Mortality Risk and its Effect on Shortfall and Risk Management in Life Insurance

AFIR 2011 Colloquium, Madrid
June 22\textsuperscript{nd}, 2011

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Introduction

Motivation

- Recently there has been growing interest in mortality risk and its management, especially due to the demographic development.

- Therefore, several alternative instruments for managing demographic risk have been proposed and discussed, e.g.
  - transferring mortality or longevity risk to the capital market or
  - natural hedging.

- To analyze the effectiveness of these alternative risk management strategies comprehensively, mortality risk can be divided into three components.
Introduction

Motivation

- Mortality risk components
  - Unsystematic mortality risk: individual time of death is a random variable with a certain probability distribution
  - Systematic mortality risk: probability distribution of the time of death is subject to sudden unexpected change
  - Adverse selection: which here refers to the fact that the mortality rate differs for different groups of insured, i.e. the mortality rate for annuitants is lower than for the population as a whole

- All of these mortality components might have considerable impact on the risk situation and risk management of a life insurance company
Introduction

Aim of paper

1. Study the interactions between different types of mortality risk with respect to the risk situation of an insurance company
   - explicitly modeling unsystematic mortality risk, systematic mortality risk and adverse selection

2. Analyze the impact of mortality risk components on the effectiveness of different risk management tools, namely
   - purchasing Mortality Contingent Bonds (MCB)
   - natural hedging, i.e. hedging systematic mortality risk through portfolio composition
Model framework
Modeling and forecasting unsystematic mortality


\[ D_{x,t} \sim Poisson \ E_{x,t} \cdot \mu_{x,t} \ \text{with} \ \mu_{x,t} = e^{a_x + b_x \cdot k_t} \]

with

• \( D_{x,t} \) poisson-distributed number of deaths,
• \( E_{x,t} \) exposure at risk
• \( a_x \) and \( b_x \) indicating the general shape of mortality over age
• \( k_t \) indicating the general level of mortality in the population

• Forecasting of \( k_t \), respectively \( \mu_x(t) \)

⇒ ARIMA process for estimated time series of \( k_t \)
Model framework
Modeling mortality basis risk and systematic mortality risk

• Adverse selection:
  • extension of the brass-type relational model by Brouhns, Denuit and Vermunt (2002)

\[
\ln \mu_{x,t}^{annu} = \alpha + \beta_1 \cdot \ln \mu_{x,t}^{pop} + \beta_2 \cdot \ln \mu_{x,t}^{pop} \cdot t + \varepsilon_{x,t}.
\]

→ Implies a different level and trend of annuitant mortality

• Systematic mortality risk:
  • modeled through a change in the drift of the time trend of \( k_t \)

→ Leads to an unexpected change in the level and in the future development of mortality
Model framework
Model of a life insurance company

- Simplified balance sheet of a two-product life insurance company:

<table>
<thead>
<tr>
<th>Assets $A(t)$</th>
<th>Liabilities $L(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{high}(t)$</td>
<td>$M_A(t)$</td>
</tr>
<tr>
<td>$A_{low}(t)$</td>
<td>$M_L(t)$</td>
</tr>
<tr>
<td>$M_{bond}(t)$</td>
<td>$E(t)$</td>
</tr>
</tbody>
</table>

- $A_i(t)$: market value of assets at time $t$ for $i =$ high risk, low risk
- $M_{bond}(t)$: value of mortality contingent bond (MCB) at time $t$
- $M_i(t)$: value of liabilities at time $t$ for $i =$ annuities, life insurance
- $E(t)$: equity in time $t$

- Default of insurance company, if $L(t) > A(t)$
Model framework

Liabilities

• Premiums and benefits are calculated using the actuarial equivalence principle and risk-neutral valuation

• Based on this, the value of liabilities for both insurance products is defined as the net of future payment obligations for the insurance company

• The mortality rates used in the calculation
  – are those forecasted stochastically using the BDV model and
  – differ for life insurance policyholder and annuitants
Model framework

Assets

- Value of assets in $t = 0$
  \[ A(0) = E(0) + n_A \cdot SP_A + n_L \cdot P_L - \Pi_{x,d} \]
  with $\Pi_{x,d}$ premium of MCB

- $A(t)$ can be calculated via
  \[ A(t) = A_{\text{high}}(t) + A_{\text{low}}(t) + n_L \cdot P_L \cdot n_A \cdot a - d_L \cdot t \cdot DB + X(t) - \text{div}(t) \]
  where
  - $A_{\text{low}}(t) = \alpha \cdot A(t)$, i.e. a constant fraction $\alpha$ is invested in low risk assets
  - $X(t)$ is the coupon payment of the MCB in time $t$ and
  - $\text{div}(t)$ is the dividend paid to shareholders in return for their investment
Model framework
Mortality Contingent Bond (MCB)

- Proposed by Blake and Burrows (2001) under the name “survivor bond”

- In return for a premium paid in advance, the insurance company receives a variable coupon payment $X(t)$ at the end of year $t$
  - The coupon payment $X(t)$ depends on the number of survivors in the reference population $n_{ref}(t)$
    \[ X_t = \frac{n_{ref}^t}{n_{ref}^0} \cdot C \]

- Mortality in the reference population is thereby equal to population mortality, which differs from annuitant mortality (= adverse selection) ➞ Basis risk
Model framework
Risk measurement – default risk measures

• Probability of default (PD)

\[ PD = P \left( T_d \leq T \right) \]

with \( T_d = \inf \{ t : A_t < L_t \}, t = 1, \ldots, T. \)

• Mean Loss (ML)

\[ ML = E \left[ L \left( T_d - A T_d \right) \cdot 1 + r^{-T_d} \cdot 1 \left( T_d \leq T \right) \right] \]
Numerical results

Estimation of mortality risk

- Estimation of mortality of the population (Switzerland)

\[
\ln \mu_{x,t}^{\text{annu}} = -0.3197 + 1.0747 \cdot \ln \mu_{x,t}^{\text{pop}} - 0.0004 \cdot \ln \mu_{x,t}^{\text{pop}} \cdot t
\]

ARIMA (1,1,0) process

- Estimation of adverse selection
Numerical results
Impact of adverse selection on the risk situation

- Two assumptions concerning adverse selection

Information about the generally lower mortality of annuitants is partly hidden (asymmetric information) → adverse selection mispriced

Insurance company is able to forecast adverse selection completely (e.g. through experience rating) → adverse selection perfectly priced

### Numerical Results

**Impact of Adverse Selection on the Risk Situation**

<table>
<thead>
<tr>
<th>Unsystematic Risk</th>
<th>Unsystematic Risk + Adverse Selection</th>
<th>Systematic Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>+100%</td>
<td>+4%</td>
<td></td>
</tr>
</tbody>
</table>

**Graphs**

- **Probability of Default**
  - X: Probability of default in %
  - Y: Fraction of life insurance $f_L$ in %

- **Mean Loss in T**
  - X: Fraction of life insurance $f_L$ in T
  - Y: Mean loss in T

**Legend**

- Red: Unsystematic risk
- Blue: Unsystematic risk + Adverse selection
- Green: Unsystematic risk + Adverse selection + Systematic risk
Numerical results

Effectiveness of MCBs

Effectiveness of MCBs for reducing the impact of systematic mortality risk

here: measured through the change in the riskiness of an insurance company in response to a change in mortality

<table>
<thead>
<tr>
<th>For a portfolio with only annuities $f_L = 0$</th>
<th>Without adverse selection (no basis risk)</th>
<th>With adverse selection (in the presence of basis risk)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Loss</td>
<td>Mean Loss</td>
</tr>
<tr>
<td>Without MCB</td>
<td>1,369 T</td>
<td>2,442 T</td>
</tr>
<tr>
<td>With MCB</td>
<td>749 T</td>
<td>1,631 T</td>
</tr>
<tr>
<td>Relative reduction through MCB</td>
<td>45.3%</td>
<td>33.2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>44.0%</td>
</tr>
</tbody>
</table>

The relative reduction is defined as $\frac{ML_{without\ MCB} - ML_{with\ MCB}}{ML_{without\ MCB}}$. 

Gatzert/Wesker “Mortality Risk and its Effect on Shortfall and Risk Management in Life Insurance”
### Numerical results

#### Natural hedging under adverse selection

**Mean Loss in T**

<table>
<thead>
<tr>
<th>Without adverse selection</th>
<th>With adverse selection</th>
</tr>
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<tbody>
<tr>
<td><strong>misestimated</strong></td>
<td><strong>Perfectly estimated</strong></td>
</tr>
<tr>
<td>ML</td>
<td>ML</td>
</tr>
</tbody>
</table>

| Fraction of life insurance | 13.1% | 12.4% | 11.8% |

**Without adverse selection**

**With adverse selection**

- **misestimated**
- **Perfectly estimated**
Summary

• Our results show an increase in the risk of an insurance company through adverse selection for all portfolios, even if it can be perfectly forecasted
  – This effect is stronger when considering mixed portfolios as compared to a portfolio consisting only of annuities

• That adverse selection does not impair the effectiveness of natural hedging, however it does affect the immunizing portfolio composition

• In terms of hedging against systematic mortality risk, the effectiveness of MCBs is decreased slightly, given that adverse selection can be properly forecasted and is taken into account in pricing
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Thank you very much for your attention!

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