The Impact of Policyholder Behavior on Pricing, Hedging, and Hedge Efficiency of Withdrawal Benefit Guarantees in Variable Annuities

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Research Purpose

- **Variable Annuities are fund-linked annuities**
  - the policyholder typically pays a single premium, which is invested in one or several mutual funds
  - several guarantee riders available on top of this

- **“Guaranteed Lifetime Withdrawal Benefits” (GLWB)**
  - the policyholder is guaranteed lifelong minimum withdrawals
  - the invested capital is not annuitized
    - fund assets remain accessible to the policyholder
  - withdrawals are deducted from the policyholder’s account value as long as it has not been depleted
    - afterwards, the insurer has to compensate for the guaranteed withdrawals until the insured’s death
  - in return for this guarantee, the insurer receives guarantee fees deducted from the policyholder’s fund assets

→ combination of policyholder behavior, longevity and market risk that is difficult to hedge
Research Purpose – Previous Paper

  • analysis of the impact of stochastic equity volatility on pricing and hedging of GLWB riders
    – similar framework used as in Bauer et al. (2008)
  • comparison of different hedging strategies and their efficiency
    – no hedge, delta hedge, and delta-vega hedge
  • focus lies on model risk (financial market)
    – hedging model vs. data-generating model
  • policyholder behavior modeled deterministically

⇒ starting point of current paper
Research Questions

- The focus of this paper lies on behavioral risk in the context of GLWB riders. Its key question is:

  What is the impact of policyholder behavior on pricing and hedging of GLWB riders?

- What is the magnitude of potential losses if assumptions about future policyholder behavior prove to be wrong?

- What effect does the product design of the GLWB rider have on the results?
Main findings

- Policyholder behavior that differs from pricing / hedging assumptions may lead to significant losses to the insurer.
  - assuming optimal (financially rational) behavior, however, may lead to products that are not competitive

➤ Use product design to reduce sensitivity to policyholder behavior.
  - insurer may then assume optimal behavior without losing (too much) competitiveness
  - but: such product designs may have other disadvantages
    - harder to hedge and higher sensitivity to changes in volatility

- In order to assess their behavioral assumptions, insurers should know what ‘optimal’ policyholder behavior would be.
  - modeled behavior in more elaborate models may still be far from ‘optimal’
Agenda

– Product designs
– Market models
– Models of the policyholder behavior
– Pricing results
– Hedging results
– Outlook
Product designs of the GLWB option

- All considered designs guarantee an annual minimum withdrawal amount for the lifetime of the insured.
  - surrender benefit = account value less surrender fees
  - death benefit = account value

- Depending on the product design, the guaranteed withdrawal amount increases if the fund performs well.

⇒ Three different ratchet mechanisms considered:

1) No Ratchet
   - the guaranteed withdrawal amount remains constant

2) Lookback Ratchet
   - the guaranteed withdrawal amount is calculated as a percentage of the highest account value at all past policy anniversaries

3) Remaining Withdrawal Benefit Base (WBB) Ratchet
   - if the account value exceeds a certain reference value, the surplus is used to increase the guaranteed withdrawal amount for all following payments
Market models used for pricing, hedging and simulation

- constant interest rates
- no spreads / no transaction costs
- The dynamics of the contract’s underlying fund is given by the following SDEs:
  - **Black-Scholes (1973)**
    \[
    dS(t) = \mu S(t) dt + \sigma_{BS} S(t) dW(t), \quad S(0) \geq 0
    \]
  - **Heston (1993)**
    \[
    dS(t) = \mu S(t) dt + \sqrt{V(t)} S(t) dW_1(t), \quad S(0) \geq 0
    \]
    \[
    dV(t) = \kappa (\theta - V(t)) dt + \sigma_v \sqrt{V(t)} dW_2(t), \quad V(0) \geq 0
    \]
  - with
    - \(\mu\) - drift
    - \(\sigma_{BS}\) - Black-Scholes volatility
    - \(V(t)\) - local variance at time \(t\)
    - \(\kappa\) - speed of mean reversion
    - \(\theta\) - long-term average variance
    - \(\sigma_v\) - “volatility of volatility”
    - \(W_{1/2}\) - Wiener processes
    - \(\rho\) - correlation between \(W_1\) and \(W_2\)
Policyholder Behavior

- Only two options considered:
  - policyholder withdraws guaranteed amount or
  - policyholder withdraws all of the remaining fund assets
    ➔ full surrender

- Behavior within a pool of policies modeled as annual percentage indicating the portion of policyholders who surrender their contract each year
Policyholder Behavior – Considered Models

1) Deterministic behavior
   - each year, a deterministic but time-dependent percentage of the policyholders perform full surrender

2) Optimal (financially rational) behavior
   - approximated via Least-Squares-MC approach (LSMC)

3) Moneyness approach
   - practitioner’s approach
   - use deterministic behavior as base
   - determine factor between 1/3 and 5 depending on the ‘moneyness’ of the guarantee
   - we use the ratio between surrender value and the NPV of an immediate annuity as ‘moneyness’

4) (→ paper)
Selected Pricing Results

Potential loss from mispricing: assumed vs. actual
  • assumed: Black-Scholes, $\sigma_{BS} = 22\%$, deterministic behavior

→ actual option value at inception (in % of single premium):

<table>
<thead>
<tr>
<th></th>
<th>No Ratchet</th>
<th>Lookback</th>
<th>Rem. WBB</th>
</tr>
</thead>
<tbody>
<tr>
<td>correct</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Moneyness</td>
<td>-2.4</td>
<td>-1.3</td>
<td>-1.2</td>
</tr>
<tr>
<td>Optimal</td>
<td>-4.6</td>
<td>-2.4</td>
<td>-2.3</td>
</tr>
<tr>
<td>Vol 25 %</td>
<td>-1.1</td>
<td>-2.1</td>
<td>-2.1</td>
</tr>
</tbody>
</table>
## Selected Hedging Results

- homogeneous pool of policies
- Black-Scholes delta hedge
  - monthly rebalancing of hedge portfolio

- What if actual behavior differs from pricing and hedging assumptions?
  - data-generating model Black-Scholes
  - values represent (relative changes in) the risk measure
    - CTE90 of final P&L in % of single premium

<table>
<thead>
<tr>
<th>Assumed / Actual</th>
<th>No Ratchet</th>
<th>Lookback</th>
<th>Rem. WBB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Det. / Det.</td>
<td>1.2</td>
<td>2.6</td>
<td>2.4</td>
</tr>
<tr>
<td>Det. / Moneyness</td>
<td>8.2</td>
<td>3.9</td>
<td>3.6</td>
</tr>
<tr>
<td>(+583%)</td>
<td>(+50%)</td>
<td>(+50%)</td>
<td></td>
</tr>
<tr>
<td>Det. / Optimal</td>
<td>12.5</td>
<td>6.2</td>
<td>5.4</td>
</tr>
<tr>
<td>(+942%)</td>
<td>(+138%)</td>
<td>(+125%)</td>
<td></td>
</tr>
</tbody>
</table>
Selected Hedging Results

- data-generating model: Black-Scholes ➔ Heston
  - constant equity volatility ➔ stochastic equity volatility
  - relative change in risk (range given for different behavioral models):
    - No Ratchet:  + 20% to + 27%
    - Lookback:   + 43% to + 45%
    - Rem. WBB:   + 44% to + 48%
Outlook / Future Research

- interest rate risk
- include more options for the policyholder (e.g. withdrawing nothing)
- heterogeneous pool of policies
- use LSMC approach for hedging / Greeks calculation
- ...
Thank you for your attention!

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