

IT-Supported Solutions for Asset-Liability Management: The Alm-Max Approach

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Abstract

Alm-max is a technology for banks' asset-liability management (ALM), a probability-based decision support tool, to control and manage interest rate risk aimed to protect banks' interest rate sensitive balance sheet positions against capital markets' volatilities. The stochastic tool is based on the single-factor Cox, Ingersoll and Ross model. In this context, ALM is stated as a problem which involves the overall operational structure of the intermediary. From a theoretical standpoint, the technology is based on the results of semideterministic and stochastic immunization theory. Three main problems are considered: outstanding asset-liability portfolio analysis, what-if analysis, optimal portfolio selection and rebalancing. Each problem has been solved by a modular software based on routines for financial calculus and optimization, which runs under WINDOWS. The traditional gap analysis is extended, especially for floating rate contracts. Solutions can be derived only after the reference market model has been identified. The market model, in particular, is characterized as an ancillary block of the overall ALM structure. In the paper, the foundations, the basic structure and some crucial methodological aspects of the Alm-max approach are analysed. Within this technology, an evolutionary approach to what-if analysis and the problem of optimal choices under uncertainty are considered.

Keywords

IT-supported solutions, asset-liability management, portfolio selection, mapping, Value-at-Risk technique.

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1. Foundations of the Alm-max approach

The general framework on asset-liability management – In general, asset-liability management (ALM) models are aimed to control and manage financial risks in the intermediation activity ¹.

Classifying a risk-management model as an ALM model traditionally means to consider a "weak" classification: an ALM model is only characterized by the fact that it refers to the two components, assets and liabilities (or, in other terms investments and fund-raising contracts) of the financial activity, i.e. as a "two sided model" ².

Under the "ALM model" terminology is considered differently and not directly comparable with methodological approaches for financial analysis³. The financial control schemes may have strategic or operational purposes. They may be set up referring to a static or to a dynamic management framework (just to "describe" the outstanding situation of the intermediary or to support the financial decision making process). They can use the "balance-sheet" language (defining, for example, aggregation rules of the different instruments from an accounting point of view) or the financial language (representing each security by its stream of payments). In any case, the ALM control schemes always move from the traditional idea that "assets and liabilities must be considered together" ⁴. In this framework, for example, the JP Morgan catch-phrase "from ALM to VaR" ⁵ has to be reinterpreted. The Value-at-Risk techniques allow to define, in fact, financial risk measures which can be used in a general ALM model.

Value-sensitivity hedging and cash-flow matching – The eclectic nature of asset-liability management was already recognized in the origins of the theory of finance and in its applications ⁶. The results by Redington (1952) and by Haynes and Kirton (1952) may be considered path-breaking even in this sense. Both Redington's "immunization" and Haynes and Kirton's "insulation" are strategies aimed to characterize a time distribution of assets and liabilities "in such a way that the existing business is immune to a general change in the rate of interest" ⁷. But it is relevant, for the sake of theory and for the practical effects, to remark upon the difference in the approach to the ALM problem: the Haynes and Kirton paper "dealt primarily with matching, valuation being a by-product", while Redington's paper "dealt primarily with valuation, matching being a by product" ⁸.

The strategies aimed to control the time distribution of the cash-flow are inspired by the first approach. The second approach recognizes the central role of pricing theory and of risk measures. In general, an ALM methodology has to consider both these approaches, because it allows consideration different but equally relevant aspects of financial risk. Both these approaches can be set up only after the relevant market model has been defined.

Methodological premise on interest rate risk – From a methodological point of view, interest rate risk differs from that inherent in most risky securities. Interest rate sensitive (IRS) contracts, in fact, usually have known cash flow profiles (amounts and timing of cash payments) and this gives to interest rate risk an "inherently intertemporal" dimension, related to investor's preferences on the op-

timal wealth allocation, which is difficult to specify if other securities (equities) are considered⁹. It follows that the typical schemes for the management of equity portfolios, based on the diversification rule, cannot be used in the case of IRS portfolios. In the traditional Markowitz portfolio selection theory the security risk – risk of price's fluctuations – is controlled by diversification considering a "sufficiently large" number of securities with "little" correlated returns – the less the correlation among security returns, the greater the impact of diversification on reducing variability. Things are quite different if we consider IRS portfolios, in which the values (prices) of the contracts depends on the level of interest rate. In this case, in fact, the risks are highly correlated, hence the risk of investment portfolios cannot be reduced by diversification. In the case of intermediation portfolios (with assets and liabilities), interest rate risk can be controlled by selecting contracts as most "similar" as possible (in the sense of immunization) with opposite account sign (perfect matching is the extreme case, with perfectly negative correlation). The asset-liability management models represent then, a simple but relevant way to profit by negative correlated opportunities.

All these aspects must be considered to set up IRS portfolio selection schemes¹⁰. They also suggest how it can be difficult to define economically meaningful Value-at-Risk measures for IRS contracts.

2. The basic structure of the Alm-max technology

The ALM problem: a "minimalist" solution – The Alm-max technology is a solution to the ALM problem. The basic structure of the Alm-max technology is based on problems and, within each problem, on functions and sub-functions (as illustrated in table 1). The general ALM scheme can be simplified (and in this sense we characterize a "minimalist" solution) by referring to three main problems: outstanding asset-liability portfolio analysis, what-if analysis, optimal choices. Solutions can be derived only after the reference market model has been identified. The market model, in particular, is characterized as an ancillary block of the overall ALM structure.

The structure of asset-liability portfolio – An outstanding intermediation portfolio can be formally represented at time t by the matrix $\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n\}$ of the n asset contractual vectors and by the matrix $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n\}$ of the n liability contractual vectors (the fact of considering the same number of assets and liabilities does not involve any loss of generality). The time structure of the intermediation portfolio is referred to the time period $[t, t_m]$; in particular, it is defined by the time vector $\mathbf{t} = \{t_1, t_2, \dots, t_m\}$. If the portfolio consists of contracts with deterministic cash-flows, \mathbf{A} is a $(n \times m)$ matrix of non-negative elements, where the generic element $a_{i,k}$ represents the amount to be received at time t_k for the i -th asset. Similarly \mathbf{B} is a $(n \times m)$ matrix whose generic element $b_{j,h}$ represents the amount to be paid at time t_h for the j -th asset. For the analysis of some relevant ALM problems, in matrix \mathbf{A} and in matrix \mathbf{B} , could be also included the deterministic cash-flows defined as "equivalent" to the stochastic payments of variable-rate instruments and of derivative contracts.

Table 1

\oplus <i>problems</i>	\bullet <i>functions</i>	\otimes <i>sub-functions</i>
\oplus outstanding asset-liability portfolio analysis		
<ul style="list-style-type: none"> \bullet mapping <ul style="list-style-type: none"> \otimes expected cash-flow \otimes risk-equivalent cash-flow \bullet gap analysis <ul style="list-style-type: none"> \otimes principal \otimes interest \otimes repricing (risk decomposition) \bullet risk measures <ul style="list-style-type: none"> \otimes value \otimes duration \otimes second order risk measures \bullet capital requirements <ul style="list-style-type: none"> \otimes Basle Committee \otimes Bank of Italy \bullet VaR measures \bullet surplus stream 		
\oplus what-if analysis		
<ul style="list-style-type: none"> \bullet scenarios on the distributional parameters of the stochastic market model \bullet scenarios on new business areas and volumes \bullet same functions and sub-functions of outstanding portfolio analysis 		
\oplus optimal choices		
<ul style="list-style-type: none"> \bullet investment/fund raising decision making support tool \bullet trade-off between strategic control variables <ul style="list-style-type: none"> \otimes risk-volume \otimes risk-liquidity 		
\oplus term structure of interest rates		
<ul style="list-style-type: none"> \bullet by spot rates \bullet by forward rates \bullet by swap rates \bullet by the Cox, Ingersoll and Ross model 		

Let $\mathbf{q}^{(a)} = \{q_1^{(a)}, q_2^{(a)}, \dots, q_n^{(a)}\}$ and $\mathbf{q}^{(b)} = \{q_1^{(b)}, q_2^{(b)}, \dots, q_n^{(b)}\}$ represent, at time t , the vectors of the market prices of the assets and of the liabilities, respectively. The vectors $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \dots, \beta_n)$ give, respectively, the shares of the contracts of basket **A** and of basket **B** held in the asset and liability portfolios. If contracts involving deterministic payments are considered, the asset portfolio will have cash flow \mathbf{x} , with individual components due at time t_k given by $x_k = \sum_{i=1}^n \alpha_i a_{ik}$, $k = 1, 2, \dots, m$; the cash flow \mathbf{y} of the liability portfolio will be defined in a similar way. Let C be cost function and C_a and C_b the volume (cost)

of the assets and the liabilities. The pairs $(\bar{\alpha}_i, \bar{\alpha}_i)$ and $(\bar{\beta}_i, \bar{\beta}_i)$, $i = 1, 2, \dots, n$ can be used to indicate the asset and liability portfolios composition constraints, assumed to be non-negative (i.e. ruling out short selling), where we may have $\bar{\alpha}_i = +\infty$, $\bar{\beta}_i = +\infty$.

The input-output Alm-max vision – Functions and sub-functions work on the single financial instrument. By an information reporting system the aggregation rules can be defined using an information reporting system and the analysis can be developed either on each position or business area.

In addition to market's data and to bank's outstanding portfolio, it is possible to build sub-sets of financial asset-liability opportunities and market scenarios to analyse the effects of alternative hypotheses on actual data. The structure of the Alm-max modular software system is reported in table 2.

The software environment – The Alm-max technology has been implemented by a modular software system based on programming routines for financial calculus and optimization, which runs under WINDOWS. Under this software environment, the technology provides a very easy access to banks' and capital markets' daily information. Banks' data could otherwise be extrapolated from standard information requested from banking regulators¹¹.

3. Some crucial methodological aspects

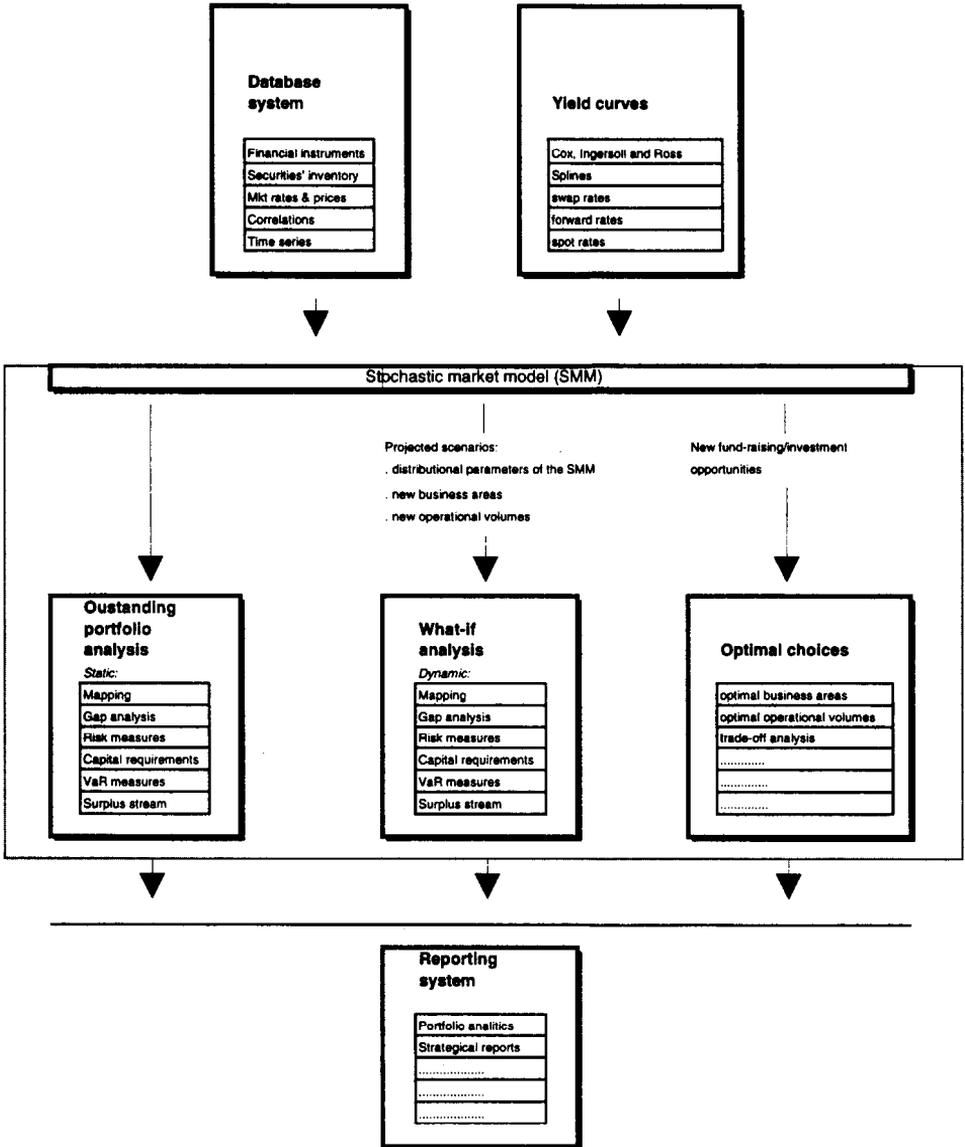
3.1 Development of maturity gap analysis

The traditional gap analysis¹² is extended, especially for floating rate contracts. The analysis is performed after having selected the time grid and the gapping period. For each time interval the asset-liability amounts (principals and coupon) under repricing are calculated¹³. If the coupon rate is determined as an average of the reference index, the principal is proportionally assigned at each relevant date. The principal is marked both referring to the corresponding indexation rules and to the reference indices (sources of uncertainty) within the same rule. The gap analysis is only based on the contractual features of the outstanding positions of the intermediary, without taking into account the market model.

3.2 The market model

The single-factor Cox, Ingersoll and Ross (CIR) model is the reference model of the Alm-max technology. It has been specified to take into account the peculiar aspects of Italian lira capital markets. On each valuation date the model is estimated on market's data by a "three step" estimation method, that combines the information provided by time series of short term rates and that obtained from the observation of successive cross-sections of medium-long term fixed rate bond prices¹⁴. This estimation approach represents an acceptable compromise in the practical use of the model¹⁵. With respect to the Brown and Dybvig estimation technique, the three step method allows identification of the single distributional parameters of the models. This allows us to characterize the expectations of future values and market rates (in particular, the term structure of expected spot

Table 2



rates)¹⁶ as well as to determine quantities such as maximum loss to be expected with a fixed probability level (Value-at-Risk measures). The stochastic model is used for pricing derivatives. In particular, calibration techniques on the capital markets relevant for banking activity are considered, to

improve the fitting between equilibrium (model) prices and market prices¹⁷. The three step estimation method also allows calculation of the exercise probability of options¹⁸.

3.3 Mapping algorithms

To perform the analysis of some relevant ALM functions, in particular to calculate the expected cash-flows (i.e. liquidity analysis) and to evaluate the intermediation margins, the so-called "mapping" process ("describing all instruments approximatively as combinations of standard building blocks"¹⁹) has to be defined. Mapping is part of the general valuation process, aimed to calculate values and risk measures of the different instruments (or of portfolio of instruments). Both mapping and valuation require the identification of the market model.

Fixed-rate instruments – For these contracts (bullet bonds, fixed rate loans, etc.) the cash-flow map derives directly from the distribution of the instruments' money flows over time (contractual payments), considering principal and coupons.

Variable-rate instruments – Floating-rate instruments have future payments which are, in general, unknown at the valuation time. In this case, "equivalent" deterministic payment streams are defined, by referring to criteria of "equivalence" between certain and uncertain pay-offs²⁰.

Example. In table 3 are illustrated four different deterministic streams which are "equivalent" (under different criteria of "equivalence") to the random payments of an Italian Treasury Credit Certificate (*CCT*)²¹. The so called ex-ante value of each uncertain coupon is given by the single-factor CIR model. The ex-ante coupon is defined as the certain amount that, when substituted for the coupon, has the same equilibrium price calculated on the basis of the model.

In addition to ex-ante coupons and flat coupons, which are all equal to the first known coupon, we consider forward coupons (which are obtained using the forward rate implicit in the estimated term structure) and par coupons (calculated on the basis of the par yield, that is the coupon rate of a bullet bond with an equilibrium price equal to par)²². In the table, "price" is the model present value and "duration" is the stochastic duration²³.

In the so-called "synchronous" indexation (in which each fixing of the reference index coincides with the start of coupon entitlement and the coupon interval is equal to the maturity of the reference security) the indexed instrument perfectly replicates, over an horizon equal to its maturity, a roll-over of the nominal capital in the underlying zero-coupon bond. It can be shown, as a direct consequence of the arbitrage argument, that the stream of indexed coupons, referred to a unit amount of nominal capital, is equivalent (in terms both of value and of sensitivity of the value to changes in the interest rate) to a pair of deterministic unit zero-coupon bonds: an asset with maturity on the next coupon payment date and a liability with maturity on the last date of the flow²⁴. This result is important (also in a "minimalist" ALM scheme) because it allows the risk associated with the uncertain indexed payment stream to be measured by calculating the traditional duration of the equivalent deterministic stream, which is found to be negative.

Table 3

payment date	forward rate	Fixed rate				Variable rate
		forward coupon	par coupon	ex ante coupon	flat coupon	coupon
26/05/92	0.0548	5.4763	5.4891	5.4763	5.4763	5.4763
26/11/92	0.0549	5.4880	5.4891	5.4843	5.4763	I ₂
26/05/93	0.0549	5.4939	5.4891	5.4905	5.4763	I ₃
26/11/93	0.0550	5.4960	5.4891	5.4928	5.4763	I ₄
26/05/94	0.0550	5.4955	5.4891	5.4925	5.4763	I ₅
26/11/94	0.0549	5.4937	5.4891	5.4905	5.4763	I ₆
26/05/95	0.0549	5.4914	5.4891	5.4880	5.4763	I ₇
26/11/95	0.0549	5.4886	5.4891	5.4852	5.4763	I ₈
26/05/96	0.0549	5.4855	5.4891	5.4821	5.4763	I ₉
26/11/96	0.0548	5.4825	5.4891	5.4791	5.4763	I ₁₀
	Price	100.0000	100.0000	99.9781	99.9036	99.9781
	Duration	3.5060	3.5057	3.5063	3.5073	0.4061

Using this method all the indexation rules typical of loans and of "private" bonds, i.e. non-bullet principal amortizing or indexation rules linked to market rates other than the short-rate (on the Italian market, for example: TUS, RIBOR, RENDISTATO, ROLINT, RENDIOB, etc.), can be analysed. It is relevant, in this case, to estimate correctly the correlations between the reference rates (of the indexation mechanism) and the interest rates which be considered to drive the market ²⁵.

Options – The mapping process for options can be set up by considering the payment stream of the equivalent portfolio determined on the basis of the reference model.

Example. Let us consider a long position in a European bond option, in particular, a call option on Italian Treasury Bonds (BTP-option). The cash flow of the option-equivalent portfolio can be derived by the formula for options' value given, for example, by the single-factor CIR model. The equivalent payment stream is obtained weighting the exercise price (on the liability side) and the pay-offs of the underlying instruments (on the asset side) with the chi-squared probability distribution estimated on the market ²⁶. A put option can be analysed in a similar way, by put-call parity. Obviously, an option on a zero coupon bond is a particular case. Let us consider, for example, a call option written on

the payment stream $\mathbf{x} = \{5.25, 5.25, 5.25, 5.25, 105.25\}$ (ITL) having maturities $\mathbf{t} = \{2, 2.5, 3, 3.5, 4, 4.5\}$ (years). The price of the call with exercise time $T = 1.5$ year and exercise price $K = 100$ ITL, results, after having estimated the CIR model (at the valuation time $t = 0$): $C(0; T, \mathbf{x}, K) = 0.07$ ITL. The option-equivalent payment stream is given by: $\{-3.86, 0.27, 0.26, 0.24, 0.23, 0.22, 3.96\}$ ITL on the time grid: $\{1.5, 2, 2.5, 3, 3.5, 4, 4.5\}$.

Others derivative instruments – Mapping for derivatives such as swaps, futures and FRAs can be defined following the same method previously described.

The margin stream – The payment streams derived from the mapping process are aggregated by type, in relation to the selected time grid. The margin stream is given by the difference between the aggregate asset stream and the aggregate liability stream. It is hence a vector with positive, negative or nil components. Both from the theoretical and operational standpoint, it is important that the mapping criteria of each instrument are "consistent" so to have "equivalent" streams which can be aggregate correctly. Obviously, the margin stream is not univocally determined, as it depends on the criteria of equivalence that has been selected.

4. An evolutionary approach to what-if analysis

The scenario techniques – The Alm-max technology allows to define scenarios on market's dynamic, on new business areas and volumes or "mix" scenarios, to evaluate the effects of the assumed environments on the actual situation. Market scenarios are characterized by defining alternative sets of the distributional parameters of the reference model (single-factor CIR model). The aim is to analyse the new outstanding situation under the different market hypotheses (what-if logic). The technical solution of characterizing the scenario through the distributional parameters of the stochastic model allows to overcome the restrictions of traditional what-if analysis (based on deterministic rules) and to capture correctly market's uncertainty. In fact, given the term structure of spot rates, it is possible, for example, to calculate (implicitly) the term structure of expected rates, the instruments' future values and their probability distributions, the options' exercise probability, etc..

The Alm-max technology structure allows also to consider new business scenarios (new hypothetical asset-liability instruments). In general, what-if analysis provides a preliminary solution to the problem of determining the new business area and volumes, choosing from "assigned" and alternative new operational and market environments. The evolutionary approach to what-if analysis here is, anyway, the first step towards optimal choices and dynamic ALM.

The Value-at-Risk techniques – Also the value at risk techniques can be included in the framework of scenarios. In particular, they answer to the question of what does happen to instruments' values if an event occurs with a certain probability. Given the self-government offered to banks' managers from the Basle Committee on Banking Supervision (1996) with respect to internal models against market risk, it is quite relevant to compare value at risk measures derived by different market

models. This is also because some recent studies on bond markets ²⁷ suggest, as already evidenced on equity markets, that the value at risk measure is "extremely dependent on parameters, data, assumptions and methodology" ²⁸. Besides the JP Morgan Risk-Metrics technique (1995), or other adjusted similar techniques which have been developed (to take into account the peculiar aspects of the Italian Government securities market), it is expressive to calculate the Daily Earning at Risk by comparing the portfolio's present value (at time t) with its value derived from the single-factor CIR model referring to the same distributional parameters estimated at time t and to the spot rate r^* such as: $Prob(r_{t+1day} < r^* | r_t) = 95\%$, r_t being the spot rate at time t and $Prob(\cdot)$ the chi-squared distribution function calculated with the estimated parameters (at time t) ²⁹.

Toward optimal choices – The evolutionary approach to what-if analysis, based on scenarios of the distributional parameters of the relevant stochastic model, is anyway not consistent with financial decision making under uncertainty. What-if analysis, in fact, always moves from ad-hoc hypotheses (even if on the distributional parameters), and produces a set of alternative management strategies (the basic hypothesis being incompatible) and there is no sense in seeking the "best" strategy since "the best strategy is not the best under any of the hypotheses considered but should be defined taking into account the state of uncertainty which exists between all the possible hypotheses" ³⁰. A consistent decision making support tool under uncertainty has to be defined referring to stochastic market models (to describe the dynamics of the markets' variables) and to optimization schemes which specify the bank's objective functions and constraints, both internal (imposed by strategic management plans) and external (determined by the market conditions).

5. Optimal choices

The asset-liability matrix – The asset-liability matrix is the basis of the problem of optimal choices. The matrix **A** and the matrix **B** "describe" the outstanding asset-liability portfolio not used for trading (each instrument has share 1 in the α or β vectors), the trading positions and the new instruments which represent the bank's fund-raising and investment opportunities (whose shares are the unknowns in the optimization problem). The same scheme can be used to perform the analysis of particular gap management policies and rebalancing. For example, it can be used to select and manage an investment portfolio (a bond portfolio) dedicated to cover a fixed liability stream. In the case of a single liability, $L > 0$, to be paid at time T ($t < T < t_m$) the variable L assumes the strategic role of a "target" variable (planned value of the portfolio at the end of the holding period).

Objective functions and constraints: some examples – A relevant theoretical framework is given by the stochastic immunization theorem ³¹. This framework states the theoretical constraints that the asset and liability prices (values) and risk (semi-elasticity of the price with respect to the spot rate) must be equal (balance-sheet and duration constraints). Even the objective functions can be defined in

the practical implementation of the theory. Fulfilling at time t conditions on prices and risks only guarantees that the asset-liability portfolio is instantaneously riskless on that date. Since portfolio rebalancing can be performed only at discrete times, actual immunization strategies are only an approximation to the idealized immunization achieved by continuous rebalancing. In many applications the rebalancing period $\tau > 0$ is fixed and known at time t . It is then possible to define as an objective function the expectation on some portfolio characteristic at the next rebalancing time $T = t + \tau$. For the sake of simplicity, let us assume that the next portfolio adjustment is performed before the first payment date. The balance-sheet constraint (at time t) requires the net present value of assets and liabilities to be equal to zero: $P_N(t) = 0$. The fact that the present value constraint is satisfied at time t does not imply, in general, that the expectation of future net value be equal zero: $E_t[P_N(T)] = 0$. Therefore it is possible to apply immunization trying to maximize this expectation, $\max E_t[P_N(T)]$. The selection problems which are inspired by this approach are linear both in the objective function and in the constraints.

Stochastic immunization strategies oriented toward perfect matching can be also considered by introducing minimum risk objectives. In particular, one can choose the objective $\min E_t[P_N^2(T)]$ obtaining a quadratic programming problem.

In the problem with the objective $\max E_t[P_N(T)]$, the selected portfolio will be the one which, among all the instantaneously riskless portfolios, maximizes the expectation (at the evaluation time t) of the net value at the fixed future time $T = t + \tau$. This is the same as if the portfolio were selected to be instantaneously riskless at time t and such that it would emerge "in the best condition as possible" (in the sense of expected value) at the next market "opening". It is relevant to notice that the variable τ , size of the rebalancing horizon, assumes an important strategic role. Its specification generally depends on management and market factors (operating costs of rebalancing, transaction costs, portfolio's maturities distribution, shocks on leading market variables, administrative deadlines, etc.). Obviously, the maximum-value objective does not allow to control on the future level of risk. The procedure with the objective $\min E_t[P_N^2(T)]$, instead minimizes the probability (evaluated at time t) that the net absolute value of the portfolio (and therefore the present value mismatching) at the next fixed rebalancing time T will be greater than a "threshold" given at time t .

Referring to these ALM schemes, gap management plans of the outstanding intermediation portfolio can be set up (to cover value and duration gaps). The two procedures allow to identification of the efficient frontier of the possible intermediation portfolios: this leads to a redefinition of the traditional "mean-variance" approach in terms of stochastic immunization ³².

A tool for surplus time allocation: the surplus stream – The concept of surplus stream was defined to answer management needs for surplus time allocation ³³. It can be also used to define optimal profit-sharing policies. In order to cover the outstanding liability stream, an asset portfolio is selected from the outstanding asset portfolio which meets (by assumption) the present value and the duration constraints and carries minimum risk. The surplus stream is the residual asset

stream after immunization. It has non-negative cash flows and its present value is equal to the net present value of the outstanding intermediation portfolio: it can therefore be interpreted as an "appropriation" stream (consisting of margin, equity capital and yield on equity capital) which is the surplus stream "disposable" after an optimally immunized nucleus of the outstanding portfolio has been guaranteed ³⁴. Obviously, with heavily unbalanced intermediation portfolios the surplus stream may not be identifiable: this occurs when the optimization problem has no solutions (infeasible problem).

The value of interest rate insurance – To guarantee interest rate risk on IRS portfolios a put option can be purchased allowing the holder to sell the portfolio at a future time T (exercise time) at a fixed value L (strike price) ³⁵. In general, the problem can be analysed referring to path-dependent options (asian option, in particular); in the case of fixed-income instruments, it can be operationally analysed in terms of bond options, either with deterministic and stochastic exercise price. Closed form solutions for the value of the guarantee can be derived in the single-factor CIR model ³⁶. This value can be interpreted as a second-order-like measure and can be used to define ALM strategies as it allows to characterize a price for interest rate risk of an immunized portfolio.

Endnotes

- ¹ In this sense, the theoretical schemes, debates and some recent applications (Revell, 1975; Maisel, 1981; Toevs, Haney, 1986; Fabozzi, Konishi, 1991; it is relevant for Italian banks ABI, 1995) confirm the famous original positions (Samuelson, 1945).
- ² Santonomero (1984), p.596.
- ³ A recent operational outline is in Risk (1995).
- ⁴ Daykin, Hey (1990), p.179. For a formal characterization of asset-liability management models based on semideterministic and stochastic immunization theory, see Castellani, De Felice, Moriconi (1992).
- ⁵ JP Morgan (1985), p.7.
- ⁶ For the references to the actuarial tradition on ALM, see De Felice (1995).
- ⁷ Redington (1952), p.289.
- ⁸ Redington (1952), p.316.
- ⁹ Interest rate risk is "inherently intertemporal" in the sense that "an unexpected change in the interest rate now affects all future returns, so interest rate risk compounds over time. On the other hand, security risk is the contemporaneous resolution of returns" (Ingersoll, 1987, p. 404).
- ¹⁰ The standard method of banking regulators are inspired by this approach (see Basle Committee, 1996, and Bank of Italy, 1994).
- ¹¹ In Italy, for example, the PUMA data base, which is periodically monitored by the Bank of Italy.
- ¹² See Toevs, Haney (1986), in particular pp. 284-296.
- ¹³ Rate-sensitive assets are those that can experience contractual changes in interest rates during the gapping period. All financial contracts that mature within the gapping period are rate-sensitive. Variable-rate instruments repriced during the gapping period are also rate-sensitive regardless of their maturity dates. Interest income and the periodic return of principal (mortgage, for example) are also rate-sensitive if these flows are invested in new instruments during the period. Rate-sensitive liabilities are similarly defined.
- ¹⁴ See De Felice, Moriconi (1991) and De Felice, Moriconi, Salvemini (1993), in particular footnote 26. Applications of the single-factor CIR model to the Italian Government securities market, to eurodollar and eurolira bond markets, are proposed in Castellani, De Felice, Moriconi, Mottura (1993). See also Moriconi (1995), pp.30-33.
- ¹⁵ In the sense that "this process of routines reparametrization is theoretically inconsistent . . . but seems to some degree unavoidable", Duffie (1992), p. 216.
- ¹⁶ See Castellani, De Felice, Moriconi, Mottura (1993), p. 123.
- ¹⁷ Two calibration techniques for Italian *CCTs* (Italian Treasury Crediti Certificati) are analyzed in De Felice, Moriconi, Salvemini (1993), pp.152-157.
- ¹⁸ See Castellani, De Felice, Moriconi, Mottura (1993), p. 176.

¹⁹ JP Morgan (1995), p.107.

²⁰ For a theoretical pricing scheme for indexed securities, see Castellani, De Felice, Moriconi (1990).

²¹ It is the case analysed in De Felice, Moriconi, Salvemini (1993), pp. 135-140. With reference to the situation observed in the screen-based market (*Mercato Telematico*) on 26 November 1991, we have analyzed a hypothetical five-year CCT with a semi-annual coupon linked to the auction prices of six-month Treasury bills in the two months ending one month before the day on which coupon entitlement begins. We assume that the security is issued on the valuation date and ignore taxation and the spread (adding a spread of 0.50 points would result in this hypothetical security being very similar to some actual CCTs, e.g. that with ABI code 13208).

²² Let us consider some remarks on mapping, referring to the different equivalent cash flows. The ex-ante coupons differ from the forward coupons because the indexation lag of around seven months does not coincide with the coupon interval (or with the maturity of the underlying security). The ex-ante coupons are systematically lower than the corresponding forward coupons and the model price of the ex-ante security is below par. The model price of the flat-coupon security does not diverge from the equilibrium price only because the slope of the relevant yield curve is not very large. The notable difference between the risk (and the durations) of the deterministic cash-flow with ex-ante coupons compared with those of the indexed security is a measure of the risk-reduction effect of the indexation mechanism for securities that (by construction) have the same time to maturity and the same model price.

²³ See Castellani, De Felice, Moriconi (1990) for definitions and derivation.

²⁴ For the proof, see De Felice, Moriconi, Salvemini (1993), Appendix 1.

²⁵ Details and applications to Italian market are given in Castellani, De Felice, Moriconi, Mottura (1996).

²⁶ See Castellani, De Felice, Moriconi, Mottura (1993), pp. 177-178.

²⁷ See Castellani, De Felice, Moriconi, Mottura (1996).

²⁸ See Beder (1995), p.12.

²⁹ Value-at-risk measures calculated under different stochastic market models are proposed in Castellani, De Felice, Moriconi, Mottura (1996).

³⁰ The argument is used by de Finetti (1967), p.7, in his comment on "pathological" responses to uncertainty.

³¹ For details and comments see De Felice (1995), pp. 71-72.

³² The theoretical analysis for portfolio selection is in Castellani, De Angelis (1996). For an application to Italian Government securities market with respect to supervisory rules see Mottura, Ristuccia (1996).

³³ The idea was originally introduced in the semi-deterministic framework, see De Felice, Moriconi (1990).

³⁴ See Mottura (1993) for an analysis of surplus stream applied to the management of profit-sharing policies.

³⁵ The "isomorphic" relationship between the guarantee and common stock put option was introduced by Merton (1977), referring to the problem of deposit insurance (see also Merton, Perold, 1993). Within the same framework, the RARORAC index has been recently defined.

³⁶ See Mottura (1995).

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