

Market Efficiency

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Abstract

This paper commences by defining market efficiency and asking why the subject should arouse such heated controversy within the actuarial profession. To answer this we first consider the implications of efficiency or otherwise from the perspective of both investment managers and traditional actuarial practice. We then go on to examine the quantitative evidence in favour of efficiency (after allowing for transaction costs) and critique some of the most celebrated contrary evidence (in particular the size effect). However, accepting that simple (testable) mathematical rules are unlikely to capture sophisticated investment skill we also consider whether more complex processes could add value.

We then show how market efficiency fits into a hierarchy of market hypotheses from full equilibrium to the basic law of one price and consider again the role of "transaction costs" in blurring the overall picture and dictating which models might be appropriate for practical applications.

Finally we address the twin conundrums (which we link) posed against market efficiency: Why is the equity risk premium so high? Why are the equity markets so volatile (for example the 1987) if they are responding only to new information?

Résumé

Ce papier commence par définir l'efficacité du marché et de s'interroger pour savoir pourquoi le sujet provoquait tant de controverse échaudées parmi la profession des actuaires. Pour y répondre nous prenons en compte en premier lieu, les implications de l'efficacité, ou le contraire, du point de vue, tant des gérants, que de la profession actuarielle traditionnelle.

Ensuite, nous examinons les preuves quantitatives en faveur de l'efficacité (en tenant compte des coûts de transactions) et nous jetons un regard critique sur certains des argumentaires les plus connus (en particulier l'effet de taille). Néanmoins, en reconnaissant que des règles mathématiques simples (démonstrables) sont peu susceptibles d'appréhender un processus d'investissement sophistiqué, nous examinons la valeur ajoutée d'approches plus complexes.

Nous démontrons ensuite comment l'efficacité de marché s'inscrit dans une hiérarchie d'hypothèses de marché de l'équilibre plein, à la loi fondamentale d'un prix, et nous considérons de nouveau comment les coûts de transactions affectent le scénario d'ensemble, et dictent les modèles utilisables pour des applications pratiques.

En dernier lieu nous répondons aux deux dilemmes (que nous relient) contre l'efficacité de marché. Pourquoi la prime de risque sur les actions est si élevée? Pourquoi le marché des actions est si volatil (par exemple en 1987) s'il réagit seulement à des informations nouvelles.

Keywords

Efficient markets, market hypotheses, investment skill, transaction costs, size effect, risk premium, information.

1 - INTRODUCTION

1.1 *What is Market Efficiency?*

1.1.1 Although often confused with specific equilibrium pricing theories (such as the CAPM, for example) and 'random walk' models of security price movements, the theory of capital market efficiency itself is only an assertion that *current prices* of securities *fully* and *instantaneously* reflect *all available information*.

1.1.2 Given this apparently rather innocuous definition it is difficult to see why the concept should have aroused such heated controversy, not least from within the actuarial profession, where almost every non trivial item in the basic statement appears to be questioned. For example Smith (1996) explains how even the idea that a market *price* might be a useful piece of information is seriously questioned by traditional methodologies. Furthermore, the widely adopted model proposed by Wilkie (1984) suggests that changes in gilt prices neither fully nor instantaneously reflect the changes in available information based on his autoregressive inflation model (and instead assumes that a rolling past average of recent inflation is a good predictor of future gilt yield changes). Clarkson (1996) goes further to suggest that the whole idea of any rational behaviour of markets should be questioned and replaced by ideas of chaos and irrationality.

1.1.3 Much of this actuarial opinion appears to be based on anecdotal evidence rather than detailed analysis. By contrast, as we shall see below, most academic research appears to support the idea that markets are broadly 'efficient' and instead seeks a finer dissection of the concept of 'available information'. There is a form of duality here - if prices reflect a certain information set then use of that information set cannot lead to excess returns. Efficient markets are then defined, more or less interchangeably, either in terms of time series properties of prices or in terms of trading opportunities. In particular, Fama (1970, 1976) defined three forms of efficient market hypothesis ("EMH"):

(1) *weak form efficiency*, which suggests that no investor can earn excess returns by developing trading rules based on historical price or return information

(2) *semi-strong form efficiency*, which goes further to argue that no investor can earn excess returns from trading rules based on *any* publicly available information, or

(3) *strong form efficiency*, which implies that no investor can earn excess returns using any information, whether publicly available or not.

As suggested above, the existence or otherwise of weak form efficiency would appear to be particularly relevant to common actuarial ideas of 'value' which are often based on what are essentially averages of historic data.

1.1.4 The remainder of this paper is laid out as follows:

Section 2 considers the implications of market efficiency or otherwise from the perspective of asset managers and also for more general actuarial modelling purposes.

Section 3 examines some of the academic literature testing for market inefficiencies. Most of these tests start by suggesting a specific (usually simple) form of trading opportunity and then investigate whether it would have made money or not.

Section 4 goes beyond the simple rules which can be back-tested mechanically, and asks whether the more complex processes employed by active managers have added value in the past.

Section 5 considers attempts by economists to construct and test market models which are consistent with efficiency.

Section 6 concludes with an examination of some features of historic returns which may seem puzzling in the context of efficient market models.

2 IMPLICATIONS OF MARKET (IN)EFFICIENCY

2.1 *The Investment Perspective*

2.1.1 In practice, only a relatively small (but growing) proportion of institutional assets are invested in tracker funds in the UK. This reflects a widely held belief in exploitable inefficiencies. There is also a widely held belief that heavy objects fall faster than lighter ones. Both these beliefs appear to have some empirical support, but merit more detailed investigation. Would-be physicists may test simplistic theories by dropping an open Sunday Times and a glass marble simultaneously from a tall building. This paper attempts similar experimentation to examine the EMH.

2.1.2 There is an interesting parallel between the efficient market hypothesis and the Mendeleev's periodic table of the elements. Since ancient times, alchemists had

attempted to turn lead into gold. If one were to believe Mendeleev's theory, then such attempts are fruitless. As the theory of the elements took hold, alchemists gradually abandoned their efforts to make gold, instead diverting their considerable intellects towards areas of chemistry which have turned out to be far more productive. The few alchemists who continue in their original tasks are conventionally regarded as cranks. In an interesting twist to the tale, modern nuclear physics does give us the possibility of turning one element into another, although nobody has yet turned this into a commercial proposition.

- 2.1.3 It is not immediately clear to us why there should in fact be such strong and vociferous objections to the efficient market hypotheses. There may be a natural inclination of active investment managers to be dismissive. However, this should not be so. After all, such professionals manage the vast majority of international assets and if one active manager is to argue that markets are grossly inefficient, we must assume a corollary that the vast majority of his peers are irrational and not maximising their (or their clients') wealth.
- 2.1.4 Furthermore, if active management really was as simple as buying when markets are low and selling when they are high, we would question why active managers should earn significant (or any) fees for their endeavours. It would seem far more flattering for active managers to argue that markets are indeed generally efficient *because* of their endeavours, rather than rejecting the hypothesis outright.
- 2.1.5 This interpretation of the role of active management is very close to the conclusion of the market equilibrium theorists, since it has long been argued that there would be no incentive for fund management firms to acquire and analyse information if they could not by doing so outperform a random (or 'index') stock picker. The argument that competent fund management leads to efficient markets is sometimes used to claim that markets could not be efficient even in principle for sustained periods of time - once the efficiency became generally accepted, investment managers would abandon any attempts at security analysis and markets would become inefficient again. However, the argument for such cycles does not stand up to close scrutiny (nor to comparison with historic data). Instead, as noted by Copeland & Weston, it is easy to demonstrate the existence of an equilibrium between this random stock picker and active manager, if allowance is made for the costs of gathering, processing and analysing information (and the risks associated with setting up and capitalising a fund management venture).
- 2.1.6 The fact that fund managers cannot (and do not) easily achieve either consistent outperformance (and associated client acquisition and retention) or boundless profits from proprietary trading deserves some explanation. Once again, it would seem that the most flattering theory, from the managers perspective, is that, despite his expertise and investment skill, efficient markets prevent him from making excess returns. The alternative is that, although opportunities exist for exploitation, the manager is consistently missing them: a theory which would call his skill and expertise into question.

2.2 *Implications for Actuarial Valuations*

- 2.2.1 While relatively few actuaries work exclusively in investment, an assessment of investment returns is implicit whenever an actuary considers a discount rate. It is therefore of interest to consider what difference a perception of market inefficiency could, or should, make to those discount rates.
- 2.2.2 A naive assessment of discount rates would relate them to the expected returns on assets. This would seem to justify the use of very high discount rates, commensurate with the excess returns to be obtained from exploiting inefficiencies. If, alternatively, one continues to use a conservative discount rate, a discounted cash flow valuation of assets should give a value considerably in excess of market value, which would have a similar effect on the overall solvency level.
- 2.2.3 Some caution needs to be applied when taking credit for future exploitation of inefficiencies in current valuations. In order to exploit future opportunities, a number of conditions ought to apply, including:
- The perceived inefficiencies must exist
 - The client must intend to exploit these inefficiencies
 - The client must be able to exploit the inefficiencies from an operational standpoint
 - Other investors must be constrained so as to prevent the opportunity being eliminated by market forces
 - The model employed should capture the inefficiencies accurately
- 2.2.4 More sophisticated approaches to the problem of discount rates do not relate specifically to the assets held, but rather to the level of 'systematic' risk in the liability cash flows. The assessment of systematic risk is more or less equivalent to identifying a 'matching' portfolio. The latter exercise can be carried out whether or not markets are efficient. Of course, this methodology would lead to all assets being priced at market. If markets are believed to be inefficient then liability cash flows would be deliberately 'mispriced', but in a consistent way to the assets. The principle at work here is that the 'law of one price' is a higher ideal than a clean relationship between risk and return. This is discussed further in section 5.
- 2.2.5 This procedure for calculating discount rates is internally consistent, but also has the advantage of robustness. If our perceived inefficiency turns out to be mistaken, then, provided we have applied this consistently to assets and liabilities, the effect of the mistake will all come out in the wash. Thus, it makes little difference whether the models used for valuation purposes turn out to represent efficient markets or not. Efficient market models are usually simpler, and behave in a more predictable fashion, than models based on inefficient markets. Since the end result may not rely on market efficiency, there would seem to be a strong case for using an efficient market model for valuation purposes even if the real market is believed to be inefficient.

3 TESTING FOR MARKET EFFICIENCY

3.1 *Nature of the Tests Applied*

- 3.1.1 One of the strongest arguments in favour of market efficiency is not empirical or statistical but rather arises from the way in which the theory developed. Some theories start on paper and then researchers go in search of empirical evidence to support them. The theory of efficient markets arose the other way round. When researchers failed to verify empirically what were believed to be foolproof methods of making money from trading, EMH was developed to explain their lack of success.
- 3.1.2 Ever since the theory was developed, the search for anomalies which disprove EMH has been fertile ground. Many of these investigations have suggested patterns in prices which appear not to be consistent with a full and instantaneous reaction to new information, as would be required by EMH. If these patterns can be used to forecast outperformance of one asset over another, then the changes in asset prices cannot be ascribed solely to *new* information. Some patterns which have been proposed are outlined below.
- 3.1.3 Before we consider the most celebrated of these tests, it is worthwhile to highlight some methodological issues which recur in many studies:

(1) *Risk adjustment of returns.* The orthodox view is that excess mean returns may legitimately be obtained by taking extra risk. Equivalently, this means that when market prices 'reflect' new information, this implicitly allows for some form of risk adjustment. The simplest tests of EMH compare long positions to short positions in the same asset, so that, if variance is used as a measure of risk, then the two portfolios have the same risk and so, it is argued, no mean return differential should exist. More complex tests attempt to adjust returns for some definition of risk before comparing means, and to reflect the observation that risk for any asset class may not be constant over time.

(2) *Estimation of expected returns.* Any analysis of 'excess returns' hinges critically on the difference between the actual outcome and expectation. Whatever the choice of model we usually have problems in estimating this expectation, for example, because of choice of samples. In addition, if risk adjustments are applied, then it must be borne in mind that the risk estimate may be biased.

(3) *Cumulation of returns.* Most theories of market efficiency consider expected returns in an arithmetic mean sense, since this combines linearly across different investment classes. As a result, some studies have chosen to add (rather than compound), returns to establish cumulative outperformance. For example, De Bond & Thaler (1985&1987) produced empirical evidence of overreaction by stock markets, claiming abnormal returns from a contrarian strategy. This was refuted by Conrad & Kaul (1993) who suggested that the outperformance was upwardly

biased because of the method of cumulation and that true returns were not associated with overreaction.

(4) *Cause-effect phenomena.* It is important to distinguish between the cause of a phenomenon and its effect. For example, are abnormal returns prior to a stock split anticipating the split or does the abnormal performance prompt the decision to split? More generally, one has to deal with so-called 'survivorship bias' which may mean that stocks which have performed in a certain manner may be more likely to appear in the sample data, for example, unsuccessful firms may drop out of an index.

(5) *Price data effects.* Academic studies need to consider the reliability of quoted stock prices and the time series structure of actual transactions. Would it have been possible to trade in large amounts at quoted prices? As an example, Niederhoffer and Osborn (1966), using actual transaction data, suggested that a reversal of price change was two or three times more likely than a continuation. It turned out that this effect was entirely caused by unexecuted sell and buy orders being triggered in turn.

3.2 *Tests of Weak Form Efficiency on Single Stocks*

- 3.2.1 An often expressed view is that one can tell whether markets are 'high' or 'low', for example after a major market rise or fall or when certain market indicators, for instance yields, P/E ratios or indicators of value relative to bond markets, fall outside some historic range. The difficulty with acting upon this view is that no rule is specified as to when to buy or sell and it is difficult to know whether markets will continue to rise or fall substantially, remain broadly unchanged or exhibit some reversal. However, it is possible to test a variety of very simple rules on single stocks.
- 3.2.2 Such tests have tended to focus on so called 'filter rules' as developed by Alexander (1961) and Fama and Blume (1966). These tests make no assumption about the distribution of returns but establish whether passive 'buy and hold' strategy can be beaten by a simple rule of the form 'hold the stock until it falls $x\%$, then sell and go short till it rises $x\%$ and repeat the process. This was tried for various filters ($x\%$). Their results showed that filters ($x\%$) of greater than 1.5% could not outperform the simple passive strategy but that smaller (positive) filters could beat the market with frequent trading. However, the trading profits were insufficient to cover the transaction costs involved in implementing the strategy. Further empirical tests of EMH were carried out by Jennergren & Korsvold (1975), who found relatively strong serial dependency in the Norwegian market, but again, this serial dependency was not large enough to permit profitable trading after allowing for the relatively high transaction costs.

3.3 *Mechanical Tests of Multi-Factor Weak Efficiency*

- 3.3.1 The most celebrated work which appears to violate the EMH is the so called size effect. Banz (1981) published the earliest and most often quoted empirical study into this phenomenon using the techniques developed by Fama & MacBeth (1973) to test the CAPM. In Banz' study, the size effect appeared to have roughly the same statistical significance as did β (the CAPM risk measure), with massive differentials (19.8% pa) between buying very small firms versus very large firms. Other anomalies which have been suggested include the January positive return effect, and excess returns attributable to investment in shares which have high yields, low P/E ratios or have a high "value" compared with "growth" characteristics.
- 3.3.2 Several authors have questioned the estimation of β used in calculating the size effect. Christie & Hertz (1981) have presented evidence that risk is understated for small size firms, while Roll (1981) and Reinganum (1981) have argued that the Beta for small firms will be biased downwards. However, even after adjustment for these effects, the size effect still appears to persist.
- 3.3.3 Another problem with the original work is the effect of transaction costs, since daily rebalancing was assumed. Roll (1982) and Blume & Stambaugh (1983) show that the size effect is cut in half if annual rebalancing is assumed. However, this again appears to offer only a partial explanation.
- 3.3.4 In a UK context two additional observations can be made. Firstly, although this may be a quirk of timing, the size effect has been conspicuous by its absence since it was discovered; small capitalisation managers have faced a tough time over recent years. Secondly, most such managers find the small capitalisation index itself a hard target to match. This suggests that it may not be possible to invest in this index in substantial amounts at the prices quoted (most studies use notional portfolios based on quoted prices, not actual portfolios)
- 3.3.5 A variety of other explanations for the size and other anomalies have also been advanced. In the interests of brevity, the following restrict attention to the size effect:
- (1) *Liquidity*. Some studies have suggested that the size effect is in fact a liquidity effect and note that the shares of small companies typically are less marketable and result in additional transaction costs. Further, the authors submit that there is often a substantial loss of value when markets crash so that their systematic risk is also high. Indeed, the evidence from 1987 in the UK was that the price decline for small companies was much higher than for main line stocks, and that beta ratios as conventionally calculated dramatically understated small company sector risk.
 - (2) *Agency effects*. The management of small companies are less easy to monitor as they are out of the limelight of public scrutiny. Managers may exercise a greater degree of control of the company with the risk that the interests of external

shareholders may be overlooked. The authors advance the possibility that this effect could be particularly serious in the event of a market downturn (1974) when managers could take action to protect themselves at the expense of external shareholders. Again, the implication is that systematic risk is high.

(3) *Valuation effects.* The size effect is usually measured by ranking companies according to market capitalisation. An alternative is to rank them according to some other characteristic, for example capital employed. This procedure usually shows a much smaller return premium. The argument is that naturally companies for which the market is demanding a high return will have a low market value and therefore a study which shows an association between value and subsequent return is inherently biased.

3.3.6 Whether these effects are real market anomalies, or due to mis specification of risk in the models used or unreliable price data (or whether they were real anomalies which are now arbitrated away), only time will tell.

3.4 *Testing of Analysts' Forecasts*

3.4.1 Since most of the above analysis was concerned with the weak form efficiency, we should mention that even the stronger versions of the EMH have gained support from empirical studies. For example, analysis of both published and unpublished analysts' forecasts have been undertaken:

(1) Davies & Canes (1978) analysed analysts' opinions published in *The Wall Street Journal*. They found that the publication of the information did have an impact on returns, but net of transaction costs the information could not generate excess returns.

(2) Dimson & Marsh (1984) analysed unpublished analysts forecasts made available to UK fund managers. Again they found that these forecasts could generate excess returns, but only sufficient to justify the costs (including a fair profit) of the analysts.

3.5 *Impact of Transaction Costs*

3.5.1 We have seen how, if trades could be accomplished at quoted prices, a number of trading rules and forecasting techniques could potentially generate excess returns. Strictly speaking, this contradicts efficient markets, as prices do not appear to have reacted fully and instantaneously to all available information.

3.5.2 However, in order to implement a strategy a number of costs must be incurred. These start with the dealing costs - brokerage, stamp duty, bid-offer spread and market impact. If the forecasts are the results of fundamental research, then information has to be collected, analysed and communicated to managers. When the costs of such exercises are taken into account, most of the apparent cases of outperformance become insignificant.

- 3.5.3 It would, however, be premature to conclude that investors should act as if markets are efficient just because active management, based on the existing evidence, appears not to have resulted in outperformance. If one has to dispose of assets, for example to pay benefits to insurance policyholders, then an ability to forecast returns could be of considerable assistance with timing. Essentially, at this stage of research, one has to recognise that this aspect of EMH has neither been proven nor disproven, and take a prudent view.
- 3.5.4 One might suppose that a skilled investment manager could, by good timing, achieve higher returns than an unskilled manager. A relevant factor in assessing this skill would be the subsequent performance of investments which were sold. Interestingly, this is not currently measured by the major actuarial performance measurement services.
- 3.6 *Other Issues*
- 3.6.1 Most other studies which claim the existence of market inefficiencies can be analysed as above. In particular, studies into alleged delays in market response to dividend and earnings announcements claim the existence of inefficiencies inconsistent with the semi strong hypothesis. These studies involve a complex series of hypotheses. For example, not only must the abnormal returns be hypothesised relative to a particular market model, but the dividend or earnings 'surprise' must also be hypothesised against some model for the expectation.
- 3.6.2 There is a final hurdle which no empirical study can really address. This is because the actual hypothesis in practical application is concerned with whether today's prices reflect all available information, a set which includes all publicly (or possibly otherwise) available analysis of that same set. Hence, even if past inefficiencies could be identified, it must be established whether these are now priced or whether they are persistent.
- 3.6.3 For example, suppose that Government produces inflation forecasts which systematically understate inflation. Initially, it may be rational to believe these forecasts. However, by the time it is possible to establish the existence of the systematic error, the efficient market hypothesis says that this bias will already be built into market expectations and a policy of selling gilts in the belief that the Governments inflation forecasts are too low will not earn any excess return.
- 3.6.4 In this context any theory of inefficiency needs to be harnessed to a convincing theory as to why the discovery of any anomaly will not lead to its demise, since it would appear that efficiency, like entropy, can only ever really increase over time if investors are rational expected utility maximisers. This issue is perhaps particularly appropriate to 'long term' actuarial models, where the older data may relate to markets either with very high transaction costs of where it is even unclear whether the price data itself is reliable for actual executions. Clarkson (1996) suggests that inefficiencies can persist because investors are irrational, although as

we discuss above, there appears to be no firm empirical evidence to support such persistency. Furthermore, in that event it would seem inevitable that the rational investors will ultimately dominate the market due to the cumulation of superior returns.

4. INVESTMENT MANAGER OUTPERFORMANCE

4.1 *Testing for Manager Outperformance*

4.1.1 In section 3, we considered strategies which were predominantly mechanical, and could in principle be applied by any market participant. However, it could be argued that market inefficiencies are far too subtle to be detected by these crude techniques. Indeed, true excess returns are only obtained by managers using highly sophisticated techniques. Understandably, such managers would keep these techniques close to their chests in order to protect their value.

4.1.2 One approach to evaluating such techniques would be to compare the performance of the so called 'quant' fund managers to the more traditional houses who claim to have a greater degree of subjective input. However, there is a danger that the significance of any performance differential may be overstated. Given the notorious herd instincts of managers with similar styles, it would be expected that *either* quant managers would outperform judgmental managers *en masse*, or vice versa, depending on which style ended up holding the lucky numbers over a particular measurement period. Furthermore, there is no guarantee that any manager's claims to sophistication are reflected in the actual stock selection process they employ.

4.1.3 It is in fact rather difficult to construct a test which could reveal traces of market inefficiency in manager outperformance. For example, we might consider whether active managers have outperformed various market indices. However, the majority of UK institutional funds are under active management. In effect, active managers are most of the market, so the few passive investors would have to perform spectacularly badly in order for active managers to beat the index by a small amount. Furthermore, even if passive index fund managers become a significant proportion of the market, then, by the index construction, the passive managers will roughly hold capitalisation weighted portfolios, so, by subtraction, active managers must hold more or less the same aggregate portfolio and so little performance differential will be detected between the two camps.

4.2. *Investment Managers Have Skill*

4.2.1 Although most studies have shown that investment managers do not outperform their target benchmarks and that past under or over performance is no guide to the future, there is some evidence that investment skill does exist. For example, various studies have shown that, in the absence of transaction costs, there is value in broker recommendations and that investment managers as a whole do outperform a random or buy/hold strategy. These observations cease to hold once the costs of acquiring and using the information or service are taken into account. Given that

investment skill does exist the question arises as to whether there are any investment managers who are able to outperform benchmark returns once transaction costs are taken into account.

- 4.2.2 In fact, one well known fund manager, Warren Buffett, has achieved substantial outperformance relative to the S&P 500 over many years. It has been suggested that this achievement proves that markets are inefficient. Of course, it is difficult to prove inefficiency: there is always some possibility that the result was achieved by chance rather than skill. Likewise, it is difficult to prove efficiency. However, sustained year-in year-out good performance of this magnitude is unlikely to have been achieved solely by luck and the question arises as to whether there are other explanations also consistent with efficient markets.
- 4.2.3 In addition, the micro structure of some investment markets means that certain organisations appear to have monopoly or oligopoly positions. For example, in the UK there is just one bank who has a significant presence both as a market maker in FTSE stocks and also as a trader on LIFFE, the exchange which trades FTSE futures. Thus, although there are sometimes discernible anomalies between futures prices versus the theoretical futures based off the cash index, there is a single organisation whose cost structure is such that these anomalies can be arbitrated. The limited depth of the two markets constrains in absolute terms the arbitrage profit that can be made. Similar monopolies or oligopolies apply in other world stock markets.
- 4.3. *Outperformance is not Inconsistent With EMH*
- 4.3.1 Even where an investment manager achieves substantial sustained good performance, EMH may not be disproved. Instead, one needs to consider whether more sophisticated definitions of risk and return apply.
- 4.3.2 Under standard CAPM, expected returns on a portfolio depend on portfolio risk. If the portfolio has a beta of 1.4, and the risk premium on equities is 5% pa, prior expectations would lead to outperformance of 2% pa. Furthermore, if utility functions are more diverse than is consistent with the CAPM, there may be possible opportunities arising from investor's different definitions of risks and returns. Even if overall portfolio beta was equal to that for the market as a whole, a manager could still outperform if CAPM does not strictly apply and there are a number of factors, maybe not universally seen as risk indicators, which influence expected returns.
- 4.3.3 While some banks appear to make arbitrage profits of limited size because of privileged oligopoly positions, these positions are in effect the outcome of past expenditure on infrastructure. There are significant costs associated with setting up the necessary trading capabilities - this factor deters other banks from joining the oligopoly. In essence, the arbitrage profits may be considered a return on the infrastructure investment rather than excess returns in financial markets.

4.4 *Statistical Flukes may give the Impression of Outperformance*

- 4.4.1 An important question relates to survivorship bias. Of all the investment managers following any given market, there will by definition be some who have outperformed over thirty years. Since managers are typically selected on the basis of a much shorter period of good performance, a manager who has outperformed over thirty years is likely to have accumulated substantial funds and become very prominent. Many fund managers run a large number of funds; this strategy maximises the chance of having one outlying fund which appears to have spectacularly outperformed. Given these biases, past performance may provide no guide as to the future.
- 4.4.2 With a relatively small number of sizeable stakes, the chance of sustained extraordinary performance over many years in the absence of investment skill is substantially increased (from negligible), even if entrepreneurial and managerial skills (as discussed below) are also absent.
- 4.4.3 Another possibility is that the manager with a given style invests in shares with particular characteristics, such as small size, low P/E, high yield or high “value”. To the extent that the returns on shares with similar attributes are correlated, the apparent odds of outstanding performance will be understated relative to a standard test which assumes independence.

4.5 *Managers can Create Shareholder Value*

- 4.5.1 It is sometimes thought to be ideal that managers should focus on shareholder value creation. Corporate governance is such that, in theory, shareholders can remove managers who fail to do this. Furthermore, incentive systems may attempt to align the financial interests of managers more closely with that of shareholders.
- 4.5.2 It could be argued that many such incentive systems are doomed to failure, as the manager may be forced to take a significant stake in one company, and is then denied the diversification opportunities other shareholders are afforded. By contrast, executive share options distort incentives by only giving participation in the upside. It is sometimes easier to manipulate a bonus system than to generate genuine added value for shareholders.
- 4.5.3 It is reasonable to suppose that markets anticipate the failure of managers to maximise value. The market price of a share will reflect expected cash flows, taking into account any management tendency to put their own personal interests above those of the company. Traded stocks will cover a spectrum from those where company management has interests closely aligned with shareholders, to those where strong conflicts of interest exist.

- 4.5.4 However, it is possible for investors to play a more active part in management, by company visits, use of voting rights, or threats of corporate activity. This may encourage wayward managers to act more in the interests of shareholders than they had originally planned (or the market had anticipated). This will lead to outperformance of these stocks, because the suboptimal managerial behaviour initially factored into the price will have failed to materialise. To the extent that the investment manager also has such managerial or entrepreneurial skill, he may contribute value to his investment and thereby enhance his overall investment performance. The EMH would not deny the existence of managerial or entrepreneurial skill. Such skill would give rise to the possibility of the sustained outperformance observed, with a likely implication being that the manager takes a limited number of strategic positions and conducts ongoing regular discussions with the management of companies in which he has positions. We believe that the points raised in this and previous sections could well be major driver behind Warren Buffett's spectacular performance.
- 4.5.5 If fund managers exist with exceptional ability to trade profitably, they will do so at the expense of other market participants. However, an active participation in corporate governance can lead to superior investment performance via better corporate management, which also has wider social and economic benefits. It seems to us that this is just as productive and as beneficial an application of investment managers' analytical skills as trading shares (which also results in the welfare benefits of liquid and efficient markets).

5 BUILDING AND TESTING MODELS

5.1 *Statistical Time Series Models*

- 5.1.1 The simplest form of economic models describe the probabilities of different economic scenarios. Each scenario would include a time series description of prices (either in continuous or discrete time).
- 5.1.2 In order to be able to assess market efficiency, the model also needs to describe an information structure. This would include a statement of the accuracy with which some future economic quantity may be forecast. It can then be ascertained whether an excess return is due to the arrival of new information (as would be required by EMH) or, alternatively, if relative returns may be anticipated to some degree.
- 5.1.3 The *law of one price* states that a specified cash flow will have a single economic value irrespective of the legal structure that delivers it. Actuarial practice is not always consistent with this law - for example, when liabilities are discounted at an expected rate of return on some (essentially arbitrary) asset portfolio
- 5.1.4 Assuming the law of one price, we might also require the following *positivity principle*: if one cash flow dominates another (in the sense of always being at least as big, and possibly being strictly bigger) then the dominating flow has the higher economic value. An *arbitrage* opportunity is a trading opportunity which will, at

worst, provide the risk-free rate of interest, but may do better - this would plainly violate the positivity principle. Absence of arbitrage would be a consequence of EMH.

- 5.1.5 However, the converse does not hold, that is, we can construct models which violate EMH but still do not permit arbitrage. For example, under some models it may be possible to construct portfolios which outperform the risk free rate with a very high probability and limited downside. However, if the odds in favour of outperformance are anything short of 100%, absence of arbitrage tells us nothing about the pricing of such portfolios. A *small* probability of under-performance may characterise a trading opportunity but is not arbitrage. Two models with different probability laws but the same possible events will be equivalent from the perspective of arbitrage opportunities. However, it is quite possible that one may describe an efficient market while the other does not.
- 5.1.6 Even given the full information structure and probability law of an investment model, it is difficult to state analytically exactly what is meant by market efficiency. We cannot even define the EMH without some prior economic assumptions about what a full reaction to information would entail, that is, without some pricing theory. The difficulty of measuring efficiency of real markets is compounded by the uncertainty surrounding the probability law.

5.2 *Models of Investor Behaviour*

- 5.2.1 One way of approaching the EMH involves modelling what might be thought of as *rational behaviour* by investors. The consequence of rational behaviour can then be interpreted as a definition of market efficiency. A time series price model can be said to satisfy the EMH if the prices are consistent with rational behaviour of market participants.
- 5.2.2 It is not necessary for all investors to behave rationally in order for EMH to hold. All that is required is that price processes are consistent with rationality, that is, that investor irrationality should have a negligible effect on observed prices. This might happen in the large even if micro level transactions are not strictly rational. One analogy is that the behaviour of gases can be predicted from fluid dynamics, even if the individual molecules form a complex multi-particle system, influenced not only by deterministic laws but even quantum randomness. Specific examples of investor misjudgement do not disprove EMH.
- 5.2.3 A degree of subjective judgement still remains in the definition of efficiency, as, even if an observed price feature may be explained in terms of optimising investor behaviour, we have not defined how we determine whether such optimising behaviour is *rational*. The boundaries are blurred. For example, the performance of most fund managers in the UK is measured relative to a benchmark index such as, for example, the FTA All Share Index for UK equities. For most fund managers, a diversified equity portfolio means low risk (of under-performance), while holdings in more 'secure' assets such as cash or gilts are perceived as high risk, because of

the tracking error versus the benchmark. If such players came to dominate the market, then one might expect cash to enjoy a risk premium over equities on account of its perceived higher risk! This contradicts conventional wisdom (and also, substantial bodies of empirical evidence) but would such an economy violate the definition of EMH?

- 5.2.3 Incidentally, EMH needs to be distinguished from efficient frontier or mean-variance efficiency (Markowitz, 1959). The latter refers to choice of portfolios with the least variance for a given expected return, or maximum expected return for a given variance. Any investor can construct an efficient portfolio in the Markowitz sense whether or not the EMH holds.

5.3 *Fundamentals of Optimisation*

- 5.3.1 So far, we have discussed market efficiency in purely verbal terms. It is difficult to get much further in rigorous model classification without a fair quantity of mathematics. This section contains what we consider the simplest possible approach to constructing models of investor optimisation. It is unlikely that a simplistic example such as this would constitute the best possible description of reality - it is intended for illustrative purposes only to show how financial economists go about building such models. More realistic descriptions do, of course, entail more complex mathematics. We think our simple case still gives considerable insight into the relationships between different forms of efficiency.

- 5.3.2 We consider investment over a finite time interval from 0 to some fixed consumption date 1. We let W_t denote the market value of a possible investment at time t , for $0 \leq t \leq 1$. Any investment must satisfy a *budget constraint* which fixes the initial wealth W_0 . The investor wishes to maximise the expected utility, that is $E[U(W_t)]$ for some suitable concave increasing function U .

- 5.3.3 We assume that this market has no transaction costs. We denote the market value of the optimal investment by W_t^* . For reasons which will later become clear, we define a process D_t by:

$$D_t = \begin{cases} \frac{E_t[W_1^* U'(W_1^*)]}{W_t^*} & t < 1 \\ U'(W_1^*) & t = 1 \end{cases}$$

where E_t denotes conditional expectation given information at time t . We can see that, as the utility function is assumed increasing, the deflator is always positive.

- 5.3.4 Let us suppose that the following relationship holds for the market value W_t of any portfolio:

$$D_t W_t = E_t[D_1 W_1] \quad \text{deflator condition}$$

where, as in 5.3.3, E_t denotes conditional expectation given information at time t . We call this the *deflator condition*. This says the conditional expected value today of some future observation of DW is exactly its current position (which plainly

holds by construction when W is the optimal strategy W^*). The conditional mean of future movements is zero, which is the same as saying that DW is a *martingale*. This is an example of a *pricing theory* - we now have one way of describing mathematically what we mean by 'prices fully and instantaneously reflecting new information'.

5.3.5 We can see that the deflator condition implies the absence of arbitrage. Let us consider two investments V and W such that $V_T \geq W_T$ always, with some chance of strict inequality. In order for this to be an arbitrage opportunity, we would need $V_0 = W_0$ but this would imply $\mathbf{E}[D_T(V_T - W_T)] = 0$, contradicting the positivity of D_T .

5.3.6 Now let W^* be an arbitrary investment process. We now demonstrate that if we can show that D as defined in 5.3.3 is a deflator, then it follows that W^* is optimal. We start by considering an alternative investment W . We model switching rules by supposing that, having started with the portfolio W^* , at some later time t we will switch a proportion P from W^* into W . The quantity P need not be deterministic - we will instead allow it to depend on information available at time t , to model the possibility of using price history to time the switch. The final wealth from this process will be

$$PW_t^* \frac{W_1}{W_t} + (1-P)W_1^*$$

5.3.7 We now claim that the deflator condition implies that W^* is at least as good as the dynamic switching strategy. We start by observing that a concave function always lies below a tangent, so that

$$U \left[PW_t^* \frac{W_1}{W_t} + (1-P)W_1^* \right] \leq U(W_1^*) + P \left[W_t^* \frac{W_1}{W_t} - W_1^* \right] D_T$$

Now, taking conditional expectations at time t , and applying the deflator condition, the last term becomes zero, leaving

$$\mathbf{E}_t U \left[PW_t^* \frac{W_1}{W_t} + (1-P)W_1^* \right] \leq \mathbf{E}_t U(W_1^*)$$

The tower law of conditional expectation then kicks the result into touch showing that W^* is indeed optimal as claimed.

5.3.8 It can be shown, under mild regularity conditions on U , that the deflator condition is not only sufficient for optimality, but is also necessary. In other words, if utility maximisation is possible, then a deflator must exist. The example here is a special case of more general results which relax hypotheses regarding the form of utility (or even its existence), and also provide more flexibility in terms of multiple horizons.

5.3.9 We can define a market to be in *equilibrium* if the aggregate demand for different investment vehicles is equal to the aggregate supply. This would imply that for an investor with an 'average' utility function, the optimal wealth process W should

correspond broadly to a capitalisation-weighted index, and the efficient market hypothesis would imply a specific functional form for the deflator expressed as a function of that index. Thus, substituting the index for W^* in the definition of D , the equilibrium concept enables us to determine a quantitative relationship between investors' utility functions and likely risk premia in the market.

- 5.3.10 Plainly, any equilibrium model will be consistent with market efficiency in the sense of section 5.2.1. On the other hand, despite the fact that the deflator condition superficially resembles the EMH, the existence of a deflator does not imply EMH. Instead, in order for EMH to hold, the deflator needs to have the specific form of a utility gradient specified above. It is not obvious to us what the time series implications are for this, but it does seem as though many of the tests in section 3 are strictly tests of the deflator condition rather than EMH.
- 5.3.11 In summary, we can outline a hierarchy of market hypotheses in decreasing order of strength, as follows:

| Economic Condition | Technical formulation |
|------------------------------|--|
| Equilibrium | All investors behave rationally |
| Efficient Markets Hypothesis | Prices fully and instantaneously reflect new information |
| Single agent optimality | Deflator condition |
| Law of one price | Absence of arbitrage |

5.4 *Impact of Transaction Costs*

- 5.4.1 While there are many beautiful models of equilibrium markets, these would nearly all imply EMH. However, as seen in sections 3 and 4, the real world seems to lie in an uneasy no-man's land where

(1) Strict EMH does not hold - there is evidence of consistent but small anomalies which suggest that prices do not react fully and instantaneously to new information.

(2) It does seem, however, that investment managers are constrained not to benefit from these anomalies because of the costs involved in exploiting them.

- 5.4.2 It would in principle be possible to construct an equilibrium model which captured all the effects above, but such a model would need to consider the costs of infrastructure, information collection, transaction costs and market impact. Some prototype models are in existence which examine the marginal effects of each of these individually, but we are not aware of any fully integrated approach.

- 5.4.3 It is of interest to consider how other financial institutions manage these issues. For example, derivative traders often mark their books to market without setting up a contingency reserve for future transaction costs. This may seem naive, but, in effect, the model is assuming that any future transaction costs will be recouped from proprietary trading. A major reason for the possible existence of price anomalies is transaction costs, and so it seems reasonable to us that for a moderately skilled trader these two items should more or less cancel out.
- 5.4.4 We believe that stochastic models for actuarial use could do a lot worse than adopt EMH wholesale. This is consistent with our observation that markets are close to being efficient, and the economic significance of the remaining anomalies is small. However, second order adjustments to investment expenses may be appropriate, to the extent that these expenses are incurred in the pursuit of excess gross returns.
- 5.5.5 One possible way forward in this direction could be to examine the investment trust discount (or premium) to net assets, which is more or less determined by perceived differentials between future performance and management fees levied. As suggested in Mehta *et al* (1996), a deeper investigation of investment trust pricing could provide profound insight into the appropriate treatment of other entities such as insurance companies or pension funds which have substantial managed assets on their balance sheet.

6 ARE MARKET RETURNS AND VOLATILITIES TOO HIGH?

6.1 *The Risk Premium Puzzle*

- 6.1.1 This section considers the equity and gilt risk/return puzzle: can we explain expected returns on equities and gilts? If markets are efficient, there should be a reasonable correspondence between risk and reward.
- 6.1.2 Many studies have found that equities have achieved returns of the order of 7% pa or 8% pa in excess of the returns on cash (using an arithmetic mean), in the main developed economies and over many years. These returns can be viewed in the context of an annualised standard deviation of around 15% in many equity markets. Implied volatilities of options provide further support for this latter figure.
- 6.1.3 On the other hand, many actuarial papers suggest that equities will achieve a lower rate of return, perhaps reflecting a perception that in the long term equities are likely to out-perform other asset classes and that the overall level of risk does not support a higher return. Various studies have explored whether achieved returns on equities have been too “high” relative to risks borne.

6.2 *The Volatility Puzzle*

- 6.2.1 If EMH holds, then prices will only move in response to new information. There is a widespread perception that, in fact, prices also move for all sorts of other reasons, generally associated with some allegation of trader irrationality, which

supposedly generates noise in market prices. This would suggest that, to some degree, past price returns may be able to predict future returns.

- 6.2.2 If returns on earlier periods cannot be used to forecast later periods, then successive returns must be uncorrelated. Positive auto-correlation would suggest that information is reflected slowly in market prices, while negative autocorrelation would imply that markets overreact to new information and then correct. This aspect is capable of empirical testing.
- 6.2.3 The empirical evidence on autocorrelation is mixed. The results seem to depend on the period over which returns are measured. Typical results are:
- (1) Over daily or weekly time scales, there is some evidence of positive autocorrelation, which would imply the existence of momentum effects.
 - (2) When annual or less frequent returns are examined, some studies suggest there is negative autocorrelation.
- 6.2.4 A simple interpretation of these results would be that markets take a few weeks to react to new information, but having reacted, then continue to overshoot until an ultimate correction some years down the line. This would be inconsistent with the EMH; it is of interest to ascertain whether other explanations can be found.
- 6.2.5 One possibility is that the short term momentum effect is real, but, as ever, not realistically exploitable because of transaction costs. For practical purposes it is particularly the apparent long term mean reversion which requires some explanation.
- 6.2.6 The estimated longer term negative autocorrelation is highly influenced by single events, such as the 1974 crash. The question arises as to whether such extreme returns are necessarily offset by positive returns in the preceding or succeeding years as may appear to be the case historically. The limited number of such observations makes analysis difficult, and also analysis with the benefit of hindsight may give rise to bias. The UK economy has survived throughout the period; in contrast, other economies (France and Germany, for example) have undergone catastrophes which resulted in total or near total losses to investors. Examination of the UK market crash in 1974 is also instructive - at the time some market participants were of the view that the state of government interference in the economy (expropriation of wealth) was such that the market would continue to decline. Ultimately, the market recovered, following moves to modify the basis of corporate taxation and, in particular, the treatment of stock appreciation.
- 6.2.7 Some investigations have shown that negative autocorrelation tends to persist even when extreme events are excluded from the analysis. In other words, there is an apparent tendency of the market mildly to overreact even in 'normal' market conditions.

6.3 *A proposed modelling framework*

6.3.1 We propose to demonstrate that the above examples can, in fact, be consistent with efficient markets, by constructing an equilibrium model which exhibits these features. We adopt the optimisation framework of section 5.3 to construct equilibrium prices.

6.3.2 We construct the model by specifying three items

- The emergence of new information
- The distribution of a final investment amount (the ‘dividend’, a proxy for a capital weighted index) on the consumption date 1.
- The utility function of a representative investor.

The price of the investment is determined by invoking the equilibrium assumption that the representative investor invests optimally in the dividend, and not in other competing investments, such as cash. Without loss of generality (and by changing the accounting currency), we may assume that the ‘risk-free’ rate of interest is zero.

6.3.3 We assume that new information reaches the market from the observation of two underlying independent processes, which are:

- a Gaussian random walk z_t , that is, a Brownian motion
- a Poisson process p_t which makes upward jumps of 1 with a frequency λ

It is very important for our purposes to have the Poisson uncertainty as well as the random walk. This enables some information to arrive in lumps, in addition to the continuous dribble conventionally assumed by financial economists.

6.3.4 We denote a capitalisation weighted total return index by s_t . At present, we only define the distribution of the terminal value s_1 . We will then use an equilibrium argument to deduce values for all $t < 1$.

We start by defining a variance process v_t which determines the extent to which Gaussian random noise affects the final index value. This process has a minimum value v_{\min} to which it tends asymptotically at a rate η . However, this asymptotic convergence is interrupted by occasional upward bursts of size α every time the Poisson process jumps. The variance process therefore satisfies the differential/difference equation:

$$dv = -\eta(v - v_{\min})dt + \alpha dp$$

We now define a further process by adding the Poisson jumps (with size β) to the appropriately scaled increments of the Gaussian process:

$$x_t = \beta p_t + \int_0^t \sqrt{v} dz$$

We can see that while the increments of x_t are not independent (because of the jump effect on the variance term), they do at least have constant conditional mean, and therefore are uncorrelated, so in some sense, x_t is a generalised random walk. We now perform an exponential transformation to get to the final index value:

$$s_1 = \exp(x_1)$$

This formula is in some sense arbitrary - we just made it up without any empirical backing. However, it does give us the flexibility to derive some standard theories as special cases, and also, we shall see, provides us with the means of explaining excess volatility and mean reversion.

- 6.3.5 It so happens that the above expression for the final index value, despite its apparent complexity, does have some analytical tractability. In particular, we can calculate conditional expectations; it can be shown that

$$\mathbf{E}_t(s_1^k) = \mathbf{E}_t \exp(kx_1) = g_k(t) \exp \left[\begin{aligned} &kx_t + \frac{k^2}{2}(1-t)v_{\min} + \frac{k^2}{2\eta}(1 - e^{-\eta(1-t)})(v_t - v_{\min}) \\ &+ \lambda(1-t)[\exp(k\beta) - 1] \end{aligned} \right]$$

where the constant $g_k(t)$ is given by

$$\frac{\log g_k(t)}{\lambda} = \int_0^{1-t} \exp \left[\frac{\alpha k^2}{2\eta} (1 - e^{-\eta\xi}) \right] d\xi - (1-t)$$

(Hint for budding probabilists who want to prove this - it suffices to check that the first expression is a martingale, so work out the conditional mean increments using Ito's formula. For the less ambitious, check the simple case $\alpha=0$ and convince yourself that $g_k(t)$ behaves sensibly for $\alpha > 0$.)

- 6.3.6 We restrict ourselves, in the main, to power law utility functions, that is, utility functions of the form

$$U(W) = \frac{W^{1-\kappa}}{1-\kappa}$$

However, we will also consider mixtures of such functions. Using the equilibrium argument from section 5.3, we can write down the condition that the optimal investment is 100% in the shares. This involves equating S_t to the optimal investment W^* and then applying the deflator condition to the risk free investment, which gives:

$$S_t = \frac{\mathbf{E}_t[s_1 U'(s_1)]}{\mathbf{E}_t[U'(s_1)]}$$

This is a bit like an discounted cash flow valuation (remember our risk free discount rate was zero), but the expected cash flows are *weighted* by the marginal utility $U'(s_1)$. This makes sense - if U is concave then greater weight will be given

to scenarios where s performs badly. Fortunately, we can easily calculate the necessary expectations for our choice of such functions using the results in 6.3.5.

6.4 *The classical approach*

- 6.4.1 For the sake of completeness, we reproduce here the classical equilibrium model, due to Merton, based purely on the random walk with volatility of 15%. This corresponds to the following parameters:

$$\lambda = 0 \text{ (no jumps)}$$

$$\alpha, \beta, \eta \text{ not defined (because there are no jumps)}$$

$$\nu_0 = \nu_{\min} = 0.0225 \text{ (} = 15\% ^2 \text{)}$$

- 6.4.2 Given a power law utility function, we can determine the expected return on the share. Conversely, given an expected return - we have assumed 7% above cash - we can solve for the value of κ in the utility exponent. This gives $\kappa = 3.007$, corresponding to utility of the form $-W_1^{-2.007}$.
- 6.4.3 This value of α suggests only very limited utility for the upside because the utility tends quickly to zero for large W , which offends our gut feeling of what the utility ought to look like. More objectively, Mehra & Prescott (1985) rule out the possibility that a consistent utility function may simultaneously explain consumption patterns and also the equity risk premium within this framework. This is rather unsatisfactory. We will now show how this paradox can be resolved by considering more general models.

6.5 *A simple model with jumps*

- 6.5.1 An examination of historic returns and price increases on equities over a long period (from 1700) suggests that returns can be better represented by a normal distribution overlaid by an occasional extreme negative return. The standard deviation of price changes was approximately 15% pa. However, there were three returns (of the order of minus two thirds each) during the three hundred year period which were larger than the extremes that might be expected based on a normal distribution with standard deviation of 15% pa.
- 6.5.2 We can redefine the x process accordingly, to incorporate jumps, and repeat the analysis. The new parameters are

$$\lambda = 1\% \text{ (one jump per century)}$$

$$\alpha = 0 \text{ (jumps do not feed into volatility)}$$

$$\beta = -1.09861, \text{ the natural log of } 1/3, \text{ so that the jump is of order } 2/3$$

$$\eta \text{ not defined (because jumps do not feed into volatility)}$$

$$\nu_0 = \nu_{\min} = 0.0225 \text{ (} = 15\% ^2 \text{)}$$

- 6.5.3 Solving for the utility which gives a required return of 7%, we obtain $\kappa = 1.596$, corresponding to utility of the form $-W_1^{-0.596}$, which is rather more plausible than what we had before. Using this value of κ , we can see what the expected return would have been if the jumps were ignored. It turns out that, of the 7% risk premium, 3.34% can be ascribed to the jumps.
- 6.5.4 We have constructed the final value s_1 of the index to depend exponentially on x_1 . It is of interest to consider how intermediate values of s_t are determined by the information structure. In the current case, with power law utility and $\alpha = 0$. It is simple to evaluate the relevant expressions to verify that the exponential link applies for all t , with a linear adjustment to x for $t < 1$. Thus, in essence, the log index performs a random walk with drift. In particular, if $\exp(x)$ falls by 2/3 then s also falls by 2/3. Here, it is easy to see that the market (fall in s) has reacted fully and instantaneously to the new information (fall in $\exp[x]$) and EMH is directly verifiable.

6.6 *Alternative Utility Functions*

- 6.6.1 We now consider some alternative utility functions. The power law functions considered so far all have the property that $U(W)$ tends to zero as W tends to infinity, so that a low utility is given to the upside. We now consider instead a utility family where, over a large range, the investor is fairly risk tolerant, but for which there is a much sharper aversion to the extreme downside. Using the same stochastic parameter values as in 6.5, we consider the utility function:

$$U(W) = W - \frac{c}{W^3}$$

The constant c is determined, dependent on the initial wealth W_0 so that the equilibrium expected return is 7%.

- 6.6.2 We can again investigate the effect on the market price when $\exp(x)$ falls by 2/3. This time, the effect on s is not proportional, because the market fall pushes investors from risk tolerant behaviour into risk averse behaviour. The singularity in the utility function comes into play, and investors now require a prospective mean return of 75% over the one-year time horizon being considered in order to persuade them to stay in equities. The market thus falls by more than 2/3 - in fact, the fall under this model is 79.7%. However, with future dividends discounted at 75% p.a., there is a strong likelihood that the market will, in fact, recover from the crash as the discount rate unwinds. This 75% may seem large, but the effect is a bounce-back to 1/3 of prior expectations, as would be expected.
- 6.6.3 It could be argued that if the underlying random walk generating the information falls by 2/3, then a 'full and instantaneous reaction' would be for the market to fall by exactly 2/3. However, the apparent overreaction of our model does give some insight into why markets might seem more volatile than is justified by changes in expected future cash flows. Furthermore, as the effective discount rate may change

with the rate of return, so we have an explanation for some bounce back after the overreaction, that is, mean reversion. However, our model is consistent with rational investor behaviour, and so with EMH.

6.7 *Stochastic Volatility*

6.7.1 Equity market (and other market) volatility changes over time. There is now significant evidence that expected returns are higher when volatility is high, and that volatility increases during market falls. These observations have important consequences for the level of market volatility, and would tend to accentuate the effect noted in 6.6.

6.7.2 Within our framework, this corresponds to a positive value of α . We therefore reran our model, with the following final set of parameters

$\lambda = 1\%$ (one jump per century)

$\alpha = 0.04$ (a jump causes volatility to rise from 15% to 25%; $0.04 = 0.25^2 - 0.15^2$)

$\beta = -1.09861$, the natural log of $1/3$, so that the jump is of order $2/3$

$\eta = 1$, the excess volatility from a jump lasting for around 1 year

$v_0 = v_{\min} = 0.0225$ ($= 15\%^2$)

6.7.3 Here, we find that a fall in $\exp(x)$ of $2/3$ results in a market fall of 81.4%. This is even deeper than the market fall which would have arisen from our previous models. In the event where a jump occurs, not only are investors thrown into the risk-averse region of their utility function, but also market volatility has gone up to 25%, compounding the effect on the required discount rate, which now comes out at 94% p.a. At the end of 1974, for example, equity risk might have been viewed as enormous, requiring a 94% return, and indeed, this is consistent with what happened.

6.7.4 As with the previous model, this also produces excess volatility and mean reversion within an efficient market. The effect of stochastic volatility is to demonstrate these features yet more strongly.

6.7.5 Our example is consistent with EMH if our utility function represents 'rational' behaviour. However, in this model, a risk-tolerant investor could potentially achieve high returns from a simple switching strategy. This is not, to our mind, evidence of inefficiency, but rather a synergy benefit arising from the fact that different investors have different utility functions. Another example of such synergies arise for insurers matching annuities - the matching (and hence low risk) asset, which are long debt securities, get a better mean return than cash because the rest of the market regards long bonds as risky. We now consider this bond premium in more detail.

6.8 *Government Bond Returns*

- 6.8.1 There has been some research which suggests that long dated government bonds are expected to achieve returns of the order of 2% pa in excess of returns on cash. A main risk attributable to investment in long dated bonds is that of an unexpected increase in future inflation expectations, since this will lead to a reduction in the price of bonds but will have no effect on holdings in cash. An excess return of 2% pa appears high in relation to the consequences of an increase in inflation and some actuaries have suggested that the existence of investors with liabilities fixed in monetary amounts could lead to a preference for conventional bonds (that is, no risk premium).
- 6.8.2 The possibility of exceptional losses due to hyper inflation or confiscation as a result of war can also be used to explain such studies. For example, a 1% probability of a 50% loss might correspond to a return requirement of approximately 1½% pa. The balance could be attributed to reward for bearing a normal level of inflation risk. Once again, jump risk combined with singular utility functions provide an acceptable explanation consistent with EMH.

6.9 *Conclusions*

- 6.9.1 The analyses set out in this section suggest that the 'high' return requirements on equities and gilts determined in many studies can be explained once account is taken of investors' aversion to the possibility of catastrophic losses due to economic collapse. The illustrations used suggest that this factor could account for over 3% pa of the return risk premium on equities and 1½% pa of the risk premium on long-term government bonds. These numbers are intended to be illustrative; a more detailed application of utility theory, analysis of option prices and of alternative return models would be desirable.
- 6.9.2 The analyses also provides an explanation of why equity and gilt returns appear to be more volatile than might be justified by a consideration of the underlying cash flows generating value. A reduction in fundamental value increases the likelihood that further declines in value will result in total investor wealth declining below minimum 'subsistence' levels. Further, market falls tend to be associated with increases in market volatility, with this volatility itself requiring compensation. For both reasons, required returns on risky asset categories increase as squeezed investors consider switches to risk-free alternatives such as cash. A significant part of equity market volatility may be to be attributable to changes in the required rate of return rather than from cash flow effects.
- 6.9.3 It would be surprising, given the weight of money and research into the gilt and equity markets, if anecdotal evidence for market inefficiency could be readily proven to be valid. This section provides explanations consistent with EMH for two types of substantial market inefficiency which have been proposed.

