The Impact of Natural Hedging on a Life Insurer’s Risk Situation

AFIR 2011 Colloquium
June 22, 2011

Nadine Gatzert and Hannah Wesker
University of Erlangen-Nürnberg
Introduction
Motivation

• Demographic risk can significantly impact a life insurer’s solvency level
  ➢ Increase in life expectancy poses serious problems to life insurers selling annuities
  ➢ However, risk of unexpected high mortality (e.g. due to pandemics) has increased as well; problem for term life

• But: Hedging instruments are still scarce
  ➢ “Natural Hedge” between term life insurance (death benefit) and annuities (lifelong survival benefits) is effective alternative
  ➢ Use opposed reaction of term life insurance and annuities towards shocks to mortality
  ➢ Hedge shocks to mortality internally through portfolio composition
Introduction
Aim of paper

• Previous literature:

• Aim of this paper:
1. Quantify impact of natural hedging on a life insurance company’s insolvency risk
   ➢ Holistic model, take into account dynamic interaction between assets and liabilities for a two-product life insurer
2. Simultaneously *immunize* an insurer’s solvency situation against changes in mortality and *fix the absolute level of risk*
   ➢ Use investment strategy
Model framework
Modeling and forecasting mortality


\[ D_{x,t} \sim \text{Poisson}(E_{x,t} \cdot \mu_x(t)) \]
\[ \mu_x(t) = \exp(a_x + b_x \cdot k_t) \]
\[ q_x(t) = 1 - \exp(-\mu_x(t)) \]

- \( D_{x,t} \) Poisson-distributed number of deaths, \( E_{x,t} \) exposure at risk
- \( a_x \) and \( b_x \) indicating the general shape of mortality over age
- \( k_t \) indicating the general level of mortality in the population (with negative drift)
- Forecasting of \( k_t \) (and \( \mu_x(t) \)) by ARIMA process for estimated time series of \( k_t \)
Model framework

Modeling systematic mortality risk

- Analyze systematic mortality risk in two ways:

  1. Shock to (decreasing) mortality time trend: $e^*k_t$
     - Leads to an unexpected change in the level and future development of mortality
     - Shocks $e > 1$: mortality rates decrease (longevity scenario)
     - Shocks $e < 1$: mortality rates increase (pandemic scenario)
     - How to compose a portfolio of term life and annuities in order to immunize the portfolio against shocks to mortality?

  2. Use empirically observed changes in mortality
     - Analyze usefulness of natural hedging under realized changes in mortality
     - Similar results
Model framework
Model of a life insurance company

- Simplified balance sheet:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(t)$</td>
<td>$E(t)$</td>
</tr>
<tr>
<td></td>
<td>$B_A(t)$</td>
</tr>
<tr>
<td></td>
<td>$B_L(t)$</td>
</tr>
<tr>
<td></td>
<td>$L(t)$</td>
</tr>
</tbody>
</table>

- $A(t)$: market value of assets at time $t$
- $B_A(t)$: book value of liabilities for annuities at time $t$
- $B_L(t)$: book value of liabilities for term life insurance at time $t$
- $E(t)$: equity at time $t$

- Default of the insurance company, if $L(t) = B_L(t) + B_A(t) > A(t)$
Model framework
Liabilities – Premium and benefit calculation

• Premiums and benefits: use actuarial equivalence principle

  ➢ Term life insurance

  \[
  \sum_{t=0}^{T-1} P \cdot p_x \cdot (1 + r)^{-t} = \sum_{t=0}^{T-1} DB \cdot p_x \cdot q_{x+t} \cdot (1 + r)^{-(t+1)}
  \]

  ➢ Life-long immediate annuity

  \[
  SP = \sum_{t=0}^{T-1} a \cdot p_x \cdot (1 + r)^{-(t+1)}
  \]

• Improve comparability and isolate effect of natural hedging:

  ➢ Calibrate input parameters such that \textit{volume} of both contract types is identical at inception

  ➢ Fix the \textit{number} of contracts sold
Model framework
Liabilities – Book value of liabilities

- Use actuarial reserve to determine book value of liabilities

\[ B_L(t) = \sum_{s=0}^{T-t-1} \left[ DB \cdot s \cdot p_{x+t}(e) \cdot q_{s+x+t}(e) \cdot (1+i)^{-(s+1)} - P \cdot s \cdot p_{x+t}(e) \cdot (1+i)^{-s} \right] \]

- Value of one annuity:

\[ B_A(t) = \sum_{s=0}^{T-t-1} a \cdot s \cdot p_{x+t}(e) \cdot (1+i)^{-(s+1)} \]

- Mortality rates are subject to shock e

- Value of liabilities \( L(t) \):

\[ L(t) = n_A(t) \cdot B_A(t) + n_L(t) \cdot B_L(t) \]
**Model framework**

**Assets**

- Assets follow a geometric Brownian motion:

\[ dA(t) = \mu \cdot A(t) \cdot dt + \sigma \cdot A(t) \cdot dW^P(t) \]

- Development of asset base depends on cash-flows of insurance portfolio

\[
\begin{array}{cccccc}
& t = 0^+ & t = 1^- & t = 1^+ & t = 2^- & \ldots \\
+ E_0 & - n_A(1) \cdot a & + n_L(1) \cdot P & - n_A(2) \cdot a & \ldots \\
+ n_A(0) \cdot SP & - d_L(0) \cdot DB & & - d_L(1) \cdot DB & \\
+ n_L(0) \cdot P & - div & & - div & \\
\end{array}
\]

- Number of annuity contracts active in \( t = 1 \)
- Constant dividend to shareholders
- Number of life insurance contracts active in \( t = 1 \)
- Number of life insurance policyholders who died during \( t = 1 \)
- Constant dividend to shareholders
Model framework

Risk measurement

- Probability of default (PD): \( PD = P(T_d \leq T) \)
  with \( T_d = (T + 1) \vee \inf \{ t : A(t) < L(t) \}, t = 1, \ldots, T \).

- Mean Loss (ML): \( ML = E \left( \max \left( (L(T_d) - A(T_d)) \cdot (1 + r)^{-T_d}, 0 \right) \cdot 1\{T_d \leq T\} \right) \)

- Expected Shortfall (ES): \( ES = \frac{ML}{PD} \)

- Contractual Payment Obligations (CP)
  \[
  CP = n_L(0) \cdot \sum_{t=0}^{T-1} DB \cdot t \cdot p_x(e) \cdot q_{x+t}(e) \cdot (1 + r)^{-(t+1)} + n_A(0) \cdot \sum_{t=0}^{T-1} a_t \cdot p_x(e) \cdot (1 + r)^{-(t+1)}
  \]
  - Only liability side
  - Linear in portfolio composition
# Numerical results

## Input parameters

### Liabilities

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age at inception of term life</td>
<td>30</td>
</tr>
<tr>
<td>Max. duration of term life</td>
<td>35</td>
</tr>
<tr>
<td>Age at inception of annuity</td>
<td>65</td>
</tr>
<tr>
<td>Premium for life insurance ($P$)</td>
<td>417</td>
</tr>
<tr>
<td>Single premium for annuity ($SP$)</td>
<td>10,000</td>
</tr>
<tr>
<td>Yearly annuity ($a$)</td>
<td>725</td>
</tr>
<tr>
<td>Death benefit ($DB$)</td>
<td>88,724</td>
</tr>
<tr>
<td>Total number of contracts sold</td>
<td>10,000</td>
</tr>
</tbody>
</table>

### Assets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drift of assets ($\mu$)</td>
<td>6%</td>
</tr>
<tr>
<td>Volatility of assets ($\sigma$)</td>
<td>10%</td>
</tr>
<tr>
<td>Risk-free interest rate ($r$)</td>
<td>3%</td>
</tr>
</tbody>
</table>
Expected Shortfall (ES)

fraction of life insurance d

in Mio.

Gatzert/Wesker “The Impact of Natural Hedging on a Life Insurer’s Risk Situation”
\[ \mu = 4\% , \quad 5\% , \quad 6\% , \quad 7\% , \quad 8\% , \quad 9\% , \quad 10\% , \quad 11.5\% , \quad 12.5\% , \quad 13.5\% , \quad 15\% , \quad 17.5\% , \quad 20\% \]

\[ \text{Investment strategy} \]

\[ \ln \% \]

\[ \ln T \]
Summary

- Results show: Natural hedging can considerably reduce absolute risk level of an insurer and immunize it against shocks to mortality
  - Optimal portfolio composition depends on risk measure
  - Holistic consideration of mortality risk with respect to insurer’s overall risk level is vital (focus on liability side only underestimates risk)
- Investment strategy can have substantial impact on the effectiveness of natural hedging
  - Use investment strategy to simultaneously fix a risk level and immunize the portfolio against shocks to mortality
  - Changing the investment strategy requires adjustment of portfolio mix to immunize portfolio against changes in mortality
The Impact of Natural Hedging on a Life Insurer’s Risk Situation

Thank you very much for your attention!

AFIR 2011 Colloquium, Madrid
June 22, 2011

Nadine Gatzert and Hannah Wesker
University of Erlangen-Nürnberg
Gatzert/Wesker “The Impact of Natural Hedging on a Life Insurer’s Risk Situation”
Model framework

Risk measurement – liability side

• Contractual Payout Obligations (CP)

\[
CP = n_L(0) \cdot \sum_{t=0}^{T-1} DB \cdot t \cdot p_x(e) \cdot q_{x+t}(e) \cdot (1 + r)^{-(t+1)}
\]

\[
+ n_A(0) \cdot \sum_{t=0}^{T-1} a \cdot t \cdot p_x(e) \cdot (1 + r)^{-(t+1)}
\]

- take into account only liabilities – focus of previous literature
- linear in portfolio composition

• Here, additional consideration of default risk measures
Contractual Payout Obligations (CP)

in Mio.

fraction of life insurance $d$

Gatzert/Wesker “The Impact of Natural Hedging on a Life Insurer’s Risk Situation”
Gatzert/Wesker “The Impact of Natural Hedging on a Life Insurer’s Risk Situation”
Probability of Default (PD)

fraction of life insurance $d$

PD in %

0.0 0.2 0.4 0.6 0.8 1.0

0.5 0.4 0.3 0.2 0.1 0.0
Model framework
Risk measurement – liability side

- Contractual Payout Obligations (CP)

\[
CP = n_L(0) \cdot \sum_{t=0}^{T-1} DB \cdot p_x(e) \cdot q_{x+t}(e) \cdot (1 + r)^{-(t+1)} \\
+ n_A(0) \cdot \sum_{t=0}^{T-1} a \cdot p_x(e) \cdot (1 + r)^{-(t+1)}
\]

- take into account only liabilities – focus of previous literature
- linear in portfolio composition

- Here, additional consideration of default risk measures