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# CONTROLLING SOLVENCY AND MAXIMISING POLICYHOLDERS' RETURNS: A COMPARISON OF MANAGEMENT STRATEGIES FOR ACCUMULATING WITH-PROFITS LONG-TERM INSURANCE BUSINESS

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*Abstract*

A stochastic model of a mutual life office, which is assumed to issue accumulating with-profits contracts, is used to assess the relative effectiveness of various long-term and short-term (crisis) management strategies in the control of policyholder returns, including the control of statutory solvency. The different strategies considered are (long-term): the asset allocation strategy, bonus philosophy and capital control, and (short-term): a dynamic crisis investment strategy and a dynamic crisis bonus (profit) distribution strategy. The effectiveness of these strategies is judged in terms of the simulated profiles of policyholder returns, including the frequencies of statutory and actual insolvency, the distribution of the time to ruin and a number of measures of the distribution of policy payouts. A special feature of the methodology is the use of a single objective measure of the collective value of the maturity payouts made over the whole projection period to the existing policyholders. This quantity is also sensitive, in an intuitive way, to the statutory insolvency risk. The mean, standard deviation and expected utility of the accumulated payout are used in conjunction with the measures of solvency to assess the relative merits of the return profiles. The methodology appears to lead to entirely intuitive results. It is found that long term asset allocation and bonus distribution strategies are of paramount importance to producing optimal return profiles. The crisis bonus strategy is useful in improving all aspects of policy returns where capital is low and where policy guarantees are not onerous. The assumed crisis investment strategy, while improving solvency, never appears to create better profiles of returns than obtained from the optimal long-term asset allocation strategy in which the crisis response is excluded. A strategy of capital replacement, funded by deductions to policy asset shares, is shown to be an effective control when existing capital is scarce. The level of the existing policy guarantees (in a stationary fund) is found to be considerably less important in determining the ultimate return profile in comparison with the continuing bonus philosophy, including the level of any guaranteed rates of increase to the policyholders' fund benefits.

## 1. *Introduction*

1.1. The management of a life insurance company aims to provide its investors, be they policyholders or shareholders, with an acceptable profile of returns from their investment. This paper examines the relative effects of a range of life insurance management strategies upon the profile of returns experienced by participating (or *with-profits)* life insurance policyholders. Whether a return profile is acceptable or not is a matter of judgement and will be a function also of the risk preferences of the policyholders themselves.

1.2. Solvency is an important aspect of the profile of returns to policyholders. When an insurer becomes insolvent a discontinuity occurs in the investment process particularly where, for example, the life fund is wound up and residual assets are distributed amongst the existing policyholders. While in practice closed funds tend to be more often acquired and managed by a new insurer, this remains a useful theoretical representation of the insolvency event. In this case, therefore, it is not just *whether* an

insurer becomes insolvent that is important, but *when* it becomes insolvent. Hence insolvency can usefully be measured by the *probability of ruin* (or the expected frequency of ruin), and by the *distribution of the time to ruin*, of which the mean and variance would be useful measures.

1.3. The overall distribution of returns is traditionally summarised by its first two moments: that is its mean and variance. Markovitz (1991) considers variance to be a suitable proxy for risk, and hence according to modern portfolio theory an investor will always prefer an investment with higher mean returns for the same variance of return, or one with a lower variance for the same mean.

1.4. Utility theory leads to the assertion that investors will prefer an investment which maximises their expected utility of returns, based on the premise that individuals prefer more wealth to less, and that the marginal preference for more wealth decreases as wealth increases. Booth, Chadburn and Ong (1997) used the maximisation of expected utility to determine optimal asset strategies for non-profit life insurance companies.

1.5. In this paper the profile of returns will be measured by the following output variables.

(i) The frequency of statutory ruin (as defined in section 4.3).
(ii) The mean and standard deviation of the time to statutory ruin.
(iii) The mean and standard deviations of policy payouts (as defined in sections 4.5 and 4.6).
(iv) The expected utility of policy payouts (see section 4.9).

*Management strategies for controlling the profile of returns*

1.6. The relevant management strategies can be grouped into two generic types.

(i) Long-term, or non-crisis, strategies.
(ii) Crisis strategies.

1.7. Long-term strategies are the normal approaches that the insurer will apply to its management, given that no solvency-threatening crisis occurs. Naturally these would be designed to avoid the risk of any crisis occurring but, should solvency become threatened, a change from this normal strategy which is geared to the avoidance of the crisis would occur. The latter would therefore be a case of 'crisis management'.

1.8. In this paper a range of both types of strategy are considered, and the effects on the profile of policyholder returns will be considered and compared, with particular aims of:

(i) identifying the circumstances under which particular crisis strategies would be preferred above others; and
(ii) identifying the long-term strategies which appear to be of greatest importance to policyholders.

*Crisis management strategies*

1.8. A with-profits life insurance fund has a number of potential candidates for crisis management. Two quite distinct strategies will be considered:

(i) a crisis investment strategy; and
(ii) a crisis bonus distribution strategy.

*Crisis investment strategy*

1.9. A method which appears to be adopted in practice and which is often assumed in stochastic modelling work is to assume that a life office will switch investments from a high equity mix into one with a higher proportion of fixed interest assets (see Ross, 1989). Companies which experience difficulties with their statutory valuation can generally increase their solvency margin by this approach, as this has the effect of increasing the rate of interest that can be used to value the liabilities, at least for conventional with-profits and non-profit business. This should also apply to other business, particularly where significant guarantees exist.

*Crisis bonus distribution strategy*

1.10. If a crisis of solvency exists, then the office can reduce its risk by witholding bonus distributions which it would otherwise have made, and only reinstating those bonuses if and when solvency improves subsequently.

1.11. Chadburn (1997) has presented some preliminary results of investigations into the effectiveness of these two strategies in controlling solvency, for offices with different liability profiles. The main conclusions were that the crisis bonus strategy was very effective at managing solvency for companies with low free assets, while the crisis investment response was more helpful at higher levels. The investment strategy tended to produce lower ruin frequencies, means and standard deviations of returns, suggesting that it might form a useful solvency management strategy for suitably risk averse investors. It was shown that both strategies were less effective where high guarantees were involved, particularly where these involved guarantees on surrender.

1.12. One of the shortfalls of this earlier work was that it failed to recognise adequately the effect of insolvency on the projected distribution of overall returns. The present paper includes an attempt to remedy this.

*Long-term strategies*

1.13. The following aspects of long-term management strategy are considered in this paper.

(i) The ongoing level of capital (the free assets).
(ii) The bonus distribution philosophy.
(iii) The asset allocation strategy.

*The level of capital*

1.14. The holding of capital is an integral part of any overall strategy. All else being equal, the more capital held, the lower will be the risk of insolvency. However, an important consideration is the cost (ultimately to the policyholders) of providing the capital.

1.15. The consideration of the provision and use of capital is dependent upon who has ownership of it. For simplicity a mutual office will be assumed. There are two considerations here. First is the question of the exisiting capital. Unless it is a new office, the exisiting capital of a mutual fund has been accumulated from contributions (retained profits) from previous generations of policyholders. Provided no increase in

capital occurs over time, then this capital does not incur any overall cost to the fund. Theoretically, a charge may be made for the use of the capital, for example by making deductions from the policy asset shares each year, but then any profit from their use is repaid to the policyholders through higher profit distributions. Hence it is simpler to assume that the use of the existing capital incurs no cost to the policyholders: it will be assumed that the exisiting assets at the start of the projection are the fortuitous benefits of history and no additional cost for their use will be assumed in the model.

1.16. Second, however, is the question of any changes to the level of existing capital. This may be seen as part of a long-term strategy, to reflect changing volumes of business and/or changes in the risk-profile of the liabilities. In a mutual company any increase in capital has to be provided by a deduction from the policyholders' asset shares, that is by withholding profits.

*Bonus distribution philosophy*

1.17. Participating policyholders in the UK share in the profits of the company through a combination of regular and terminal bonus additions to the policies' contracual benefits. The value of the benefit payments can be generally considered to be some smoothed asset share (see Needleman and Roff, 1995; Booth *et al*, 1998, pending) though it may not be communicated as such to the policyholders. The bonus distribution philosophy is the approach that companies take to their distributions, in particular:

(i) the extent of any guaranteed returns;

(ii) the relative proportion of total payout which is in the form of terminal bonus; and

(iii) the overall degree of smoothing employed, in relation to the underlying asset share.

Variations in these three aspects of bonus philosophy all contribute to the overall level of guarantee provided under contracts in force at any point in time, hence affecting the solvency risk and consequently the overall profile of returns. The smoothing itself is, of course, an important element of the policyholders' profile of returns. Bearing in mind that with-profits liabilites tend to be heavily equity-backed (solvency permitting), then the overall return is of the nature of a smoothed equity return (that is with reduced volatility), and further subject to some minimum guaranteed level.

*Asset allocation strategy*

1.18. The choice of assets in which the policyholders' money is invested has a significant impact on the profile of investment returns earned by the company, and hence upon the profile of returns generated for the company's policyholders. Booth, Chadburn and Ong (1997) demonstrated the importance of asset allocation in the context of a non-participating proprietary life insurance fund. They showed that it is theoretically possible to find optimal asset allocations which maximise the expected utility of the shareholders' returns, on the basis of a given stochastic investment model and for an assumed utility function for the investors. This approach is adapted to the case of a mutual with-profits fund in this paper.

## 2. *Accumulating With-Profits Contracts*

2.1. A model life office consisting of a mature tranche of Accumulating With-Profits (AWP) liabilities was used for the investigations. AWP was chosed for the analysis because of the rapidly increasing importance of the business in the UK market, at the expense of conventional with-profits business, while less investigative work has been published in this area. The general nature of with-profits life insurance in the UK was described in section 1; however see also Chadburn (1993) and MacDonald (1993), while the reader can refer to Booth *et al* (1998, pending) for a description of AWP business. A brief summary will now be given here.

2.2. Under AWP contracts, policyholders' returns are passed back to them in the form of a notionally accumulating fund, augmented by discretionary bonus additions. The precise form taken by AWP contracts varies from company to company (see Headdon *et al*, 1996a). The following describes a simplified benefit structure which retains the main features of the AWP contracts met in practice. Certain simplifying assumptions have been made. These include:

(1) premiums are assumed to be paid annually at the start of each year, while in reality they are often paid monthly or as irregular 'single premium' contributions;

(2) charges are assumed to be deducted at the start of each policy year while, again, in practice they would probably be deducted at more frequent intervals; and

(3) fund increases are assumed to take place annually at the end of each year, while in practice they may take place monthly or, often, daily.

*Contractual benefit structure*

2.3. If $a$ denotes the exact time of policy issue, the contractual benefit payable under an AWP contract for a given history of premium contributions exactly $t$ integer years after the issue date can be taken to be:

$$B_{a,t} = F_{a,t}(1 + TB_{a,t}) \tag{1}$$

where $F_{a,t}$ = the policy's accumulated fund to time $a+t$

and $TB_{a,t}$ = the terminal bonus rate payable on a contractual claim under the contract at time $a+t$.

2.4. The policyholder's fund accumulates according to the following relation:

$$F_{a,t} = (F_{a,t-1} + P_{a,t} - CH_{a,t})(1 + g)(1 + RB_{a+t}) \tag{2}$$

where $P_{a,t}$ = amount of premium received at the beginning of policy year $t$

$CH_{a,t}$ = amount of charges deducted at the beginning of policy year $t$

$g$ = guaranteed annual rate of fund increase

and $RB_{a+t}$ = discretionary annual rate of fund increase for the year ending at time $a+t$.

The charges cover the cost of meeting any additional insurance benefits and expenses.

2.5. The value of $TB_{a,t}$ is at the discretion of the company, and can be raised or lowered from time to time. Changes to $TB_{a,t}$ are usually made at discrete time intervals (two or three times a year or often less), and negative terminal bonus rates are not permitted. The nature of the terminal bonus is therefore a final single discretionary payment, made only at the date of claim, at the rate then currently payable by the company.

2.6. The value of $RB_{a+t}$ is also discretionary but, once the bonus increase has been made to the fund, it cannot be removed. The same rates of regular bonus are generally applied to all policies in force at the time of the bonus declaration, irrespective of policy duration. The nature of the regular bonus is therefore a regular discretionary increase, at rates which are not guaranteed in advance, but which once added serve to increase the level of guaranteed benefit payable under the contract. The value of $F_{a,t}$ is the amount of guaranteed benefit payable on a contractual claim at time $a+t$.

2.7. Under AWP it is the total contractual claim benefit $B_{a,t}$ (equation 1) which equates to the smoothed policy asset share referred to earlier in section 1.17.

## 3. *The Model Office*

*Investment experience assumptions*

3.1. Investment returns and inflation were generated stochastically using a slightly amended version of the model of Wilkie (1995). The amendment was to reduce parameter YSD of the equity model to 75% of the value recommended by Wilkie (1995), which reduces the standard deviation of the simulated equity returns to about 83% of its original value. The adjustment was determined by visual inspection of the simulated unadjusted equity returns, which appeared to be intuitively too variable.

3.2. This investment model is one of several that could have been used for this investigation (see Smith, 1996). It was chosen because of its familiarity both in applications (for example Ross, 1989; Hardy, 1993, 1997; MacDonald, 1993; Booth & Ong, 1994; and Booth, Chadburn & Ong, 1997), and also due to the fact that its properties have now been considerably researched (see Geoghehan *et al*, 1992; Kitts, 1988, 1990; Wilkie, 1995; Smith, 1996; and particularly Huber, 1996). One of the potential problems with the model may derive from the observation of Huber (1996) and others, that a temporally stationary model for inflation had been fitted to a non-stationary time series. As a result the model appears to predict too much variability for most of the time, but to understate the magnitude of shocks or large jumps which occur relatively infrequently. This inability of the model to maintain periods of relative stability at levels other than at the long-term mean results in undergeneration of sustained periods of price inflation or deflation (Kitts, 1990). The models for equities and consols display similar mean-reverting characteristics. This may contribute to the warning voiced in Geoghehan *et al* (1992), that the simulated frequency distributions of model-dependent functions are likely to be particularly unreliable at their tails. This should be borne in mind in the interpretation of the results presented in this paper.

*Other experience assumptions*

3.3. Deterministic mortality was assumed according to the A1967/70 ultimate table. Annual per-policy expenses were assumed, of £24 per annum as at the projection date, inflating according to the simulated rate of inflation produced by the investment model each year. (The projection date is the point in time which represents the current conditions of the model life office, from which future outcomes are projected). For simplicity charges were always assumed to equal expenses, and surrenders were ignored, so that no expense or surrender profit could arise.

*Historical experience assumptions and the existing liabilities*

3.4. 10000 15-year term level annual premium policies of identical size, in real terms, were assumed to have been issued each year over the past 15 years to policyholders aged 35 at entry. Assets were assumed to have earned a constant annual rate of return in the past, equal to the mean rate of return from the stochastic investment model assuming an asset mix of 75% equities and 25% consols. New premiums, expenses and charges were assumed to have increased at a constant annual rate of 4.91%, which is the average annual rate of inflation predicted by the Wilkie (1995) model. Two versions of the existing profile of policy liabilities are assumed. In the first case, referred to as the 'high' existing liability office, policyholders' funds are assumed to have been augmented at a rate of 7% per annum, which comprises both guaranteed and bonus annual increases. The overall result was to produce an existing profile of liabilities which is stationary in real terms as at the projection date, providing benefits which currently incorporate an overall terminal bonus of about 13%. In the second version (the 'low' existing liability office) the funds are assumed to have been augmented at a rate of 5% per annum, producing an overall terminal bonus proportion of about 22% of total benefit levels. It should be noted that most UK companies would probably have terminal bonus proportions currently somewhat less even than 13%, although terminal bonuses on conventional contracts might be significantly higher. This is mainly due to the relative lack of maturity of AWP business in the UK at the present time, but also partly due to a different bonus philosophy for AWP in which, it appears, a very high proportion of the earned rate of return is being distributed through annual fund increases rather than as terminal bonus (Headdon *et al*, 1996a).

*Future new business*

3.5. The model office is assumed to be closed to new business from one year after the projection date, with the last tranche of business being issued at the start of the projection period. This appears a reasonable assumption to make, bearing in mind that this investigation will not be concerned with the influence of future new business on the company's solvency or capital requirements. It is also the approach taken for the statutory valuation of long-term business insurers in the UK.

*Bonus philosophy*

3.6. Policyholders' funds are assumed to accumulate according to equation (2). The rate of declared bonus ($RB_{a,t}$) each year in each simulation is calculated according to the rules set out in the Appendix. This approach was developed by an Institute of

Actuaries working party on Unitised With-Profits to represent a 'realistic' model of a 'typical' company's behaviour with respect to its bonus philosophy (see Headdon *et al*, 1996b). The qualities 'realistic' and 'typical' are subjective, but the model allows for variation in the choice of parameter values to reflect different philosophies when required.

3.7. The main features of the assumed bonus philosophy are to declare a regular bonus $RB_{a,t}$ which, together with any guaranteed rate of fund increase, broadly reflects the yield on consols, subject to the policyholders' funds remaining lower than the value of a 'reduced policy asset share' ($RAS_{a,t}$). $RAS_{a,t}$ accumulates at some proportion $X$ ($0 < X \leqslant 1$) of the total rate of return on assets, after charges. Should $F_{a,t}$ exceed $RAS_{a,t}$, pressure to cut the bonus rate will be generated in the model. Hence the difference between $RAS_{a,t}$ and the full policy asset share ($AS_{a,t}$) effectively represents a target minimum value for the terminal bonus payable under the contract. Constraints are also enforced over the extent to which the declared bonus can change from year to year, resulting in a considerable smoothing of bonus rates over time.

3.8. The calculation of the total benefit payout is also described in the Appendix. The benefit is essentially the value of a smoothed asset share, or the value of $F_{a,t}$, if higher. The smoothed asset share is calculated by accumulating policy cash-flows (premiums less charges) at the geometric average of the rates of return earned over the previous three years.

*Investment policy*

3.9. The assets were assumed to be a mix of equities and consols. The proportions in the two classes normally remain fixed throughout the projection period, at a level which is fixed from outset. Rebalancing of assets is assumed at the beginning of each year in order to re-establish the fixed proportion at the start of each year. The long-term proportion invested in fixed interest is varied in order to test the effect of different asset allocation strategies upon the profile of returns.

3.10. This temporally static asset allocation strategy would not appear to be consistent with a closed fund of liabilities. As a result, frequencies of insolvency, in particular of actual insolvency (that is, when negative assets remain at the time when all liabilities have terminated) are likely to be mis-represented. However, as the emphasis here will be to consider relative results, not their absolute values, then useful comparisons will be possible despite these simplifying assumptions made.

*Calculation of statutory surplus*

3.11. The total assets at time $t$ years after the projection date ($A_t$) were generated by the following recurrence relation:

$$A_t = (A_{t-1} + P_t - E_t)(1 + roa_t) - C_t \qquad (3)$$

where $P_t =$ total premiums received at the start of year $t$ under all policies in force
$E_t =$ total expenses incurred at the start of year $t$
$roa_t =$ total return on assets over the year $[t\text{-}1, t]$
and $C_t =$ total benefits paid under maturity and death claims during the year $[t\text{-}1, t]$.

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3.12. For simplicity, premiums and expenses were assumed to be incurred at the beginning of each year, with claims incurred at the end of each year. The statutory value of the assets in the UK is essentially a market valuation: hence $A_t$ is also taken to be the statutory value of the assets at time $t$.

3.13. The statutory valuation of AWP liabilities in the UK has been the subject of considerable debate and review over the past few years, with papers presented on the subject by Scott *et al* (1996) and by Wright *et al* (1998, pending). While not wishing to anticipate the final outcome from these considerations, the method adopted in this investigation is consistent with that of Wright *et al*.

3.14. Companies are required to hold reserves which are at least as large as those calculated according to a minimum standard basis set out in the regulations. A key element of this basis is the definition of the maximum rate of interest which can be used to value the liabilities. This is essentially a weighted average of the redemption yield of the company's fixed interest assets, and the running yield for all other assets, with an overall maximum rate of 6% per annum.

3.15. Companies are also required to hold sufficient assets such that, should there be a sudden adverse change in investment conditions, they will still be able to satisfy the valuation regulations. This is referred to as resilience testing and companies therefore hold 'resilience reserves', where necessary, to meet these contingencies.

3.16. In a stochastic investigation it is not altogether clear whether or not resilience reserves should be allowed for. Other workers have adopted different views: for example Ross (1989) argues against their inclusion while others (for example Hardy, 1993; MacDonald, 1993) have not excluded them. I would argue that the inclusion of resilience reserves in a stochastic model is very likely to overstate the insolvency risk to a considerable degree. This is because, should a potentially insolvent situation arise in a stochastic projection, this will generally coincide with unusually depressed market valuations of the company's assets. Should such an event arise in practice, it is very likely that the UK regulators would require companies to hold lower than normal resilience reserves, or possibly even none at all. While this is debateable, it is clear that the regulator's requirements to hold resilience reserves in extreme conditions cannot be predicted with any confidence. Also, at the time of writing, the UK resilience test methodology was under review. It was therefore decided to ignore resilience reserves in this investigation, as this involved less conjecture and required fewer assumptions; the omission should, however, be borne in mind when interpreting the results.

3.17. For the present investigation the statutory valuation of the liabilities therefore reduces to considerng the higher of the following two values as at the valuation date:

(i) the lower of:
   - (a) the policy fund value; and
   - (b) the amount that policyholders' would reasonably expect to be currently payable on surrender;

and

(ii) the prospective value of the guaranteed benefit on a future contractual claim, at the valuation rate of interest (as calculated according to paragraph 3.13).

For modelling purposes the surrender value in (i)(b) is assumed to be equal to the policy's asset share.

3.18. The resulting total value of the liabilities is denoted by $V_t$. The statutory surplus at time $t$ is therefore:

$$S_t = A_t - V_t \qquad (4)$$

3.19. Statutory insolvency is assumed to occur where $S_t < 0$. In practice regulations require companies to hold a minimum amount of statutory surplus, which for this business would be essentially equal to 4% of $V_t$, according to the relevant European Union Directives. It was felt that incorporation of this detail into the model was not material to the validity of this investigation, again remembering that only relative results are important here.

## 4. *Methodology*

4.1. The effect of variations in long-term management strategies upon the profile of returns produced by offices insuring different liability types, and under different initial conditions, was investigated. The effect of adopting different crisis management strategies was also considered.

*Output measures*

1. *Frequency of actual insolvency*

4.2. A projection of the model life office which produces negative assets at the time when liabilities have run-off (that is after 15 years) is defined as resulting in actual insolvency. The frequency of actual insolvency is the proportion of the 1000 simulations which produce this outcome.

2. *Frequency of statutory insolvency*

4.3. A projection of the model office in which statutory insolvency occurs at least once over the projection period is defined as resulting in statutory insolvency. This naturally includes all projections which remain solvent over the whole 15 years but are actually insolvent at the end.

3. *Time to first ruin*

4.4. Statutory insolvency will affect a greater proportion of policyholders, and each to a greater extent, the earlier the insolvency occurs. An important statistic is therefore the time to statutory insolvency. Two versions of this will be measured.

(i) $T$: the time to insolvency, assuming a value of $T = 15$ for all projections which do *not* result in insolvency. $T$ is then the total 'survival time' of the fund.

(ii) $T1$: the time to insolvency, conditional upon insolvency occurring.

Hence while the simulated mean of $T1$ (with respect to just the insolvent simulations) gives an estimate of the expected time to ruin *if* ruin occurs, the mean of $T$ (with respect to all simulations) gives an estimate of the expected future lifetime of the office. The distribution of $T$ is hence a function both of the distribution of $T1$ and of the probability of ruin. $T$ might indeed be considered to be the more important statistic than either of its components, something which has not always been considered in investigations such as these.

3. *The distribution of policyholders' benefits*

4.5. The effect of management decisions upon the value of the policy benefits is of paramount interest to the policyholders; indeed it is logical to assume that such decisions should "be based upon producing an expected distribution of future returns which is considered by the investors to be optimal" (Booth, Chadburn & Ong, 1997, page 1). The distribution of payouts made to all policyholder cohorts should be considered in this assessment. One approach would be to examine the distribution of maturity values paid under policies which mature in each year. The problem with this approach is that it is difficult to make objective comparisons using a series of values. Booth, Chadburn and Ong (1997) overcame a similar problem, in which they wished to assess the aggregate value of a dividend stream to shareholders, by accumulating projected dividends in the equity market until a common horizon date. A parallel approach will be adopted here. Assuming that the AWP liabilities are supporting personal pension policies, policyholders will be purchasing immediate annuities when their contracts mature. This implies that maturity values are reinvested (for the investors' benefit) in fixed interest assets. Hence in this investigation it is assumed that maturities are reinvested in consols and accumulated (according to the given stochastic investment model) to the end of the projection period. This produces a single value of policy returns made to all the existing policyholders in force at the start of the period. The value of death benefits, which are relatively small in size, have been ignored. Surrenders (as explained in paragraph 3.3) have also been ignored for simplicity. The inclusion of surrender values would give an interesting additional dimension to the consideration of the profile of policy returns: this is an area for future investigation.

4.6. Where a simulation leads to a statutory insolvency at a time $t_s$, say, then the total of the existing assets are assumed to be distributed to poicyholders at that point (noting that, at insolvency, these must have less value than the policyholders' contractual benefits). This amount is accumulated (along with all previous years' maturities) to the 15-year horizon date. However, no future contributions are credited beyond the date of insolvency, and hence the loss of the investment service due to the closure of the company is translated into lost policy benefits in the model. While this approach is somewhat simplistic, the implied penalty caused by insolvency and reflected in the measure of policy payouts is appealing.

4.7. Following on from considerations introduced in section 1.5, it was decided to measure the followng attributes of the distribution of these accumulated payouts.

(i) The simulated mean value.

(ii) The simulated standard deviation.

(iii) The simulated mean value of the utility of the accumulated payout, using the logarithmic utility function.

4.8. Item (iii) is an estimate of the expected utility of the payouts to the policyholders. The utility function of an investor defines the relative preference for risk (variability) and return (expected value) of that investor (see Booth, Chadburn & Ong, 1997). Booth, Chadburn & Ong argue that management decisions should be sought which maximise the expected utility of the investors' returns, thereby identifying the optimal decision. These authors found that using utility maximisation criteria resulted in asset allocation decisions which were generally mean-variance efficient (Levy & Markovitz, 1979), even where solvency constraints were imposed. In this investigation therefore, the effect of decisions upon the expected utility of the policyholders' returns will be considered an important measure of the relative values of these decisions to the policyholders.

4.9. The utility of an amount of wealth $x$, according to the logarithmic utility function, is deined as :

$$U(x) = \ln(x) \tag{6}$$

(see Booth, 1997). The logarithmic utility function was chosen because of its property of displaying a constant relative risk aversion, defined as:

$$\rho(x) = \frac{-U''(x)}{U'(x)} x = 1 \tag{7}$$

4.10. This implies that decisions will depend upon the *proportion* of policyholders' wealth invested (for example, 10% of wealth will have the same utility to an investor regardless of the absolute wealth of that investor). Booth, Chadburn & Ong (1997) argue that this makes the assumption of a logarithmic utility function ideally suited for application to an aggregation of like investors, such as a group of with-profits policyholders, who while they may have very different absolute levels of personal wealth may be expected to invest similar proportions of their wealth in their policies.

*Model variations*

*Variations in bonus philosophy*

4.11. Three different versions are considered in this paper, in conjunction with the two versions of the existing liablility profile described in section 3.4. These will be referred to as follows.

*The 'low/low' office*: the low existing liability profile, with a bonus philosophy where $g = 0$ (see section 2.4), and $X = .75$ (see section 3.7).

*The 'low/high' office*: the low existing liability profile with $g = .06$ and $X = .9$.

*The 'high/low' office*: the high existing liability profile with $g = .03$ and $X = .75$.

*The 'high/high' office*: the high liability profile with $g = .06$ and $X = .9$.

These combinations were chosen to produce a broad range of risk profiles over time. Figure 1 illustrates the means (over all 1000 simulations) of the ratio of total in-force unit funds (UF) to total in-force asset shares (AS) at the end of each year. It can be seen that the low/high and high/high models ultimately reach identical levels of this ratio, having begun from very different starting values. The low/low and high/low models do not converge upon the same ultimate level, which reflects (only) the different levels of value for $g$ assumed in the two cases. It is interesting that, despite quite extreme assumptions for the high philosophy models, the high initial conditions canot be maintained under the dynamic distribution assumptions of this model. Hence while Headdon *et al* (1996) may have considered their assumptions to be reasonable, they may not be realistic if they cannot maintain the status quo. Of course, an alternative (or contributory) reason for the effect may be the possibly exaggerated variability of returns projected by the investment model, although it should be noted that these were reduced from those of the original Wilkie (1995) model (see section 3.2).

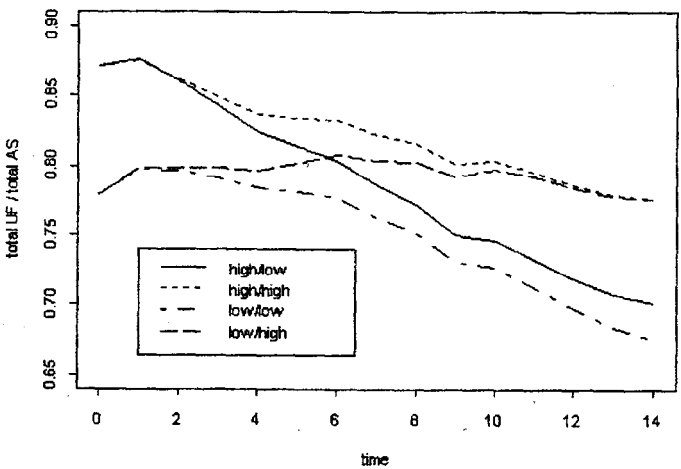


**Figure 1: simulated mean ratios between unit fund size and asset shares for each of the four version of the model**

*Variations to the level of existing capital*

4.12. Two different levels of initial capital were investigated for each of the four models described above. The levels were chosen so that the lower level produced a relatively high insolvency risk, while the upper level had relatively little solvency risk. The chosen levels were as follows.

(i) For the high/high office: 15% and 25%.

(ii) For all other offices: 5% and 15%.

The above percentages are defined as the excess of assets over total asset shares as a proportion of the total asset shares, as at the start of the projection.

4.13. It will be noted that all versions are investigated at capital levels of 15%.

*Decision variables*

4.14. The investigation examines the effect of the following decision variables on the distribution of policy returns.

(i) The long-term proportion invested in consols, denoted by *PC*.

(ii) The annual amount of deduction from policy asset shares, expressed as a proportion *CD* of asset shares, which will have the effect of increasing the level of capital provision. This strategy, when used, is referred to as the 'capital replacement' (or CR) strategy.

(iii) The use of a dynamic crisis investment strategy (the CI strategy).

(iv) The use of a dynamic crisis bonus distribution strategy (the CB strategy).

4.15. Incorporating the dynamic crisis investment strategy will imply that, if at the end of a year the ratio of the value of the assets to the statutory value of the liabilities (the 'solvency ratio') is less than 1.15, equities are progressively moved into consols until there is 100% investment in consols at a solvency ratio or 1.05 or less. This rule, when it applies, overrides the normal rebalancing rule, described in paragraph 3.9. This is similar to the assumption first suggested by Ross (1989), except that under his assumptions the strategy took effect over the range of solvency values down from 1.25. The lower value of 1.15 was chosen here so that, even where the CI strategy is incorporated, at least in some cases the strategy will not be activated for all of the time.

4.16. A similarly simple yet reasonably realistic mechanism was required for the crisis bonus strategy. (This mechanism was first used and described in Chadburn, 1997). It was felt that policyholders would reasonably expect normal bonuses to be witheld only when solvency was becoming critical. It was therefore decided that the crisis bonus response would be effective only where the solvency ratio was less than 1.04. The assumed response is:

(1) to remove all terminal bonuses from benefit payments; and

(2) to withold the addition of the regular bonus that would otherwise have been declared if there were no solvency crisis.

As soon as the crisis is resolved, the total benefit levels return to the normal levels, and the next year's regular bonus is calculated as if there had been no interruption. However, the missed bonus (or bonuses) are not replaced, resulting in a lower level of guarantee

existing for some years after the crisis has been resolved. The crisis bonus policy will therefore have both immediate and longer term benefits for the solvency of the office.

## 5. *Results*

*High/low office, initial capital = 15%*

5.1. By varying the long-term proportion invested in consols (*PC*), and ignoring all other strategies, it was found that (to the nearest 5%) a value of *PC* = 30% maximised the expected utility of policy payouts (column (1) of Table 1).

5.2. The introduction of the CB strategy produced the output shown in column (2). Compared with column (1), it can be seen that the CB strategy reduces the frequencies of statutory and absolute ruin, although not by a huge extent. The expected lifetime of the fund E(*T*) has increased marginally. However, an important feature is that the mean return has been increased, the standard deviation of return decreased, and the expected utility of return has increased. Hence the use of the CB strategy has benefited all aspects of the profile of policy returns.

5.3. With the CB strategy included, it was found that a higher expected utility was obtained at *PC* = 25% (column (3) of table 1). The higher utility reflects the increase in mean return obtained, although (compared with *PC* = 30%) other aspects of the distribution are made worse, as would be expected from increasing the equity proportion. It should be remembered that the relative preference for (3) compared with (2) is determined by the assumed utility function, whereas the preference for (2) compared with (1) (ie due to the CB strategy) is independent of that assumption.

5.4. The effect of incorporating the CI strategy is shown in column (4), and of the CR strategy, with *CD* = 1%, in column (5), both assuming *PC* = 30%. Columns (4) and (5) should therefore be compared with columns (1) and (2) of table 1.

5.5. Column (4) shows that the CI strategy has a somewhat bigger effect in reducing statutory insolvency than CB, both in terms of reducing the ruin frequency and of increasing the conditional expected time to ruin, E(*T*1). However, a considerable penalty is incurred upon he expected utility of returns. Compared with column (1), mean returns have fallen by 4% and the standard deviation of returns by only 10%, a combination which does not merit higher utility given the assumed utility function. Investors would have to be considerably more risk averse than that implied by the logarithmic utility function in order to prefer the CI strategy in this case.

5.6. Column (5) shows that capital replacement at 1% per annum makes the largest single impact upon the frequency of statutory ruin, although E(*T*) is only marginally increased compared with column (4). Means, standard deviations and utility of returns are only marginally different from (4). It can also be observed how the mean value of the residual assets at the end of the projection period, E(*A*), has been increased by the strategy, to approximately double previous levels.

5.7. An interesting contrast is provided by considering the use of a combination of both the CB and CR (*CD* = 1%) strategies (column (6)). This has the same expected utility as the CI strategy, but in almost all other respects shows some improvement upon that strategy, most notably in the frequency of ruin. It therefore appears that, for this

level of initial capital for the high/low office, capital replacement at as much as 1% of asset shares per annum is no more detrimental to policy returns than incorporating a crisis investment response, while having a somewhat greater beneficial effect upon solvency.

*Utility maximisation subject to constraints*

5.8. The optimum represented by column (3) of table 1, while optimal on the basis of the implied utility function of the investors, may not be the most desirable on the basis of other criteria. Hence what individuals might subjectively consider to be optimal is not truly represented by the assumed utility function. In particular the column (3) strategy has a very high probability of statutory ruin (21.5%). While, as has been said earlier, this absolute value is probably meaningless, it would be interesting to see how the 'optimal' strategies would be affected by requiring a maximum probability of ruin. An arbitrary maximum of 5% was chosen.

5.9. The maximum utility result is obtained using the CB strategy with $PC$ = 60%, and is shown in column (7) of table 1. What is particularly striking is the fact that all other aspects of the distribution of policy returns are improved compared with the attempts shown in columns (4), (5) and (6) of the same table. While the expected returns in column (7) are similar to those in the other cases (in fact, slightly increased), the standard deviation of returns is considerably reduced. The value of E(T1) is also considerably higher. The expected utility of returns is also higher for $PC$ = 60% compared with any of the attempts at managing solvency when $PC$ = 30%. This quite strongly suggests that solvency is more effectively managed through an appropriate choice of long-term investment strategy, rather than by choosing a strategy with higher equity-backing and trying to avert insolvency using either the CI or CR strategies.

5.10. Attempts were made to see if improvements to returns could be made, within the 5% ruin constraint, by incorporating the CI and CR strategies, and hence allowing a higher proportion to be invested in equities. However, all attempts produced less favourable return profiles than that shown in column (7), as the observations of the previous paragraph would suggest.

*High/low office, initial capital = 15%*

5.11. Trials indicated that, at this level of capital, the CB strategy made almost no difference to the profile of returns. This strategy was therefore not considered further for this particular model office.

5.12. The unconstrained optimal result was obtained with $PC$ = 10%, and shown as column (1) of table 2. Comparison can be made with the unconstrained optimum for this office with initial capital of 5% (column (3) of table 1).

5.13. It is instructive to consider the mechanism by which the new optimum has been reached , beginnning from the original optimal point (with $PC$ = 25%). The existence of more capital naturally reduces the probability of ruin (column (2) of table 2). The lower frequency of ruin increases the expected value of the accumulated payouts and reduces their variability, thereby increasing utility. Now the company can further increase expected utility by increasing the proportion invested in equities, because the increased returns so generated are not sufficiently offset by the increase in insolvencies which accompany it. The latter only becomes sufficiently onerous beyond an equity

proportion of 90%, hence 90% equities forms the new optimal long-term investment strategy. This forms an intuitively reasonable explanation of the process, and highlights the importance both of the level of capital and of the frequency of ruin in the decision-making process.

5.14. The constrained maximum utility decision is, coincidentally, the case where $PC = 25\%$, and which is therefore as shown in column (2) of table 2. The significant reduction in variability of payouts and in the probability of ruin, with an increase to the expected time to first ruin (both $T$ and $T1$) indicates that this could well be a more acceptable profile of returns to a more risk averse investor. Comparison with column (7) of table 1 shows the beneficial effects of the higher capital upon the expected value and expected utility of payouts in the constrained case. Similar arguments to paragraph 5.13 apply here also.

5.15. Attempts were again made to see if the incorporation of the CI and/or CR strategies could lead to increased utility of returns within the required solvency constraint, by allowing a higher long-term equity-backing. It was found that PC can be reduced to 20% when the CI strategy is incorporated, leading to the profile of returns shown in column (3). In column (4) the results of incorporating a CR strategy, with $CD = 0.75\%$, are shown. This produces a rather more dramatic fall in expected payouts and expected utility, and this therefore seems an even less effective option than the CI strategy. These observations provide further support for the conclusions of paragraph 5.9. In addition, at this level of capital, the results suggest that the effectiveness of the CR strategy seems to be reduced relative to the CI strategy. It seems intuitive that the relative effect of providing more capital will be greater the less the level of capital that is already available.

*The high/high office, initial capital = 15%.*

5.16. As in the previous section, the CB strategy produced minimal effects on returns and was therefore ignored. In table 3 column (1) represents the unconstrained optimum, at $PC = 35\%$, while the utility maximising result subject to the ruin constraint is shown in column (2), for $PC = 61\%$.

5.17. Comparison with the unconstrained optimum of the high/low office at the same level of capital can be made (column (1) of table 2). Notice first of all that the optimal asset mix has changed from $PC = 10\%$ to $PC = 35\%$ as a result of the higher guarantees. Secondly, while the probability of ruin has only increased marginally, the mean of the policy payouts has reduced by about 5%, and the standard deviation by about 20%. The overall utility of returns has also reduced considerably. Hence, even without constraints, the effect of increasing the level of policy guarantees has been to cut the expected utlity of returns considerably. Investors would clearly need to be significantly more risk averse than that implied by the logarithmic utility function before they would prefer the high guarantee as opposed to the low guarantee office.

5.18. To compare the outcomes under the 5% ruin constraint, column (2) of each of tables 2 and 3 should be compared. Nearly all aspects of the return profile, including the time to first ruin, have been made worse by the existence of the higher guarantees, with even higher reductions to the expected level of returns being obtained than for the

unconstrained case. Note, however, as with the unconstrained case, the variabilty of payouts has been considerably reduced by the existence of the higher guarantees.

5.19. As before, attempts were made to investigate whether alternative strategies could provide better outcomes under constraint for this office. This proved not to be possible using the CR strategy. The results for the CI strategy are shown in column (3) of table 3, relating to $PC$ =56% (the lowest value consistent with the ruin constraint). There is clearly very little to choose between the outcomes shown in columns (2) and (3), and this marks an improvement in the relative effectiveness of the CI strategy in managing solvency where contracts offer higher guarantees (compare with the results described in paragraph 5.15).

*High//high office, capital = 25%*

5.20. The unconstrained and constrained utility maximising results are shown in columns (4) and (5) of table 3, which relate to asset mixes of 5% and 32% consols respectively. The expected utility of returns has now increased as a result of the increased capital (compare with columns (1) and (2) of table 3). However comparison with columns (1) and (2) of table 2 shows that the return profiles are still not as favourable compared with the high/low office even when holding 10% less capital. In particular the unconstrained optimum has a higher standard deviation of returns, lower expected time to first ruin, and a considerably higher expected frequency of ruin. This is particularly interesting if it is noted that the unconstrained optimal asset mix for the high/low office was 10% consols (as compared to 50% for the high/high office), so that the observed outcome is entirely due to the higher guarantees provided by the high/high office.

5.21. The CI strategy made very little difference to the utility maximising outcome under constraint. The expected utility was reduced for very little reduction in ruin frequency. The relative reduction in impact of this strategy at higher levels of capital may be expected, due to the fact that the CI strategy only takes effect when free assets fall to less that 5% of reserves, so that the strategy will operate on fewer occasions and to a lesser degree.

*Low/high office, capital = 15%*

5.22. Results for this office are shown in table 4. Unconstrained and constrained results are shown in columns (1) and (2), representing $PC$ = 15% and $PC$ =50 % respectively.

5.23. Comparison with table 3 (columns (1) and (2)) show the effect of the different initial starting conditions for offices providing the same long-term guarantees, and with the same level of capital. As might be expected, the asset allocations for the initially low office consists of higher proportions in equities, which results in higher policy returns and higher utility of those returns in both the unconstrained and constrained cases. The CI strategy was very ineffective at reducing the risk of insolvency (for example, at $PC$ = 45%, the CI strategy caused the frequency of ruin to fall by just 0.1% from 6.2% to 6.1%, while the expected utility of returns fell from 22.4709 to 22.4567). This probably reflects the lower reserves required by the low/high office, at least over the first few projection years, thereby effectively increasing the free asset ratio

and hence reducing the number of occasions where the CI strategy will be effective. The CB strategy was similarly ineffective in this case.

*Low/high office, capital = 5%*

5.24. The unconstrained optimal result for the low/high office, with capital of 5%, is shown in column (1) of table 5. This is produced for $PC = 40\%$. Incorporation of the CB strategy has a fairly small but nevertheless beneficial effect upon returns, including solvency, as shown in column (2). The lowest probability of ruin obtained from any asset mix is that produed where PC = 75%, the results of which (with the CB strategy incorporated) are shown in column (3). This falls far in excess of the required 5% maximum ruin constraint.

5.25. Column (4) shows the effect of incorporating the CI stategy. This actually increases the probability of ruin which, on reflection, is actually an intuitive result. Incorporation of the CI strategy will result in fixed interest proportions either equal to or in excess of 75%, as this is the assumed value of *PC*. As ruin frequencies actually increase for asset mixes in excess of this value (as shown by the fact that the PC value of 75% is the asset mix at which the ruin frequency is at a minimum), then the observed increase in ruin frequency naturally followed. If this phenomenon applies generally, then the CI strategy (as assumed) becomes counter-productive whenever it generates asset mixes with proportions in consols which exceed the optimal value (where in this case 'optimal' refers to the value which generates the lowest ruin frequencies). This may well explain the relative ineffectiveness of the CI strategy in some other cases reported in this paper.

5.26. The result of incorporating the CR strategy, with the CB strategy, with $PC = 75\%$, is shown in column (5). The figures shown represent the lowest rate of capital replacement (2.25% per annum) which resulted in a ruin frequency below 5%. Compared with the unconstrained optimum column (2) there is a considerable reduction in expected utility. However, this is perfectly reasonable considering the rather severe situation that this office initially finds itself in: with inadequate capital for the level of guarantee implied by the high bonus philosophy, despite the low initial starting conditions.

*Low/low office, capital = 5%*

5.27. It should be noted that, when comparing these results with those of the high/low office, the bonus strategy in the low/low case actually implies lower guarantees than that of the high/low case. (This is graphically illustrated in figure 1).

5.28. Table 6 column (1) shows the unconstrained optimal result, which is obtained at $PC = 15\%$, without the CB strategy. Incororating the CB strategy results in the figures shown in column (2). The improvements brought about by the CB strategy are considerable, more so than for any other liability structure tested at this level of capital. Improvements occur to the mean and standard deviation of returns, but most markedly in reducing the frequency of statutory ruin.

5.29. Column (3) shows the effect of the CI strategy, in addition to CB, still at $PC = 15\%$, while column (4) shows the effect of the CR strategy (again in addition to CB), with $CD = 0.75\%$ per annum. Clearly both the CI and CR strategies are very effective at reducing the frequency of insolvencyy for this low guarantee office. While

both strategies (in this example) lead to the same expected utility, the CR strategy is marginally better in other respects.

5.30. The utility maximising result subject to constraint is shown in column (5), while column (6) additionally incorporates the CI strategy. The effect of the latter is to allow the proportion invested in consols to be reduced from 44% to 25%, but the effect on the overall return profile is not an improvement, having rather higher variability of returns in particular.

*Low/low office, capital = 15%*

5.31. In this case, the unconstrained and constrained cases are obtained at $PC = 0$ and at $PC = 5\%$ respectively (columns (1) and (2) of table 7). Probabilities of ruin are very low, and returns very high. Neither the CI nor CR strategies provide any viable alternative strategies in this case (values not shown).

## Key to all tables

| | |
|---|---|
| E(MV) | Mean value of the the accumulated policy maturity payouts. |
| SD(MV) | Standard deviation of the accumulated policy maturity payouts. |
| E(U) | Mean value of the utility of the accumulated policy maturity payouts. |
| N(SR) | Frequency of statutory ruin. |
| E(T) | Expected lifetime of the fund. |
| E(T1) | Mean time to first ruin, conditional on insolvency occurring. |
| SD(T1) | Standard deviation of T1. |
| N(AR) | Frequency of actual ruin. |
| E(A) | Mean amount of residual assets in the fund after expiry of liabilities. |

| | | |
|---|---|---|
| Units: | Rows 1, 2 and 9: | £million. |
| | Rows 4 and 8: | percent. |
| | Rows 5,6 and 7: | years. |
| | Row 3: | arbitrary. |

## Table 1

Profile of policy returns for high/low office, capital = 5%

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| E(MV) | 5886.86 | 5894.24 | 5921.03 | 5659.99 | 5658.96 | 5660.84 | 5662.36 |
| SD(MV) | 1482.17 | 1466.58 | 1562.10 | 1341.33 | 1328.46 | 1326.57 | 1029.14 |
| E(U) | 22.4632 | 22.4654 | 22.4658 | 22.4292 | 22.4288 | 22.4292 | 22.4409 |
| N(SR) | 21.0% | 18.7% | 21.5% | 15.2% | 9.6% | 9.4% | 4.8% |
| E(T) | 13.38 | 13.54 | 13.29 | 13.91 | 14.10 | 14.12 | 14.69 |
| E(T1) | 7.3 | 7.21 | 7.03 | 7.86 | 5.62 | 5.69 | 8.37 |
| SD(T1) | 3.64 | 3.55 | 3.63 | 3.59 | 2.97 | 2.97 | 3.30 |
| N(AR) | 6.0% | 3.6% | 4.6% | 6.2% | 1.3% | 1.1% | 0.5% |
| E(A) | 264.44 | 289.70 | 304.53 | 317.58 | 634.31 | 6636.95 | 217.28 |

## Key to columns

1. Optimal strategy without constraint; $PC = 30\%$
2. As (1) but including CB strategy; $PC = 30\%$.
3. As (2) but for $PC = 25\%$ (optimal)
4. $PC = 30\%$, with CI.
5. $PC = 30\%$, with CR ($CD = 1\%$ p.a.)
6. $PC = 30\%$, with CB and CR ($CD = 1\%$ p.a.)
7. $PC = 60\%$, with CB (maximum utility with constraint)

## Table 2

Profile of policy returns for high/low office, capital = 15%

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| E(MV) | 6215.54 | 6097.04 | 6043.53 | 5913.13 |
| SD(MV) | 1707.73 | 1429.45 | 1465.32 | 1428.91 |
| E(U) | 22.5121 | 22.5039 | 22.4934 | 22.4717 |
| N(SR) | 11.1% | 4.8% | 4.8% | 4.7% |
| E(T) | 14.00 | 14.60 | 14.60 | 14.52 |
| E(T1) | 6.03 | 6.75 | 6.78 | 6.22 |
| SD(T1) | 3.02 | 3.09 | 3.08 | 3.03 |
| N(AR) | 2.0% | 0.7% | 1.4% | 0.6% |
| E(A) | 837.78 | 762.11 | 891.54 | 1077.54 |

**Key to columns**

1. Optimal strategy without constraint; $PC = 10\%$
2. $PC = 25\%$: maximum utility subject to constraint.
3. $PC = 20\%$, with CI.
4. $PC = 20\%$, with CR ($CD = 0.75\%$).

## Table 3

Profile of policy returns for high/high office

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| E(MV) | 5883.10 | 5657.11 | 5661.84 | 6218.09 | 6024.57 |
| SD(MV) | 1368.99 | 1014.30 | 1047.80 | 1826.31 | 1325.08 |
| E(U) | 22.4680 | 22.4405 | 22.4403 | 22.5072 | 2204953 |
| N(SR) | 14.4% | 4.9% | 4.8% | 18.5% | 4.9% |
| E(T) | 13.63 | 14.56 | 14.56 | 13.18 | 14.54 |
| E(T1) | 5.47 | 6.04 | 5.88 | 5.19 | 5.71 |
| SD(T1) | 2.83 | 3.01 | 2.91 | 2.98 | 2.64 |
| N(AR) | 2.1% | 0.1% | 0.1% | 3.1% | 0.5% |
| E(A) | 696.27 | 594.40 | 681.03 | 1363.92 | 1175.16 |

**Key to columns**

1. Optimal strategy without constraint; $PC = 35\%$, capital 15%.
2. $PC = 61\%$: maximum utility subject to constraint, capital 15%.
3. $PC = 56\%$, with CI, capital 25%.
4. Optimal strategy without constraint; $PC = 5\%$, capital 25%.
5. $PC = 32\%$: maximum utility subject to constraint, capital 25%.

## Table 4

Profile of policy returns for low/high office, capital = 15%.

| | (1) | (2) |
|---|---|---|
| E(MV) | 6059.62 | 5803.61 |
| SD(MV) | 1702.60 | 1115.60 |
| E(U) | 22.4845 | 22.4636 |
| N(SR) | 22.0% | 4.7% |
| E(T) | 13.03 | 14.66 |
| E(T1) | 6.03 | 7.68 |
| SD(T1) | 3.06 | 3.05 |
| N(AR) | 3.9% | 0.2% |
| E(A) | 792.08 | 640.63 |

**Key to columns**

1. Optimal strategy without constraint; $PC = 15\%$.
2. $PC = 50\%$: maximum utility subject to constraint.

## Table 5

Profile of policy returns for low/high office, capital = 5%.

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| E(MV) | 5671.93 | 5674.55 | 5391.82 | 5320.45 | 4918.01 |
| SD(MV) | 1395.21 | 1386.54 | 918.93 | 882.33 | 791.48 |
| E(U) | 22.4278 | 22.4287 | 22.3939 | 22.3813 | 22.3036 |
| N(SR) | 32.6% | 31.1% | 18.9% | 21.4% | 4.5% |
| E(T) | 12.35 | 12.46 | 13.53 | 13.31 | 14.55 |
| E(T1) | 6.88 | 6.83 | 7.22 | 7.11 | 5.04 |
| SD(T1) | 3.23 | 3.25 | 3.01 | 2.84 | 2.20 |
| N(AR) | 8.7% | 6.2% | 1.0% | 2.3% | 0 |
| E(A) | 232.76 | 260.14 | 493.20 | 213.48 | 793.32 |

**Key to columns**

1. $PC = 40\%$.
2. Optimal strategy without constraint; $PC = 40\%$, with CB.
3. $PC = 75\%$, with CB.
4. $PC = 75\%$, with CB and CI.
5. $PC = 75\%$, with CB, CI and CR ($CD = 2.25\%$ p.a.)

## Table 6

Profile of policy returns for low/low office, capital = 5%

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| E(MV) | 6059.79 | 6116.95 | 5951.54 | 5952.98 | 5871.94 | 5889.43 |
| SD(MV) | 1711.05 | 1662.85 | 1539.40 | 1518.74 | 1180.71 | 1385.56 |
| E(U) | 22.4836 | 22.4965 | 22.4747 | 22.4747 | 22.4737 | 22.4696 |
| N(SR) | 23.8% | 16.7% | 7.4% | 6.2% | 4.6% | 4.6% |
| E(T) | 13.36 | 13.90 | 14.59 | 14.52 | 14.79 | 14.75 |
| E(T1) | 8.09 | 8.39 | 9.51 | 7.26 | 10.48 | 9.63 |
| SD(T1) | 3.87 | 3.87 | 3.60 | 3.32 | 3.35 | 3.49 |
| N(AR) | 10.8% | 4.5% | 2.7% | 1.2% | 1.1% | 1.3% |
| E(A) | 309.27 | 340.16 | 416.26 | 618.02 | 245.97 | 379.85 |

**Key to columns**

1. Optimal strategy without constraint; $PC = 15\%$
2. As (1) but including CB strategy; $PC = 15\%$.
3. $PC = 15\%$, with CI.
4. $PC = 15\%$, with CR ($CD = 0.75\%$ p.a.)
5. $PC = 44\%$: maximum utility subject to constraint.
6. $PC = 25\%$, with CI.

## Table 7

Profile of policy returns for low/low office, capital = 15%.

| | (1) | (2) |
|---|---|---|
| E(MV) | 6403.72 | 6353.67 |
| SD(MV) | 1809.52 | 1715.03 |
| E(U) | 22.5409 | 22.5366 |
| N(SR) | 5.5% | 4.3% |
| E(T) | 14.59 | 14.69 |
| E(T1) | 7.47 | 7.84 |
| SD(T1) | 3.12 | 3.11 |
| N(AR) | 1.4% | 1.3% |
| E(A) | 909.54 | 879.38 |

**Key to columns**

1. Optimal strategy without constraint; $PC = 0$.
2. $PC = 5\%$: maximum utility subject to constraint.

## 6. *Summary and discussion of the results*

6.1. The CB strategy improves all aspects of the return profile where capital is low (5%). This is particularly the case where guarantees are low, presumably due to the greater discretionary element in the benefits payable, and which therefore lends itself to control using the CB strategy. It appears initially curious that withholding bonus distributions can actually increase the overall expected value of policy returns. This is almost certainly due to a key assumption of the model, in which insolvency leads to reduced accumulated value to the policyholders considered as a whole. The reduction in insolvencies due to the CB strategy thereby increases the aggregate return. In effect, increased solvency is translated into increased value to the policyholders in this model. This is theoretically rational, but an individual policyholder may need some convincing that a withholding of bonus is beneficial to ultimate returns. An appreciation of the real dis-benefit of insurer insolvency on policyholder returns would be important in making a convincing argument.

6.2. The CI strategy is moderately effective in controlling solvency at levels of capital of 15% or less, in part reflecting the choice of upper limit in the strategy chosen. Again the most marked reductions to ruin frequency were obtained for offices with low levels of guarantee and at low levels of capital, but in no single case was determined as unequivocally the best option. A contribution to this may be a fault in the assumption made for this strategy (see paragraph 5.25). The strategy would probably be more effective if the upper limit for the proportion invested in consols was limited to the value of *PC* for which the probability of ruin was a minimum.

6.3. A strategy of capital replacement produces useful reductions to the frequency of ruin where capital is initially low. It is shown that there are cases in which this strategy is no more detrimental to policy returns than the CI strategy, and is usually superior to that strategy, at least in the latter's currently assumed form.

6.4. The most important single strategy for the optimisation of returns is the long-term investment strategy. Faced with an unacceptable solvency risk with the company's given level of capital, it should seek to make a long-term switch to a more optimal asset mix rather than incorporate short-term solutions (such as the CI strategy) while trying to maintain a too heavily equity-backed portfolio.

6.5. A logical mechanism by which the presence of existing capital contributes to improving the profile of policy returns, through the process of utility maximisation, is described in paragraph 5.13. When considering the practical implications of this, the assumption that the *initial* capital incurs no cost upon the existing policyholders will not always be reasonable, particularly if the office is proprietary.

6.6. For a given level of capital and for a given existing level of guarantee, the existence of high long-term policy guarantees makes worse all aspects of the profile of policy returns *except* for the variability of returns. It is argued that investors would need to be significantly more risk averse than that implied by the logarithmic utility function in order to prefer the high guarantee to the low guarantee office, at least according to the assumptions made here.

6.7. The level of initial guarantee appears to be less significant in its effect on the profile of policy returns than is the long-term bonus philosophy and the levels of

guaranteed return. Clearly, in practice, the level of the existing guarantees must increase in importance the more mature is the fund.

## 7. *Conclusions*

7.1. In this paper some attempt has been made to identify the relationships between long-term and short term management strategies, in the control of the profile of returns for AWP policyholders. An important feature of the methodology was the incorporation of a single objective function which represented the aggregate amount of policy maturities from all policies in force at the start of the projection period. The fact that this quantity was sensitive to the disbenefit from insolvency appeared to be important in generating rational outcomes from the model.

7.2. The results illustrate that it is more important to make appropriate long-term decisions than to attempt to compensate for inappropriate long-term choices through crisis management strategies. Two aspects of long-term strategy are shown to be particularly important to the profile of returns: the long-term investment strategy, and the long-term bonus distribution philosophy (at least in relation to the regular bonus distributions). If policy guarantees develop at a high rate as a result of this philosophy, then the profile of returns (including the risk of insolvency) could be significantly prejudiced. Greater knowledge regarding the utility function of with-profits investors would be helpful in judging the appropriateness of any assumed bonus philosophy.

7.3. There is scope for considerable extension of this work. One way would be to consider more sophisticated investment policies, including a larger range of asset classes including derivatives. The latter are often used in practice to cover guarantees.

7.4. The work could also be extended by incorporating surrender claims into the model. The conflicts caused by adverse selection producing short term policyholder gains but longer term increased solvency risks would form an interesting challenge to resolve, and would provide useful information with regard to the extent to which the imposition of market value adjusters on surrender may be to the benefit or to the detriment of policyholders' returns.

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## APPENDIX

*Model office routines for calculating benefits*

(Subscript $t$ replaces $a,t$ as used in the main text, for convenience)

*Calculating total benefit levels*

Let $roa_t$ = the return on assets between times $[t\text{-}1, t]$, where $t$=0 is the projection date;

$i_t$ = the return attributed to policyholders over $[t\text{-}1, t]$
= $roa_t - CD$

where $CD$ is the annual contribution to capital: see paragraph 4.14.

Let $ir_t$ = notional reduced return over $[t\text{-}1, t]$
= $X \times i_t$ for $i_t > 0$
= $i_t / X$ for $i_t < 0$

where $(0 < X \leqslant 1)$: see paragraph 3.7.

$is_t$ = the geometrically smoothed policyholders' return over times $[t\text{-}1,t]$, calculated as the geometric mean of $i_t$ over the previous 3 years, ie over times $[t\text{-}3, t]$.

Then $AS_t$ = policy asset share at time $t$ (calculated by accumulating cash-flows over times $[0, t]$ at the rates $i_k$, $k = 1,2, \ldots t$);

$RAS_t$ = reduced policy asset share at time $t$ (calculated by accumulating cash-flows over times $[0, t]$ at the rates $ir_k$, $k = 1,2, \ldots t$);

$SAS_t$ = smoothed policy asset share at time $t$ (calculated by accumulating cash-flows over times $[0, t]$ at the rates $is_k$, $k = 1,2, \ldots t$);

$F_t$ = amount of policyholder's fund at time $t$ (calculated by accumulating cash-flows over times $[0, t]$ at the rates $(1 + g)(1 + RB_k) - 1$, $k = 1,2, \ldots t$);

where $g$ = guaranteed annual rate of fund increase ; and

$RB_k$ = declared regular bonus fund increase over $[k\text{-}1, k]$, declared annually in advance.

The total benefit level at time $t$ is calculated as:

$$B_t = \max\{SAS_t, F_t\}$$

so that the terminal bonus is:

$$TB_t = B_t - F_t$$

*Determining the regular bonus*

Define $DRT_{t-1} = RB_t - RB_{t-1}$

The regular bonus rate for the year $[t, t+1]$ is determined at time $t$ as follows.

1. Calculate: $$DRB_t = 0.5 \times \left\{ \max\left( \frac{ic_t - g}{1+g}, 0 \right) - RB_t \right\}$$

   and $$DRU_t = 0.25 \times \min\left( \frac{RAS_t - F_t}{RAS_t}, 0 \right)$$

   where $ic_t$ is the yield on consols at time $t$.

2. Calculate: $drt_t = DRB_t + DRU_t$
   rounded down to the next lower .0025.

3. $drt_t$ is then constrained to lie within [-.02, +.01].

4. If $drt_t$ is non-zero, compare it with $DRT_{t-1}$:
   - if $drt_t$ and $DRT_{t-1}$ are of the same sign, then $DRT_t = drt_t$
   - if $drt_t$ and $DRT_{t-1}$ are of different sign, then $DRT_t = 0$
   - if $DRT_{t-1} = 0$, then compare $drt_t$ with $DRT_{t-2}$:
     - if $drt_t$ and $DRT_{t-2}$ are of different sign, then .005 is deducted from the absolute value of $drt_t$ to produce $DRT_t$, subject to retaining the same sign or zero.
     - otherwise $DRT_t = drt_t$

5. If the return on assets is negative, then $DRT_t$ is constrained not to exceed zero.

6. Calculate $RB_{t+1} = RB_t + DRT_t$
   with a minimum of zero.

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