Plan

• Aim
• Preliminaries
• Case Study: model + data
• Decomposing correlation
Motivation

- Pension plan wishes to hedge its exposure to longevity risk

- Options:
  - Customised hedge $\Rightarrow \sim 100\%$ hedge effectiveness
  - Index-based hedge $\Rightarrow$ basis risk

- Aim: to understand contributors to:
  - basis risk, hedge effectiveness and correlation
Key quantities

- $T =$ future liability valuation date
- $a_k(T, x) =$
  - annuity value at $T$
  - life annuity of 1 per annum
  - for an individual aged $x$ at $T$, in population $k$
  - allowing for future mortality improvements
Simple example

- Liability value $L(T) = a_2(T, 65)$
- Hedging instrument: deferred longevity swap
  
  $$H(T) = a_k(T, x) - \hat{a}_k^{\text{fbd}}(0, T, x)$$
  
  $\hat{a}_k^{\text{fbd}}(0, T, x) = \text{value at } T \text{ of swap fixed leg}$

- $k = 2 \Rightarrow \text{CUSTOMISED hedge}$
- $k = 1 \Rightarrow \text{INDEX hedge}$
Dynamic Models: Two stages

1. Simulation from 0 to $T$ using model $M_1(0)$

2. Valuation at $T$:
   - Why? Valuation regulations; Accounting standards
   - Requirement: a valuation model at $T$: $M_2(T)$
What is model $M_2(T)$?

- $M_2(T) \neq M_1(0)$
  - (Re-)calibration using data up to $T \Rightarrow$ realistic!
  - Valuers just observe historical mortality plus one future sample path of mortality from 0 to $T$
    $\Rightarrow$ they do not know the “true” $M_1(0)$
  - Using $M_1(0) \Rightarrow$ too optimistic (??)  c.f. Black-Scholes
Hedge Effectiveness: basic idea

- $L = \text{liability value}$
- $H = \text{value of hedging instrument}$

Hedge effectiveness depends on: $\rho = \text{cor}(L, H)$

(if “risk” = S.D. or Variance; or if $(L, H) \sim \text{multivariate normal}$)
Case Study

- Population 1: England and Wales males
- Population 2: UK CMI assured lives, males
- 1961–2005; ages 50-89
- Here: 2-population model (Cairns et al., 2010)
- **Model here: just one example**
  
  (simple model: but both period and cohort effects)
Age-Period-Cohort model (APC) (M3-2 pops)

\[ m_k(t, x) = \text{population } k \text{ death rate} \]

\[
\log m_k(t, x) = \beta^{(k)}(x) + \kappa^{(k)}(t) + \gamma^{(k)}(t - x)
\]

\( \beta^{(1)}(x), \beta^{(2)}(x) \) population 1 and 2 age effects
\( \kappa^{(1)}(t), \kappa^{(2)}(t) \) period effects
\( \gamma^{(1)}(c), \gamma^{(2)}(c) \) cohort effects
$M1(0)$: simulation model – key features

- For each age $x$, $m_1(t, x)/m_2(t, x)$ does not diverge over time

- Bayesian approach + MCMC
  - $\Rightarrow$ full posterior for process params + latent state variables
  - $\Rightarrow$ easy to incorporate parameter uncertainty

- Simulation up to $T$:
  - with/without parameter uncertainty
  - with/without Poisson risk in death counts
M2(T): valuation model – key features

• Simple deterministic approximation
  fast & accurate (+ reality!)

• Consistent population 1 and 2 projections

\[
\hat{\kappa}^{(1)}(T + s) = \kappa^{(1)}(T) + \mu s
\]
\[
\hat{\kappa}^{(2)}(T + s) = \kappa^{(2)}(T) + \mu s
\]

i.e. median of a random walk with common drift \(\mu\)
Variants

- Hedging with population 1 or population 2
- Full parameter uncertainty (PU) \((\rightarrow \mathcal{M}_1(0))\)
  - \(\mathcal{M}_2(T)\) recalibrated in 2015 using latest data
- Full parameter certainty (PC):
  - PC version of \(\mathcal{M}_1(0)\) for simulation
  - \(\mathcal{M}_2(T)\) calibration fixed in 2005
- Partial PC:
  - PC version of \(\mathcal{M}_1(0)\)
  - \(\mathcal{M}_2(T)\) recalibrated in 2015 using latest data
- With and without Poisson Risk
Customised hedge; PC; No Poisson

Hedging Instrument Reference Age, y

Correlation

Cohort effects knownable in 2005

A: Full PC, no recalibration of mu1
Index hedge; PC; No Poisson
Index hedge; Partial PC (recalibration risk); No Poisson
Recalibration risk explained

- $L \equiv a_2(T, x; \kappa^{(2)}(T), \mu, \ldots)$
- $H \equiv a_1(T, x; \kappa^{(1)}(T), \mu, \ldots) - \hat{a}_1^{\text{fd}}(0, T, x)$

- Random $\mu \Rightarrow$ extra risk,

  BUT also higher correlation
Index hedge; Full PU (recalibration risk); No Poisson

B: PPC, customised hedge
C: PU, customised hedge
G: PPC, index hedge
I: PU, index hedge

Correlation

Hedging Instrument Reference Age, y

55 60 65 70 75 80 85
0.70 0.75 0.80 0.85 0.90 0.95 1.00
Index hedge; Full PU (recalibration risk); + Poisson

Hedging Instrument Reference Age, y

Correlation

C: PU, customised hedge
D: PU, customised hedge + Poisson risk
I: PU, index hedge
J: PU, index hedge + Poisson risk
Recalibration window $W = 20$ or $35$; Knightian Uncertainty!
Conclusions

- Population basis risk is only part of the story
- Parameter uncertainty is significant
  - especially recalibration risk, recalibration window
- Dependence on an uncertain but common trend ⇒
  index hedges are more effective than you might think
Discussion + questions

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Further comments + work

• Robustness of optimal hedge ratios
  – Impact of sub-optimal allocation
  – Sensitivity to PC/PU etc.

• Vega hedging;
  Use of more than one hedging instrument

• Use of more recent EW data

• Models with more complex correlation structure
Simple hedging problem (?)

liability \( L = \mu + \sigma \left( \rho Z_1 + \sqrt{1 - \rho^2} Z_2 \right) \)

hedging instrument \( H = \mu + \sigma Z_1 \)

hedged portfolio \( P(h) = L + h \times H \)

\( Z_1 \) and \( Z_2 \) are i.i.d. \( \sim N(0, 1) \)

Q1: What is \( \text{cor}(L, H) ? \) \( \rho ? \)

Q2: What \( h \) minimises \( \text{Var}(P(h)) ? \) \( -\rho ? \)

Q3: What is the hedge effectiveness? \( \rho^2 ? \)
Relevance

NEWS: 31 January 2011 (Professional Pensions)

*Pall scheme completes world’s first longevity hedge for non-retired members*

First for:

index-based longevity hedge (10-year q-forwards)

+ pre-retirement pension plan members
What is $M2(T)$?

- $M2(T)$ calibrated using data from $T - W$ to $T$
  
  - otherwise $\implies$ poor risk management
  
  - here: calibration window $W = 20$ years
    
    * relatively short?
    
    * but market practice to capture recent “trend”
    
    * source of Knightian Uncertainty
life or 25-year annuity

Hedging Instrument Reference Age, y

Correlation

J: PU + Poisson risk, W=20, life annuity
K: PU + Poisson risk, W=20, temporary annuity