Value-at-Risk: A Risk Theoretical Perspective with Focus on Applications in the Insurance Industry

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Abstract

The value-at-risk conception is becoming more and more a global standard for the measurement and control of market risk of banks active in trading, paying special consideration to the use of derivative instruments. Therefore the question is timely, if and under which conditions a corresponding application of this conception is possible within the framework of risk control of the asset portfolio of an insurance company.

To accomplish this the present paper first develops the basic conception of the value-at-risk approach. The structural parallelity to the risk theoretical approach to solvency control is emphasized. Next the applications in the bank case and the bank specific realization of the VaR-conception are discussed. This forms the basis to contrast the insurance case and to draw conclusions on the specific necessities of realising the VaR-conception in the insurance case.

Résumé

La conception "valeur au risque" devient de plus en plus un standard globale pour le mesurage et contrôle du risque des banques qui sont actifs sur le marché financier. Cette approche tient particulièrement compte des engagements dans des produits dérivés. Pour cette raison, il se pose la question si et à quelle condition une application parallele de cette conception soit possible dans le cadre du contrôle du risque d'un portefeuille des actifs financiers d'une compagnie d'assurance.

Pour répondre à cette question, la contribution présente développe premièrement la conception de base du approche "valeur au risque". Nous mettons l'accent sur le parallelisme structural à l'approche risque théorique du contrôle de la solvabilité. Puis, la discussion se concentre sur l'application du cas des banques et sur la réalisation de la conception "valeur au risque" spécifique à la banque. Cela établit les bases de l'opposer le cas de l'assurance et de formuler des conséquences à l'égard des nécessités spécifiques de la réalisation de la conception "valeur au risque" dans le cas de l'assurance.

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1. Introduction

The Value-at-Risk conception[1] is becoming more and more a global standard[2] for the measurement and control of the market risk for banks active in trading, with special consideration of the use of derivative instruments. Therefore the question is timely[3], if and under which conditions a corresponding application of this conception is possible within the framework of risk control of the asset portfolio of an insurance company.

For this purpose the present paper first develops the basic conception of the Value-at-Risk approach. The structural parallelity to the risk theoretical approach to solvency control is emphasized. Then the applications in the bank case and the bank specific realization of the VaR-conception are discussed. This forms the basis to contrast the insurance case and to draw conclusions to the specific necessities of realizing the VaR-conception in the insurance case.

2. Value-at-Risk: A Risk Theoretical Analysis
2.1 Basic Construction: VaR as a Quantile Value

Let denote V_t the future (random) value of a financial asset resp. a portfolio of financial assets at time t and v_0 the corresponding (known) value at present. Let denote $F_t(x)$ the distribution function of V_t. We assume for the following that F_t possesses a density function $f_t(x)$.

Let in addition denote $0 < \alpha < 1$ a prescribed safety level as well as $VaR_\alpha(t)$ the *Value-at-Risk at confidence level α for the time horizon t*. The basic structural

property of $VaR_\alpha(t)$ then is given by

$$P(V_t - v_0 \leq -VaR_\alpha(t)) = \alpha \qquad \text{(2.1a)}$$

resp. equivalently

$$P(v_0 - V_t \geq VaR_\alpha(t)) = \alpha \quad . \qquad \text{(2.1b)}$$

The quantity to be controlled is the change in value $\Delta V(t) := V_t - v_0$ of the financial asset resp. the portfolio during $[0, t]$. $VaR_\alpha(t)$ is identical to the negative value of the α-quantile of the distribution of $V_t - v_0$. Equivalently to this $VaR_\alpha(t)$ is identical to the $(1-\alpha)$-quantile of the distribution of $v_0 - V_t$, i.e. the Value-at-Risk at the confidence level α is that realization of the loss amount $v_0 - V_t$, which will not be exceeded with probability $1 - \alpha$, as illustrated in the following figure.
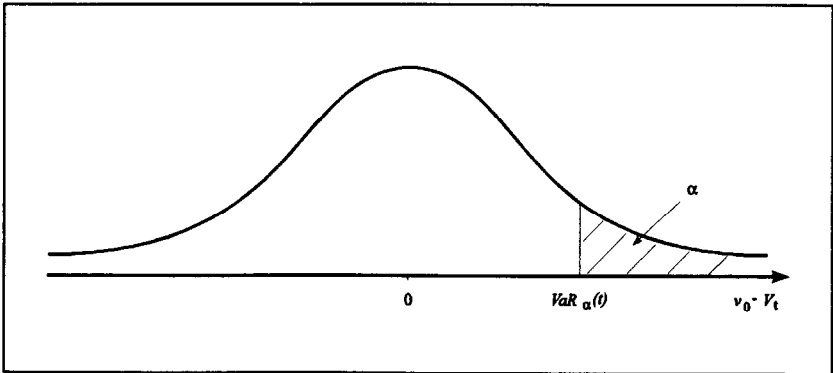
Value-at-Risk as the $(1-\alpha)$-quantile of the distribution of $v_0 - V_t$

With respect to the distribution function $F_t(x)$ of V_t $VaR_\alpha(t)$ can in addition be characterized as follows. Let denote $V_\alpha(t) := F_t^{-1}(\alpha)$ the α-quantile of the distribution of V_t, i.e. we have $P(V_t \leq V_\alpha(t)) = \alpha$. From this we get $P(v_0 - V_t \geq v_0 - V_\alpha(t)) = \alpha$

and therefore

$$VaR_\alpha(t) = v_0 - V_\alpha(t) \quad . \tag{2.2}$$

To prepare the further analysis we split up $V_\alpha(t)$ into the expected value part $E(V_t)$ as well as into the excess part $Z_\alpha(V_t) := E(V_t) - V_\alpha(t)$, which depends on the assumed distribution of V_t. Expression (2.2) then changes to

$$VaR_\alpha(t) = [v_0 - E(V_t)] + Z_\alpha(t) \quad . \tag{2.3}$$

In case we make the assumption $E(V_t) \approx v_0$, which is likely to hold approximately for very short time horizons (a day, a week), cf. *Beckström/Campbell* (1995b, p. 41), we obtain

$$VaR_\alpha(t) \approx Z_\alpha(t) \quad , \tag{2.4}$$

i.e. the Value-at-Risk corresponds to a measure for the deviation of the distribution of V_t from its expected value. At the same time this way of proceeding reduces the model risk of assuming unjustly an expected increase in value, which would result in a reduction of VaR according to (2.3). The assumption $E(V_t) \approx v_0$ implies a determination of the Value-at-Risk which is on the safe side. At the same time in this case we can interpret the Value-at-Risk directly as a measure of risk.

Example 1:

Suppose V_t is following a normal distribution, $V_t^* := [V_t - E(V_t)] / \sigma(V_t)$ is following a standard normal distribution. In this case we have $N_\alpha = -N_{1-\alpha}$, and obtain $P(V_t^* \leq -N_{1-\alpha}) = \alpha$. From this we derive $\alpha = P(V_t \leq E(V_t) - N_{1-\alpha} \sigma(V_t)) = P(v_0 - V_t \geq$

$v_0 - E(V_t) + N_{1-\alpha} \sigma(V_t)$), i.e. the Value-at-Risk is given by

$$VaR_\alpha(t) = v_0 - E(V_t) + N_{1-\alpha}\sigma(V_t) \quad . \tag{2.5}$$

If we are willing to assume $E(V_t) \approx v_0$, we obtain

$$VaR_\alpha(t) \approx N_{1-\alpha}\sigma(V_t) \quad , \tag{2.6}$$

i.e. the Value-at-Risk is proportional to the risk measure standard deviation.

An alternative way of presenting the VaR-conception can be obtained by intro-ducing the quantity Return-on-Value-at-Risk *RoVaR$_\alpha$(t)* defined by

$$RoVaR_\alpha(t) := \frac{V_t - v_0}{VaR_\alpha(t)} \quad . \tag{2.7}$$

The structural property (2.1a) then equivalently reads

$$P[RoVaR_\alpha(t) \leq -1] = \alpha \tag{2.8a}$$

resp.

$$P[RoVaR_\alpha(t) > -1] = 1 - \alpha \quad , \tag{2.8b}$$

i.e. the probability of not exhausting the Value-at-Risk by the return (related to the VaR) of capital investment is under control and high.

2.2 How to realize the VaR-conception

To calculate a VaR the decision maker needs to obtain a forecast of the probability distribution of the future value of V_t. This means that the decision maker must create a finance model that describes value. A standard way of proceeding in this connection is to relate the values V_t^i of the financial assets to relatively few funda-mental factors, e.g. on the basis of linear factor models - cf. in general e.g. *Al-*

brecht/Maurer/Mayser (1995) - for $\Delta V_t^i := V_t^i - v_0^i$ or for $\ln \Delta V_t^i$. In the first case the modelling assumption reads (assuming m factors, $j = 1, .., m$)

$$\Delta V_t^i = a_i(t) + \sum_{j=1}^{m} b_{ij}(t) \, F_j(t) + \varepsilon_i(t) \quad , \tag{2.9}$$

with the standard assumptions $E[\varepsilon_i(t)] = 0$, $i = 1, ..., I$; $Cov[\varepsilon_i(t), \varepsilon_{i'}(t)] = 0$ for $i \neq i'$, and $Cov[F_j(t), \varepsilon_i(t)] = 0$ for all i, j.

To calculate the portfolio-VaR in this case (ignoring the time parameter) only an identification of the sensitivities a_i resp. b_{ij}, the expected values $f_j = E(F_j)$, the covariances $f_{ij} = Cov(F_i, F_j)$ as well as the variances $s_i^2 = Var(\varepsilon_i)$ is necessary.

In case of non-linear pay-offs and respective valuations the usual approach is to linearize the non-linear pricing models through the use of the Taylor expansion (this leads to the sensitivity models for the VaR). In case of option and option-like securities, however, one works directly with (non-linear) option pricing models.

To determine the Value-at-Risk of a portfolio $V_\alpha^P(t)$ on the basis of one of the preceding models there are the following possibilities:

- analytical determination on the basis of a distributional assumption, usually in conjunction with variance-covariance-estimations,
- Monte Carlo-simulation,
- historical simulation.

As for the purpose of this contribution primarily the conceptual framework of the VaR-approach is of relevance, we refer to *Beckström/Campbell* (1995b) and *Wilson* (1994) for more details of realizing the VaR-conception.

2.3   Portfolio Effects

In case of a homogeneous class of financial assets VaR is proportional to the number k of assets,

$$VaR_\alpha^P(t) = k\, VaR_\alpha(t) \qquad .\qquad\qquad (2.10)$$

This is easily seen on the basis of (2.1a) using the property $P(V_t - v_0 \le -VaR_\alpha(t))$ $= P(k\,(V_t - v_0) \le -k\,VaR_\alpha(t))$.

We now take a look at a heterogeneous portfolio consisting of n asset classes. The value of class i at time t is denoted by V_t^i, similarly V_t^P denotes the corresponding portfolio value. Obviously for $\Delta V_P(t) := V_t^P - v_0^P$ we have

$$\Delta V_P(t) = \sum_{i=1}^{n} \Delta V_i(t) \qquad\qquad (2.11)$$

and the Value-at-Risk of the portfolio $VaR_\alpha^P(t)$ is defined by

$$P(\Delta V_P(t) \le -VaR_\alpha^P(t)) = \alpha \qquad .\qquad\qquad (2.12)$$

Example 2:

Carrying on with example 1, we assume that all V_t^i follow a normal distribution. We define $\mu_i(t) := E(V_t^i)$, $\sigma_i(t) := \sigma(V_t^i)$ and $\rho_{ij}(t) := \rho(V_t^i, V_t^j)$. We then have V_t^P

$\sim N\,(\mu_P(t),\, \sigma_P(t))$ where $\mu_P(t) = \sum_{i=1}^{n} \mu_i(t)$ and $\sigma_P(t) = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij}(t)\, \sigma_i(t)\, \sigma_j(t)\right]^{\frac{1}{2}}$.

For the Value-at-Risk of the portfolio we therefore obtain

$$VaR_\alpha^P(t) = \sum_{i=1}^{n} [v_0^i - \mu_i(t)]$$

$$+ N_{1-\alpha}\left[\sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}(t)\,\sigma_i(t)\,\sigma_j(t)\right]^{\frac{1}{2}} . \qquad (2.13)$$

This means that for a heterogeneous portfolio consisting of different asset classes the Value-at-Risk is <u>not</u> necessarily additive. In case of example 2 for instance the sum of the Value-at-Risk amounts of each single asset class is given by $\sum_{i=1}^{n} VaR_\alpha^i(t) = \sum_{i=1}^{n}\left[v_0^i - \mu_i(t)\right] + N_{1-\alpha}\sum_{i=1}^{n} \sigma_i(t)$. With the exception of the case of a

perfect positive correlation between all asset classes, i.e. $\rho_{ij} = 1$ for all $i \neq j$, the collective Value-at-Risk according to (2.13) is always lower than the sum of the Value-at-Risk amounts of the asset classes, i.e.

$$VaR_\alpha^P(t) < \sum_{i=1}^{n} VaR_\alpha^i(t) . \qquad (2.14)$$

In case (still keeping the assumption of a normal distribution) of not all the asset classes being perfectly positively correlated, we consequently have a collective reduction effect when calculating the portfolio-VaR. This clarifies the effect that the VaR always has to be understood relative to the portfolio of assets to be analyzed. This especially gains importance, when we interpret the VaR as a required risk based capital, cf. part 3.1 of this paper.

2.4 Time Effects

To be able to identify possible time effects on the VaR we have to specify a specific type of stochastic process $\{V_t;\ t \geq 0\}$ for the development of the value of a

financial asset or a portfolio of such assets. To be in accordance with examples 1 and 2 we, in the following analysis, assume a Brownian motion (Wiener process with drift μ and diffussion σ), the standard model of a time continuous random walk (with drift). The starting point of the random walk is $V_0 = v_0$. The general development of the Brownian motion can be put in the following form

$$V_t = v_0 + \mu t + \sigma\sqrt{t}\, Z \quad , \tag{2.15}$$

where Z is a standard normally distributed random variable, i.e. $Z \sim N(0, 1)$. From (2.15) we have

$$E(V_t) = v_0 + \mu t \quad , \quad \sigma(V_t) = \sigma\sqrt{t} \quad . \tag{2.16}$$

The corresponding Value-at-Risk according to (2.5) now is

$$VaR_\alpha(t) = N_{1-\alpha}\sigma\sqrt{t} - \mu t \quad . \tag{2.17}$$

In this case the development of the VaR possesses a linear as well as a degressive part. If one is willing to assume a random walk without drift ($\mu = 0$), then the term (2.17) reduces to

$$VaR_\alpha(t) = N_{1-\alpha}\sigma\sqrt{t} \quad , \tag{2.18}$$

which means that the VaR is degressively dependent on the length of the time horizon.

2.5 Value-at-Risk and Shortfall-Probability

The shortfall-probability of a general random variable X with respect to a target z is given by

$$LPM_0(z;X) = P(X \le z) \quad . \tag{2.19}$$

If we denote Q_α as the α-quantile of the distribution of X we generally have $Q_\alpha =$

sup $\{z; LPM_0(z; X) \le \alpha\}$. With respect to the Value-at-Risk the following relations are valid

$$VaR_\alpha(t) = -sup \{v; LPM_0(v; V_t - v_0) \le \alpha\}$$
$$= inf \{-v; LPM_0(v; V_t - v_0) \le \alpha\} \tag{2.20a}$$

resp.

$$VaR_\alpha(t) = v_0 - sup \{v; LPM_0(v; V_t) \le \alpha\} \quad . \tag{2.20b}$$

In case X possesses a density function we are able to obtain more specific relations, particularly Q_α is the solution of the equation $LPM_0(Q_\alpha; X) = \alpha$; because $LPM_0(v; X)$ is strictly monotonous in v we therefore obtain $Q_\alpha = LPM_0^{-1}(\bullet; X)(\alpha)$. According to that (2.20) now reads

$$VaR_\alpha(t) = -LPM_0^{-1}(\bullet; V_t - v_0)(\alpha) \tag{2.21a}$$

resp.

$$VaR_\alpha(t) = v_0 - LPM_0^{-1}(\bullet; V_t)(\alpha) \quad . \tag{2.21b}$$

2.6   The Risk Theoretical Dimension of the VaR: Solvency Control

The interpretation of VaR as a reference quantity to determine the required risk based capital, cf. part 3.1, clarifies the connection between risk control of a portfolio of assets on the one hand and a portfolio of insurance risks on the other. Let denote S the accumulated claim amount of a portfolio of insurance risks for one period (typically one year) and π the collective (risk) premiums earned at the beginning of the period. Now the standard risk theoretical way to determine the necessary solvency capital SC with respect to the collective of insured risks is to keep the loss probability under control, i.e. to maintain a tolerance level ε. The control

criterion chosen in this case reads:

$$P(S \geq SC + \pi) = \varepsilon \quad . \tag{2.22}$$

The version of (2.22), which is equivalent to (2.1a), obviously is $P(\pi - S \leq -SC)$ $= \varepsilon$, the version equivalent to (2.1b) is $P(S - \pi \geq SC) = \varepsilon$. The change in value $\Delta V_P(1)$, which in case of the Value-at-Risk reflects the one-period profit on investment, corresponds to the technical profit/loss-position $\pi - S$ in case of the risk theoretical solvency control. Note in addition that the determination of the required solvency capital is always related to the entire collective of insured risks and not to the sub-collectives (which would correspond to asset classes) and usually is not done for shorter periods $[0, t]$ with $t < 1$. This is indicating some main distinctions between the insurance and the bank case, which we are going to deal with in part 4.1.

2.7   A Generalized VaR-Conception on the Basis of the Shortfall-Expectation

The VaR-conception based on the relationship (2.1) is criticized[4] because it only considers loss probabilities and not e.g. the average loss amount. A corresponding generalization is proposed by *Schröder* (1996) for a VaR on return basis. A similar proposal is presented in *Albrecht/König/Maurer/Schradin* (1996), who control the expected excess-value of the random variable (discounted) loss amount in the framework of determining the required risk based capital in case of default risk.[5] In the following we will discuss this approach in order to obtain a generalized VaR-conception under more general aspects.[6]

To do this we use the conception of the Return-on-Value-at-Risk according to (2.7). The property (2.8a), being a version of (2.1a), then can be equivalenty stated in a form that the shortfall-probability of RoVaR with respect to the target return $r_T = -1$ is under control. As shown in *Albrecht* (1994, S. 93) a general class of

measures of shortfall-risk of a return quantity R with respect to the target return r_T is given by

$$LPM_n(r_T) = E\left[\max(r_T - R, 0)^n\right] \quad , \qquad (2.23)$$

i.e. it corresponds to the lower partial moments of order n of R with regard to r_T. The case n = 0 corresponds to the shortfall-probability, being the control criterion underlying (2.8a). The case n = 1 corresponds to the shortfall-expectation, which will be the basis of our further analysis. The control of the generalized Value-at-Risk $VaR_\alpha^*(t)$ is now done on the basis of limiting the shortfall-expectation with respect to $r_T = -1$ to a small quantity α (e.g. 1% or 5%). Formally the property is

$$E\left[\max(-1 - RoVaR_\alpha^*(t), 0)\right] = \alpha \qquad (2.24a)$$

resp. equivalently

$$E\left[\max\left(\frac{v_0 - V_t}{VaR_\alpha^*(t)} - 1, 0\right)\right] = \alpha \quad . \qquad (2.24b)$$

As in general we have E[max $(kX, 0)$] = k E[max $(X, 0)$], this consequently is equivalent to

$$E\left[\max\left(v_0 - V_t - VaR_\alpha^*(t), 0\right)\right] = \alpha\, VaR_\alpha^*(t) \quad . \qquad (2.24c)$$

Obviously the relation (2.24c) has the following interpretation. The expected value of loss amount $v_0 - V_t$ in excess of the (generalized) VaR is limited to a defined fraction of the VaR.

2.8   Value-at-Risk and Required Risk Based Capital in Case of Default Risk

If one suceeds in specifying a probability distribution of the credit loss[7] CL, a de-

termination of the required risk based capital again is possible on the basis of a VaR-conception. Examples of such probability distributions can be found in *Albrecht/König/Maurer/Schradin* (1996) or in *Iben/Brotherton-Ratcliffe* (1994). The VaR_{CL} for a specific position subject to default risk is then determined according to the relation

$$P\big(CL > VaR_{\alpha}\big) = \varepsilon \quad . \tag{2.25}$$

The generalized VaR-conception of part 2.7 in the default risk-framework is discussed in *Albrecht/König/Maurer/Schradin* (1996).

3. Applications of the VaR-Conception for Risk Control and Risk Based Profit Control (Bank Case)

3.1 Risk Based Capital Requirements

The application of the VaR-conception to determine the capital requirements for the trading operations of a bank with respect to market risk within the framework of a bank internal risk measurement model is supported by several institutions, e.g. The Group of Thirty, The Derivatives Policy Group and in Europe the Basle Committee on Banking Supervision. The required capital on a daily basis for trading operations proposed by the Basle Committee is the VaR[8] multiplied by a factor three or more (to take account for the inherent "model risk"). The time horizon over which the VaR is calculated amounts to $t = 10$ days.

For the following analysis we assume that the probability distribution of V_t is correctly specified, i.e. there is no model risk. If we now denote $RAC^{req}(t)$ to be the required risk based capital for a time horizon of length t, then the following

relation has to be fulfilled

$$RAC^{req}(t) \geq VaR_\alpha^P(t) \quad . \tag{3.1}$$

From (2.12) we therefore obtain

$$P(RAC^{req}(t) + \Delta V_t^P < 0) \leq \alpha \quad . \tag{3.2}$$

This means that the probability that the change in value of the considered portfolio will exhaust the required capital over a time horizon of length t is limited by α.

Because of the collective effects described in part 2.3 one obtains capital requirements which are unnecessarily high in case one uses the isolated Value-at-Risk amounts for the single asset classes and not for the entire collective of assets.

3.2. Prescription of Risk Limits

In contrast to part 3.1, where one takes the positions in financial assets as given and tries to determine the required capital, the situation is different, when constructing risk limits to be prescribed. Here one assumes an amount of risk based capital $RAC_P^{is}(t)$ as given for a time interval $[0, t]$. The possible investment strategies are now limited by the requirement that with confidence level α a change in value of the resulting portfolio will not exhaust the given RAC. If we denote formally ϑ as a decision parameter, which determines the distribution of V_t^P, i.e. $V_t^P = V_t^P(\vartheta)$, and therefore as well determines the Value-at-Risk at confidence level α, i.e. $VaR_\alpha^P(t) = VaR_\alpha^P(t, \vartheta)$, then the following relation has to be fulfilled:

$$VaR_\alpha^P(t, \vartheta) \leq RAC_P^{is}(t) \quad . \tag{3.3}$$

In case that V_t^P is normally distributed the possible investment strategies ϑ would determine the accessible (μ, σ)-combinations $(E(V_t^P), \sigma(V_t^P)) = (E_\vartheta(V_t^P), \sigma_\vartheta(V_t^P))$

and, because of that, one would have the restriction:

$$v_0 - E_\vartheta\!\left(V_t^P\right) + N_{1-\alpha}\sigma_\vartheta\!\left(V_t^P\right) \leq RAC^{is}(t) \quad . \tag{3.4}$$

The setting of risk limits described so far is relating to the portfolio level. To be able to control sub-positions, too, risk limits for sub-positions (e.g. for asset classes) are as well of interest. This leads to the non-trivial tasks of splitting up the entire VaR and of distributing it risk adequately over the sub-positions considered.

3.3   Risk Adjusted Performance Control

So far only the control of risk was the subject of our analysis. Beyond that the return (performance) of financial operations is of central interest.

As a simultaneous optimization of the activities under the restrictions (3.3) resp. (3.4) is practically infeasible one has to take more simple approaches into account. As a starting point we use the Return-on-Value-at-Risk according to (2.7), and we assume that a minimum return of $r_0(t)$ is the target for the desired (risk adjusted) return. We can formalize that, either by requiring that the target return has to be achieved in expectation only, i.e.

$$E\!\left[RoVaR_\alpha^P(t)\right] \geq r_0(t) \quad , \tag{3.5}$$

or alternatively with a high confidence level, i.e.

$$P\!\left[RoVaR_\alpha^P(t) \geq r_0(t)\right] \geq 1 - \varepsilon \quad . \tag{3.6}$$

4.   VaR for Insurance Companies: Possibilities and Problems

4.1   Preliminary Remarks on the Peculiarities of the Insurance Case

The analysis of part 3 pertains to the <u>specific</u> realization of the general VaR-conception of part 2 to the bank case being characterized by

-    a focus on market risk
-    a focus on short-term horizons and
-    a focus on trading operations.

The insurance case is significantly different. First of all in the insurance case the purpose of holding financial positions is different. Financial instruments are not held for trading purposes but for investment purposes accompanied by risk management activities. Investment is not done for its own sake but to be able to fulfill the liabilites of the insurance company, i.e. it has to be seen in an asset/liability-framework. As insurance liabilities are of middle or of long term, this has consequences for the relevant time horizon. As the investment in financial assets has to be seen under middle-to-long term aspects the exclusive considerations of market values loses its importance and other categories of value (e.g. accounting values, book values) have to be considered, too. This especially goes for countries (e.g. Germany), where accounting is not done on a market value basis. Summing up, we have the following important differences compared to the bank case:

-    the purpose of holding financial positions,
-    the relevant term of the time horizon,
-    the categories of value to be considered as well as
-    the general importance of the liabilities when controlling the investment process.

This does, however, <u>not</u> imply that the VaR-conception will be not of use for insu-

rance companies. We see the possible applications primarily in risk control[9] and we will discuss that in the following sections. But, the general VaR-conception has to be realized in the insurance case in a different way. This, however, as well means that VaR-systems specifically designed for the bank case are not necessarily suitable for the insurance case. At least they have to be modified to be able to meet the special requirements of the insurance case. Because of the longer term and the asset/liability-context we believe, that actuarial stochastic investment models will be of decisive importance in such a modified VaR-system.

4.2   Risk Based Capital Requirements

A regulation of capital requirements in the insurance field which explicitly takes into account investment risk are the US-RBC (risk based capital)-standards proposed by the NAIC. A study of the RBC-standards again reveals the important differences between the insurance and the bank case consisting of

- the use of balance sheet values,
- the one year time horizon and
- the embedding of the Asset-RBC-requirements in a general system of risk categories, containing especially underwriting risk.

But there are further important differences to the VaR-conception. There is no consideration of the possible application of derivative instruments. Basically the approach in calculating the Asset-RBC is only very loosely connected to the models and results of modern capital market theory, which in contrast play a fundamental role in the VaR-conception. In addition the RBC-standard requires the insurers to use a single, rigid approach whereas the Basle Committee allows the banks to use their own internal models to calculate the VaR (subject to the provision that certain

"quality" requirements are fulfilled).[10]

4.3   Control of Loss- and Depreciation-Potential

A further possible application of the VaR-conception in the insurance case is loss control with respect to the asset side. The Value-at-Risk $VaR_\alpha(t)$ is a measure for the maximum loss in value $v_0 - V_t$ at confidence level α, i.e. only in 100 $\alpha\%$ of the cases over a time horizon of length t a larger loss than $VaR_\alpha(t)$ will have to be realized. With regard to loss control one has to distinguish between loss on a market value basis and loss on a book value basis (depreciation) depending on the relevant accounting rules.

With respect to the control of loss and/or depreciation it is recommendable to complete Value-at-Risk considerations by using stress-tests, i.e. the consequences of worst-case-scenarios on the loss/depreciation potential. This takes into account the consideration that Value-at-Risk calculations are typically based on average changes in value on a statistical basis and are not adequately able to cope with the consequences of exceptional developments on the capital markets.

An extension of stress-tests to include the liability side of the insurance operations are the *Resilience Tests* applied in the United Kingdom.

5.   Conclusions

The VaR-conception discussed in part 2 of the present paper is to be understood as a general approach to measure and to control the risk potential of a portfolio of financial assets. So far this general conception has only been specifically realized for the requirements of the trading operations of banks. The insurance case is different

in many instances and therefore a genuine realization of the VaR-conception has to be adapted to be able to meet adequately the specific requirements of insurance companies. Actuarial stochastic investment models should prove to be the key in accomplishing this task.

Endnotes

1)   Cf. e.g. in general *Beckström/Campbell* (1995a).

2)   Cf. e.g. the latest publication of the *Basle Committee on Banking Supervision* (1996).

3)   Cf. e.g. in addition *Ufer* (1996).

4)   Cf. e.g. *Boudoukh et al.* (1995) or *Schröder* (1996).

5)   The development of a VaR-conception in case of default risk is done in part 2.8.

6)   *Boudoukh et al.* (1995) propose in contrast to that to control the expected "worst-case-scenario-risk", formally measured by E [min $(Z_1, ..., Z_n)$], where the Z_i correspond to the changes in value of successive periods.

7)   One correspondingly has to work with a discounted credit loss, when the risk of default pertains over several periods of time.

8)   More precisely the higher value of the VaR of the last day and the average of the daily VaR-values over the last 60 days has to be taken.

9)   We believe that the applications in the domain of risk adjusted performance control discussed in part 3.3 are primarily of relevance for trading operations only.

10)   *Styblo Beder* (1995, p. 23) emphasizes the difficult trade-offs between a single, rigid approach and internal VaR-models.

References

Albrecht, P. (1994): Shortfall Returns and Shortfall Risk, in: Actuarial Approach for Financial Risks, Proceedings of the 4th AFIR International Colloquium, Orlando, Vol. I, 87 - 110

Albrecht, P., A. *König*, R. *Maurer*, H. R. *Schradin* (1996): An Actuarial Approach to Determine the Required Capital for Portfolios of Options with Default Risk, Contribution to the 6th AFIR International Colloquium, Nürnberg

Albrecht, P., R. *Maurer*, J. *Mayser* (1995): Analyse und Steuerung von Aktien-Portefeuilles auf der Grundlage von Faktorenmodellen, Transactions of the 25th International Congress of Actuaries, Brussels, Vol. 3, 21 - 40

Basle Committee on Banking Supervision (1996): Amendment to the Capital Accord to Incorporate Market Risks, Basle, January 1996

Beckström, R.A., A. *Campbell* (Eds.; 1995a): *An Introduction to VAR*, Palo Alto

Beckström, R.A., A. *Campbell* (1995b): Value-at-Risk (VAR): Theoretical Foundations, in: *Beckström*, R.A., A. *Campbell* (1995a), 31 - 76

Boudouhk, J., M. *Richardson*, R. *Whitelaw* (1995): Expect the Worst, RISK 8, September 1995, 100 - 101

Iben, B., R. *Brotherton-Ratcliffe* (1994): Credit Loss Distributions and Required Capital for Derivatives Portfolios, The Journal of Fixed Income 4, June 1994, 6 - 14

Schröder, M. (1996): The Value at Risk Approach: Proposals on a Generalization, Contribution to the 6th AFIR International Colloquium, Nürnberg

Styblo Beder, T. (1995): VAR: Seductive but Dangerous, Financial Analysts Journal, September/October 1995, 12 - 24

Ufer, W. (1996): The "Value at Risk" Concept for Insurance Companies, Contribution to the 6th AFIR International Colloquium, Nürnberg

Wilson, T. (1994): Plugging the Gap, RISK 7, October 1994, 75 - 80