Analyzing Surplus Appropriation Schemes in Participating Life Insurance from the Insurer’s and the Policyholder’s Perspective

AFIR Colloquium
Madrid, Spain
June 22, 2011

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Introduction
Motivation

• Participating life insurance contracts:
  – Important product design in German life insurance market
  – Include interest rate guarantees and bonus mechanisms through which profits are distributed and appropriated to the policyholders

• Focus on:
  – Analysis of surplus appropriation schemes, i.e. different ways of how a given amount of surplus, determined by a reserve based distribution system, can be credited to the policyholders’ contracts
Introduction
Aim of paper

- Examine surplus appropriation schemes often inherent in participating life insurance contracts:
  - Bonus system: surplus increases death and survival benefit
  - Interest-bearing accumulation: accumulates surplus on a separate account, death benefit is kept constant
  - Shortening the contract term: death and survival benefit is kept constant, survival benefit is paid earlier

With respect to their impact on:
  - Insurer’s shortfall risk
  - Net present value from a policyholder’s viewpoint

- Conduct this analysis by considering mortality risk as well as market risk
Model framework
Insurance contract and modeling mortality probabilities

- Pool of traditional participating life insurance products:
  - Actuarially priced based on a mortality table (DAV 2008 T)

- Constant annual premium is given by equivalence principle:

\[ P \cdot \bar{a}_{x:n} = S_1 \cdot A_{x:n} \]

with \( A_{x:n} = \sum_{k=0}^{n-1} v^{k+1} \cdot p_{x+k} \cdot q_{x+k} + v^n \cdot n \cdot p_x \) and \( \bar{a}_{x:n} = \sum_{k=0}^{n-1} v^k \cdot p_x \)

- Actual mortality rates for risk measurement derived by Lee-Carter (1992) model:

\[
\ln \left[ \mu_x, \tau \right] = a_x + b_x \cdot k \tau + \varepsilon_{x,\tau} \Leftrightarrow \mu_x, \tau = e^{a_x + b_x \cdot k \tau + \varepsilon_{x,\tau}}
\]

- Modification by Brouhns, Denuit, and Vermunt (2002):

\[ D_{x,\tau} \sim \text{Poisson } E_{x,\tau} \cdot \mu_x, \tau \] with \( \mu_x, \tau = e^{a_x + b_x \cdot k \tau} \)
Model framework

Policy reserves

- Actuarial reserve for individual contract is given by
  \[ V_x = S_{t+1} \cdot A_{x+t:n-t} \cdot P \cdot \ddot{a}_{x+t:n-t} \]

- Total portfolio policy reserve is determined by
  \[ PR_t = (N - \sum_{i=1}^{t} d_i) \cdot V_x \quad \text{where } N = \text{initial number of contracts sold, } \sum_{i=1}^{t} d_i = \text{number of deaths until year } t \]

- Development of payments over time

<table>
<thead>
<tr>
<th></th>
<th>- 0</th>
<th>+ 1</th>
<th>...</th>
<th>- t</th>
<th>...</th>
<th>- n-1</th>
<th>- n</th>
<th>time</th>
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<tbody>
<tr>
<td>age</td>
<td>x</td>
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<td>Dec. 31\textsuperscript{st}</td>
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<td>Jan. 1\textsuperscript{st}</td>
<td>x+n-1</td>
<td>x+n</td>
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<td>31\textsuperscript{st}</td>
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<td>0</td>
<td>S_{n-1}</td>
<td>S_n</td>
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<td>premium</td>
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<td>P_0=P</td>
<td>P_1=P</td>
<td>0</td>
<td>P_t=P</td>
<td>0</td>
<td>P_{n-1}=P</td>
<td>0</td>
</tr>
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<td>dividend</td>
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<td>D_1</td>
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<td>D_t</td>
<td>0</td>
<td>D_{n-1}</td>
<td>D_n</td>
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</tr>
</tbody>
</table>

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Model framework
Development of the asset base

• Asset portfolio follows a geometric Brownian motion

\[ dA(t) = \mu \cdot A(t) \cdot dt + \sigma \cdot A(t) \cdot dW^P(t) \]

• Portfolio is composed of bonds and stocks, with a continuous one-period return of the portfolio, given by

\[ r_t = a \cdot r_S + 1 - a \cdot r_B, \text{ with } E(r_t) = m = \mu - 0.5\sigma^2 \]

• Assets at the end of year \( t \), after accounting for decrements in the portfolio of policyholders due to death, results to

\[ A_t^- = A_{t-1}^+ \cdot \exp \ [r_t - S_t \cdot d_t], \text{ with } A_0^- = 0, A_0^+ = E_0 + P \cdot N \]

payment of death benefits,

\( S_t \) = sum insured, depends on surplus scheme,
\( d_t \) = number of deaths in year \( t \)
Model framework

Surplus appropriation schemes

- Actual policy interest rate credited to the policyholders for period \( t-1 \) until \( t \), based on a smoothing scheme by Grosen and Jørgensen (2000), is given by

\[
  r_t^P = \max \left\{ r^G, \alpha \cdot \left( \frac{B_{t-1}^+}{PR_{t-1}^- + IA_{t-1}^- + RD_{t-1}^-} - \gamma \right) \right\}
\]

where

- \( \alpha = \) surplus distribution ratio
- \( \gamma = \) target buffer ratio
- \( r^G = \) guaranteed interest rate

- Surplus for the \( t \)-th year results to

\[
  PR_{t-1}^- \cdot r_t^P - r^G
\]

amount is used differently within each of the 3 companies depending on the concrete appropriation scheme.
Model framework

Appropriation scheme: bonus system

1. Bonus system:
   - Surplus is used to increase the initially guaranteed sum insured $S_1$ (death and survival benefit)
   - Done by using the surplus as a single premium for an additional contract of the same type with same maturity:

$$\Delta S_t \cdot A_{x+t}^{n-t} = PR_t \cdot r_t^P - r_t^G \left/ N - \sum_{i=1}^{n} d_i \right.$$

Increased sum insured is given by $S_{t+1} = S_t + \Delta S_t$
Model framework

Appropriation scheme: interest-bearing accumulation

2. Interest-bearing accumulation:
   - Sum insured is kept constant, i.e. $S_t = S_1$, $\forall t = 1, \ldots, T$
   - Surplus is accumulated on a separate account, $IA_t$

   - Forward projection of the interest-bearing accumulation account is given by
     \[
     IA_t = IA_{t-1} \cdot (1 + r^{IA}) \cdot \left(1 - d_t / N - \sum_{i=1}^{t-1} d_i\right) + PR_{t-1} \cdot r^p_t - r^G
     \]

     Adjustment for death: funds that belonged to policyholders that died within the $t$-th year, are passed to the collectivity of policyholders

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Model framework

Appropriation scheme: shortening the contract term

3. Shortening the contract term:
   - Surplus is used to decrement the remaining years to maturity (contract term $n(t)$ is a function of time $t$)
   - Reduce the contract term for full years only
     
     $$RD_t = RD_{t-1^+} \cdot \left(1 + r^{RD} \right) \cdot 1 - d_t / \left[ N - \sum_{i=1}^{t-1} d_i + PR_{t-1^-} \cdot r^P_t - r^G_t \right], \quad RD_0 = 0$$
     
   - Policy reserve incl. surplus for an individual insured
     
     $$V_{x \up{surplus}} n \ t - 1 = V_x n \ t - 1 + RD_{t-1^-} / \left[ N - \sum_{i=1}^{t-1} d_i \right]$$
     
   - Determine the years to reduce the contract term
     
     $$k_{\max t} = \max_{k \in K t} k: V_{x \up{surplus}} n \ t - 1 - V_x n \ t - 1 - k \geq 0$$

     with $K t = 0, \ldots, n t - 1 - t$

     new policy period is given by

     $$n t = n t - 1 - k_{\max t}$$

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Model framework
Evaluating the surplus appropriation schemes

• Shortfall probability (assets not sufficient to cover liabilities):

$$SP = P \quad T_s \leq T \quad \text{with} \quad T_s = \inf \quad t \quad A_t^{-} < PR_t^{-} + IA_t^{-} + RD_t^{-} , \quad t = 1, \ldots, T$$

• Net present value from a policyholder’s viewpoint = expected value of insurance benefits - premiums

$$NPV = E^{Q} \left( \sum_{t=0}^{T-1} t p_x' \cdot q'_{x+t} \cdot S_{t+1} \cdot e^{-t+1 \cdot r_f} - t p_x' \cdot P \cdot e^{-t \cdot r_f} \cdot 1 \quad T_s > T \right)$$

$$+ E^{Q} \left( TP_x' \cdot S_T + IA_T^{-} + RD_T^{-} + TB_T \cdot \frac{1}{N - \sum_{i=1}^{t} d_i} \cdot e^{-T \cdot r_f} \cdot 1 \quad T_s > T \right)$$

$$+ E^{Q} \left( \sum_{t=0}^{T-1} t p_x' \cdot A_t^{-} \cdot e^{r_{t+1}} \cdot 1 - c \cdot \frac{1}{N - \sum_{i=1}^{t} d_i} \cdot e^{-t+1 \cdot r_f} - t p_x' \cdot P \cdot e^{-t \cdot r_f} \right) \cdot 1 \quad T = t + 1$$
### Numerical results

#### Input parameters

**Assets**

<table>
<thead>
<tr>
<th></th>
<th>stocks</th>
<th>bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected one-period returns</td>
<td>8.00%</td>
<td>6.02%</td>
</tr>
<tr>
<td>Volatility</td>
<td>21.95%</td>
<td>3.30%</td>
</tr>
<tr>
<td>Correlation between stocks and bonds</td>
<td>-0.1648</td>
<td></td>
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<tr>
<td>Stock portion</td>
<td>10%</td>
<td></td>
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<tr>
<td>Risk-free rate</td>
<td>3%</td>
<td></td>
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</table>

**Liabilities**

<table>
<thead>
<tr>
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<th>$r^G$</th>
<th>$r^A$</th>
<th>$r^{RD}$</th>
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<tbody>
<tr>
<td>Rate of interest</td>
<td>2.25%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Number of contracts sold</td>
<td>100,000</td>
<td></td>
<td></td>
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<tr>
<td>Sum insured in $t = 0$</td>
<td>1</td>
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<tr>
<td>Level premium for $T = 30$</td>
<td>0.0247</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contract term</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age of the policyholders in $t = 0$</td>
<td>35</td>
<td></td>
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</table>
Numerical results

Shortfall risk

- $SP$ as a function of stock portion and shock to mortality

$SP = 0.05$ for:
- Bonus system $a = 19.81\%$
- Interest-bearing accum. $a = 21.11\%$
- Shortening contract term $a = 20.54\%$

**Diagram:**

- **shortfall probability**
  - x-axis: stock portion $a$
  - y-axis: shortfall probability
  - Colors and markers indicate different strategies:
    - Red circle: bonus system
    - Green cross: interest-bearing accumulation
    - Blue line: shortening the contract term

- **shortfall probability**
  - x-axis: shock to mortality $\delta$
  - y-axis: shortfall probability
  - Markers indicate the effect of increasing shock to mortality by +50%
Numerical results

Shortfall risk

- $SP$ as a function of contract term $T$

<table>
<thead>
<tr>
<th>Shortfall probability for $a = 10%$</th>
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<tbody>
<tr>
<td>Contract term $T$</td>
</tr>
<tr>
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</tr>
<tr>
<td>30</td>
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<tr>
<td>35</td>
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<tr>
<td>40</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Shortfall probability for $a = 25%$</th>
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<tbody>
<tr>
<td>Contract term $T$</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>35</td>
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<tr>
<td>40</td>
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</tbody>
</table>

- bonus system
- interest-bearing accumulation
- shortening the contract term

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Numerical results
Net present value

- $NPV$ as a function of stock portion and shock to mortality

![Graph showing NPV as a function of stock portion and shock to mortality]
Summary

• Results show: Even if the smoothing surplus distribution scheme is the same, the impact of the concrete surplus appropriation (with respect to guaranteed death/survival benefits) differs substantially:

  - Insurer’s risk situation, from highest to lowest: 1) bonus system – 2) shortening contract term – 3) interest-bearing accumulation
  - Net present value from policyholder’s viewpoint: 1) shortening contract term – 2) interest-bearing accumulation – 3) bonus system

• Increasing gap in shortfall risk between 3 schemes for higher stock portions and higher distributed surplus

• In contrast: shock to mortality implies similar increase in risk

• Risk reduction for longer contract periods not as effective in case of the (most common) bonus system, especially for high stock portion
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Thank you very much for your attention!

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