cost of providing the predetermined benefits depends on the actual experience in terms of the investment performance, salary growths and the demographic factors, and it would not be possible to achieve a constant contribution rate except for by coincidence, unless the scheme is a defined contribution scheme.

4.5. On the other hand, the company might close down suddenly without any assets for the members. To prevent such a case, it would be prudent to try to hold a certain level of discontinuance solvency. The level required would be the level at which all the accrued rights can be fully met out of the existing fund.

This aim may need to be balanced with the above because pursuing discontinuance solvency excessively could make the continuation of the scheme difficult.

Also, this is a likely area where statutory restrictions, such as minimum funding requirements, may be imposed for the protection of employees. If there is one, the managers need to satisfy the requirement. (It becomes very important for continuation of scheme if the possible penalty is a closure of the scheme.)

4.6. The on-going solvency target may be set as the funding level equal to (or exceeding with a certain level of margin to) the standard fund, which may include allowance for future salary increases. The amount equal to the actuarial liability of the Projected Unit Method under a set of realistic assumptions may be considered as the minimum funding level which satisfy this condition.

This target on-going solvency level can be higher than target discontinuance solvency level, and by maintaining the on-going solvency, the discontinuance solvency may also be achieved at the same time.

To stabilise, or to make future contribution levels comparatively predictable, it is desirable to follow a funding plan which makes the contribution rates stable at least in theory. Such funding plans tend to require a high level of funding. On-going solvency is defined in connexion to this level. In this sense, this aim can be a proxy to stable contributions.

Therefore, maintaining on-going solvency may not be an objective by itself. However, it can be a proxy to two other objectives.

4.7. Priority among the objectives may be affected by regulations. (e.g. strengthening of penalty against solvency requirement could push solvency the top agenda, because maintaining a certain level of solvency is necessary to continue the scheme.)

4.8. In the case of life assurance offices, the objectives would depend on

- the constitution of the office; proprietary or mutual,
- in the case of proprietary companies, degree of importance of with-profit policies among their portfolio, and
- the position of each party involved; managers, shareholders, with-profit policyholders or non-profit policyholders.

Booth et al(1993) applied the utility function introduced by Sheris(1992) to further investigations on the asset allocation problem for investors with and without specific liabilities, and an application to

various types of life assurance companies was discussed.

We shall briefly discuss the objectives of managers of proprietary life assurance companies.

4.9. Managers are responsible primarily to protect and enhance the shareholders' interest. Shareholders would wish good return from the business. The managers may seek this objective by trying to sell profitable products, to limit expenses, to increase sales, and so on.

To the shareholders, main risk to care might be a loss in the value of capital which they have invested in the business, and volatility in their returns.

4.10. In one extreme, if the office closes down, the shareholders would lose possible future gains, and also may not be able to recover enough money from the residue. The latter is particularly so, if statutory protections on policyholders' rights are strongly exercised.

So, the closure, other than sale of the company to other parties, should be avoided. If this situation is considered serious, the measurement of risk should be down-side type.

4.11. On a closure, policyholders are also likely to lose by receiving less money than the potential value of their policies. Therefore, for the protection of existing and potential policyholders, regulations which re-

quire the companies to maintain a certain level of solvency may be imposed.

Failing to maintain such statutory requirements might trigger a closure before worsening the losses to policyholders, but shareholders may be more likely to suffer a loss on such occasions.

If the failure is not serious, a penalty might be imposed. This would directly damage the profitability of the business, and more importantly, the company's reputation may be damaged.

Therefore, following regulations would become an important objective to the managers.

4.12. From 4.10. and 4.11., importance to maintain an adequate level of solvency may be concluded.

Furthermore, once the company goes into a financial difficulty, the company would need to raise an extra capital to increase the solvency. However, this could be difficult because existing and potential shareholders may not be confident about the future profitability of the investment into the company. This is different from pension fund which could obtain extra resources from the sponsoring employers.

In this sense, the managers would need early warnings on the danger of insolvency.

At the same time, existing and potential policyholders would like to see a good level of solvency, and for the company, maintaining a good standard of solvency is important to remain in business successfully.

4.13. Now, we shall consider the other risk of volatile investment return to shareholders. This may be caused by volatile share prices and volatile dividend payouts. Here, dividend payouts can be stabilised to some extent by building up some buffer fund, and this risk may not be practically important.

Volatile share prices would be caused by moves in market average of share prices, prospects of the life assurance industry, the company's own achievements and prospects, and various other factors. Many of them are difficult to control, but the managers may have some control over their business to stabilise profits emerging from it.

4.14. Risk of insolvency, absolute or statutory, is always important. However, the relative importance of this compared with the risk of volatile business achievements depends on the circumstances, and it is difficult to draw a simple conclusion on this matter.

The situation would be dependent on at least the following two factors.

The first is the degree of risk of insolvency. If the company currently holds sufficient free assets and the risk of insolvency seems remote, the

managers may care other risks, such as volatile profits, more.

The second is the degree of preference towards stable profit emergence by the shareholders. Each investors may have different views on this matter, and the average attitude might only be guessed through the movements in share prices.

The final decision what priority or weight should be put on each risk would rest on the managers.

5.Application to Matching

5.1. The phrase ‘matching of assets and liabilities’ has been widely used in academic and practical contexts with various meanings, and the precise meaning may not be clear. ‘Matching’ is a concept which is closely related to ‘risk’, and in this section, we shall discuss the meaning of ‘matching’ in relation to risk.

5.2. Firstly, it can mean equalising the future incomes from the existing assets and outgoes from the existing liabilities at any time in the future. This guarantees the payment of liabilities as they fall due without the need for extra resources, disposals of assets or any future reinvestments. Such matching is sometimes referred to as ‘absolute matching’, the concept of which comes from Haynes & Kirton(1952).

5.3. The phrase ‘matching by duration’ is also often used. This matching emphasizes the matching of the timings of the cashflows, although

the amounts of outgoes may not be the same as those of incomes. In this sense, the 'matching by duration' may be regarded as a weakened form of 'absolute matching'.

5.4. Next, it can mean equalising the degree of increases/decreases of the future incomes from the assets and the future outgoes from the liabilities. This is sometimes referred to as 'matching by type'. This does not guarantee full repayment of liabilities. Rather, the phrase can be used even if there is apparent shortage in assets compared with the liabilities.

The value of liabilities held by a financial institution may increase or decrease because of the changes in economic environment. If the matching in this meaning is achieved, the value of assets will increase or decrease according to the changes in the value of liabilities to the same extent. Then, the ratio of the value of assets to the value of liabilities will be unchanged by such changes in the economy. Thus, a financial institution can reduce the volatility in the difference between liabilities and assets, and stabilise its finance.

5.5. As a variant to matching, Redington(1952) introduced the concept of 'immunization' for fixed liabilities. (the concept of duration which is essential to immunization theory was known to Macaulay in 1930s. (see Shiu(1988))) Immunization is an arrangement of the distribution of the term of the assets which reduces the possibility of loss arising from a change in interest rates, under certain assumptions. Redington(1952)

noted that immunization is important for a financial entity to remain solvent.

5.6. Here, it must be remembered that immunization only deals with fixed liabilities and fixed interest assets, while the majority of assets and liabilities of pension funds tend to produce cashflow which are more or less linked to inflation (particularly, in the U.K.).

Further works (e.g. Boyle(1978) and Milgrom(1986) have been carried out to generalise the idea of immunization under various yield curve models.

5.7. Wise(1984a,b) gave an interpretation of the meaning of ‘matching’ as the state where the square of the residual assets after meeting all the liability-outgoes is minimum in his stochastic model. This is equivalent to the standard deviation of the residue is 0 as shown by Wilkie(1985).

5.8. Now, we shall consider the meaning of ‘matching’ further. Broadly speaking, by achieving a matched position, a financial institution is exposed to less volatility in the amount of residual assets after meeting all the liability outgoes, or less probability of insolvency than otherwise. In a sense, matching is an attempt to minimise risk. (or, to achieve a risk-free asset-liability combination)

In this sense, matching depends on the type of risk to be minimised.

5.9. Absolute matching avoids any volatility and insolvency, while matching by duration minimises the ratio of value of assets and liabilities under varying interest rates. Matching by type minimises the volatility of the ratio of values of assets and liabilities under relative changes in value of different types of assets and liabilities. These matching other than absolute matching contribute primarily to reduce the variability of the ratio, but they do not necessarily require sufficient assets to cover the liabilities. If assets held are sufficient, these matching contribute to reduce the risk of insolvency, too.

Wise(1984a)'s definition of 'matching' may be an example of matching which intends to minimise the risk of volatility. This matching may not directly reduce the risk of insolvency.

5.10. Immunization requires a certain amount of assets to cover the liabilities, and immunization contributes to avoid insolvency at least in theory. However, it may not reduce volatility because immunization in Redington's sense may produce small surpluses which are unlimited in theory.

5.11. Here, as pointed out in section 2.12., reducing volatility with positive expected surplus would reduce the probability of ruin. At the same time, reducing insolvency would normally limit possibilities of high returns, and would result in reducing volatility.

However, the two types of risk are different as discussed in section

2.13.. For example, risk of volatility does not depend on the relative size of the assets compared with the size of the liabilities, while risk of insolvency does.

This difference may be well illustrated by considering the concept of mis-matching reserve, or free assets. The existence of 'mis-matching reserve' is supposed to compensate the risk caused by a mis-matching of liabilities and the assets which are tied to them. Here, risk means the risk of insolvency because increasing the amount of assets may reduce the risk of insolvency, but may not the risk of volatility unless the asset mix of 'mis-matching reserve' mitigates the degree of mis-matching by type or duration in the assets corresponding to the liabilities.

It is important to consider an appropriate measurement of risk when we discuss matching. Otherwise, inappropriate definition of matching is given, and conclusions drawn from that definition might direct a financial entity to a wrong course of action.

5.12. As we have discussed in sections 2-4, risk depends on each financial entity's objectives and the exact role with the entity of the person who uses the projection results. Therefore, the type of matching (or immunization) which should be considered for a particular financial institution depends on the objectives.

For example, if maintaining stable contribution rates is important to a pension fund, matching to be considered would be matching by dura-

tion, matching by type, or other matching which intend to minimise volatility. On the other hand, if remaining solvent is important objective for a life assurance company, matching to be considered would be immunization, or other matching which intend to minimise probability of insolvency.

6. Conclusion

6.1. As we have seen in section 2., measurements of risk which are frequently used in past works for the evaluation of stochastic projections can be categorised into two; 'volatility type' and 'down-side risk type'. They have different characteristics. (see section 2.13.)

6.2. The choice of the measurement is important to make appropriate judgments, and the choice should be consistent with the objectives of the person who use the cashflow projection.

6.3. Various parties of different financial entities can have different objectives, or different priority on each objective. In the case of pension fund trustees of a defined benefit scheme in section 4., stabilising contribution rates and maintaining discontinuance solvency may be the most important objectives. In the case of life assurance companies, because of relative difficulty in obtaining extra funds once the company's finances becomes weak, requirements for maintaining solvency may be more emphasized than pension funds. However, the final choice would be a matter of judgment, and there may not be any definite answers.

6.4. In section 5., we have discussed that matching can be taken as the state of minimum risk. Therefore, exact meaning of matching depends on the definition of risk in mind. Immunization may be interpreted as a type of matching which minimises the risk of insolvency, but not necessarily the volatility of financial results.

6.5. Discussion on the objectives of financial entities may be difficult, and would not have definite answers. However, I believe that such discussion is as important as the discussion on cashflow projection techniques, and I hope more discussion is made in this area.

Finally, I would like to thank Professor Haberman (City University, U.K.) for his comments.

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