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SELECTING AN OPTIMAL INVESTMENT PORTFOLIO FOR A PENSION INSURANCE COMPANY

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Summary

This article presents a portfolio management model based on multi-stage stochastic optimization. The model determines an optimal investment portfolio for a Finnish pension insurance company operating under a statutory pension system. The stochastic elements include inflation, wage index, various interest rates, equity prices, dividend yields, property values and rental yields.

The ultimate goal of the model is to avoid unfavorably low levels of working capital. If the working capital falls below the minimum level prescribed in the Finnish Insurance Companies Act, the company will need to take special measures to recover an appropriate level of working capital. Unable to do so, the company may lose its license to manage pensions.

For the evaluation of the results, we calculate probabilities that the company's working capital is on various zones prescribed in the Finnish Insurance Companies Act. We also compare the results with two other models, a simulation model and a single-stage stochastic optimization model. According to the results, the multi-stage model clearly outperforms the other approaches.

1. Introduction

An important aspect of the pension insurance business is the management of the investment portfolio. A crucial feature of pensions is that the last benefits are generally paid decades after the payment of the first premiums, so that during this time quite substantial funds accumulate. This means that the long term development of the investment returns has a considerable influence on the overall result of the company. In addition to the assets covering the technical reserves for its future liabilities, the company must also have some extra assets as a buffer against adverse investment results. In Finnish insurance companies the working capital acts as this kind of buffer. An insurance company needs to invest its assets in a way that produces sufficient return to cope with its liabilities and, in addition, to maintain its working capital on an adequate level.

The Finnish pension insurance companies managing pensions according to the statutory employees' pension system TEL are subject to the Finnish Insurance Companies Act. It prescribes a minimum level for the working capital. If the working capital falls below this level, the company will be set under the special supervision of the Ministry of Social Affairs and Health. In the case of disability to recover its solvency, the company will eventually lose its license to transact insurance business. Therefore it is essential for a TEL company to have effective tools for the portfolio management.

Markowitz initiated the work on portfolio management problems. He developed a portfolio theory according to which the mean and the variance of the portfolio return are sufficient measures of the uncertainty for the portfolio selection purposes. Since its introduction in the 1950s the theory has been utilized in several applications. Most of these applications are static allowing only one time period but also dynamic models have been developed. However, dynamic models have faced resistance due to the computational difficulties and vast data requirements as well as the sensitivity of the results to errors in input parameters (Hakala [6]).

Another approach to portfolio selection is the use of the utility theory of von Neumann and Morgenstern (see e.g. Owen [10]). A basic result of this axiomatic theory is that rational decisions under uncertainty are based on comparisons of expected utilities of alternative outcomes. A review of the consistency between the results of the two theories is provided by Hakala [6].

As mentioned earlier, the early applications were static ignoring completely the time beyond the horizon of the model. In practice, however, portfolio problems are dynamic in nature so that possible future courses of action are not irrelevant to current decision problem. This has led to the development of multi-period models. The fast development of the computer technology and optimization methods has vastly reduced the computational difficulties faced earlier and encouraged the use of more realistic multi-period models.

Cariño et al. [2] present an asset/liability model for a Japanese Insurance

Company. They use multistage stochastic optimization to determine an optimal investment strategy that incorporates the uncertainty in future assets and liabilities as well as complex regulations imposed by Japanese insurance laws and practices. Their model bears resemblance to the one presented in this paper.

The primary goal of our model is to maintain a pension insurance company's solvency at a desired level. The model should incorporate the regulations imposed by insurance laws and take into account the multi-period nature and the uncertainties of the decision environment. We have chosen a multistage stochastic optimization approach to the problem. This approach takes the uncertainty into account through stochastic elements. It also allows the initial decisions to be reconsidered as time passes by and new information is revealed. The objective function, which is formulated as a function of the working capital, optimizes the results with regard to solvency. Together with various constraints it also enforces legal restrictions.

In Section 2 we present basic definitions and ideas of stochastic optimization. Section 3 provides an overview of the model formulation. Scenario generation is discussed in Section 4. Preliminary results of the model and comparisons to other approaches are presented in Section 5.

2. Preliminaries

We shall now introduce the general convex optimization problem with reference to its solution techniques. Thereafter, it is specialized to dynamic problems under uncertainty. The resulting model is the multi-stage stochastic optimization problem, which is subsequently employed in our asset management model.

2.1 Convex optimization

Consider the following convex programming problem:

$$\max_{x \in X} f(x) \tag{2.1}$$

subject to

$$g_i(x) \geq 0 \quad \text{for} \quad i = 1, \ldots, k \tag{2.2}$$

$$h_j(x) = 0 \quad \text{for} \quad j = 1, \ldots, l \tag{2.3}$$

where X is a convex and closed subset of R^n, f and g_i are concave and h_j affine functions defined on X. The function f is the objective function. The vector x is an n-vector of decision variables. The problem is solved for the values of the variables $x_1, \ldots, x_n$ that satisfy the constraints and maximize the function f.

A vector $x \in X$ satisfying all the constraints (2.2)-(2.3) is called a feasible solution to the problem and the union of all feasible solutions is the feasible region. Under our assumptions, the feasible region is a convex set. Hence, the problem is to find a feasible point $\bar{x}$ such that $f(\bar{x}) \geq f(x)$ for all feasible solutions x. If such a solution $\bar{x}$ exists, it is an optimal solution to the problem. If more than one optimal solution exist, they are called alternative optimal solutions.

For standard theory and methods for convex optimization, see Rockafellar [13], and Bazaraa and Shetty [1]. Practical methods for solving such problems of large-scale are presented by Murtagh and Saunders [9], by Kallio and Salo [8], and by Kallio and Rosa [7].

2.2 Multistage stochastic programming

A deterministic model described above produces an optimal solution based on one data set. If the model involves parameters whose values will be only known in the future, this set is an approximation of all possible data sets. As such it does not take the uncertainty explicitly into account.

In order to model the uncertainty more accurately, we follow the line of thought introduced by Dantzig and Madansky [3]. In the stochastic optimization approach, the uncertainty is modelled as a scenario tree. For an example, see Figure 1. The tree displays the timing and sequence of uncertain events. Each scenario ω, $\omega = 1, 2, \ldots, \Omega$, begins from the same node (on the left of Figure 1) representing the situation at the initial stage. The branches emanating from the node represent the realizations of uncertain events at that stage. At the next stage each branch diverges further. The process continues until one has reached the end of the time horizon. Thus, each scenario is a path through the tree representing a possible sequence of realizations of stochastic elements. Associated with each scenario ω is a probability p_ω of occurrence.

The branching points of the scenario tree divide our finite time horizon into several stages t, $t = 0, 1, 2, \ldots, T$. Thereby we subdivide the time horizon into a number of periods in each scenario. Such a subdivision of the time horizon may be scenario dependent. However, for the sake of simplicity, we assume that the time periods t ($t = 0, 1, 2, \ldots, T$) are the same in each scenario. The beginning of each time period represents an opportunity to make a decision and a decision vector will be associated with such points in time.

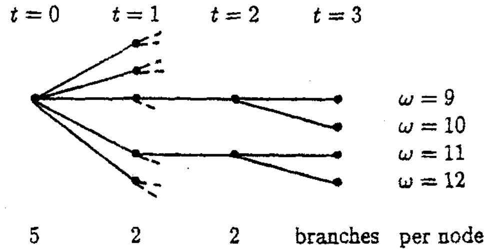

Figure 1. A scenario tree. Total number of scenarios $\Omega = 20$.

In the beginning the decision maker does not know in which scenario he is. Therefore, the decisions for all scenarios have to be identical. That is, the information at hand is the same for all scenarios and there is no reason for different decisions. The constraints enforcing this principle are called nonanticipativity constraints. Likewise, at stage two the decisions for scenarios 9 and 10 must be the same, as well as the decisions for scenarios 11 and 12.

To generalize this idea, we define for all t and ω, an information set I_ω^t as the set of scenarios, for which the path in the scenario tree from the beginning of the time horizon to the beginning of period t matches such a path associated with scenario ω. Therefore, if two scenarios ω and ν belong to I_ω^t, then the information available for decision at the beginning of period t is the same for both scenarios, and hence, there is no reason for different decisions at this stage in the two scenarios.

Let x_ω^t denote the vector of decision variables for period t in scenario ω, and let $x \in R^n$ be the overall decision vector consisting of decision variables in x_ω^t taking into account all t and ω. As in Section 2.1, we may define an objective function $f(x)$ for evaluating the performance of alternative decisions x. For practical applications again, concavity of f is essential. However, for most applications, two additional assumptions are justifiable. First, we define the objective function as the expected value of scenario-wise objective functions. Second, scenario-wise objectives are separable in time steps; in other words, if $f^t(x_\omega^t)$ denotes the objective function for period t, then the scenario-wise objective is $\sum_t f^t(x_\omega^t)$. With these assumptions and with scenario probabilities p_ω, the overall multi-stage stochastic optimization problem is to

$$\max \sum_\omega p_\omega \sum_t f^t(x_\omega^t) \tag{2.4}$$

subject to

$$A_\omega^t x_\omega^{t-1} + B_\omega^t x_\omega^t = b_\omega^t \quad \forall\ \omega \text{ and } t, \quad (\text{with } x_\omega^{-1} \equiv 0) \tag{2.5}$$

$$l_\omega^t \le x_\omega^t \le u_\omega^t \quad \forall\ \omega \text{ and } t \tag{2.6}$$

$$x_\omega^t = x_\nu^t \quad \forall\ \omega \text{ and } \nu \in I_\omega^t \tag{2.7}$$

where A_ω^t and B_ω^t are data matrices (of suitable dimensions), and b_ω^t, l_ω^t and u_ω^t are vectors of problem data, for all t and ω.

Often the objective function is chosen to represent the expected utility, while the scenario-wise objectives represent discounted sums of utility over time. As functions f^t are independent of scenarios, our formulation refers to the usual state-independent utility. However, the complexity of the optimization problem does not increase, if a state-dependent utility would be employed instead.

Sometimes, the objective function may be interpreted as (the negative of) a penalty function. This is appropriate, for instance, for our portfolio selection; see Section 3.2 below. Such an objective function tends to penalize violations of goals. The interdependence between the penalty and the violation depends on the mathematical form of the function f^t.

Our stochastic optimization problem (2.4)-(2.7) is a special case of the convex optimization problem (2.1)-(2.3) of Section 2.1. It is often called the deterministic equivalent. The dynamics constraints (2.5) represent affine constraints of (2.3), while the convex set X is defined by simple bounds (2.6) and by nonanticipativity constraints (2.7). At present, the deterministic equivalent does not include concave inequality constraints (2.2). However, if desired, such constraints may be included without too much extra computational difficulty in solving the problem.

3. The asset management model

3.1 Scenario tree

The stochastic elements of the model are inflation, wage index and various income and price returns for investments. Using a simulation model developed by Ranne [11], we generate a scenario tree representing possible realizations of the stochastic elements. The structure of the tree depends on the specifications given for the scenario generator. These specifications determine the subdivision of the time horizon into time periods as well as the number of branches at each stage. As there is no theoretical rule for the structure of the scenario tree, we find it useful to test different structures. For the illustrative results presented later in this paper we use a tree that consists of five periods (Figure 2). The lengths of the periods are $2 + 3 + 5 + 10 + 10$ years, giving the total time horizon of 30 years. The number of branches per node by stage is 16 × 4 × 4 × 2 × 2, which result in 1024 scenarios in total. For the purpose of a suitable objective function, the data for the model is calculated yearly, even though decision variables are defined for the beginning of each time period only.

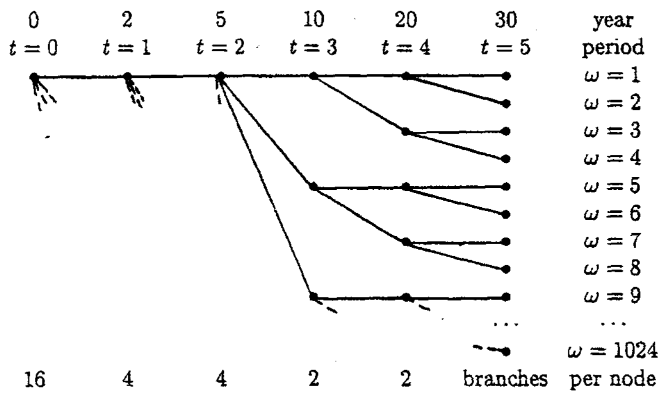

Figure 2. The structure of the scenario tree with $\Omega = 1024$.

3.2 Objective and constraints

The decision vector x^t_ω consists of nine decision variables: the allocations of the total market value $z^t_{a\omega}$ of the company's assets among seven asset classes $a = 1, 2, \ldots, 7$, reserves v^t_ω (including technical reserves and bonus funds) and the working capital c^t_ω. The asset classes are premium loans, investment loans, cash (short term deposits), bonds, equity and property. In addition, there is a small proportion of transitory items, called other assets, that do not produce income. The decision variables can be interpreted to represent an insurance company's balance sheet:

Assets	Liabilities
$z^t_{1\omega}$ =Cash	v^t_ω =Reserves
$z^t_{2\omega}$ =Premium loans	c^t_ω =Working capital
$z^t_{3\omega}$ =Investment loans	
$z^t_{4\omega}$ =Bonds	
$z^t_{5\omega}$ =Equity	
$z^t_{6\omega}$ =Property	
$z^t_{7\omega}$ =Other assets	

The objective is to maximize the expected utility over the time horizon of the model. The chosen utility or (the negative of a) penalty function is a concave exponential function of the working capital defined for each year τ of the time horizon.

$$f^\tau_\omega = -\exp(-\gamma c^\tau_\omega),$$

where γ is the risk-aversion coefficient and c^τ_ω is the working capital in year τ and scenario ω. The objective of the stochastic optimization problem is to

$$\max \sum_\omega p_\omega \sum_\tau f^\tau_\omega.$$

A concave utility function represents the behavior of a risk-averse decision maker. In other words, near the insolvency point even a small improvement in the working capital results in a significant increase in the utility whereas at high levels of the working capital the similar improvement has only marginal impact on the utility.

The decision vector x^t_ω refers to the situation in the beginning of period t. The optimization procedure takes care of the reallocation of assets only in the beginning of each period. However, each period consists of several years τ and the changes in the market values of different asset classes and the liabilities need to be handled yearly. The changes are due to income and price returns, pension payments, customer bonuses, etc. As a result, in the end of the year we may have a surplus that can be used to new investments or a deficit that needs to be recovered by selling the assets.

For years other than the initial one in each time period, we use myopic rules to determine which assets to buy or sell in order to keep the assets and the liabilities in balance. For the demonstration, we simply allocate the surplus in fixed proportions to various assets. This rule is only a temporary solution to the problem and will be modified later. The basic desirable property for the myopic rule is that both working capital and reserves on annual basis will be represented as affine transformations of the decision variables of the stochastic optimization model.

The constraints of the problem enforce the rules for entering a new period. The working capital in the end of period t is equal to the working capital in the beginning of period $t+1$. The same is true for the reserves. The budget constraints ensure that the assets are equal to the liabilities. In addition, there are legal restrictions on the allocations of assets. For example, the law prescribes that at most 50 percent of the total assets may be invested in equities. These restrictions can be tightened to correspond the company policy which might state even stricter limits to the riskiest assets. Furthermore, the company may have special constraints like a minimum level for cash to maintain its liquidity.

4. Scenario generation

Stochastic insurance company models have been studied by actuaries since the beginning of the 1980's. The earliest published works were by the Finnish (1981) and the British (1984) solvency working parties. The aim of these studies were to investigate the solvency requirements of an insurance company by simulation methods. The stochastic variables of these kinds of models can include, for instance, the amount of claims, the premium income, the operating expenses and the investment income. A general description of stochastic insurance company models can be found e.g. in the book by Daykin et al. [4].

For this study, we adopt the model developed by Ranne [11] and [12] describing a Finnish pension insurance company that operates according to the statutory employees' pension system (TEL). It was developed to simulate how the yearly fluctuation of the company's investment income affects its solvency. Because of this restricted aim, the main stochastic component of the model relates to investments. All the other parameters of the model are calculated deterministically except for the effect of changing inflation and wage indices.

This model structure means that only part of the factors affecting the solvency of the company are taken into account. This is possible because the solvency margin of a Finnish pension insurance company consists of two parts: the equalization reserve and the working capital. The purpose of the equalization reserve is to take care of the random fluctuation of the underwriting result, and so the main role left for the working capital is to act as a buffer for adverse investment results. For this reason useful results about the working capital can be achieved by considering only investment variables and ignoring the underwriting profits or losses.

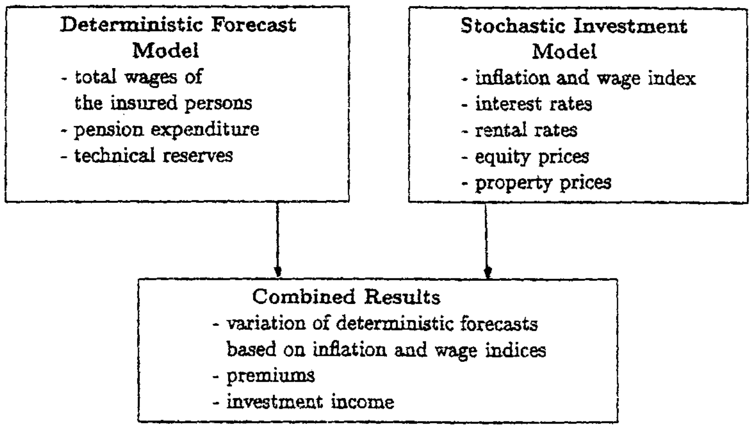

Figure 3. The main structure of the pension company model.

The main structure of the model is shown in Figure 3. The model has three main parts: the deterministic forecast of pension expenditure and reserves, the stochastic investment model and the combining of these results, where most of the data for the model is calculated.

The deterministic forecasts are produced prior to the simulations. For different purposes, the forecasts can be calculated to correspond the development of a particular pension company or, as in the examples of this paper, a theoretical company. Here the pension expenditure and reserves are calculated to represent the average development of Finnish pension companies from the year 1995 onwards.

These forecasts depend, among other assumptions, on the growth of prices and wages. Because these variables are calculated stochastically in the investment model, in principle different forecast are needed for every realization. This is, however, not practical because the deterministic model requires a great deal of computer time. For this reason approximate formulas have been estimated by which the forecast can be modified to follow the fluctuation of prices and wages. In this manner only one forecast is needed and it can be calculated in advance.

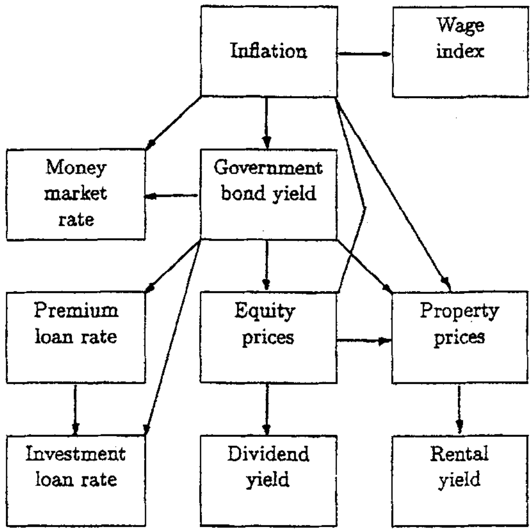

Figure 4. The flow chart of the stochastic investment model.

The company's investment classes are cash (short term deposits), premium loans, investment loans, bonds, equity and property. We assume that the balance sheet contains a small fixed proportion of other assets (transitory items that do not produce income).

Premium loans are an arrangement by which policyholders have the right to reborrow part of their paid premiums within certain limits. The company can not refuse these loans if the guarantees are sufficient, and so the amount invested in the premium loans is not decided by the company. In the model the ratio of the premium loans to the reserves is a stochastic variable. The amounts of all the other classes of investments can in principle be chosen freely, although in practice there may be problems connected with supply and demand in the market.

The structure of the stochastic investment model is shown in Figure 4. The model is based on the work of the Finnish Insurance Modelling Group (see e.g. FIM Group [5]) and its general structure is somewhat similar to the widely used model developed by David Wilkie in the United Kingdom (Wilkie [14]).

The variables of the investment model are discrete and calculated yearly. Because of the long time horizon of the pension business, the length of the model's realizations is usually many years, even decades. For simplicity, all the dependencies between various variables go only to one direction. A primary process is inflation which influences all the other variables. The equations and the parameters of the model have been determined using financial statistics from Finland and some other industrial countries. The principal aim has not been to produce accurate short term forecasts, but to generate investment returns whose fluctuations are realistic in the long run.

In the combined model the investment income of the company depends on the relevant stochastic yield variables. Part of the profit of a company is distributed to the policyholders as bonuses. Dividends paid to the shareholders have not been included in the model since their amount is negligible in the Finnish mandatory pension insurance. Depending on its financial status the company decides the amount it moves into a bonus fund from which 30 percent is yearly paid to the customers. In the model described here the payments to the bonus fund and the distribution of new investments can be considered to form the company's strategy. In different applications these variables can be calculated according to predecided rules or, can be among the decision variables in an optimization model.

5. Computational illustration

5.1 Acceptable level of the working capital

The working capital is the most important of the model's results. If it cannot be held continuously positive, the company is bankrupted. For this reason, the company should have its working capital at a safe level, and there are also statutory lower limits. Also a high level of working capital enables the company to pay more bonuses and so perhaps gain a bigger market share. On the other hand, there is also a statutory upper limit for the working capital, which the company is allowed to exceed only temporarily. The ability of the model company to survive during the model horizon depends essentially on the level of its working capital at the beginning. In the examples below, the working capital at the start is assumed to be near the lower border of the target zone.

The Finnish Insurance Companies Act prescribes the rules for the calculation of a lower border for the working capital, called the solvency limit. This lower border depends on the structure of the investment portfolio. The riskier the assets, the higher the solvency limit. The equation for the solvency limit $\bar{c}_\omega^\tau$ is

$$\bar{c}_\omega^\tau = h_\omega^\tau v_\omega^\tau$$

where the share h_ω^τ is determined with weights w_j on asset shares as follows

$$h_\omega^\tau = \sum_j w_j [z_{j\omega}^\tau / \sum_k z_{k\omega}^\tau]$$

The riskier the asset, the heavier the weight, which enforces the rule that the companies with risky investments should have high minimum working capital to ensure the continuance of the activities during unfavorable fluctuations of the investment returns. The Act prescribes different zones that describe the company's solvency. The limits for the zones are based on the solvency limit as follows:

Zone	Limits		
Breakdown		c^T	< 0
Crisis	$0 \leq$	c^T	$< \bar{c}^T$
Below target	$\bar{c}^T \leq$	c^T	$< 2\bar{c}^T$
Target	$2\bar{c}^T \leq$	c^T	$< 4\bar{c}^T$
Above target	$4\bar{c}^T \leq$	c^T	

The working capital should stay well above the breakdown limit, which means the bankruptcy of the company. To prevent such a situation, the crisis zone acts as a warning signal. If the working capital falls to this zone, the authorities will intervene. The company should attempt to stay in the target level. Temporary violations of the target limits are not serious. If the company falls into the below-the-target zone, it can improve the situation for example by decreasing the customer bonuses or by replacing risky assets with safer ones. The opposite measures are valid for a company in the above-the-target zone. Changes in customer bonuses affect the working capital directly whereas reallocation of assets influences the zone limits through the solvency limit.

5.2 Cases in comparison

For the comparison of the results we have run the problem using three different models. The first model is a simulation model. The initial values for the decision variables are given. In the following years myopic rules determine the values of the decision variables.

The second model utilizes single-stage stochastic programming. The initial decision vector is optimized. Optimization of subsequent decisions is not allowed; instead, myopic rules determine the changes among asset allocations. In this model there is a single decision node, nine variables and nine constraints.

The third approach is the multistage stochastic programming model described earlier. The decision vectors for all periods t and for all scenarios ω are optimized simultaneously. Myopic rules determine the allocations for the years between the decision nodes. This model has 849 decision nodes, 7641 variables and 7642 constraints.

The model	Breakdown (%)	Zone proportions (%)			
		Crisis	Below	Target	Above
Simulation	19.5	10.1	26.3	42.2	19.6
Single-stage optimization	7.8	6.5	24.3	48.7	20.0
Multi-stage optimization	0.0	1.7	12.0	31.6	54.7

Table 1: The breakdown probability and the proportions of years with the working capital on specified zones.

5.3 Preliminary results

Table 1 presents the results obtained using the three different models. The zone proportions present the percentage of years where the working capital is in each zone specified in section 5.1. However, the calculation of the breakdown probability differs from the other probabilities. The company can go bankrupt only once in each scenario. Whether the working capital falls below zero again in this scenario is irrelevant, because the company exists no more. Therefore, we calculate the percentage of scenarios, instead of years, in which the working capital falls below zero. Thus, the figure represents the probability that the company will go bankrupt during the following 30 years whereas the other figures refer to the probabilities of each zone in a yearly basis.

Without optimization the probability of a bankruptcy is nearly 20 percent, which is clearly too high. Single-stage stochastic programming drops the figure to around 8 percent. Multistage stochastic programming performs distinctly better, the breakdown probability falls to zero. Another interesting zone is the crisis zone. Falling into this zone is a signal of insufficient solvency. The results are quite similar as with the breakdown zone. The multistage stochastic optimization outperforms the other approaches. However, in one sense this approach fails. Only in one third of the years the working capital is on the target zone. The reason for this is the high proportion of years where the working capital is above the target. Even though surpassing the target limit does not sound instinctively unfavorable, the law prescribes that this may only happen temporarily. The results using multistage programming imply that surpassing would happen on average every second year, which is too often to be interpreted as temporary.

5.4 Modified objective function

To avoid frequent exceeding of the target zone, we modify the objective function. We set a target for the working capital. This target lies in the middle of the target zone, i.e. the value of the target is

$$3\bar{c}_\omega^\tau = 3v_\omega^\tau/(c_\omega^\tau + v_\omega^\tau)\sum_j w_j z_{j\omega}^\tau$$

The model	Breakdown (%)	Zone proportions (%)			
		Crisis	Below	Target	Above
Simulation	19.5	10.1	26.3	42.2	19.6
Single-stage optimization	7.8	6.5	24.3	48.7	20.0
Single-stage optimization *	6.5	6.3	22.8	53.7	16.7
Multi-stage optimization	0.0	1.7	12.0	31.6	54.7
Multi-stage optimization *	0.0	1.2	14.1	72.9	11.8

Table 2: The breakdown probability and the proportions of years with the working capital on specified zones. Programming models run with two different objective functions. *Modified objective function.

To keep the numerical optimization as simple as possible, we make use of the following approximation:

$$v_\omega^\tau/(c_\omega^\tau + v_\omega^\tau) \approx 0.9$$

We formulate an additional objective as a quadratic function that penalizes the deviations from the approximative target:

$$\min \sum_\omega p_\omega \sum_\tau (c_\omega^\tau - 2.7 \sum_j w_j z_{j\omega}^\tau)^2$$

The original objective function is modified by subtracting the quadratic penalty. The optimizing models are run again to test its influence on results. Table 2 presents the results using both the original and modified objective function.

As can be seen from Table 2, the modified objective function behaves as expected. The objective is not only to pursue as high a working capital as possible but also to prevent excessive deviations from the target value. In single-stage programming the improvement is slight whereas in multistage programming the probability of being on the target zone more than doubles. Moreover, this does not happen at the expense of the more critical zones (breakdown and crisis zones). In fact, the probability of falling into these zones is slightly reduced.

5.5 Further possible developments

Although it is important for a pension insurance company to try to stay in the target zone, in reality this cannot be the only aim of the company. Generally an insurance company might, for example, try to maximize the dividends paid to the shareholders. This, however, does not apply to the Finnish pension insurance, since the amount of the dividends is quite strictly restricted by the authorities (because pension insurance is mandatory and considered to be part of the social security system). Instead a rational objective for a pension company would be to try to maximize the amount of bonuses paid to the policyholders, which would cause an increase in its market share. Therefore a natural further modification of the objective function would include a target for the amount of bonuses.

It is possible that the investment distribution in the model may change much from one year to another. In reality, it is not possible for a Finnish pension insurance company with a large market share to change the distribution of its investments too quickly. This could be taken into account in the model for example by introducing transaction costs.

6. Conclusions

We have presented a multistage stochastic programming model for the portfolio management of a Finnish pension insurance company. The most important aspects of the model are a multi-period decision horizon, stochasticity of relevant factors and the inclusion of statutory restrictions.

The results we have obtained so far seem encouraging, especially when compared to simulation and single-stage programming models that follow the same underlying principles as the multistage problem. However, the problem is still under construction and the results should be considered as preliminary.

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